1. Real Numbers

Let us Work Out 1.1

1. Question

Let us write the definition of rational numbers and also write 4 rational numbers.

Answer

A number which can be expressed in form of $\frac{p}{q}$,

Where p and q are integers and q≠0

[**Extra note**

Rational numbers include whole numbers, natural numbers, and integers.

In Decimal values either they are terminating or non-terminating and repeating]

Four rational numbers are

$\frac{4}{5};\frac{6}{7};\frac{11}{8};\frac{1}{2}$

2. Question

Is 0 a rational number? Let us express 0 in the form of p/q [Where p & q are integers and q ≠ 0 and p & q have no common factor other than 1].

Answer

Yes 0 is a rational number

As rational numbers are number which can be expressed in form of $\frac{p}{q}$,

Where p and q are integers and q≠0

⇒ 0 divides and multiply by any number gives 0

$\frac{0}{1} = 0$

As 0 has infinite number of factors

Here, p & q are integers and q ≠ 0 and p & q have no common factor other than 1

3. Question

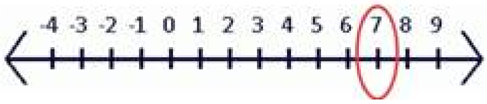
Let me place the following rational numbers on Number Line:

(i) 7, (ii) −4, (iii) $\frac{3}{5}$, (iv) $\frac{9}{2}$, (v) $\frac{2}{9}$, (vi) $\frac{11}{5}$, (vii) $-\frac{13}{4}$.

Answer

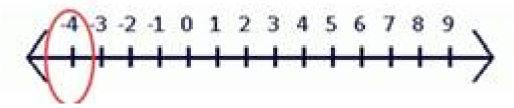
(i) Draw the number line

Place 7 on it



(ii) Draw the number line

Place -4 on it



(iii) Draw the number line

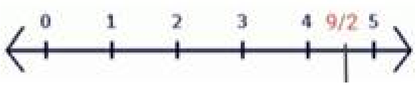
As $\frac{3}{5}$ lies on 0 and 1

Draw number line of 0 to 1 having $\frac{3}{5}$ in between

(iv) Draw the number line

As $\frac{9}{2}$ lies on 4 and 5

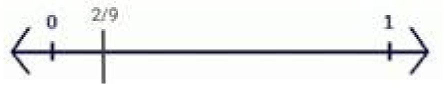
Draw number line of 0 to 5 having $\frac{9}{2}$ in between



(v) Draw the number line

As $\frac{2}{9}$ lies on 0 and 1

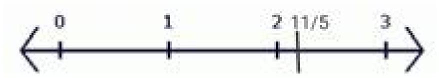
Draw number line of 0 to 1 having $\frac{2}{9}$ in between



(vi) Draw the number line

As $\frac{11}{5}$ lies on 2 and 3

Draw number line of 0 to 3 having $\frac{11}{5}$ in between



(vii) Draw the number line

As $\frac{13}{4}$ lies on 0 and 1

Draw number line of 0 to 1 having $\frac{3}{5}$ in between

4 A. Question

Let me write one rational number lying between two numbers given below and place them on Number Line.

4 & 5

Answer

Formula used.

If 2 rational number X and Y

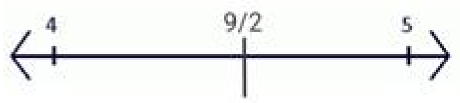
Then, $\frac{X+Y}{2}$ is a rational number lying between X and Y

Solution.

X = 4 and Y = 5

Then number lying between 2 number is

$\frac{4+5}{2} = \frac{9}{2}$



4 B. Question

Let me write one rational number lying between two numbers given below and place them on Number Line.

1 & 2

Answer

Formula used.

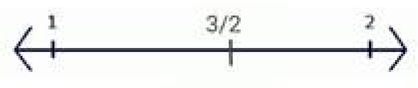
If 2 rational number X and Y

Then, $\frac{X+Y}{2}$ is a rational number lying between X and Y

X = 1 and Y = 2

Then number lying between 2 number is

$\frac{1+2}{2} = \frac{3}{2}$



4 C. Question

Let me write one rational number lying between two numbers given below and place them on Number Line.

$\frac{1}{4}$ & $\frac{1}{2}$

Answer

Formula used.

If 2 rational number X and Y

Then, $\frac{X+Y}{2}$ is a rational number lying between X and Y

X = $\frac{1}{4}$ and Y = $\frac{1}{2}$

Then number lying between 2 number is

$\frac{\frac{1}{4}+\frac{1}{2}}{2} = \frac{1+2}{4} \times \frac{1}{2} = \frac{3}{8}$

4 D. Question

Let me write one rational number lying between two numbers given below and place them on Number Line.

$-1 \,\&\, \frac{1}{2}$

Answer

Formula used.

If 2 rational number X and Y

Then, $\frac{X+Y}{2}$ is a rational number lying between X and Y

X = -1 and Y = $\frac{1}{2}$

Then number lying between 2 number is

$$\frac{-\frac{1}{1}+\frac{1}{2}}{2} = \frac{-2+1}{2} \times \frac{1}{2} = \frac{-1}{4}$$

4 E. Question

Let me write one rational number lying between two numbers given below and place them on Number Line.

$\frac{1}{4} \,\&\, \frac{1}{3}$

Answer

Formula used.

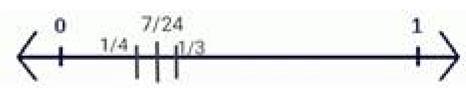
If 2 rational number X and Y

Then, $\frac{X+Y}{2}$ is a rational number lying between X and Y

X = $\frac{1}{4}$ and Y = $\frac{1}{3}$

Then number lying between 2 number is

$$\frac{\frac{1}{4}+\frac{1}{3}}{2} = \frac{4+3}{12} \times \frac{1}{2} = \frac{7}{24}$$



4 F. Question

Let me write one rational number lying between two numbers given below and place them on Number Line.

–2 & –1.

Answer

Formula used.

If 2 rational number X and Y

Then, $\frac{X+Y}{2}$ is a rational number lying between X and Y

X = -2 and Y = -1

Then number lying between 2 number is

$\frac{(-1)+(-2)}{2} = \frac{-3}{2}$

5. Question

Let me write 3 rational numbers lying between 4 & 5 and place them on Number Line.

Answer

Formula used.

If X and Y are 2 rational numbers and X<Y then n rational numbers can be taken on line between X and Y

(X+d),(X+2d),(X+3d)……,(X+nd)

Where d = $\frac{Y-X}{n+1}$

X = 4 and Y = 5

n = 3;

d = $\frac{5-4}{3+1} = \frac{1}{4}$

Then the 3 numbers are,

$(4+\frac{1}{4})$, $(4+2\times\frac{1}{4})$, $(4+3\times\frac{1}{4})$

$\frac{17}{4}$; $\frac{18}{4}$; $\frac{19}{4}$

6. Question

Let me write 6 rational numbers lying between 1 & 2 and place them on Number Line.

Answer

Formula used.

If X and Y are 2 rational numbers and X<Y then n rational numbers can be taken on line between X and Y

(X+d),(X+2d),(X+3d)……,(X+nd)

Where $d = \frac{Y-X}{n+1}$

X = 1 and Y = 2

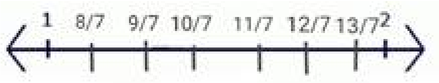
n = 6;

$d = \frac{2-1}{6+1} = \frac{1}{7}$

Then the 6 numbers are,

$(1+\frac{1}{7})$, $(1+2\times\frac{1}{7})$, $(1+3\times\frac{1}{7})$, $(1+4\times\frac{1}{7})$, $(1+5\times\frac{1}{7})$, $(1+6\times\frac{1}{7})$

$\frac{8}{7};\frac{9}{7};\frac{10}{7};\frac{11}{7};\frac{12}{7};\frac{13}{7}$



7. Question

Let me write 3 rational numbers lying between $\frac{1}{5}$ and $\frac{1}{4}$.

Answer

Formula used.

If X and Y are 2 rational numbers and X<Y then n rational numbers can be taken on line between X and Y

(X+d),(X+2d),(X+3d)……,(X+nd)

Where $d = \frac{Y-X}{n+1}$

$X = \frac{1}{5}$ and $Y = \frac{1}{4}$

n = 3;

$d = \frac{\frac{1}{4}-\frac{1}{5}}{3+1} = \frac{5-4}{20} \times \frac{1}{4} = \frac{1}{80}$

Then the 3 numbers are,

$(\frac{1}{5}+\frac{1}{80})$, $(\frac{1}{5}+2\times\frac{1}{80})$, $(\frac{1}{5}+3\times\frac{1}{80})$

$\frac{17}{80};\frac{18}{80};\frac{19}{80}$

8. Question

Let me put (T) if the statement is true and write (F) if the statement is wrong.

(i) By adding, subtracting and multiplying two integers, we get integers.

(ii) By dividing two integers, we always get an integer.

Answer

(i) True

Let's get it by example

Take 2 integers suppose 2 and 3

On adding

We get 2+3 = 5

Which is an integer

On subtracting

We get 2-3 = -1

Which is an integer

On multiplying

We get 2×3 = 6

Which is an integer

∴ On adding, subtracting and multiplying two integers, we get integer

(ii) False

Let's get it by example

Take 2 integer suppose 2 and 5

On dividing

We get $\frac{2}{5}$ = 0.4

Which is not an integer;

9. Question

Let me see and write what I will get by adding, subtracting, multiplying and dividing (divisor is non-zero) two rational numbers.

Answer

As rational numbers are number which can be expressed in form of $\frac{p}{q}$,

Where p and q are integers and q≠0

Let's take 2 rational number $\frac{x}{y}$ and $\frac{r}{s}$

On adding 2 rational numbers.

$$\frac{x}{y}+\frac{r}{s}=\frac{xs+yr}{ys}$$

Where result is in form of $\frac{p}{q}$

As multiplying and adding integers gives an integer

(ys) can't be 0 because y,s≠0

∴ Adding 2 rational number gives a rational number

On subtracting 2 rational numbers.

$$\frac{x}{y}-\frac{r}{s}=\frac{xs-yr}{ys}$$

Where result is in form of $\frac{p}{q}$

As multiplying and subtracting integers gives an integer

(ys) can't be 0 because y,s≠0

∴ Subtracting 2 rational number gives a rational number

On multiplying 2 rational numbers.

$$\frac{x}{y}\times\frac{r}{s}=\frac{xr}{ys}$$

Where result is in form of $\frac{p}{q}$

As multiplying an integers gives an integer

(ys) can't be 0 because y,s≠0

∴ Multiplying 2 rational number gives a rational number

On Dividing 2 rational numbers.

$$\frac{x}{y} \div \frac{r}{s} = \frac{xs}{yr}$$

Where result is in form of $\frac{p}{q}$

As multiplying an integer gives an integer

(yr) can be 0 because r can be 0

∴ Dividing 2 rational number doesn't gives a rational number

Let us Work Out 1.2

1. Question

Let us write the right or False statement from the following:

(i) The sum of two rational numbers will always be rational.

(ii) The sum of two irrational numbers will always be irrational.

(iii) The product of two rational numbers will always be rational.

(iv) The product of two irrational numbers will always be rational.

(v) Each rational number must be real.

(vi) Each real number must be irrational.

Answer

(i) True

We can explain this with an example.

Let the two rational numbers be $\frac{a}{b}$ and $\frac{p}{q}$.

On addition, $\frac{a}{b} + \frac{p}{q} = \frac{aq+bp}{bp}$ which is also a rational number.

(ii) True

We can explain this with an example.

Let the two irrational numbers be $\sqrt{2}$ and $\sqrt{3}$

On addition, $\sqrt{2}+\sqrt{3}$, which is also an irrational number.

(iii) True

Explanation: We can explain this with an example.

Let the two rational numbers be $\frac{a}{b}$ and $\frac{p}{q}$.

On addition, $\frac{a}{b} \times \frac{p}{q} = \frac{ap}{bq}$ which is also a rational number.

(iv) False

We can explain this with an example.

Let the two irrational numbers be $\sqrt{3}$ and $\sqrt{5}$

On multiplication, $\sqrt{3} \times \sqrt{5} = \sqrt{15}$,

which is an irrational number. So, this is not always true.

(v) True

Since a rational number can be plotted on a number line, therefore, every rational number is a real number.

(vi) False

This can be explained with an example. Let us consider any real number, say 2, 2 is a real number as it can be plotted on a number line.

We can write 2 as $\frac{2}{1}$, so 2 is a rational number.

∴ The given statement is not always true.

2. Question

What is meant by irrational numbers? —let me understand. Let me write 4 irrational numbers.

Answer

The numbers which cannot be expressed in the form of $\frac{p}{q}$, where q is not equal to zero are called Irrational Numbers.

$\sqrt{2}$, since its value 1.414… we cannot write the exact value of $\sqrt{2}$ and thereby cannot write it in $\frac{p}{q}$ form, as a result it is an irrational number.

Four irrational numbers are: $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$ and $\sqrt{7}$

3. Question

Let us write rational and irrational numbers from the following:

(i) $\sqrt{9}$ (ii) $\sqrt{225}$

(iii) $\sqrt{7}$ (iv) $\sqrt{50}$

(v) $\sqrt{100}$ (vi) $-\sqrt{81}$

(vii) $\sqrt{42}$ (viii) $\sqrt{29}$

(ix) $-\sqrt{1000}$

Answer

(i) As we know 9 is a square of 3,

$\Rightarrow \sqrt{9} = 3$, which is rational

$\therefore \sqrt{9}$ is a rational number.

(ii) As we know 225 is a square of 15,

$\Rightarrow \sqrt{225} = 15$, which is a rational number

$\therefore \sqrt{225}$ is a rational number.

(iii) As value of $\sqrt{7}$ cannot be written exactly and thereby cannot be written in $\frac{p}{q}$ form, $\sqrt{7}$ is an irrational number.

(iv) We can write,

$\sqrt{50} = 5\sqrt{2}$

As value of $\sqrt{2}$ cannot be written exactly and thereby cannot be written in $\frac{p}{q}$ form, $\sqrt{2}$ is an irrational number.

$\therefore \sqrt{50}$ is an irrational number.

(v) As we know 100 is a square of 10,

$\Rightarrow \sqrt{100} = 10$, which is a rational number

$\therefore \sqrt{100}$ is a rational number.

(vi) As we know 81 is a square of 9,

$\Rightarrow -\sqrt{100} = -9$, which is a rational number

$\therefore -\sqrt{81}$ is a rational number.

(vii) As value of $\sqrt{42}$ cannot be written exactly and thereby cannot be written in $\frac{p}{q}$ form, $\sqrt{42}$ is an irrational number.

(viii) As value of $\sqrt{29}$ cannot be written exactly and thereby cannot be written in $\frac{p}{q}$ form, $\sqrt{29}$ is an irrational number.

(ix) We can write,

$-\sqrt{1000} = -10\sqrt{10}$

As value of $\sqrt{10}$ cannot be written exactly and thereby cannot be written in $\frac{p}{q}$ form, $\sqrt{10}$ is an irrational number.

$\therefore -\sqrt{1000}$ is an irrational number.

4. Question

Let me place $\sqrt{5}$ on Number Line.

Answer

Let the point O represent 0 on the number line.

We place a point A on the number line such that OA = 2 units.

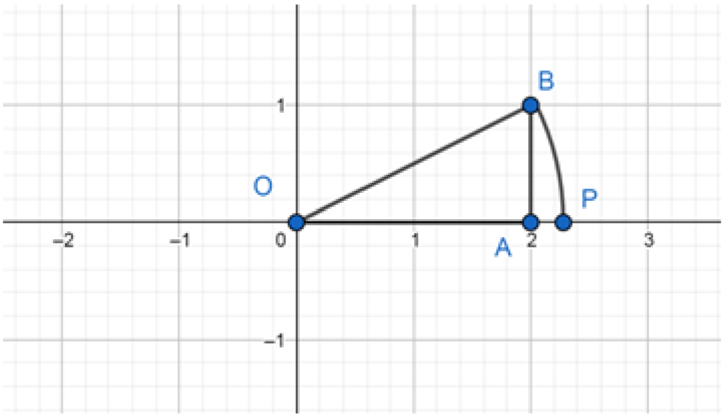
Now, we draw AB perpendicular to OA at point A such that AB = 1unit.

By Pythagoras' Theorem, we know that,

$OB^2 = OA^2 + AB^2$

$\Rightarrow OB^2 = 4 + 1 = 5$

$\Rightarrow OB = \sqrt{5}$



Now, taking O as a centre and OB as the radius, an arc is drawn which intersects the number line at P.

$\therefore OP = \sqrt{5}$ units

Hence, Point P represents $\sqrt{5}$

5. Question

Let me place $\sqrt{3}$ on Number Line.

Answer

Let the point O represent 0 on the number line.

We place a point A on the number line such that OA = 1 units.

Now, we draw AB perpendicular to OA at point A such that AB = 1unit.

By Pythagoras' Theorem, we know that,

$OB^2 = OA^2 + AB^2$

$\Rightarrow OB^2 = 1 + 1 = 2$

$\Rightarrow OB = \sqrt{2}$

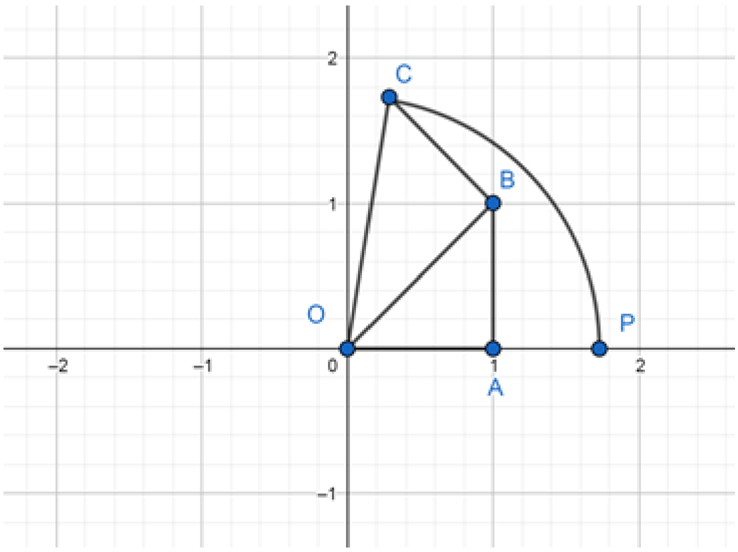
Now, we draw BC perpendicular to OB at point B such that BC = 1unit.

Again, by Pythagoras' Theorem, we know that,

$OC^2 = OB^2 + BC^2$

$\Rightarrow OC^2 = 2 + 1 = 3$

$\Rightarrow OC = \sqrt{3}$



Now, taking O as a centre and OC as the radius, an arc is drawn which intersects the number line at P.

$\therefore$ OP = $\sqrt{3}$ units

Hence, Point P represents $\sqrt{3}$

6. Question

Let me place $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $-\sqrt{6}$, $-\sqrt{8}$, $-\sqrt{11}$ on same Number Line.

Answer

Let the point O represent 0 on the number line.

We place a point A on the number line such that OA = 2 units.

Now, we draw AB perpendicular to OA at point A such that AB = 1unit.

By Pythagoras' Theorem, we know that,

$OB^2 = OA^2 + AB^2$

$\Rightarrow OB^2 = 4 + 1 = 5$

$\Rightarrow OB = \sqrt{5}$

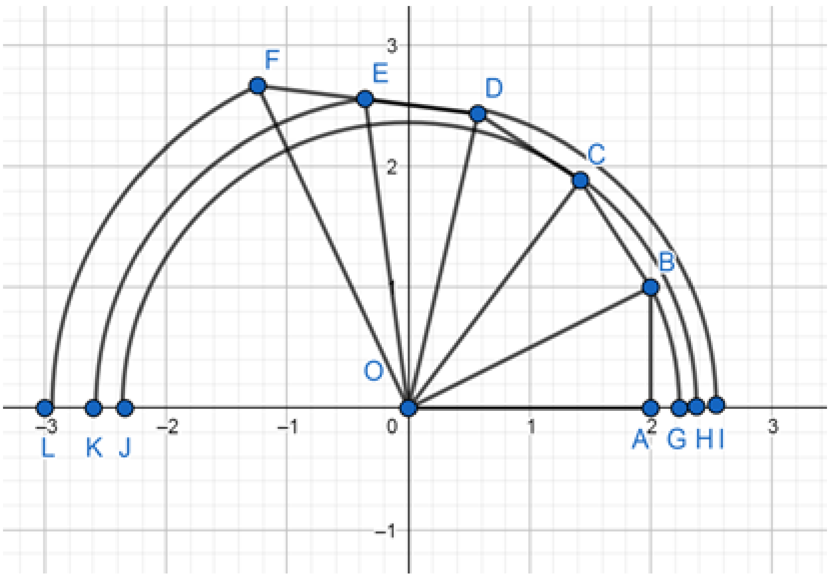
Now, we draw BC perpendicular to OB at point B such that BC = 1unit.

Again, by Pythagoras' Theorem, we know that,

$OC^2 = OB^2 + BC^2$

$\Rightarrow OC^2 = 5 + 1 = 6$

$\Rightarrow OC = \sqrt{6}$



We draw CD perpendicular to OC at point C such that CD = 1unit.

Again, by Pythagoras' Theorem, OD = $\sqrt{7}$

Again, we draw ED perpendicular to OD at point D such that ED = 1unit.

Again, by Pythagoras' Theorem, OE = $\sqrt{8}$

If we extend ED till F such that EF = 2 units and applying Pythagoras' theorem on ΔODF, we get OF = $\sqrt{11}$

Now, taking O as a centre and OB, OC, OD, OE and OF as the radius, the arcs are drawn which intersects the number line at G, H, I, K and L.

$\therefore$ OG = $\sqrt{5}$ units, OH = $\sqrt{6}$ units, OI = $\sqrt{7}$ units, OJ = $-\sqrt{6}$ units, OK = $-\sqrt{8}$ units and OL = $-\sqrt{11}$ units