TALENT & OLYMPIAD

Introduction

In previous chapter we have studied about the polynomials. Quadratic equation is also a type of polynomial of degree two. The most general form of the quadratic equation is ax^2bx+c, where a, b, c are the coefficients. Solving the quadratic equation in general form has been worked out by some of the ancient mathematicians. In fact, Brahmagupta gave an explicit method to solve the quadratic equations. Later Sridharacharya derived special formulae for solving the quadratic equations known as the quadratic formula. One of the most prominent methods used in olden days mathematics for solving the quadratic equation was given by Bhaskar-11 for solving the quadratic equation by the completing square method.

Quadratic Equations

The most general quadratic equation is $ax^2+bx+c=0$. This equation can be solved by using the Discriminant method. In this method we find the discriminant of the given quadratic equation as follows:

$D=b^2-4ac,$

(a) If D > 0, then the given equation will have real and distinct roots and we can find the roots of the given equation.

(b) If D = 0, then the given equation will have real and equal roots.

(c) If D < 0, then the given equation will have no real roots. In this case roots will be imaginary. Here we find the imaginary roots only.

In case of real roots we can find the roots by using the formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} or \frac{-b\pm\sqrt{D}}{2a}$$

The quadratic equation can have maximum of two roots.

In case of imaginary roots we can find the roots by using the relation,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} or \frac{-b\pm i\sqrt{D}}{2a}$$

Where, i denote the imaginary part of the roots.

Relations between the Roots of the Quadratic Equation

If $\alpha\, and\, \beta$ are the roots of the quadratic equation $ax^2+bx+c=0$, then the relation between the roots of the quadratic equation is given by,

Sum of the roots $=S=\alpha+\beta=-\frac{b}{a}$

Product of the roots $=P=\alpha\beta=\frac{c}{a}$

Formation of Quadratic Equations

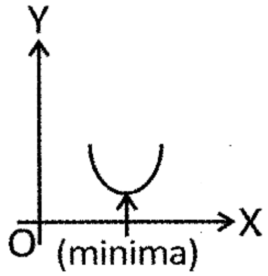
If $\alpha\, and\, \beta$ are the roots of the quadratic equation, S denotes its sum and P denotes its product, then the quadratic equation is given by:

$x^2-(S)X+P=0$

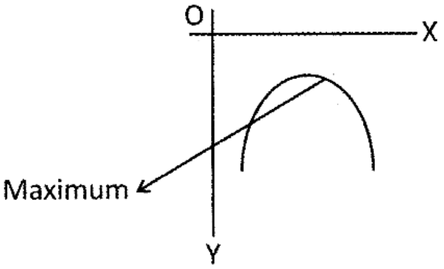
Graphical Representation of a Quadratic Equation

For the quadratic equation $ax^2bx+c=0, a\neq 0$, the nature of graph for different values of D is:

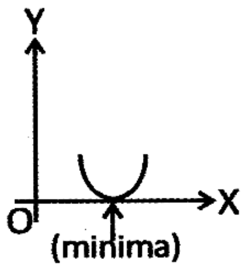
(a) If D < 0, and a > 0, then the graph is given by:



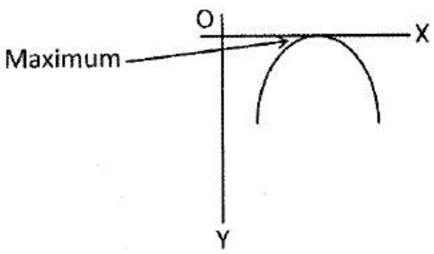
If a < 0, then the graph is given by.



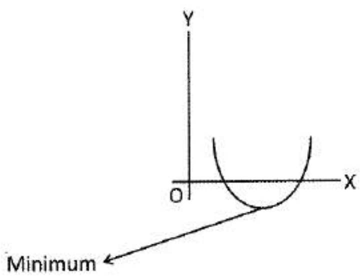
(b) If D = 0, and a > 0, then the graph of the function is given by



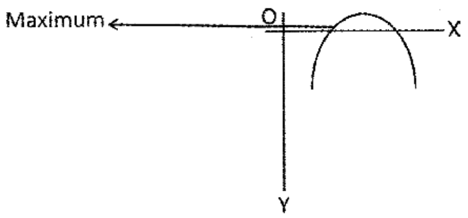
If a < 0, then the graph is given by



(d) If D > 0, and a > 0, then the graph of the function is given by,



If a < 0, then the graph of the function is given by,



Roots of Biquadratic Equation

Any biquadratic equation, $ax^4 + bx^3 + cx^2 + e = 0$, will have four roots. If a, P, Y, and 5 are its roots, then the relation between the roots is given by

Sum of the roots $\alpha + \beta + \gamma + \delta = -\frac{b}{a}$

Sum of product of two roots at a time

$= \alpha\beta + \beta\gamma + \gamma\delta + \delta\alpha = \frac{c}{a}$

Sum of product of three roots at a time

$\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta ga + \delta\alpha\beta = -\frac{d}{a}$

Product of the roots $\alpha\beta\gamma\delta = \frac{e}{a}$

Commonly Asked QUESTIONS

Find the value of $\sqrt{9+\sqrt{9+\sqrt{9+---}}}$.

(a) $\frac{1 \pm \sqrt{37}}{2}$

(b) $\frac{1 \pm \sqrt{36}}{2}$

(0 $\frac{1 \pm \sqrt{9}}{2}$

(d) $\frac{1 \pm \sqrt{8}}{2}$

(e) None of these

Answer: (a)

Explanation

Let $y = \sqrt{9+\sqrt{9+\sqrt{9+---}}}$

$\Rightarrow y = \sqrt{9+y}$

$\Rightarrow y^2 = 9 + y$

$\Rightarrow y^2 - y - 9 = 0$

$\Rightarrow y \frac{1 \pm \sqrt{37}}{2}$

A If α *and* β are the roots of the equation $4x^2+5x-6=0$, then the value of $\alpha\beta^2+\alpha^2\beta+\alpha\beta$ are.

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$

(c) $-\frac{3}{8}$ (d) $-\frac{5}{8}$

(e) None of these

Answer: (b)

Explanation

We have, $\alpha\beta^2+\alpha^2\beta+\alpha\beta(\alpha+\beta)+\alpha\beta$

$=\frac{-6}{4}\times\frac{-5}{4}+\frac{-6}{4}=\frac{3}{8}$

If α *and* β is real and α^2 *and* $-\beta^2$ is the roots of $4y^2+y+3=0$, then find the value of β^2.

(a) $\left(\frac{5}{8}\&-1\right)$ (b) $(1\&-1)$

(c) $\left(1\&\frac{3}{4}\right)$ (d) $\left(1\&\frac{-3}{4}\right)$

(e) None of these

Answer: (d)

Explanation

We have sum of the roots $=\alpha^2=\beta^2=\frac{-1}{4}$

Product of the roots $=\alpha^2\beta^2=\frac{3}{4}$

Form the above two equations we get,

$\left(\beta^2-\frac{1}{4}\right)\beta^2=\frac{3}{4}$

$\Rightarrow 4\beta^2+3=0$ $\Rightarrow \beta^2=\frac{1\pm\sqrt{49}}{8}$ $\Rightarrow \beta^2=1\&\frac{-3}{4}$

If α *and* β are the roots of $y^2+4y+1=0$ and γ *and* δ are the roots of $y^2+7y+1=0$, then find the value of

$(\alpha-\beta)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)$.

(a) 15 (b) 33

(c) 41 (d) 63

(e) None of these

Answer: (b)

If the coefficient of z in the quadratic equation $z^2+az+b=0$ is taken as 18 in place of 12 and its roots were found to be -16 and -2. The roots of the original equation are:

(a) - 12 & - 6
(b) - 14 & - 4
(c) - 16 & - 2
(d) – 8 &- 4
(e) None of these

Answer: (d)

You Must KNOW

- The demerit of the factorization method, is that it takes time to figure out the numbers within the factors.
- The first known solution of a quadratic equation is the one given in the Berlin papyrus from the Middle Kingdom (ca. 2160-1700 BC) in Egypt.
- The Hindu mathematician Aryabhata (475 or 476-550) gave a rule for the sum of a geometric series that shows knowledge of the quadratic equations with both solutions
- Viete was among the first to replace geometric methods of solution with analytic ones, although he apparently did not grasp the idea of a general quadratic equation.
- Sridhara gave the positive root of the quadratic formula, as stated by Bhaskara.

SUMMARY

- The most general quadratic equation is in the form $ax^2+bx+c=0$, where a, b, c are constants and x is the variable.
- A real number m is said to be the root of the quadratic equation if it satisfies the quadratic equation, $i.e. am^2+bm+c=0$.
- For any quadratic equation, the number of root is always two.
- Quadratic equation can be solved by factorization or using the discriminant method.
- The value of $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}, if\ D\leq 0$.
- No real root exist for D < 0.

Self Evaluation TEST

Duration 10 Minutes

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1. **The quadratic equation $\alpha z^2+\beta z+\gamma=0$ is such that one of its roots is equal to the m^ power of the other, then the value of**

   $\left(\alpha\gamma^m\right)^{\frac{1}{m+1}}+\left(\alpha^m\gamma\right)^{\frac{1}{m+1}}$ **is_____.**

   (a) α (b) β
   (c) γ (d) $-\beta$
   (e) None of these

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2. **A group of students is solving a quadratic equation in y. One among the students copied the constant term incorrectly and got the roots as 4 and 6. The other copied the constant term and coefficient of y2 as 8 and 2 respectively. The correct roots of the equation are:**

   (a) (- 2 & 2) (b) (4 & 1)
   (c) (3 & - 1) (d) (1 & - 1)
   (e) None of these

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3. **If the product of the roots of the equation $z^2-4mz+3e^{210gm}-1=0$ is 26 and m > 0, then the value of m is given by:**

   (a) ± 3 (b) - 2
   (c) ± 5 (d) ± 6
   (e) None of these

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4. **If $\alpha\, and\, \beta$ are the roots of the equation $4y^2-8y+12=0$, then the value of $(4\alpha-8)^{-2}+4(4\beta-8)^{-2}$ is given by:**

   (a) $\frac{9}{52}$ (b) 0
   (c) $-\frac{1}{72}$ (d) 1
   (e) None of these

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5. **If the roots of the equation $y^2-(3+n)y+(n^2+4n+4)=0$ are equal, then the value of n is:**

   (a) $\left(1\&\frac{5}{3}\right)$ (b) $\left(-1\&\frac{-5}{3}\right)$
   (c) $\left(3\&-\frac{5}{3}\right)$ (d) $(3\&2)$ (e) None of these

6. **If a, P, y are the roots of the equation $z^3 - 4z + 2 = 0$, then the value of $(\alpha - 3)(\beta - 3)(\gamma - 3)$ is given by:**

(a) - 9 (b) - 14
(c) - 12 (d) - 17
(e) None of these

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7. **If one of the irrational roots of the equation $z^3 - 11z^2 + 37z - 35 = 0$ is given by $\left(3 + \sqrt{2}\right)$ then one of the rational roots of the above equation is:**

(a) 5 (b) - 4
(c) - 2 (d) 17
(e) None of these

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8. **For the given equation, if we have f $f(1) = f(3) = 0$, then find f (4), if the given equation is** $z^4 + az^3 + bz^2 + \gamma z + \delta = 0$

(a) 24 (b) 14
(c) 16 (d) 18
(e) None of these

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9. **Find the value of m if the sum of the products of the three roots of the equation** $(m+1)y^4 + 25z^3 + 45z^2 - (2m-3)z + 9 = 0$.

(a) 4 (b) $\frac{3}{4}$
(c) 19 (d) 18
(e) None of these

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10. **Thomas is sitting at the top of the building with rifle in his hand aiming at the mad dog which is running towards the base of the building. If the building is 9 m high and dog is at a distance of 27 m from the base of the building, then at what distance form the base will the bullet hit the dog if the speed of dog and the bullet are same.**

(a) 10m (b) 12m
(c) 16m (d) 18m
(e) None of these

Answers – Self Evaluation Test																			
1.	D	**2.**	B	**3.**	A	**4.**	C	**5.**	B	**6.**	D	**7.**	D	**8.**	A	**9.**	C	**10.**	B

Self Evaluation Test
SOLUTIONS

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1. Let the two roots of the quadratic equation be 'a' and 'b'.
Then according to the question, $a = b^m$.
Now sum of the root = a + b = $-\dfrac{\beta}{\alpha}$ ----------- (1)
Product of the root = ab = $\dfrac{\gamma}{\alpha}$ --------- (2)
Since $a = b^m$, so we have $b^{m+1} = \dfrac{\gamma}{\alpha}$ or $b = \left(\dfrac{\gamma}{\alpha}\right)^{\frac{1}{m+1}}$
Therefore, $a = \left(\dfrac{\gamma}{\alpha}\right)^{\frac{m}{m+1}}$
Putting the value of a and b in equation (1) we get, $\left(\dfrac{\gamma}{\alpha}\right)^{\frac{m}{m+1}} + \left(\dfrac{\gamma}{\alpha}\right)^{\frac{1}{m+1}} = -\dfrac{\beta}{\alpha}$
$\Rightarrow -\beta = \alpha^{\frac{1}{m+1}}\gamma^{\frac{m}{m+1}} + \alpha^{\frac{m}{m+1}}\gamma^{\frac{1}{m+1}}$
$\Rightarrow -\beta = (\alpha\gamma^m)^{\frac{1}{m+1}} + \left(\alpha^m\gamma\right)^{\frac{1}{m+1}}$.

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2. By first condition, sum of the root = 10 and product of the root = 24.
Therefore the quadratic equation will be $= ay^2 - 10y + 24 = 0$
But in the above equation the constant term is wrong, therefore the equation is reduced to $ay^2 - 10y + c = 0$,
Now by second condition, the coefficient of y^2 is 2 and the constant term is 8.
Hence the equation is reduced to $2y^2 - 10y + 8 = 0$.
On solving the above equation we get the value of y as 4 and 1.

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3. From the above equation we have the product of the root $= 3e^{210mg} - 1$
But the product of the root is given as = 26
$\therefore 3e^{210mg} - 1 = 26$
$\Rightarrow 3m^2 = 27 \Rightarrow \pm 3$

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4. $\alpha + \beta = 2$ and $\alpha\beta = 3$
Now we, $\left(4\alpha - 8\right)^{-2} + \left(4\beta - 8\right)^{-2} = \dfrac{1}{\left(4\alpha - 8\right)^2} + \dfrac{1}{\left(4\beta - 8\right)^2}$

$$=\frac{16\alpha^2+64-64\alpha+16\beta^2+64-64\beta}{(16\alpha\beta-32\alpha-32\beta+64)^2}=\frac{\left\{(\alpha+\beta)^2-2\alpha\beta\right\}-64(\alpha+\beta)+128}{(16\alpha\beta-32(\alpha+\beta)+64)^2}$$

$$=\frac{16\alpha^2+16\beta^2-64(\alpha+\beta)+128}{(16\alpha\beta-32(\alpha+\beta)+64)^2}=\frac{-32-128+128}{48\times 48}=-\frac{1}{72}$$

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5. As the roots of the equation $y^2-(3+n)y+\left(n^2+4n+4\right)=0$ are equal, hence D = 0

$\Rightarrow b^2-4ac=0$

$\Rightarrow(3+n)^2-4\left(n^2+4n+4\right)=0$

$\Rightarrow(3+n)^2=4(n+2)^2$

$\Rightarrow(3+n)\pm 2(n+2)$

$\Rightarrow n=\left(-1\&\frac{-5}{3}\right)$
