CHAPTER-5

CONTINUITY AND DIFFERENTIABILITY

CONTINUITY

TWO MARK QUESTIONS

1. Check the continuity of the function f given by $f(x)=2x+3$ at $x=1$. (U)

2. Examine whether the function f given by $f(x)=x^2$ is continuous at $x=0$. (U)

3. Discuss the continuity of the function f given by $f(x)=|x|$ at $x=0$. (U)

4. Show that the function f given by $f(x) = \begin{cases} x^3+3, & \text{for } x\neq 0 \\ 1, & \text{for } x=0 \end{cases}$ is not continuous at $x=0$.(U)

5. Check the points where the constant function $f(x)=\mathrm{k}$ is continuous. (U)

6. Prove that the identity function on real numbers given by $f(x)=x$ is continuous at every real number. (U)

7. Is the function defined by $f(x)=|x|$, a continuous function? (U)

8. Discuss the continuity of the function f given by $f(x)=x^3+x^2-1$. (U)

9. Discuss the continuity of the function f defined by $f(x)=\frac{1}{x}$, $x\neq 0$. (U)

10. Show that every polynomial function is continuous. (U)

11. Show that every rational function is continuous. (U)

12. Prove that the function $f(x)=5x-3$ is, continuous at $x=0$.(U)

13. Prove that the function $f(x)=5x-3$ is, continuous at $x=-3$.(U)

14. Prove that the function $f(x)=5x-3$ is, continuous at $x=5$.(U)

15. Examine the continuity of the function $f(x)=2x^2-1$ at $x=3$.(U)

16. Examine the following functions for continuity: **(Each question of 2 Marks)**

a) $f(x)=x-5$ **b)** $f(x)=|x-5|$

c) $f(x)=\frac{x^2-25}{x+5}$, $x\neq -5$ **d)** $f(x)=\frac{1}{x-5}$, $x\neq 5$. (U)

17. Prove that the function $f(x)=x^n$ is continuous at $x=n$, where n is a positive integer. (U)

18. Discuss the continuity of the following functions: **(Each question is of 2 Marks)**

a) $f(x)=\sin x+\cos x$ **b)** $f(x)=\sin x-\cos x$ **c)** $f(x)=\sin x\cdot\cos x$. (U)

19. Discuss the continuity of the cosine, cosecant, secant and cotangent functions. (U)

THREE MARK QUESTIONS

1. Discuss the continuity of the function f defined by $f(x)=\begin{cases} x+2, & if \ x\le 1 \\ x-2, & if \ x>1 \end{cases}$. (U)

2. Find all the points of discontinuity of the function f defined by $f(x)=\begin{cases} x+2, & if \ x<1 \\ 0, & if \ x=1 \\ x-2, & if \ x>1 \end{cases}$. (U)

3. Discuss the continuity of the function f defined by $f(x)=\begin{cases} x+2, & if \ x<0 \\ -x+2, & if \ x>0 \end{cases}$. (U)

4. Discuss the continuity of the function f defined by $f(x)=\begin{cases} x, & if \ x\ge 0 \\ x^2, & if \ x<0 \end{cases}$.(U)

5. Discuss the continuity of the sine function. (U)

6. Prove that the function defined by $f(x)=\tan x$ is a continuous function. (U)

7. Show that the function defined by $f(x)=\sin(x^2)$ is a continuous function. (U)

8. Is the function f defined by $f(x)=\begin{cases} x, & if \ x\le 1 \\ 5, & if \ x>0 \end{cases}$ continuous at $x=0$? At $x=1$? At $x=2$?

(U)

FOUR MARK QUESTIONS

1. Find all points of discontinuity of the greatest integer function defined by $f(x)=[x]$, where $[x]$ denotes the greatest integer less than or equal to x. (U)

2. Show that the function f defined by $f(x)=|1-x+|x||$, where x is any real number, is a continuous function. (U)

3. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases} 2x+3, & if \ x\le 2 \\ 2x-3, & if \ x>2 \end{cases}$.

(U)

4. Find all points of discontinuity of f,where f is defined by: $f(x)=\begin{cases} |x|+3, & if \ x\le -3 \\ -2x, & if \ -3<x<3 \\ 6x+2, & if \ x\ge 3 \end{cases}$

.(U)

5. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases} x+1, & if \ x\ge 1 \\ x^2+1, & if \ x<1 \end{cases}$.

(U)

6. Find all points of discontinuity of f,where f is defined by: $f(x)=\begin{cases} \frac{x}{|x|}, & if \ x<0 \\ -1, & if \ x\ge 0 \end{cases}$. (U)

7. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases}\frac{|x|}{x}, & if \quad x\neq 0\\ 0, & if \quad x=0\end{cases}$. (U)

8. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases}x^3-3, & if \quad x\leq 2\\ x^2+1, & if \quad x>2\end{cases}$.

(U)

9. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases}x^{10}-1, & if \quad x\leq 1\\ x^2, & if \quad x>1\end{cases}$.

(U)

10. Is the function defined by $f(x)=\begin{cases}x+5, & if \quad x\leq 1\\ x-5, & if \quad x>1\end{cases}$ a continuous function? (U)

11. Discuss the continuity of the function f, where f is defined by:

$f(x)=\begin{cases}3, & if \quad 0\leq x\leq 1\\ 4, & if \quad 1<x<3\\ 5, & if \quad 3\leq x\leq 10\end{cases}$. (U)

12. Discuss the continuity of the function f, where f is defined by:

$f(x)=\begin{cases}2x, & if \quad x<0\\ 0, & if \quad 0\leq x\leq 1\\ 4x, & if \quad x>1\end{cases}$. (U)

13. Discuss the continuity of the function f, where f is defined by:

$f(x)=\begin{cases}-2, & if \quad x\leq -1\\ 2x, & if \quad -1<x\leq 1\\ 2, & if \quad x>1\end{cases}$. (U)

14. Find the relationship between 'a' and 'b' so that the function 'f' defined by

$f(x)=\begin{cases}ax+1, & if \quad x\leq 3\\ bx+3, & if \quad x>3\end{cases}$ is continuous at $x=3$. (U)

15. For what value of λ is the function defined by $f(x)=\begin{cases}\lambda(x^2-2x), & \text{if} \quad x\leq 0\\ 4x+1, & if \quad x>0\end{cases}$ is

continuous at $x=0$? What about continuity at $x=1$? (U)

16. Show that the function defined by $g(x)=x-[x]$ is discontinuous at all integral points. Here $[x]$ denotes the greatest integer less than or equal to x. (U)

17. Is the function defined by $f(x)=x^2-\sin x+5$ continuous at $x=\pi$? (U)

18. Find all the points of discontinuity of f, where $f(x)=\begin{cases}\frac{\sin x}{x}, & if \quad x<0\\ x+1, & if \quad x\geq 0\end{cases}$. (U)

19. Determine if f defined by $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & if\ \ x\neq 0 \\ 0, & if\ \ x=0 \end{cases}$ is continuous function? (U)

20. Examine the continuity of f , where f is defined by $f(x)=\begin{cases} \sin x-\cos x, & if\ \ x\neq 0 \\ -1, & if\ \ x=0 \end{cases}$.

(U)

21. Determine the value of k, if $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, & if\ x\neq\frac{\pi}{2} \\ 3, & if\ \ x=\frac{\pi}{2} \end{cases}$ is continuous at $x=\frac{\pi}{2}$. (U)

22. Find the value of k if $f(x)=\begin{cases} kx^2, & if\ \ x\leq 2 \\ 3, & if\ \ x>2 \end{cases}$ is continuous at $x=2$. (U)

23. Find the value of k so that the function $f(x)=\begin{cases} kx+1, & if\ \ x\leq\pi \\ \cos x, & if\ \ x>\pi \end{cases}$, is continuous at

$x=\pi$. (U)

24. Find the value of k so that the function $f(x)=\begin{cases} kx+1, & if\ \ x\leq 5 \\ 3x-5, & if\ \ x>5 \end{cases}$, at $x=5$ is a

continuous function. (U)

25. Find the values of $a\ and\ b$ such that $f(x)=\begin{cases} 5, & if\ \ x\leq 2 \\ ax+b, & if\ 2<x<10 \\ 21, & if\ \ x\geq 10 \end{cases}$ is a continuous

functions. (U)

26. Show that the function defined by $f(x)=\cos(x^2)$ is a continuous function. (U)

27. Show that the function defined by $f(x)=|\cos x|$ is a continuous function. (U)

28. Examine that $\sin|x|$ is a continuous function. (U)

29. Find all the points of discontinuity of f defined by $f(x)=|x|-|x+1|$. (U)

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DIFFERENTIABILITY

ONE MARK QUESTIONS

1. Find the derivative of $y = \tan(2x+3)$. (U)

2. If $y = \sin(x^2+5)$, find $\frac{dy}{dx}$. (U)

3. If $y = \cos(\sin x)$, find $\frac{dy}{dx}$. (U)

4. If $y = \sin(ax+b)$, find $\frac{dy}{dx}$. (U)

5. If $y = \cos(\sqrt{x})$, find $\frac{dy}{dx}$. (U)

6. Find $\frac{dy}{dx}$, if $y = \cos(1-x)$. (U)

7. If $y = \log(\sin x)$, find $\frac{dy}{dx}$. (U)

8. Find $\frac{dy}{dx}$, *if* $x - y = \pi$. (U)

9. Find $\frac{dy}{dx}$, if $y = e^{-x}$. (U)

10. Find $\frac{dy}{dx}$, if $y = \sin(\log x)$, $x > 0$. (U)

11. Find $\frac{dy}{dx}$, if $y = \cos^{-1}(e^x)$. (K)

12. If $y = e^{\cos x}$, find $\frac{dy}{dx}$. (U)

13. Find $\frac{dy}{dx}$, if $y = e^{\sin^{-1} x}$. (A)

14. Find $\frac{dy}{dx}$, if $y = e^{x^3}$. (U)

15. Find $\frac{dy}{dx}$, if $y = \log(\log x)$, $x > 0$. (U)

16. Find $\frac{dy}{dx}$, if $y = x^3 + \tan x$. (K)

17. Find $\frac{dy}{dx}$, if $y = x^2 + 3x + 2$. (K)

18. Find $\frac{dy}{dx}$, if $y = x^{20}$. (K)

19. Find $\frac{dy}{dx}$, if $y = x\cos x$. (U)

20. Find $\frac{dy}{dx}$, if $y = \log x$. (U)

21. Find $\frac{dy}{dx}$, if $y=\tan^{-1}x$. (U)

22. Find $\frac{dy}{dx}$, if $y=\sin(\log x)$. (U)

23. If $y=e^{\log x}$, prove that $\frac{dy}{dx}=1$. (A)

24. Find $\frac{dy}{dx}$, if $y=5^x$. (U)

TWO MARK QUESTIONS

1. If $y=(2x+1)^3$, find $\frac{dy}{dx}$. (K)

2. Find the derivative of the function given by $f(x)=\sin(x^2)$. (U)

3. Find $\frac{dy}{dx}$, *if* $y+\sin y=\cos x$. (U)

4. Find $\frac{dy}{dx}$, if $2x+3y=\sin x$. (U)

5. Find $\frac{dy}{dx}$, if $2x+3y=\sin y$. (U)

6. Find $\frac{dy}{dx}$, if $ax+by^2=\cos y$. (U)

7. Find $\frac{dy}{dx}$, if $x^2+xy+y^2=100$. (U)

8. Find $\frac{dy}{dx}$, if $\sin^2 x+\cos^2 y=1$. (U)

9. If $\sqrt{x}+\sqrt{y}=10$, show that $\frac{dy}{dx}+\sqrt{\frac{y}{x}}=0$. (U)

10. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$. (U)

11. Find $\frac{dy}{dx}$, if $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$. (U)

12. If $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$, find $\frac{dy}{dx}$. (U)

13. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$. (U)

14. Find $\frac{dy}{dx}$, if $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1<x<1$. (U)

15. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $\frac{-1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$. (U)

16. Find $\frac{dy}{dx}$, if $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$, $0<x<\frac{1}{\sqrt{2}}$. (U)

17. Find $\frac{dy}{dx}$, if $y=\log_a x$. (A)

18. Find $\frac{dy}{dx}$, if $y=\frac{e^x}{\sin x}$. (K)

19. Find $\frac{dy}{dx}$, if $y=\sin\left(\tan^{-1}e^{-x}\right)$. (A)

20. Find $\frac{dy}{dx}$, if $y=\log\left(\cos e^x\right)$. (A)

21. Find $\frac{dy}{dx}$, if $y=e^x+e^{x^2}+e^{x^3}.....+e^{x^5}$. (U)

22. Find $\frac{dy}{dx}$, if $y=\sqrt{e^{\sqrt{x}}}$, $x>0$. (U)

23. Find $\frac{dy}{dx}$, if $y=\frac{\cos x}{\log x}$, $x>0$. (K)

24. Find $\frac{dy}{dx}$, if $y=\cos(\log x+e^x)$, $x>0$. (U)

25. Differentiate a^x with respect to x, where a is a positive constant. (K)

26. Differentiate $x^{\sin x}$, $x>0$ with respect to x. (U)

27. Differentiate $(\log x)^{\cos x}$ with respect to x. (U)

28. If $y=x^x$, find $\frac{dy}{dx}$. (U)

29. Differentiate $\left(x+\frac{1}{x}\right)^x$ w. r. to x. (U)

30. Find $\frac{dy}{dx}$, if $y=x^{\left(x+\frac{1}{x}\right)}$. (U)

31. Find $\frac{dy}{dx}$, if $y=(\log x)^x$ *OR* $y=x^{(\log x)}$. (U)

32. Find $\frac{dy}{dx}$, if $y=(\sin x)^x$ *OR* $y=\sin^{-1}\sqrt{x}$. (U)

33. Find $\frac{dy}{dx}$, if $y=x^{\sin x}$ *OR* $y=(\sin x)^{(\cos x)}$. (U)

34. Find $\frac{dy}{dx}$, if $y=\log_7(\log x)$. (A)

35. Find $\frac{dy}{dx}$, if $y=\cos^{-1}(\sin x)$. (U)

36. Find $\frac{dy}{dx}$, if $y=(3x^2-9x+5)^9$. (U)

37. Find $\frac{dy}{dx}$, if $y=\sin^3 x+\cos^6 x$. (U)

38. Find $\frac{dy}{dx}$, if $y=(5x)^{3\cos 2x}$. (U)

39. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(x\sqrt{x}\right), 0\le x\le 1$.(K)

40. Find $\frac{dy}{dx}$, if $y=\frac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}$, $-2<x<2$. (K)

41. Find $\frac{dy}{dx}$, if $y=(\log x)^{\log x}$, $x>1$. (U)

42. Find $\frac{dy}{dx}$, if $y=\cos(a\cos x+b\sin x)$, for some constant 'a' and 'b'. (U)

43. Find $\frac{dy}{dx}$, if $y=x^3\log x$. (U)

44. Find $\frac{dy}{dx}$, if $y=e^x\sin 3x$.(U)

45. Find $\frac{dy}{dx}$, if $y=e^{6x}\cos 3x$. (U)

THREE MARK QUESTIONS

1. Differentiate $\sin(\cos(x^2))$ w. respect to x. (U)

2. If $y=\sec\left(\tan(\sqrt{x})\right)$, find $\frac{dy}{dx}$.(U)

3. If $y=\frac{\sin(ax+b)}{\cos(cx+d)}$, find $\frac{dy}{dx}$. (U)

4. If $y=\cos x^3\cdot\sin^2(x^5)$, find $\frac{dy}{dx}$. (U)

5. Prove that the function *f* given by $f(x)=|x-1|, x\in R$ is not differentiable at $x=1$. (K)

6. Prove that the greatest integer function defined by $f(x)=[x],\ 0<x<3$ is not differentiable $x=1$ *and* $x=2$. (A)

7. If $y=2\sqrt{\cot(x^2)}$, find $\frac{dy}{dx}$. (U)

8. Find $\frac{dy}{dx}$, *if* $x+\sin xy-y=0$. (K)

9. Find $\frac{dy}{dx}$, if $xy+y^2=\tan x+y$. (K)

10. Find $\frac{dy}{dx}$, if $x^3+x^2y+xy^2+y^3=81$. (K)

11. Find $\dfrac{dy}{dx}$, if $\sin^2 x + \cos xy = k$. (K)

12. Find the derivative of f given by $f(x) = \sin^{-1} x$ assuming it exists. (K)

13. Find the derivative of f given by $f(x) = \tan^{-1} x$ assuming it exists. (K)

14. Differentiate e^x w. r. to x from first principle method. (K)

15. Differentiate $\log_e x$ w. r. to x from first principle method. (K)

16. Differentiate $\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$ with respect to x. (K)

17. Find $\dfrac{dy}{dx}$, *if* $y^x + x^y + x^x = a^b$. (K)

18. Find $\dfrac{dy}{dx}$, if $y = \cos x \cdot \cos 2x \cdot \cos 3x$. (K)

19. Differentiate $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x. (K)

20. Find $\dfrac{dy}{dx}$, if $x^x - 2^{\sin x}$. (K)

21. Find $\dfrac{dy}{dx}$, if $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$. (K)

22. Find $\dfrac{dy}{dx}$, if $x^y + y^x = 1$. (U)

23. Find $\dfrac{dy}{dx}$, if $x^y = y^x$. (U)

24. Find $\dfrac{dy}{dx}$, if $xy = e^{x-y}$. (U)

26. Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$. (K)

27. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ by using product rule. (K)

28. Find $\dfrac{dy}{dx}$, if $y = x^{x\cos x}$ *OR* $y = \dfrac{x^2+1}{x^2-1}$. (U)

29. Find $\dfrac{dy}{dx}$, if $y = (x\cos x)^x$ *OR* $y = (x\sin x)^{\frac{1}{x}}$. (U)

30. If u, v and w are functions of x, then show that

$$\frac{d}{dx}(uvw) = uv\frac{d}{dx}w + vw\frac{d}{dx}u + wu\frac{d}{dx}v$$

31. Find $\dfrac{dy}{dx}$, if $x = a\cos\theta,\ y = a\sin\theta$. (U)

32. Find $\dfrac{dy}{dx}$, if $x = at^2,\ y = 2at$. (U)

33. Find $\frac{dy}{dx}$, if $x=a\left(\theta+\sin\theta\right),\ y=a\left(1-\cos\theta\right)$. (U)

34. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$. (U)

35. If $x=a\,cos^3\theta\,and\ y=a\,sin^3\theta,$ prove that $\frac{dy}{dx}=-\sqrt[3]{\frac{y}{x}}$. (A)

36. Find $\frac{dy}{dx}$, if $x=2at^2,\ y=at^4$. (U)

37. Find $\frac{dy}{dx}$, if $x=a\cos\theta,\ y=b\cos\theta$. (U)

38. Find $\frac{dy}{dx}$, if $x=sin\,t,\ y=cos\,2t$.(U)

39. Find $\frac{dy}{dx}$, if $x=4t,\ y=\frac{4}{t}$. (U)

40. Find $\frac{dy}{dx}$, if $x=\cos\theta-\cos2\theta,\ y=\sin\theta-\sin2\theta$. (U)

41. If $x=a\left(\theta-\sin\theta\right)$ and $y=a\left(1+\cos\theta\right)$ then prove that $\frac{dy}{dx}=-\cot\left(\frac{\theta}{2}\right)$. (A)

42. Find $\frac{dy}{dx}$, if $x=\frac{\sin^3 t}{\sqrt{\cos 2t}},\ y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$. (U)

43. Find $\frac{dy}{dx}$, if $x=a\left(\cos t+\log\tan\frac{t}{2}\right),\ y=a\sin t$. (A)

44. Find $\frac{dy}{dx}$, if $x=a\,sec\,\theta,\ y=b\,tan\,\theta$.(U)

45. Find $\frac{dy}{dx}$, if $x=a\left(\cos\theta+\theta\sin\theta\right),\ y=a\left(\sin\theta-\theta\cos\theta\right)$. (U)

46. If $x=\sqrt{a^{\sin^{-1}t}}$ *and* $y=\sqrt{a^{\cos^{-1}t}}$, then prove that $\frac{dy}{dx}=-\frac{y}{x}$. (A)

48. If $x=a\left(\theta+\sin\theta\right)$ *and* $y=a\left(1-\cos\theta\right)$. Prove that $\frac{dy}{dx}=\tan\left(\frac{\theta}{2}\right)$. (A)

49. Verify Rolle's Theorem for the function $y=x^2+2,\ x\in[-2,2]$. (K) **"OR"**

Verify Rolle's Theorem for the function $y=x^2+2,\ a=-2\ and\ b=2$. (K)

50. Verify Mean Value Theorem for the function $y=x^2$ in the interval $[2,4]$. (K)

51. Verify Rolle's Theorem for the function $y=x^2+2x-8,\ x\in[-4,2]$. (K)

52. Verify Mean Value theorem, if $f(x)=x^2-4x-3$ in the interval $[a,b]$, where $a=1\ and\ b=4$. (K)

53. Verify Mean Value Theorem if $f(x)=x^3-5x^2-3x$ in the interval $[1,3]$. (K)

54. Find $\frac{dy}{dx}$, if $y=\sqrt{3x+2}+\frac{1}{\sqrt{2x^2+4}}$. (U)

55. Find $\frac{dy}{dx}$, if $y=e^{\sec^2 x}+3\cos^{-1}x$. (K)

56. Find $f'(x)$, if $f(x)=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.(A)

57. Find $f'(x)$, if $f(x)=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$.(A)

58. Find $f'(x)$ if $f(x)=(\sin x)^{\sin x}$ for all $0<x<\pi$. (U)

59. For a positive constant 'a' find $\frac{dy}{dx}$, where $y=a^{t+\frac{1}{t}}$ and $y=\left(t+\frac{1}{t}\right)^a$. (U)

60. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$. (K)

61. Find $\frac{dy}{dx}$, if $y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0<x<\frac{\pi}{2}$. (A)

62. Find $\frac{dy}{dx}$, if $y=(\sin x-\cos x)^{(\sin x-\cos x)}$, $\frac{\pi}{4}<x<\frac{3\pi}{4}$. (K)

63. Find $\frac{dy}{dx}$, if $x^x+x^a+a^x+a^a$, for some fixed $a>0$ and $x>0$. (K)

64. Find $\frac{dy}{dx}$, if $y=(x)^{x^2-3}+(x-3)^{x^2}$, for $x>3$. (U)

65. Find $\frac{dy}{dx}$, if $y=12(1-\cos t)$, $x=10(t-\sin t)$, $-\frac{\pi}{2}<t<\frac{\pi}{2}$. (U)

66. Find $\frac{dy}{dx}$, if $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$, $0<x<1$. (A)

67. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for $-1<x<1$. Prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.(K)

68. If $\cos y=x\cos(a+y)$, with $\cos a\neq\pm1$ prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$. (K)

69. If $f(x)=|x|^3$, $f''(x)$ exists for all real x and find it. (U)

70. Using mathematical induction prove that $\frac{d}{dx}x^n=nx^{n-1}$ for all positive integer n .(U)

71. Using the fact that $\sin(A+B)=\sin A\cos B+\cos A\sin B$ and the differentiation, obtain the sum formula for cosines. (U)

72. Does there exists a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer. (A)

73. *If* $y=\begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that, $\frac{dy}{dx}=\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$. (S)

FIVE MARKS QUESTIONS

1. If $y=A\sin x+B\cos x$, then prove that $\frac{d^2y}{dx^2}+y=0$. (K)

2. If $y=3e^{2x}+2e^{3x}$, then prove that $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$. (K)

3. If $y=\sin^{-1}x$, then prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$. (U)

4. If $y=5\cos x-3\sin x$, then prove that $\frac{d^2y}{dx^2}+y=0$. (U)

5. If $y=\cos^{-1}x$, *find* $\frac{d^2y}{dx^2}$ in terms if y alone. (S)

6. If $y=3\cos(\log x)+4\sin(\log x)$, show that $x^2 y_2+xy_1+y=0$. (U)

7. If $y=Ae^{mx}+Be^{nx}$, prove that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+(mn)y=0$. (K)

8. If $y=500e^{7x}+600e^{-7x}$, show that $y_2=49y$. (K)

9. If $e^y(x+1)=1$, show that $y_2=y_1^2$. (K)

10. If $e^y(x+1)=1$, Prove that $\frac{dy}{dx}=-e^y$ hence prove that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$. (K)

11. If $y=\left(\tan^{-1}x\right)^2$, show that $(1+x^2)^2y_2+2x(1+x^2)y_1=2$. (U)

12. If $y=e^{a\cos^{-1}x}$, $-1\le x\le 1$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$. (A)

13. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{d^2y}{dx^2}$. (A)

14. If $(x-a)^2+(y-b)^2=c^2$, *for some* $c>0$, prove that $\frac{\left(1+y_1^{\,2}\right)^{\frac{3}{2}}}{y_2}$ is a constant independent of a and b. (A)