Previous Years Paper

8th August 2022 (Shift 3)

Q1. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{23} & a_{22} & a_{23} \\ a_{32} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is co-factor of a_{ij}, then
Value of Δ is equal to
(a) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$
(b) $a_{11}A_{31} + a_{12}A_{21} + a_{13}A_{31}$
(c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
(d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Q2. If the matrix
$\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew symmetric, then
6x + y is equal to
(a) 6
(b) 12
(c) 18
(d) 2

Q3. If $\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 5 \\ 1 & x \end{vmatrix}$ then $|x|$ is equal to
(a) $\sqrt{\frac{5}{2}}$
(b) 4
(c) $2\sqrt{2}$
(d) 2

Q4. Which of the following statements are true?
(a) A square matrix A is said to be non-singular If |A| = 0
(b) A square matrix A is invertible if and only if A is non-singular matrix.
(c) If elements of a row are multiplied with cofactors of any other row, then their sum is zero.
(d) A is square matrix of order 3 then | Adj.(A) | = $|A|^3$
Choose the correct answer from the options given below
(a) A and C only
(b) B and C only
(c) C and D only
(d) B and D only

Q5. The interval in which $y = x^2e^{2x}$ is increasing is
(a) $(-\infty, -1)$
(b) $(-1, \infty)$
(c) $(-\infty, -1) \cup (0, \infty)$
(d) $(-\infty, 0) \cup (1, \infty)$

Q6. If $x = t^3, y = t^4$ then $\frac{d^2y}{dx^2}$ at $t = 2$ is
(a) $\frac{8}{3}$
(b) $\frac{1}{9}$
(c) $\frac{2}{9}$
(d) $\frac{9}{16}$

Q7. Match List – I with List -II

List – I	List – II
A. $\int \frac{dx}{\sqrt{9x^2-16}}$	I. $\frac{1}{3}\sin^{-1}\frac{3x}{4} + C$
B. $\int \frac{dx}{\sqrt{16-9x^2}}$	II. $\log(e^x + 1) + C$
C. $\int \frac{e^{2\log_e x}+1}{e^{2\log_e x}-1}$dx	III. $\frac{1}{3}\log_e\left|3x + \sqrt{9x^2-16}\right| + C$
D. $\int \frac{1}{1+e^{-x}}dx$	IV $x + \log_e\left|\frac{x-1}{x+1}\right| + C$

Choose the correct answer from the options given
(a) A-I, B-III, C-II, D-IV
(b) A-I, B-III, C-IV, D-II
(c) A-III, B-I, C-IV, D-II
(d) A-III, B-I, C-II, D-IV

Q8. The area (in sq. units) bounded by the parabola $y^2 = 8x$ and the line $x = 2$ is
(a) $\frac{16}{3}$
(b) $\frac{32}{3}$
(c) $\frac{32\sqrt{2}}{3}$
(d) $\frac{16\sqrt{2}}{3}$

Q9. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2\pi-x}}$dx is equal to
(a) $\frac{\pi}{4}$
(b) π
(c) $\frac{\pi}{2}$
(d) 1

Q10. The solution curve $y = y(x)$ of the differential equation $ydx - xdy = 0$, passing through (2, 4), does not pass through the point
(a) $(-1, -2)$
(b) $\left(\frac{-1}{2}, -1\right)$
(c) $(-1, -2)$
(d) $\left(1, \frac{1}{2}\right)$

Q11. The solution curve of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, passing through the origin is
(a) $4e^{3x} - 3e^{-4y} - 1 = 0$
(b) $4e^{3x} + 3e^{-4y} - 7 = 0$
(c) $3e^{3x} + 4e^{-4y} - 7 = 0$
(d) $3e^{3x} - 4e^{-4y} + 1 = 0$

Q12. If the probability distribution of a random variable X is

X	1	2	3	4	5
$P(X = x)$	K	2k	3k	3k	K

Value of $P(X > 2)$ is
(a) $\frac{5}{6}$
(b) $\frac{9}{10}$

(c) $\frac{4}{5}$
(d) $\frac{7}{10}$

Q13. The curve represented by the differential equation $\frac{dy}{dx} = \frac{2(x+1)y}{x^2+2x+2}, y > 0$, passes through the point $(0, 4)$. If it also passes through $P(-1,\ k)$ and $Q(\lambda, 10)$ then $(PQ)^2$ is equal to
(a) 148
(b) 66
(c) 68
(d) 72

Q14. If $\int \frac{dx}{\sqrt{x} - \sqrt{x-1}} = \lambda\left(x^{\frac{3}{2}} + (x-1)\frac{3}{2}\ \right) + C$, then the value of λ is
(a) 2
(b) $\frac{3}{2}$
(c) 1
(d) $\frac{2}{3}$

Q15. The corner points of the feasible region of an LPP are (0, 0), (30, 0), (20, 30) and (0, 50). If the objective function maximize (z) = ax + by, where $a \neq b$ and a, b > 0, has alternate optimal solutions, then
(a) a = 2b
(b) a = 3b
(c) 2a = b
(d) 3a = b

Q16. Which of the following is not an equivalence relation on Z?
(a) a R b $\Leftrightarrow$ a + b is an even integer
(b) a R b $\Leftrightarrow$ a − b is an even integer
(c) a R b $\Leftrightarrow$ a ≤ b
(d) a R b $\Leftrightarrow$ a = b

Q17. The function f : R – {-1} → R – {1} defined by
$$f(x) = \frac{x}{x+1} is$$
(a) Both 1-1 and onto
(b) Only 1-1
(c) Only onto
(d) Neither 1-1 nor onto

Q18. If $X + Y = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 0 & 3 \\ 7 & 9 \end{bmatrix}$, then Y is Equal to
(a) $\begin{bmatrix} -1 & 1 \\ -2 & -2 \end{bmatrix}$
(b) $\begin{bmatrix} -1 & 1 \\ 2 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$

Q19. If A and B are any two matrices such that
$$(A - B)^2 = A^2 - 2AB + B^2 \ then$$
(a) $B^2A^3 + A^3B^2 = 0$
(b) $AB + BA = 0$
(c) $B^3A^4 - A^4B^3 = 0$
(d) $AB(B - A) = 0$

Q20. If A and B are two matrices such that AB = O (null matrix) then
A. A may be a null matrix, but B may not be a null matrix.
B. B may be a null matrix, but A may not be a null matrix.
C. both A and B may be null matrices.
D. both A and B may be non-null matrices.
Choose the correct answer from the option given below:
(a) A, B Only
(b) A, B, C only
(c) C, D only
(d) A, B, C, D only

Q21. $\int \frac{x^2+3x-1}{(x+1)^2} dx$ is equal to
(a) $\frac{1}{x+1} + \frac{1}{2}\log_e|x+1| + C$
(b) $x + \frac{3}{x+1} + \log_e|x+1| + C$
(c) $x - \frac{1}{x+1} + \log_e|x+1| + C$
(d) $x + \frac{3}{x+1} + \frac{1}{2}\log_e|x+1| + C$

Q22. The area (in sq. units) of the region bounded by the curve $y = x^2$ and the line y = 4 is
(a) $\frac{16}{3}$
(b) $\frac{32}{3}$
(c) 32
(d) 24

Q23. If $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = ax + b\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{x}{2}\right) + d,$
(Where d is a constant of integration) then $\frac{a+c}{b}$ is equal to
(a) $-\sqrt{3}$
(b) $\sqrt{3}$
(c) $2\sqrt{3}$
(d) −2

Q24. The function f(x)=$\begin{cases} \frac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1 \leq x < 0, \\ \frac{2x+1}{x-2} & 0 \leq x \leq 1, \end{cases}$
Is continuous in the interval [−1, 1] then p is equal to
(a) 1
(b) $-\frac{1}{2}$
(c) $\frac{1}{2}$
(d) −1

Q25. If y = $\log_e\left(\sin\left(\frac{x^2}{3} - 1\right)\right)$ $then \frac{d^2y}{dx^2}$ is equal to:
(a) $\frac{2}{3}\left(-\cot\left(\frac{x^2}{3} - 1\right) + x\, cosec^2\left(\frac{x^2}{3} - 1\right)\right)$
(b) $\frac{2}{3}\cot\left(\frac{x^2}{3} - 1\right) - \frac{4x^2}{9} cosec^2\left(\frac{x^2}{3} - 1\right)$
(c) $-\frac{2}{3}\cot\left(\frac{x^2}{3} - 1\right) + \frac{4x^2}{9} cosec^2\left(\frac{x^2}{3} - 1\right)$
(d) $\frac{2}{3}\left(\cot\left(\frac{x^2}{3} - 1\right) - x\, cosec^2\left(\frac{x^2}{3} - 1\right)\right)$

Q26. If $z = \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$ then 6z is equal to________
(a) 1/3
(b) 16
(c) 17
(d) 29

Q27. If $x = \sqrt{t}$, $y = \sin t$ and $\frac{d^2y}{dx^2} = a\cos x^2 + bx^2 \sin x^2$
Then $a^2 + b^2$ is equal to
(a) 4
(b) 16
(c) 20
(d) 48

Q28. If $x\sqrt{1+y} + y\sqrt{1+x} = 0, x \neq y$ then $2\frac{dy}{dx} + y$ at $x = -3$ is equal to
(a) –1
(b) 1
(c) 2
(d) –2

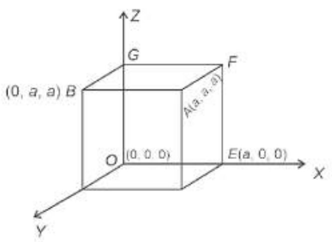
Q29. The radius of a sphere is increasing at the rate of 0.5 cm/minute. The rate of change of the surface area of (in cm^2/ minute) the sphere when the radius is 20 cm is
(a) 20π
(b) $40\ \pi$
(c) $160\ \pi$
(d) $80\ \pi$

Q30. The equation of the tangent to the curve $x(\theta) = 2\sqrt{2}\,(\cos\theta + \theta\sin\theta)$, $y(\theta) = 2\sqrt{2}\,(\sin\theta - \theta\cos\theta)$, at
$\theta = \frac{\pi}{4}$ is equal to
(a) $x - y = \frac{\pi}{2}$
(b) $x - y = \pi$
(c) $x - y = 2$
(d) $x - y = 4$

Q31. Match List – I with List – II

List – I		List – II	
A.	Distance of the point (1, –1, 1) from the plane $\vec{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 2$, is	I.	$x + y + z = 2$
B.	$\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 2,$	II.	$-x + y + z = 5$
C.	Distance of a point (1, –1, 1) From the plane $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 5, is$	III.	$\frac{1}{\sqrt{3}}$
D.	$\vec{r} \cdot (-\hat{\imath} + \hat{\jmath} + \hat{k}) = 5,$	IV.	$2\sqrt{3}$

Choose the correct answer from the options given below.
(a) A-IV, B-II, C-III, D-I
(b) A-III, B-II, C-IV, D-I
(c) A-IV, B-I, C-III, D-II
(d) A-III, B-I, C-IV, D-II

Q32.


Given figure is cuboid then the acute angle between the diagonal OA and BE is
(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(b) $\cos^{-1}\left(\frac{1}{6}\right)$
(c) $\cos^{-1}\left(\frac{2}{3}\right)$
(d) $\cos^{-1}\left(\frac{1}{3}\right)$

Q33. A soldier fires a bullet in the air and at time 't' the bullet travels along the path x = 2t, y = t – 2, z = 5 – t. Then the path of the bullet is a
(a) Parabola
(b) Straight line with direction ratios 2, 1, –1
(c) Straight line with direction ratios 2, 1, 1
(d) Ellipse centred at (0, 2, 5)

Q34. Which of the following is NOT a corner point of the feasible set $S = \{(x, y) : 3x + 4y \leq 24, 8x + 6y \leq 48,$
$x \leq 5, x \geq 0, y \geq 0\}$ of some LPP?
(a) $\left(\frac{24}{7}, \frac{24}{7}\right)$
(b) $\left(5, \frac{4}{3},\right)$
(c) $\left(\frac{24}{7}, \frac{33}{7}\right)$
(d) (0, 6)

Q35. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is
(a) $\sqrt{2}$
(b) 3
(c) $\sqrt{3}$
(d) 2

Q36. $\hat{\imath} \cdot (\hat{\jmath} \times \hat{k}) + \hat{\jmath} \cdot (\hat{k} \times \hat{\imath}) + \hat{k} \cdot (\hat{\imath} \times \hat{\jmath})$ is equal to
(a) 1
(b) 0
(c) 3
(d) -3

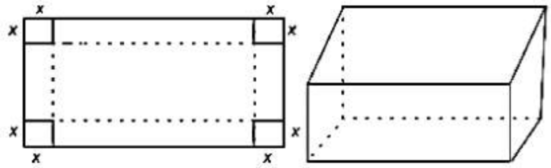
Q37. If $|\vec{a} - \vec{b}| = 10, |\vec{a}| = 8, |\vec{b}| = 6$, then the angle between $\vec{a}$ and $\vec{b}$ is:
(a) 0
(b) $\cos^{-1}\left(\frac{29}{48}\right)$
(c) $\cos^{-1}\left(\frac{5}{24}\right)$
(d) $\frac{\pi}{2}$

Q38. The linear programming problem
max. (z)= 5x + 7y
Subject to
x +y ≥ 10
$3x + 5y \leq 15, x \geq 0, y \geq 0$:
(a) has unique optimal solution
(b) is infeasible
(c) has multiple optimal solutions
(d) is feasible and unbounded

Q39. A bag contains 5 red and 7 black balls. Three balls are chosen at random one by one without replacement. What is the probability that the third ball is a Red ball given that first ball is red and second ball is black?
(a) $\frac{7}{33}$
(b) $\frac{14}{33}$
(c) $\frac{7}{66}$
(d) $\frac{20}{33}$

Q40. If two independent events A and B are such that $P(A) = \frac{1}{2}$ and $P((A' \cap B')') = \frac{3}{5}$, then P(B) is equal to
(a) $\frac{1}{10}$
(b) $\frac{1}{5}$
(c) $\frac{2}{5}$
(d) $\frac{3}{5}$

Directions 41-45
Passage:
Case - Study :



An online retail company ships its product in cartons Each carton is made by rectangular sheet of fiber board with dimension of 8 m by 3 m. While making the carton equal squares of side x metres are cut off from each corner of the rectangular sheet of fiber board. After that the resulting flaps are folded to form the carton (see that figure to identify the side of the carton).
Based on the above information, answer the following question.

Q41. The volume (v) of the carton is given by $v = f(x)$, Where $f(x)$ is:
(a) $4x^3 - 24x^2 + 22x$
(b) $4x^3 + 24x^2 - 22x$
(c) $-4x^3 + 22x^2 + 24x$
(d) $4x^3 - 22x^2 + 24x$

Q42. The derivative $f(x)$ of the volume function $f(x)$ is equal to
(a) $24 + 44x - 12x^2$
(b) $24 - 44x + 12x^2$
(c) $-22 + 48x + 12x^2$
(d) $22 - 48x + 12x^2$

Q43. The value of x (in meters) for which the volume V of carton is maximum is
(a) 3
(b) $\frac{2}{3}$
(c) 1
(d) $\frac{1}{2}$

Q44. The length (in meters) of carton box having maximum volume is
(a) 6
(b) $\frac{5}{3}$
(c) $\frac{20}{3}$
(d) $\frac{22}{3}$

Q45. The maximum volume (in m^3) of carton box is
(a) $\frac{200}{9}$
(b) 6
(c) $\frac{154}{27}$
(d) $\frac{200}{27}$

Directions 46-50
Passage:
By using equations of the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z - 3 = 0$, answer the following questions.

Q46. The acute angle between the line and the plane is
(a) $\cos^{-1}\left(\frac{8}{21}\right)$
(b) $\sin^{-1}\left(\frac{8}{21}\right)$
(c) $\sin^{-1}\left(\frac{6}{7}\right)$
(d) $\cos^{-1}\left(\frac{6}{7}\right)$

Q47. Co-ordinates of point of intersection of line and plane
(a) $\left(\frac{-33}{10}, \frac{69}{20}, \frac{39}{10}\right)$
(b) $\left(\frac{-33}{10}, \frac{-69}{20}, \frac{-39}{10}\right)$
(c) $\left(0, \frac{3}{2}, 6\right)$
(d) $\left(-2, -\frac{3}{2}, 0\right)$

Q48. Distance of plane $10x + 2y - 11z - 3 = 0$, from the origin is:
(a) 1
(b) $\frac{1}{10}$
(c) $\frac{1}{15}$
(d) $\frac{1}{5}$

Q49. The foot of perpendicular from the origin to the given plane is
(a) $\left(\frac{2}{75}, \frac{2}{15}, -\frac{11}{75}\right)$
(b) $\left(\frac{2}{15}, \frac{2}{75}, -\frac{11}{75}\right)$
(c) $\left(\frac{2}{5}, \frac{2}{25}, \frac{11}{25}\right)$
(d) $\left(\frac{2}{5}, \frac{2}{25}, -\frac{11}{25}\right)$

Q50. The plane $10x + 2y - 11z - 3 = 0$ intersect the Z-axis at the point
(a) $\left(0,0, \frac{3}{10}\right)$
(b) $\left(0,0, \frac{3}{2}\right)$
(c) $\left(0,0, \frac{3}{11}\right)$
(d) $\left(0,0, -\frac{3}{11}\right)$

SOLUTIONS

S1. **Ans. (d)**
Sol. $\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

S2. **Ans. (b)**
Sol. $y = 0$ and $3x = 6$
So, $6x + y = 12$

S3. **Ans. (c)**
Sol. $\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 5 \\ 1 & x \end{vmatrix}$
$\Rightarrow\ 11 = 2x^2 - 5$
$\Rightarrow\ x^2 = 8$
$\Rightarrow\ |x| = 2\sqrt{2}$

S4. **Ans. (b)**
Sol. Statement A is incorrect as for singular matrices
$|A| = 0$
Statement B is correct as A is invertible if $|A| \neq 0$
Statement C is correct
Statement D is incorrect as $|\text{Adj}(A)| = |A|^2$

S5. **Ans. (c)**
Sol. $\frac{dy}{dx} = x^2 \cdot 2e^{2x} + 2x \cdot e^{2x} = 2xe^{2x}(x+1)$
$\frac{dy}{dx} > 0$ for $x \in (-\infty, -1) \cup (0, \infty)$

S6. **Ans. (b)**
Sol. $\frac{dy}{dx} = \frac{4t^3}{3t^2} = \frac{4}{3}t$
$\frac{d^2y}{dx^2} = \frac{4}{3} \times \frac{dt}{dx}$
$= \frac{4}{3} \cdot \frac{1}{3t^2} = \frac{4}{9t^2}$
At $t = 2, \frac{d^2y}{dx^2} = \frac{1}{9}$

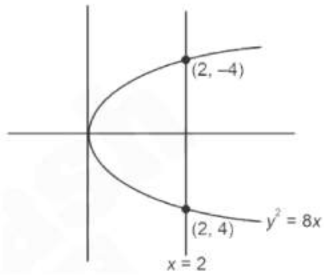
S7. **Ans. (c)**
Sol. (A) $I = \int \frac{dx}{\sqrt{9x^2-16}} = \frac{1}{3}\int \frac{dx}{\sqrt{x^2 - \frac{16}{9}}}$
$= \frac{1}{3}\left(ln\left|x + \sqrt{x^2 - \frac{16}{9}}\right|\right) + C$
$= \frac{1}{3}\left(ln\left|x + \sqrt{x^2 - \frac{16}{9}}\right| + ln3\right) + C$
$= \frac{1}{3} ln\left|3x + \sqrt{9x^2 - 16}\right| + C$

(B) $I = \int \frac{dx}{16-9x^2} = \frac{1}{3}\int \frac{dx}{\sqrt{\frac{16}{9} - x^2}} = \frac{1}{3}\sin^{-1}\frac{3x}{4} + C$

(C) $I = \int \frac{x^2+1}{x^2-1} dx = \int 1 + \frac{2}{x^2-1} dx$
$= x + ln\left|\frac{x-1}{x+1} + C\right|$

(D) $I = \int \frac{e^x}{e^x+1} dx$
Put $e^x + 1 = t, e^x dx = dt$
$I = \int \frac{dt}{t} = ln|t| + C$
$= ln(e^x + 1) + C$

S8. **Ans. (b)**
Sol.



Area bounded by parabola $= 2\int_0^2 \sqrt{8x} dx$
$= 4\sqrt{2} \left. \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right|_0^2$
$= \frac{8\sqrt{2}}{3} \cdot 2^{\frac{3}{2}} = \frac{32}{3}$ sq. units

S9. **Ans. (c)**
Sol. $I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2\pi - x}} dx$ ...(i)
$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{2\pi - x}}{\sqrt{2\pi - x} + \sqrt{x}} dx$ ...(ii)
Adding eq. (i) & (ii), we get
$2I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 dx$
$= \pi$

S10. **Ans. (d)**
Sol. $y\, dx = x\, dy$
$\Rightarrow \frac{dx}{x} = \frac{dy}{y}$
Integrating both sides
$ln\, x = ln\, y + ln\, c$
$or\ x = cy$
It passes through (2, 4)
$\Rightarrow c = \frac{1}{2}$
Or $y = 2x$
Since $\left(1, \frac{1}{2}\right)$ does not satisfy the curve. So, $y = 2x$ does not passes through the point $\left(1, \frac{1}{2}\right)$.

S11. **Ans. (b)**
Sol. $\frac{dy}{dx} = e^{3x+4y}$
$\Rightarrow \frac{dy}{e^{4y}} = e^{3x} dx$
Integrating both sides
$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$
It passes through (0, 0)
$\frac{-1}{4} = \frac{1}{3} + C$
$\Rightarrow C = \frac{-7}{12}$
$\therefore$ Equation of curve is
$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$
or $-3e^{-4y} = 4e^{3x} - 7$

S12. **Ans. (d)**
Sol. $k + 2k + 3k + 3k + k = 1$
$\Rightarrow k = \frac{1}{10}$
$P(x > 2) = 3k + 3k + k$
$= \frac{7}{10}$

S13. **Ans. (c)**
Sol. $\frac{dy}{dx} = \frac{2(x+1)y}{x^2+2x+2}$
$\Rightarrow \frac{dy}{y} = \frac{2(x+1)dx}{x^2+2x+2}$

Integrating both sides
ln y = ln $(x^2 + 2x + 2)$ + ln C
$\Rightarrow y = C(x^2 + 2x + 2)$
It passes through (0, 4)
$4 = C(0 + 0 + 2)$
$\Rightarrow C = 2$
Equation of curve is
$y = 2(x^2 + 2x + 2)$
$k = 2(1 - 2 + 2) = 2$
$10 = 2(\lambda^2 + 2\lambda + 2)$
$\Rightarrow \lambda^2 + 2\lambda - 3 = 0$
OR $\lambda = -3, 1$
So, $P(-1, 2)$ $Q(1, 10)$ OR $(-3, 10)$
$(PQ)^2 = 68$

S14. Ans. (d)
Sol. $I = \int \frac{dx}{\sqrt{x}-\sqrt{x-1}} = \int \frac{\sqrt{x}+\sqrt{x-1}}{1} dx$
$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} + C$

S15. Ans. (b)
Sol. $z_{(0,0)} = 0$
$z_{(30,0)} = 30a$
$z_{(0,50)} = 50a$
$z_{(20,30)} = 20a + 30b$
For $a = 3b, 20a + 30b = 30a$

S16. Ans. (c)
Sol. Relation in option (c) is not an equivalence relation as it is not symmetric
$a \leq$ b then $b \geq$ a
So, if (a, b) ∈ R then (b, a) ∉ R

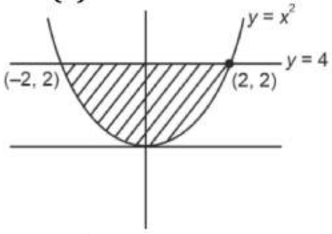
S17. Ans. (a)
Sol. $f(x) = 1 - \frac{1}{x+1}$
$f'(x) = \frac{1}{(x+1)^2} > 0$ so, f(x) is one one.
$y = \frac{x}{x+1} \Rightarrow xy + y = x$
$\Rightarrow x(y-1) = -y$
$\Rightarrow x = \frac{y}{1-y}$
So, Range is R – {1}
So, f(x) is onto

S18. Ans. (c)
Sol. Adding given equations
$2X = \begin{bmatrix} 1 & 4 \\ 10 & 14 \end{bmatrix}$
$\Rightarrow X = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$
Similarly,
$2Y = \begin{bmatrix} 2 & -2 \\ -4 & -4 \end{bmatrix}$
$Y = \begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix}$

S19. Ans. (c)
Sol. $A^2 - AB - BA + B^2 = A^2 - 2AB + B^2$
$\Rightarrow AB = BA$
$A^2B = ABA$
$A^2B^2 = BAAB$
$= BABA$
$A^2 B^2 = B^2 A^2$
$A^3 B^2 = B^2 A^3$
$A^3 B^3 = B^3 A^3$
$A^4 B^3 = A$

S20. Ans. (d)
Sol. All Statements are correct.

S21. Ans. (b)
Sol. $I = \int \frac{x^2+3x-1}{(x+1)^2} dx$
$= \int \left(\frac{x-2}{(x+1)^2}\right) dx$
$= \int \left(1 + \frac{1}{x+1} - \frac{3}{(x+1)^2}\right) dx$
$= x + \ln(x+1) + \frac{3}{(x+1)} + C$

S22. Ans. (b)
Sol.


$A = 2\int_0^2 (4 - x^2) dx$
$= 2\left(4x - \frac{x^3}{3}\right)\Big|_0^2$
$= 2\left(8 - \frac{8}{3}\right) = \frac{32}{3}$ sq. units

S23. Ans. (a)
Sol. $I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$
$= \int \left(1 - \frac{2}{x^2+3}\right)\left(1 - \frac{2}{x^2+4}\right) dx$
$= \int \left(1 - \frac{2}{x^2+3} - \frac{2}{x^2+4} + 4\left(\frac{1}{x^2+3} - \frac{1}{x^2+4}\right)\right) dx$
$= \int \left(1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}\right) dx$
$= x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{6}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$
$a = 1, b = \frac{2}{\sqrt{3}}, c = -3$
$\frac{a+c}{b} = -\sqrt{3}$

S24. Ans. (b)
Sol. lt $f(x) =$
$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}$
$= \lim_{x \to 0^-} \frac{2px}{x.2} = p$
$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} \frac{2x+1}{x-2} = \frac{-1}{2}$

S25. Ans. (b)
Sol. $\frac{dy}{dx} = \frac{1}{\sin\left(\frac{x^2}{3} - 1\right)} \cos\left(\frac{x^2}{3} - 1\right) \frac{2x}{3}$
$\frac{dy}{dx} = \frac{2x}{3} \cot\left(\frac{x^2}{3} - 1\right)$
$\frac{d^2y}{dx^2} = \left(-cosec^2\left(\frac{x^2}{3} - 1\right) \cdot \left(\frac{2x}{3}\right)^2 + \cot\left(\frac{x^2}{3} - 1\right) \times \frac{2}{3}\right)$

S26. Ans. (c)
Sol. $z = \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$
$= \frac{\tan\left(\sin^{-1}\frac{3}{5}\right) + \tan\left(\cot^{-1}\frac{3}{2}\right)}{1 - \tan\left(\sin^{-1}\frac{3}{5}\right) tan\left(\cot^{-1}\frac{3}{2}\right)}$
$= \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4}\frac{2}{3}}$
$= \frac{17/12}{6/12} = \frac{17}{6}$

S27. Ans. (c)

Sol. $y = \sin(x^2)$
$\frac{dy}{dx} = \cos(x^2)2x$
$\frac{d^2y}{dx^2} = 2x(-\sin(x^2)\,2x) + 2\cos(x^2)$
$= -4x^2\sin(x^2) + 2\cos(x^2)$
$a = 2, b = -4$
$a^2 + b^2 = 20$

S28. Ans. (d)

Sol. $x\sqrt{1+y} + y\sqrt{1+x} = 0$
$\Rightarrow\ x^2(1+y) = y^2(1+x)$
$\Rightarrow\ x^2 - y^2 + xy(x-y) = 0$
$\Rightarrow\ (x-y)(x+y+xy) = 0$
$x \neq y\ so\ x + y + xy = 0$
$1 + \frac{dy}{dx} + x\frac{dy}{dx} + y = 0$
$\Rightarrow \frac{dy}{dx}(1+x) = -1-y$
$\Rightarrow \frac{dy}{dx}(1+x) = -1-y$
$\Rightarrow \frac{dy}{dx} = -\left(\frac{1+y}{1+x}\right)$
$at\ x = -3, -3 + y - 3y = 0$
$-2y = 3$
$y = -\frac{3}{2}$
$\frac{dy}{dx} = -\frac{\left(\frac{-1}{2}\right)}{-2} = -\frac{1}{4}$
$2\frac{dy}{dx} + y = \frac{-1}{2} - \frac{3}{2} = -2$

S29. Ans. (d)

Sol. $x = 4\pi r^2$
$\frac{ds}{dt} = 8\pi r\frac{dr}{dt}$
$\therefore 8\pi \times 20 \times 0.5$
$= 80\pi\ \text{cm}^2/\text{min}$

S30. Ans. (b)

Sol. $\frac{dy}{dx} = \frac{2\sqrt{2}(\cos\theta + \theta\sin\theta - \cos\theta}{2\sqrt{2}(-\sin\theta + \theta\cos\theta + \sin\theta)}$
$= tan\,\theta$
$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = 1$
$x(\pi/4) = 2\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}\right)$
$y(\pi/4) = 2\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)$
Equation of tangent
$x - y = \pi$

S31. Ans. (d)

Sol. (a) $d = \left|\frac{1+1+1-2}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$
(b) $x + y + z = 2$
(c) $\left|\frac{-1-5}{\sqrt{3}}\right| = d$
(d) $-x + y + z = 5$

S32. Ans. (d)

Sol. $\overrightarrow{OA} = \vec{a}(\hat{\imath} + \hat{\jmath} + \hat{k})$
$\overrightarrow{BE} = \vec{a}(\hat{\imath} - \hat{\jmath} - \hat{k})$
Let angle between OA and BE is θ
$\cos\theta\ \left|\frac{1-1-1}{\sqrt{3}\sqrt{3}}\right| = \left|\frac{-1}{3}\right|$

S33. Ans. (b)

Sol. $\frac{x}{2} = \frac{y+2}{1} = \frac{5-z}{1}$
Or $\frac{x}{2} = \frac{y+2}{1} = \frac{z-5}{-1}$
So straight line with direction ratios (2, 1, –1)

S34. Ans. (c)

Sol. Clearly, we can see that $\left(\frac{24}{7}, \frac{33}{7}\right)$ is not intersection point of any 2 given conditions so it cannot be a corner point.

S35. Ans. (c)

Sol. $|\vec{a} + \vec{b}| = 1$
$(\vec{a} + \vec{b})(\vec{a} + \vec{b}) = 1$
$\Rightarrow\ \vec{a}\cdot\vec{b} = \frac{-1}{2}$
$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$
$= 3$

S36. Ans. (c)

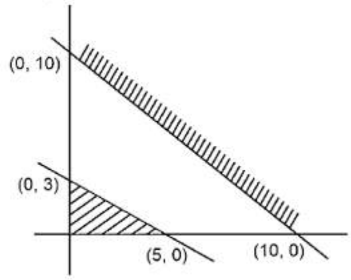
Sol. $\hat{\imath}.(\hat{\imath}) + \hat{\jmath}.(\hat{\jmath}) + \hat{k}.(\hat{k})$
$=1 + 1 + 1$
$=3$

S37. Ans. (d)

Sol. $|\vec{a} - \vec{b}| = 10$
$\Rightarrow\ |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 100$
$\Rightarrow\ \vec{a}\cdot\vec{b} = 0$

S38. Ans. (b)

Sol.



Region is infeasible

S39. Ans. (*)

Sol. After given 2 selections, 4 red and 6 black balls are present.
So required probability $= \frac{4}{10} = \frac{2}{5}$

S40. Ans. (b)

Sol. $P(A \cup B) = \frac{3}{5}$
$\Rightarrow\ P(A) + P(B) - (A \cap B) = \frac{3}{5}$
$\Rightarrow\ \frac{1}{2} + x - \frac{1}{2}\cdot x = \frac{3}{5}$
$\Rightarrow\ \frac{x}{2} = \frac{1}{10}$
$\Rightarrow\ x = \frac{1}{5}$

S41. Ans. (d)

Sol. $v(x) = (8-2x)(3-2x)x$
$= x(24 + 4x^2 - 22x)$
$= 4x^3 - 22x^2 + 24x$

S42. Ans. (b)

Sol. $f'(x) = 12x^2 - 44x + 24$

S43. Ans. (b)

Sol. $f'(x) = 12x^2 - 44x + 24$
$= 4(3x^2 - 11x + 6)$
$= 4(3x^2 - 9x - 2x + 6)$
$= 4[3x(x-3) - 2(x-3)]$
$= (3x-2)(x-3)$
$f'(x) = 0$ at $x = \frac{2}{3}, 3$

$f''(x) = (3x - 2) \times 1 + (x - 3) \times 3$
$f''(x) < 0$ for $x = \frac{2}{3}$
So, f(x) attains maxima at $x = \frac{2}{3}$

S44. Ans. (c)
Sol. Length$= 8 - 2x$
$= 8 - 2 \cdot \frac{2}{3}$
$= \frac{20}{3}$

S45. Ans. (d)
Sol. $V(x) = 4x^3 - 22x^2 + 24x$
At $x = \frac{2}{3}$
$V = 4 \cdot \frac{8}{27} - 22 \cdot \frac{4}{9} + 16$
$= \frac{32}{27} - \frac{88}{9} + 16$
$= 16 - \frac{232}{27}$
$= \frac{200}{27}$

S46. Ans. (b)
Sol. $\text{Sin}\,\theta = \left|\frac{10 \cdot 2 + 2 \cdot 3 - 11 \cdot 6}{15 \cdot 7}\right|$
$= \left|\frac{-40}{105}\right| = \frac{8}{21}$

S47. Ans. (b)
Sol. Point on line $(\equiv 2\lambda - 1, 3\lambda, 6\lambda + 3)$
So, $10(2\lambda - 1) + 2(3\lambda) - 11(6\lambda + 3) - 3 = 0$
$\Rightarrow 20\lambda - 10 + 6\lambda - 66\lambda - 33 - 3 = 0$
$\Rightarrow -40\lambda - 46 = 0$
$\Rightarrow \lambda = \frac{-23}{20}$
Point $\equiv \left(\frac{-46}{20} - 1, \frac{-69}{20}, \frac{-138}{20} + 3\right)$
$\equiv \left(\frac{-33}{10}, \frac{-69}{20}, \frac{-39}{10}\right)$

S48. Ans. (d)
Sol. $d = \left|\frac{-3}{\sqrt{10^2 + 2^2 + 11^2}}\right| = \frac{1}{5}$

S49. Ans. (b)
Sol. Let foot of perpendicular be (α, β, γ)
$\frac{\alpha - 0}{10} = \frac{\beta - 0}{2} = \frac{\gamma - 0}{-11} = \frac{-(-3)}{225}$
$(\alpha, \beta, \gamma) = \left(\frac{30}{225}, \frac{6}{225}, \frac{-33}{225}\right)$
$= \left(\frac{2}{15}, \frac{2}{75}, \frac{-11}{75}\right)$

S50. Ans. (d)
Sol. At Z-axis, x and y coordinates are zero
So, $z = -\frac{3}{11}$