13. BINOMIAL THEOREM

1. Statement of Binomial theorem : If a, b $\in$ R and n $\in$ N, then

$$(a + b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \ldots + {}^nC_r\, a^{n-r} b^r + \ldots + {}^nC_n\, a^0 b^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

2. Properties of Binomial Theorem :

(i) **General term :** $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

(ii) **Middle term (s) :**

(a) If n is even, there is only one middle term, which is $\left(\frac{n+2}{2}\right)$th term.

(b) If n is odd, there are two middle terms, which are $\left(\frac{n+1}{2}\right)$th and $\left(\frac{n+1}{2}+1\right)$th terms.

3. Multinomial Theorem : $(x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots x_k)^n = \sum_{r_1+r_2+\ldots+r_k=n} \frac{n!}{r_1!\, r_2! \ldots r_k!}\, x_1^{r_1} \cdot x_2^{r_2} \ldots x_k^{r_k}$

Here total number of terms in the expansion = ${}^{n+k-1}C_{k-1}$

4. Application of Binomial Theorem :

If $(\sqrt{A}+B)^n = I + f$ where I and n are positive integers, n being odd and $0 < f < 1$ then $(I + f)\, f = k^n$ where $A - B^2 = k > 0$ and $\sqrt{A} - B < 1$.
If n is an even integer, then $(I + f)(1 - f) = k^n$

5. Properties of Binomial Coefficients :

(i) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots\ldots + {}^nC_n = 2^n$

(ii) ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots\ldots\ldots\ldots + (-1)^n\, {}^nC_n = 0$

(iii) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$

(iv) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (v) $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

6. Binomial Theorem For Negative Integer Or Fractional Indices

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r!}x^r + \ldots, |x| < 1.$$

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)}{r!}\, x^r$$