11 Chapter

Geometry and Mensuration

The chapters of geometry and mensuration have their share of questions in the CAT and other MBA entrance exams. For doing well in questions based on this chapter, the student should familiarise himself/herself with the basic formulae and visualisations of the various shapes of solids and two-dimensional figures based on this chapter.

The following is a comprehensive collection of for- mulae based on two-dimensional and three-dimensional figures.

For the purpose of this chapter we have divided the theory in two parts:

- Part I consists of geometry and mensuration of two-dimensional figures
- Part II consists of mensuration of three-dimensional figures.

PART I: GEOMETRY

INTRODUCTION

Geometry and Mensuration are important areas in the CAT examination. In the Online CAT, the Quantitative Aptitude and Data Interpretation section has consisted by an average between 3–5 questions out of 30 from these chapters. Besides, questions from these chapters appear prominently in all major aptitude based exams like MBAs, Bank POs, etc.

Hence, the student is advised to ensure that he/she studies this chapter completely and thoroughly. Skills to be developed while studying and practising this chapter will be application of formula and visualisation of figures and solids.

The principal skill required for doing well in this chapter is the ability to apply the formulae and theorems.

The following is a comprehensive collection of formulae based on two-dimensional figures. The student is advised to remember the formulae in this chapter so that he is able to solve all the questions based on this chapter.

THEORY

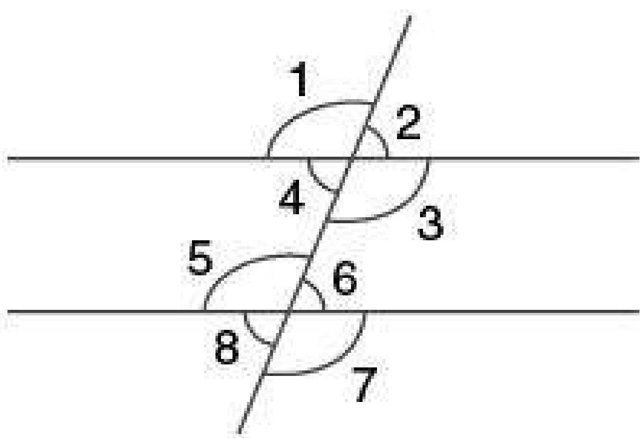
Basic Conversions			
A.	1 m = 100 cm = 1000 mm 1 km = 1000 m = 5/8 miles 1 inch = 2.54 cm	B.	1 m = 39.37 inches 1 mile = 1760 yd = 5280 ft 1 nautical mile (knot) = 6080 ft
C.	100 kg = 1 quintal 10 quintal = 1 tonne = 1000 kg 1 kg = 2.2 pounds (approx.)	D.	1 litre = 1000 cc 1 acre = 100 sq m 1 hectare = 10000 sq m

STRAIGHT LINES

Parallel Lines: Two straight lines are parallel if they lie on the same plane and do not intersect however far produced.

Transversal: It is a straight line that intersects two parallel lines. When a transversal intersects two parallel lines then

1. Corresponding angles are equal, (that is: For the following figure)
   1 = 5; 2 = 6; 4 = 8; 3 = 7
2. Alternate interior angles are equal, that is (Refer following figure)
   4 = 6; 5 = 3
3. Alternate exterior angles are equal, that is
   2 = 8; 1 = 7
4. Interior angles on the same side of transversal add up to 180°, that is
   4 + 5 = 3 + 6 = 180°



POLYGONS

Polygons are plane figures formed by a closed series of rectilinear (straight) segments. ***Example:*** Triangle, Rectangle…

Polygons can broadly be divided into two types:

(a) *Regular polygons:* Polygons with all the sides and angles equal.

(b) *Irregular polygons:* Polygons in which all the sides or angles are not of the same measure.

Polygon can also be divided as *concave* or *convex* poly-gons.

Convex polygons are the polygons in which all the diagonals lie inside the figure otherwise it's a concave polygon

Polygons can also be divided on the basis of the number of sides they have.

No. of sides	*Name of the polygon*	*Sum of all the angles*
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	500°
8	Octagon	1080°
9	Nonagon	1260°
10	Decagon	1440°

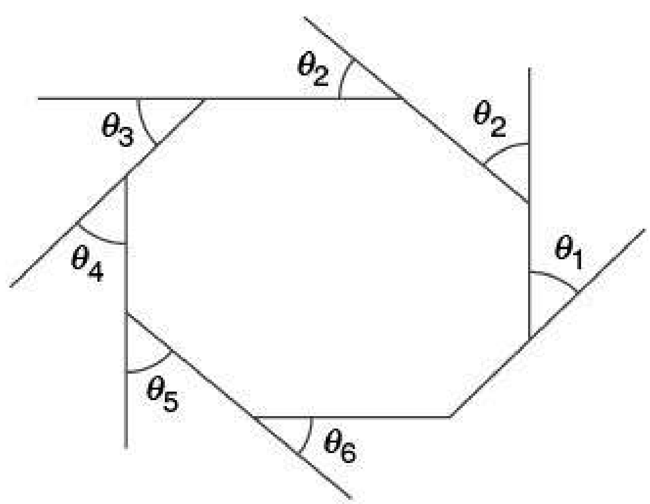
Properties

1. Sum of all the angles of a polygon with n sides = $(2n - 4)p/2$ or $(n - 2)p$ Radians = $(n - 2)$ 180° degrees
2. Sum of all exterior angles = 360°◊

   i.e. In the figure below:

   $q_1 + q_2 + ... + q_6 = 360°$

   In general, $q_1 + q_2 + ... + q_n = 360°$
3. No. of sides = 360°/exterior angle.
4. Area = $(ns^2/4) \times \cot(180/n)$; where s = length of side, n = no. of sides.
5. Perimeter = $n \times s$.



TRIANGLES (D)

A triangle is a polygon having three sides. Sum of all the angles of a triangle = 180°.

Types

1. *Acute angle triangle:* Triangles with all three angles acute (less than 90°).
2. *Obtuse angle triangle:* Triangles with one of the angles obtuse (more than 90°).

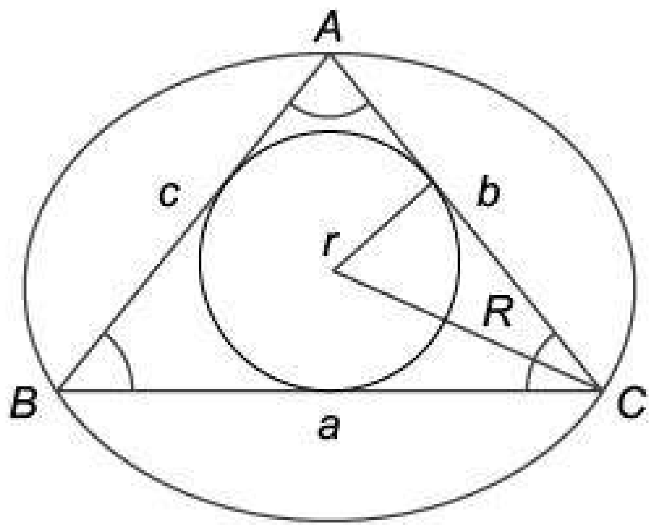
   Note: we cannot have more than one obtuse angle in a triangle.
3. *Right angle triangle:* Triangle with one of the angles equal to 90°.
4. *Equilateral triangle:* Triangle with all sides equal. All the angles in such a triangle measure 60°.
5. *Isosceles triangle:* Triangle with two of its sides equal and consequently the angles opposite the equal sides are also equal.
6. *Scalene Triangle:* Triangle with none of the sides equal to any other side.

Properties (General)

- Sum of the length of any two sides of a triangle has to be always greater than the third side.
- Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
- Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
- The sine rule: $a/\sin A = b/\sin B = c/\sin C = 2R$

  (where R = circum radius.)
- The cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

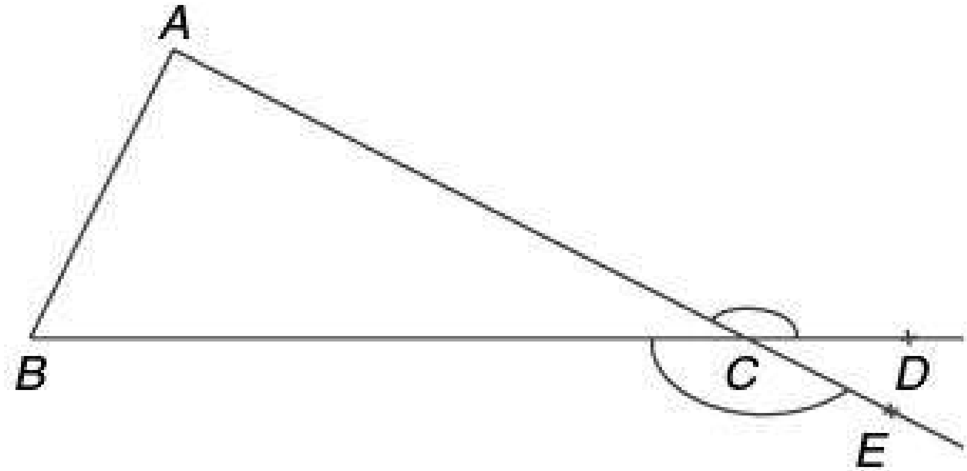
  This is true for all sides and respective angles.

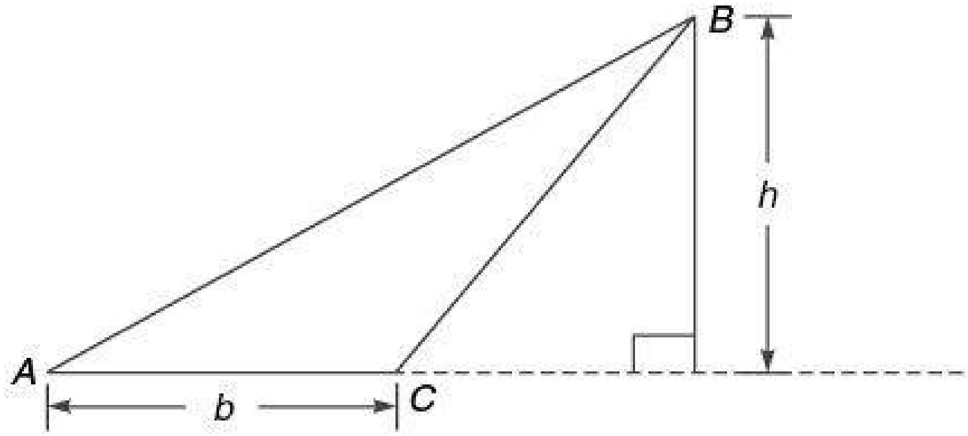


  In case of a right –, the formula reduces to $a^2 = b^2 + c^2$

  Since cos 90 = 0
- The exterior angle is equal to the sum of two interior angles not adjacent to it.

  $–ACD = –BCE = –A + –B$

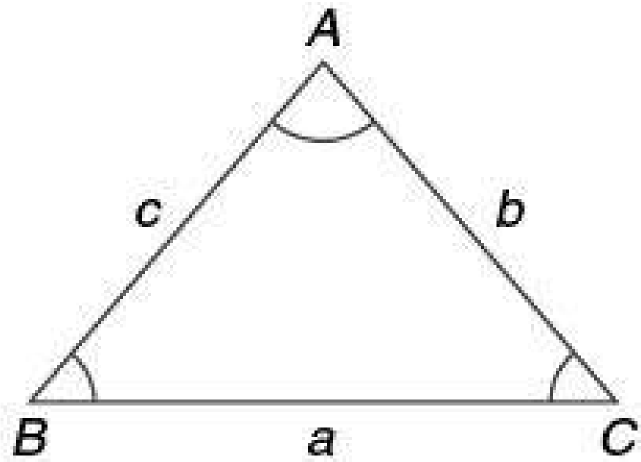




Area

1. Area = 1/2 base × height or 1/2 bh.
   Height = Perpendicular distance between the base and vertex opposite to it
2. Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (Hero's formula)

   where $S = \dfrac{a+b+c}{2}$ abc being the length of the sides)and,(
3. Area = rs (where r is in radius)
4. Area = 1/2 × product of two sides × sine of the included angle
   = 1/2 ac sin B
   = 1/2 ab sin C
   = 1/2 bc sin A

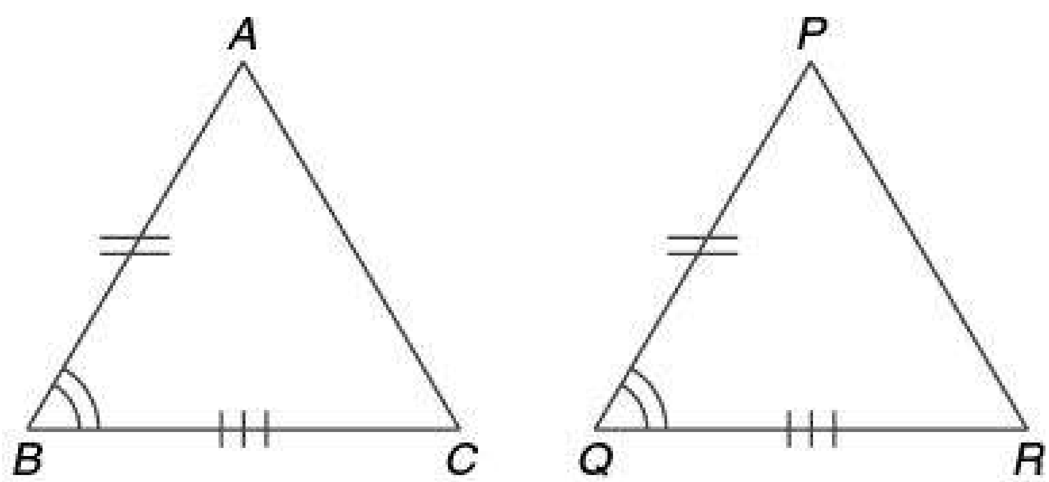


4. Area = *abc*/4*R*
   where *R* = circum radius

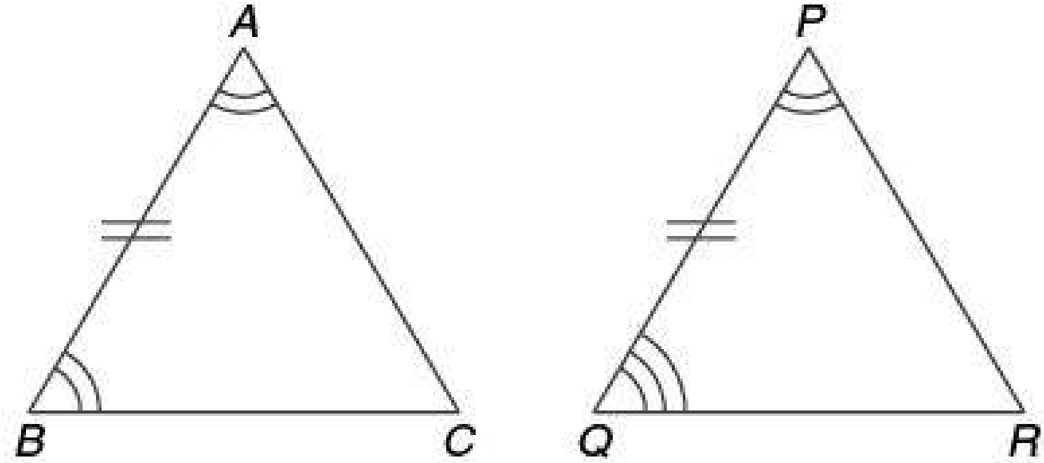
Congruency of Triangles Two triangles are congruent if all the sides of one are equal to the corresponding sides of another. It follows that all the angles of one are equal to the corresponding angles of another. The notation for congruency is (≅).

Conditions for Congruency

1. *SAS congruency*: If two sides and an included angle of one triangle are equal to two sides and an included angle of another, the two triangles are congruent. (See figure below.)
   Here, *AB* = *PQ*
   BC = *QR*
   and –*B* = –*Q*
   So D*ABC* ≅ D*PQR*



2. *ASA congruency*: If two angles and the included side of one triangle is equal to two angles and the included side of another, the triangles are congruent. (See figure below.)
   Here, –*A* = –*P*
   –*B* = –*Q*
   and *AB* = *PQ*
   So D*ABC* ≅ D*PQR*



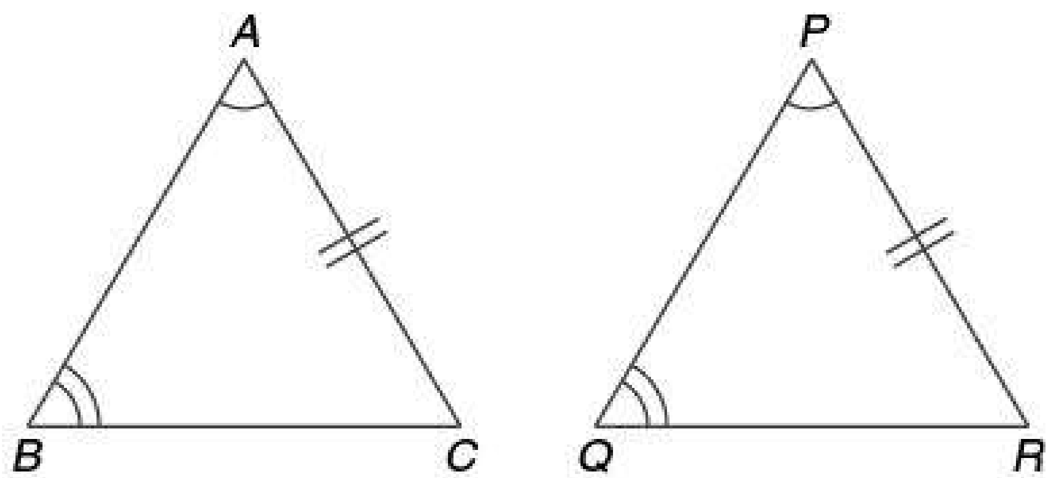
3. *AAS congruency*: If two angles and side opposite to one of the angles is equal to the corresponding angles and the side of another triangle, the triangles are congruent. In the figure below:

–*A* = –*P*

–*B* = –*Q*

and *AC* = *PR*

So D*ABC* ≅ D*PQR*



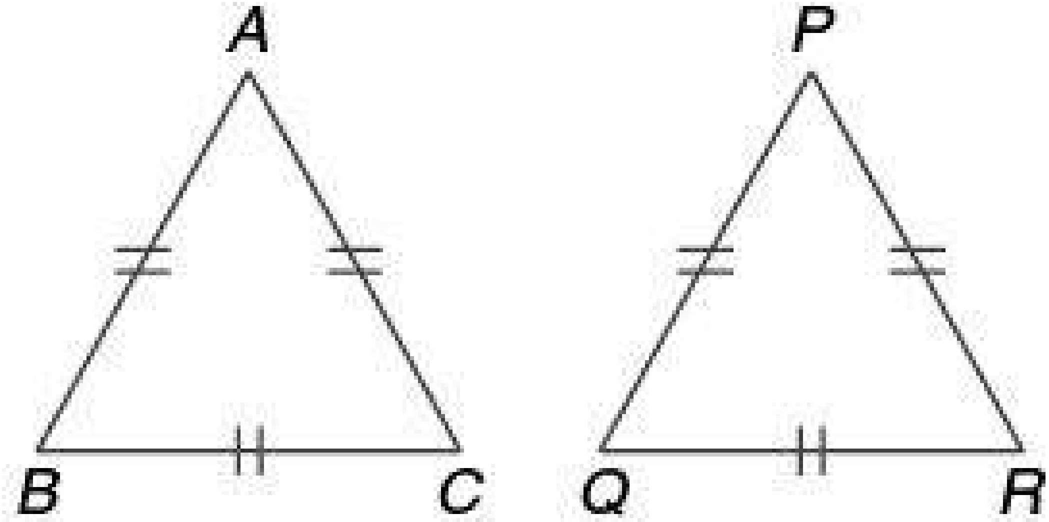
4. *SSS congruency*: If three sides of one triangle are equal to three sides of another triangle, the two triangles are congruent. In the figure below:

AB = *PQ*

BC = *QR*

AC = *PR*

\ D*ABC* ≅ D*PQR*

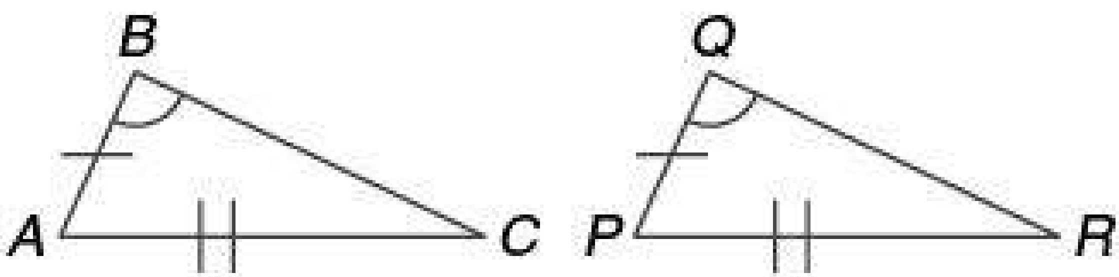


5. *SSA congruency*: If two sides and the angle opposite the greater side of one triangle are equal to the two sides and the angle opposite to the greater side of another triangle, then the triangles are congruent. The congruency doesn't hold if the equal angles lie opposite the shorter side. In the figure below, if

   AB = *PQ*

   AC = *PR*

   –*B* = –*Q*



   Then the triangles are congruent.

   i.e. D*ABC* $\cong$ D*PQR*.

Similarity of triangles Similarity of triangles is a special case where if either of the conditions of similarity of polygons holds, the other will hold automatically.

Types of Similarity

1. *AAA similarity:* If in two triangles, corresponding angles are equal, that is, the two triangles are equiangular then the triangles are similar.

   Corollary (*AA similarity*)*:* If two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar. The reason being, the third angle becomes equal automatically.

2. *SSS similarity:* If the corresponding sides of two triangles are proportional then they are similar.

   For D*ABC* to be similar to D*PQR*, *AB*/*PQ* = *BC*/*QR* = *AC*/*PR*, must hold true.

3. *SAS similarity:* If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

   D *ABC* ~D*PQR*

If $AB/BC = PQ/QR$ and $B = –Q$

Note: In similar triangles; the following identity holds:

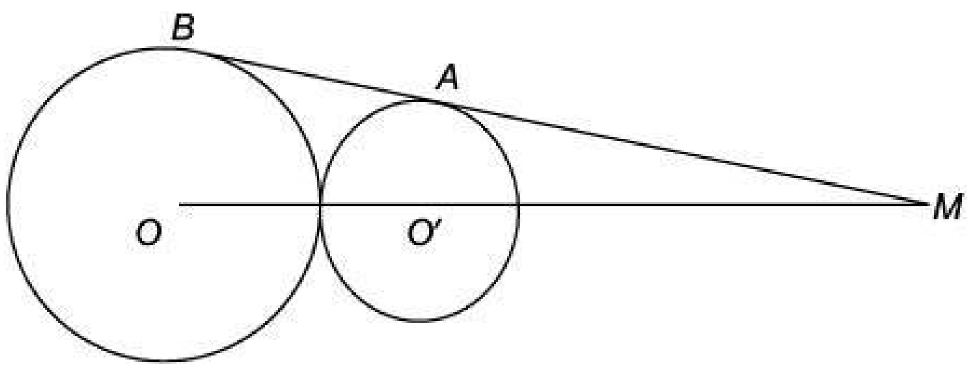
Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of inradii = Ratio of angle bisectors

While there are a lot of methods through which we see similarity of triangles, the one thing that all our Maths teachers forgot to tell us about similarity is the basic real life concept of similarity. i.e. **Two things are similar if they look similar!!**

If you have been to a toy shop lately, you would have come across models of cars or bikes which are made so that they look like the original—but are made in a different size from the original. Thus you might have seen a toy Maruti car which is built in a ratio of 1:25 of the original car. The result of this is that the toy car would look very much like the original car (of course if it is built well!!). Thus if you have ever seen a father and son looking exactly like each other, you have experienced similarity!!

You should use this principle to identify similar triangles. In a figure two triangles would be similar simply if they look like one another.

Thus, in the figure below if you were to draw the radii OB and O¢A the two triangles MOB and MO¢A will be similar to each other. Simply because they look similar. Of course, the option of using the different rules of similarity of triangles still remains with you.

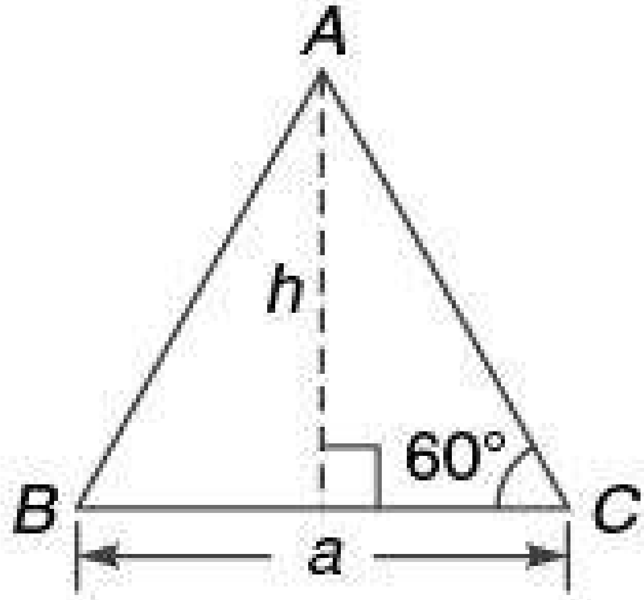


Equilateral Triangles (of side *a*):

1. (\ sin 60 = $\sqrt{3}$ /2 = h/side)

   $h = \dfrac{a\sqrt{3}}{2}$

2. Area = 1/2 (base) × (height) = $\dfrac{1}{2} \times a \times \dfrac{a\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4} a^2$



3. R (circum radius) = $\frac{2h}{3} = \frac{a}{\sqrt{3}}$.
4. r (in radius) = $\frac{h}{3} = \frac{a}{2\sqrt{3}}$.

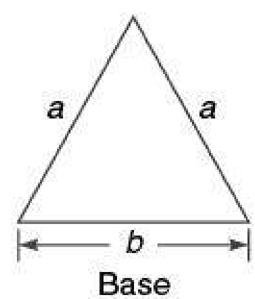
Properties

1. The incentre and circumcentre lies at a point that divides the height in the ratio 2 : 1.
2. The circum radius is always twice the in radius. [$R = 2r$.]
3. Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.
4. An equilateral triangle in a circle will have the maximum area compared to other triangles inside the same circle.

Isosceles Triangle

Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

In an isosceles triangle, the angles opposite to the equal sides are equal.



Right-Angled Triangle

Pythagoras Theorem In the case of a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In the figure below, for triangle ABC, $a^2 = b^2 + c^2$

Area = 1/2 (product of perpendicular sides)

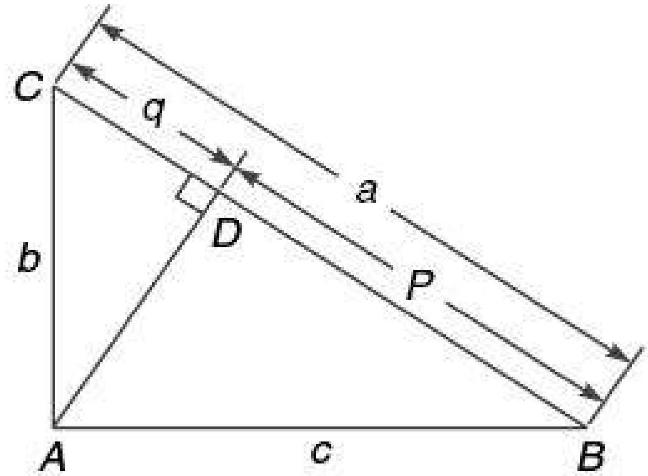
R(circumradius) = $\frac{\text{hypotenuse}}{2}$

Area = rs

(where r = in radius and $s = (a + b + c)/2$ where a, b and c are sides of the triangle)

fi$1/2\ bc = r(a + b + c)/2$

fi $r = (bc)/(a + b + c)$



In the triangle ABC,

DABC ~ DDBA ~ DDAC

(*Note:* A lot of questions are based on this figure.)

Further, we find the following identities:

1. DABC ~ DDBA

   \ $AB/BC = DB/BA$

   fi $AB^2 = DB \times BC$

   fi $c^2 = pa$

2. DABC ~ DDAC

   $AC/BC = DC/AC$

   fi $AC^2 = DC \times BC$

   fi $b^2 = qa$

3. DDBA ~ DDAC

   $DA/DB = DC/DA$

   $DA^2 = DB \times DC$

   fi $AD^2 = pq$

Basic Pythagorean Triplets

Æ 3, 4, 5 Æ 5, 12, 13 Æ 7, 24, 25 Æ 8, 15, 17 Æ 9, 40, 41 Æ 11, 60, 61 Æ 12, 35, 37 Æ 16, 63, 65 Æ 20, 21, 29 Æ 28, 45, 53. These triplets are very important since a lot of questions are based on them.

Any triplet formed by either multiplying or dividing one of the basic triplets by any positive real number will be another Pythagorean triplet.

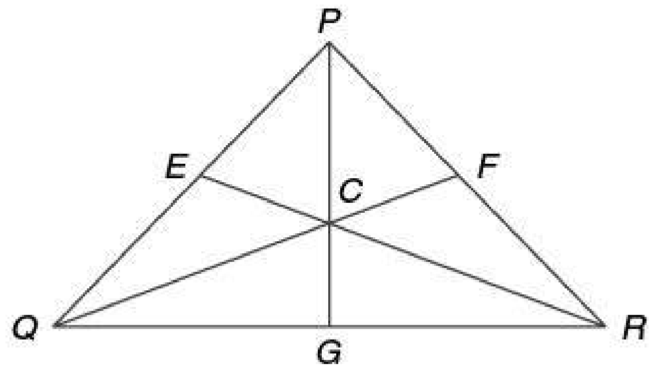
Thus, since 3, 4, 5 form a triplet so also will 6, 8 and 10 as also 3.3, 4.4 and 5.5.

Similarity of right triangles Two right triangles are similar if the hypotenuse and side of one is proportional to hypotenuse and side of another. (RHS–similarity–Right angle hypotenuse side).

Important Terms with Respect to a Triangle

1. Median A line joining the mid-point of a side of a triangle to the opposite vertex is called a median. In the figure the three medians are *PG*, *QF* and *RE* where *G*, *E* and *F* are mid-points of their respective sides.

- A median divides a triangle into two parts of equal area.
- The point where the three medians of a triangle meet is called the *centroid* of the triangle.
- The centroid of a triangle divides each median in the ratio 2 : 1.
  i.e. $PC : CG = 2 : 1 = QC : CF = RC : CE$



Important formula with respect to a median

Æ 2 × (median)^2 + 2 × (1/2 the third side)^2

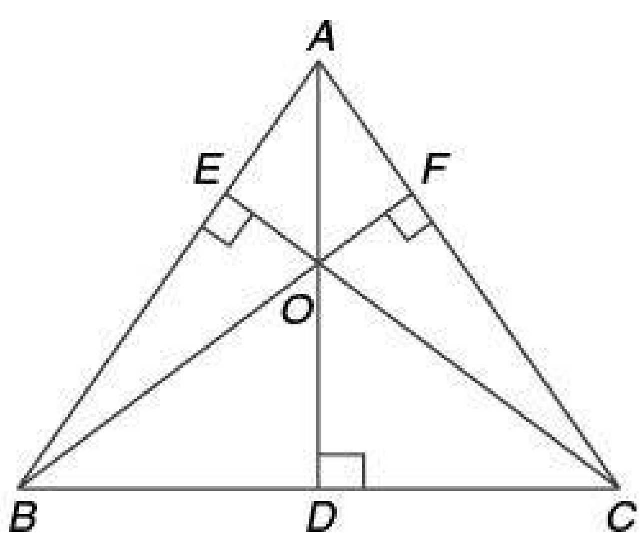
= Sum of the squares of other two sides

fi $2(PG)^2 + 2 \times \left(\frac{QR}{2}\right)^2$

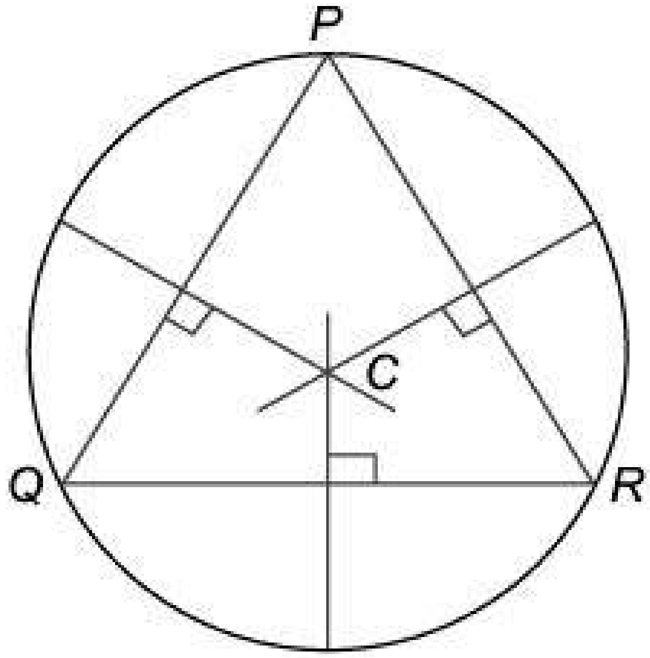
$= (PQ)^2 + (PR)^2$

2. Altitude/Height A perpendicular drawn from any vertex to the opposite side is called the *altitude*. (In the figure, *AD*, *BF* and *CE* are the altitudes of the triangles).

- All the altitudes of a triangle meet at a point called the *orthocentre* of the triangle.
- The angle made by any side at the orthocentre and the vertical angle make a supplementary pair (i.e. they both add up to 180°). In the figure below:
  –*A* + –*BOC* = 180° = –*C* + –*AOB*



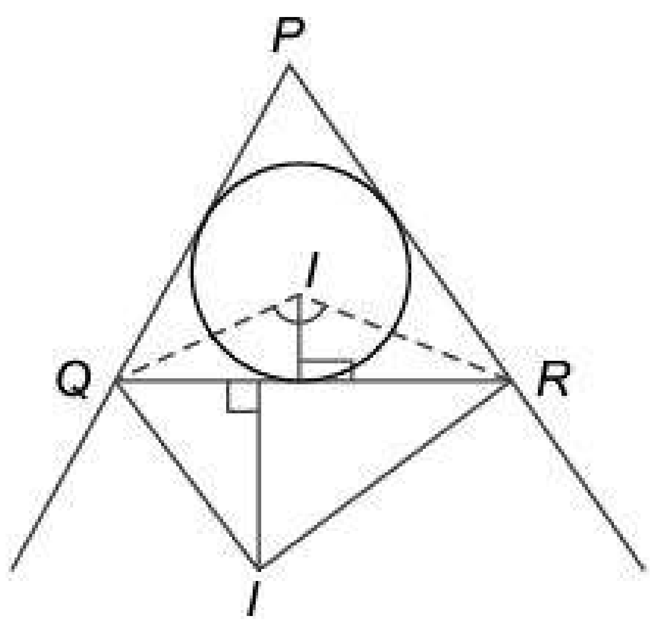
3. Perpendicular Bisectors A line that is a perpendicular to a side and bisects it is the perpendicular bisector of the side.



- The point at which the perpendicular bisectors of the sides meet is called the *circumcentre* of the triangle
- The circumcentre is the centre of the circle that circumscribes the triangle. There can be only one such circle.
- Angle formed by any side at the circumcentre is two times the vertical angle opposite to the side. This is the property of the circle whereby angles formed by an arc at the centre are twice that of the angle formed by the same arc in the opposite arc. Here we can view this as:
  –*QCR* = 2 –*QPR* (when we consider arc *QR* and it's opposite arc *QPR*)

4. Incenter

- The lines bisecting the interior angles of a triangle are the angle bisectors of that triangle.
- The angle bisectors meet at a point called the *incentre* of the triangle.
- The incentre is equidistant from all the sides of the triangle.



- From the incentre with a perpendicular drawn to any of the sides as the radius, a circle can be drawn touching all the three sides. This is called the *incircle* of the triangle. The radius of the incircle is known as *inradius*.
- The angle formed by any side at the incentre is always a right angle more than half the angle opposite to the side.

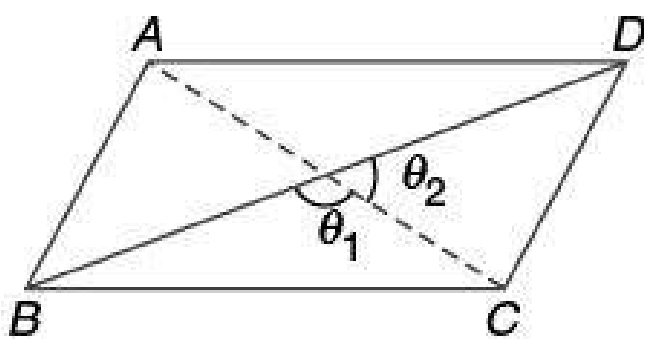
  This can be illustrated as –*QIR* = 90 + 1/2 –P
- If *QI* and *RI* be the angle bisectors of exterior angles at *Q* and *R* then,

  QIR = 90 – 1/2 –*P*.

QUADRILATERALS

Area

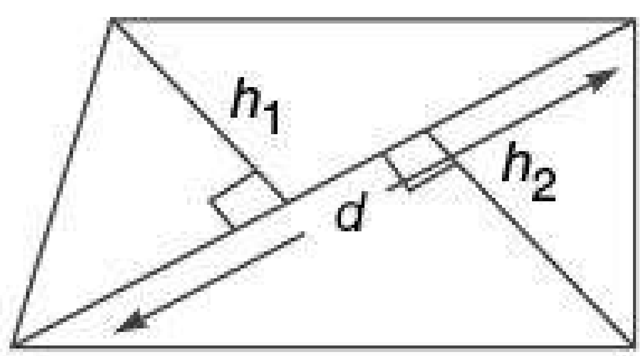
(A) Area = 1/2 (product of diagonals) × (sine of the angle between, them)



If q_1 and q_2 are the two angles made between themselves by the two diagonals, we have by the property of intersecting lines Æ $q_1 + q_2 = 180°$

Then, the area of the quadrilateral = $\frac{1}{2}\ d_1\, d_2 \sin q_1 = \frac{1}{2}\ d_1\, d_2 \sin q_2$.

(B) Area = 1/2 × diagonal × sum of the perpendiculars to it from opposite vertices = $\frac{d\,(h_1 + h_2)}{2}$.



(C) Area of a circumscribed quadrilateral

$A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$

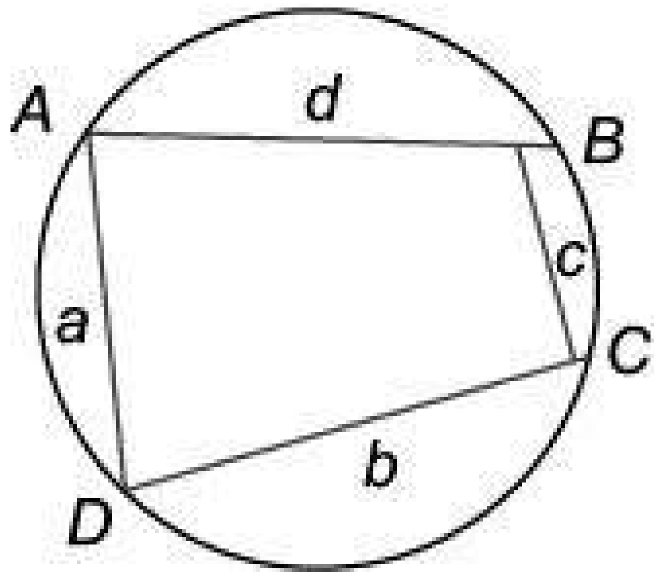
Where $S = \dfrac{a+b+c+d}{2}$

(where *a*, *b*, *c* and *d* are the lengths of the sides.)

Properties

1. In a convex quadrilateral inscribed in a circle, the product of the diagonals is equal to the sum of the products of the opposite sides. For example, in the figure below:

   $(a \times c) + (b \times d) = AC \times BD$



2. Sum of all the angles of a quadrilateral = 360°.

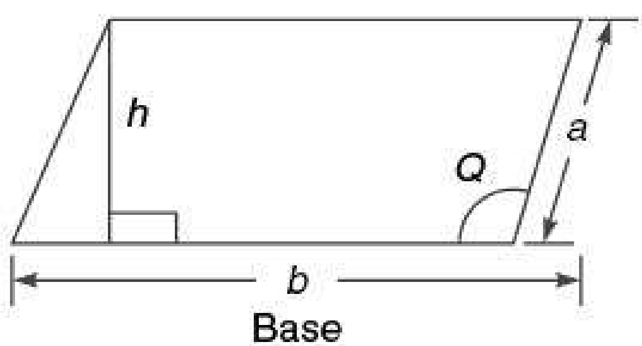
TYPES OF QUADRILATERALS

1. Parallelogram (|| gm)

A parallelogram is a quadrilateral with opposite sides parallel (as shown in the figure below.)

(A) Area = Base (b) × Height (h)

$= bh$



(B) Area = product of any two adjacent sides × sine of the included angle.

$= ab \sin Q$

(C) Perimeter = $2(a + b)$

where a and b are any two adjacent sides.

Properties

(a) Diagonals of a parallelogram bisect each other.

(b) Bisectors of the angles of a parallelogram form a rectangle.

(c) A parallelogram inscribed in a circle is a rectangle.

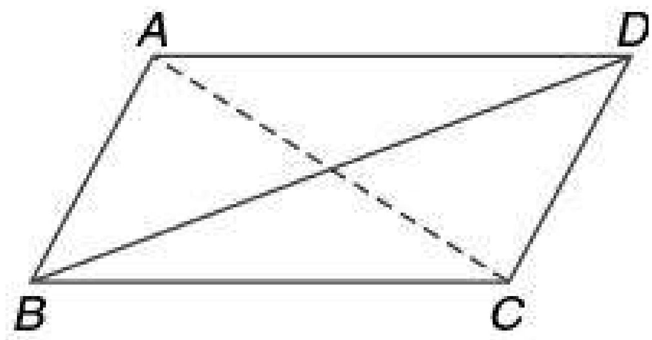
(d) A parallelogram circumscribed about a circle is a rhombus.

(e) The opposite angles in a parallelogram are equal.

(f) The sum of the squares of the diagonals is equal to the sum of the squares of the four sides in the figure:

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

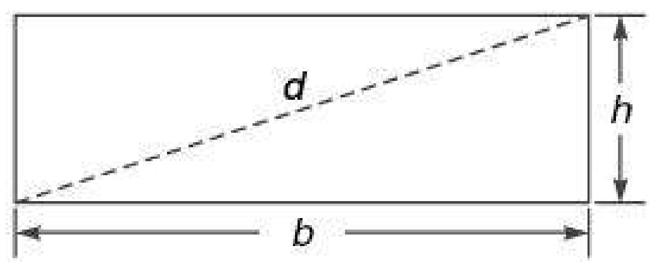
$= 2(AB^2 + BC^2)$



2. Rectangles

A rectangle is a parallelogram with all angles 90°

(A) Area = Base × Height = $b \times h$



Note: Base and height are also referred to as the length and the breadth in a rectangle.

(B) Diagonal (d) = $\sqrt{b^2 + h^2}$ Æ by Pythagoras theorem

Properties of a Rectangle

(a) Diagonals are equal and bisect each other.

(b) Bisectors of the angles of a rectangle (a parallelogram) form another rectangle.

(c) All rectangles are parallelograms but the reverse is not true.

3. Rhombus

A parallelogram having all the sides equal is a rhombus.

(A) Area = 1/2 × product of diagonals × sine of the angle between them.

$= 1/2 \times d_1 \times d_2 \sin 90°$ (Diagonals in a rhombus intersect at right angles)

$= 1/2 \times d_1 d_2$ (since sin 90° = 1)

(B) Area = product of adjacent sides × sine of the angle between them.

Properties

(a) Diagonals bisect each other at right angles.

(b) All rhombuses are parallelograms but the reverse is not true.

(c) A rhombus may or may not be a square but all squares are rhombuses.

4. Square

A square is a rectangle with adjacent sides equal or a rhombus with each angle 90°

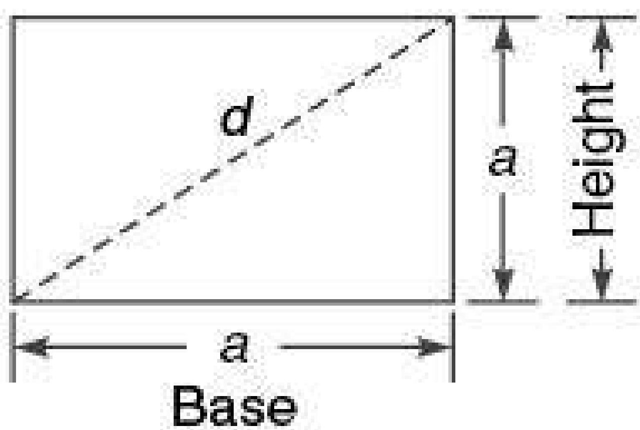
(a) Area = base × height = a^2

(b) Area = 1/2 (diagonal)^2 = 1/2 d^2 (square is a rhombus too).

(c) Perimeter = 4a (a = side of the square)

(d) Diagonal = $a\sqrt{2}$

(E) In radius = $\frac{a}{2}$



Properties

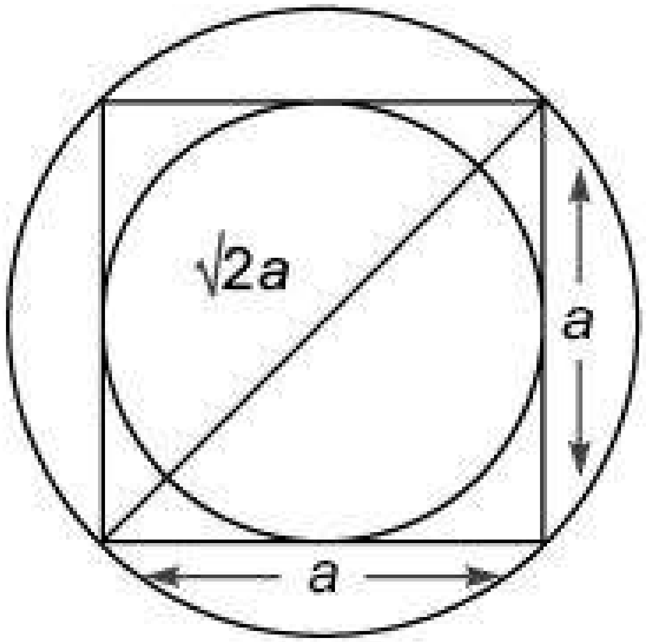
(a) Diagonals are equal and bisect each other at right angles.

(b) Side is the diameter of the inscribed circle.

(c) Diagonal is the diameter of the circumscribing circle.

fiDiameter = $a\sqrt{2}$.

Circumradius = $a/\sqrt{2}$



5. Trapezium

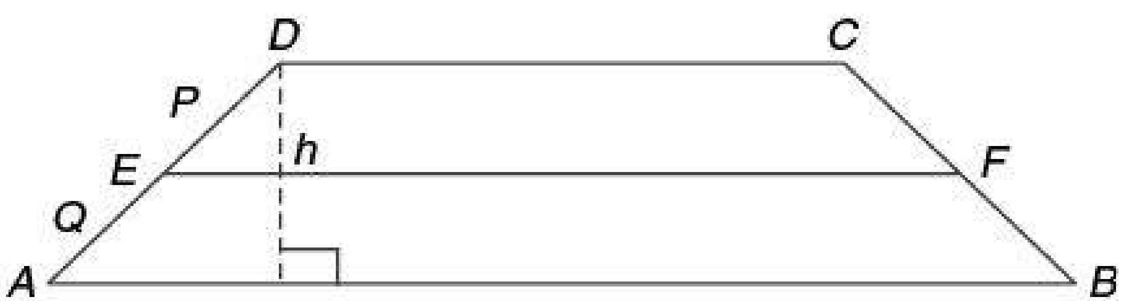
A trapezium is a quadrilateral with only two sides parallel to each other.

(a) Area = 1/2 × sum of parallel sides × height = 1/2 (AB + DC) × h—For the figure below.

(b) Median = 1/2 × sum of the parallel sides (median is the line equidistant from the parallel sides)

For any line EF parallel to AB

$$EF = \frac{\{[P \times (AB)] + [Q \times (DC)]\}}{AD}$$



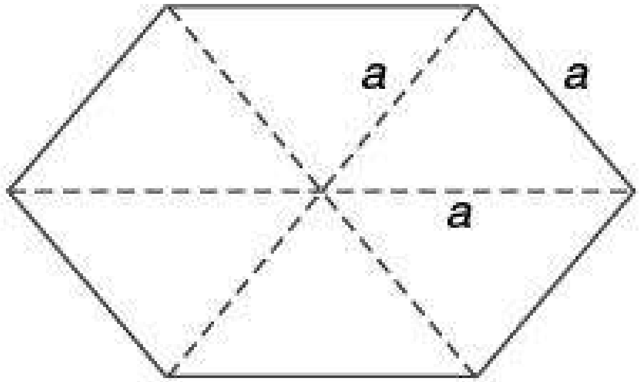
Properties

(A) If the non-parallel sides are equal then diagonals will be equal too.

REGULAR HEXAGON

(a) Area = [(3$\sqrt{3}$)/2] (side)^2

$$= \frac{3\sqrt{3} \times a^2}{2}$$



(b) A regular hexagon is actually a combination of 6 equilateral triangles all of side '*a*'.

Hence, the area is also given by: 6 × side of equilateral triangles = $6 \times \frac{\sqrt{3}}{4} a^2$

(c) If you look at the figure closely it will not be difficult to realise that circumradius (R) = a; i.e the side of the hexagon is equal to the circumradius of the same.

CIRCLES

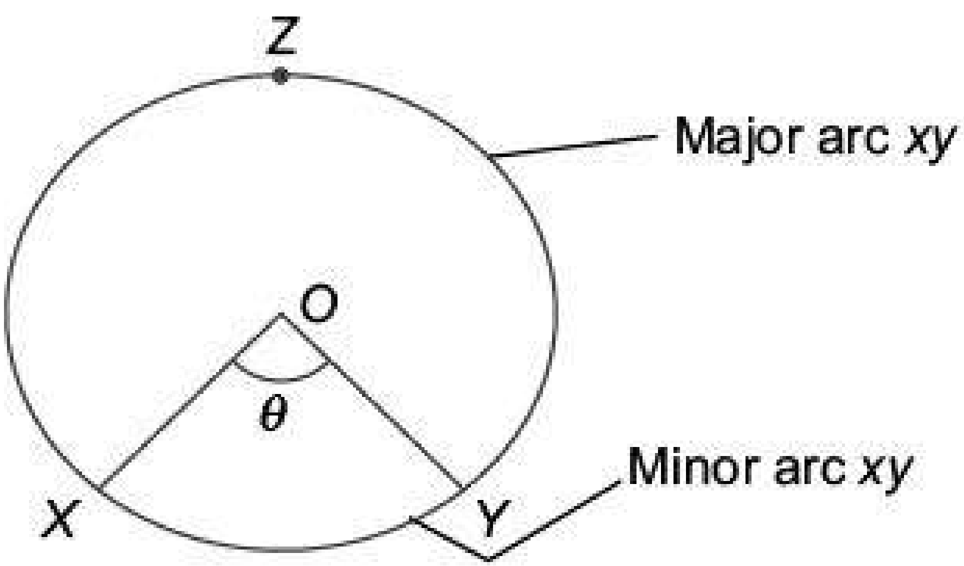
(a) Area = pr^2

(b) Circumference = 2 pr = (r = radius)

(c) Area = 1/2 × circumference × r

Arc: It is a part of the circumference of the circle. The bigger one is called the *major arc* and the smaller one the *minor arc*.

(d) Length (Arc XY) = $\frac{\theta}{360} \times 2pr$



(e) **Sector of a circle** is a part of the area of a circle between two radii.

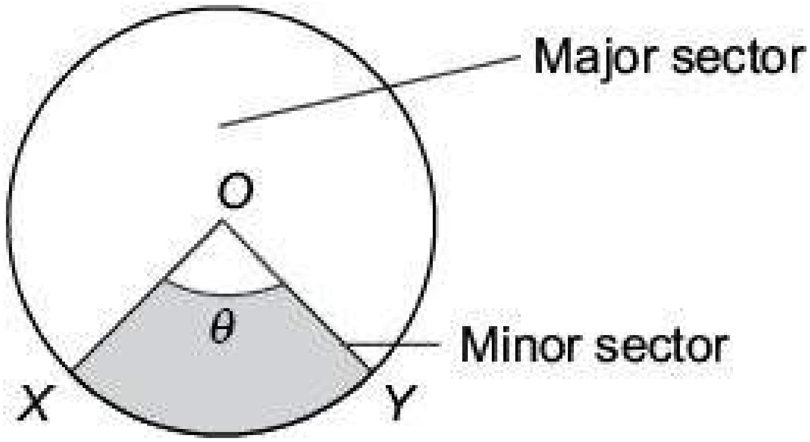
(f) Area of a sector = $\frac{\theta}{360} \times pr^2$

(where q is the angle between two radii)

= (1/2)r × length (arc xy)

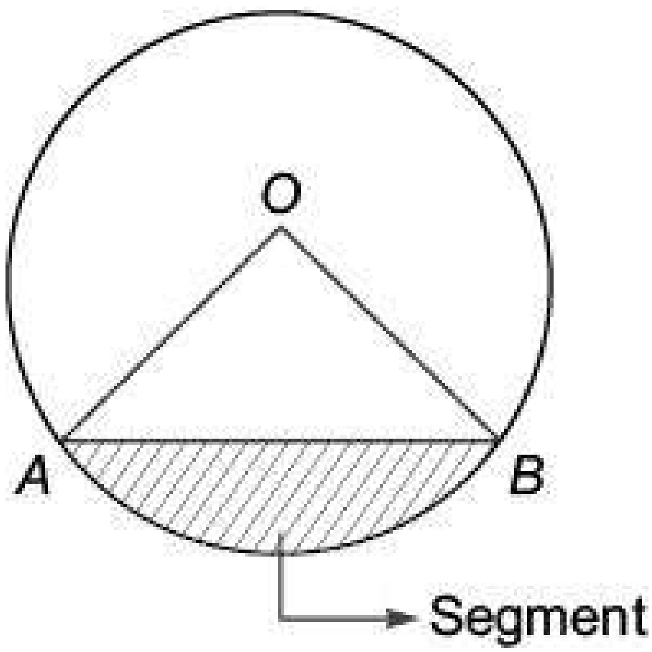
(\ $pr q/180$ = length arc xy)

$$= \frac{1}{2} \times r \times \frac{\pi r\theta}{360}$$



(g) **Segment:** A sector minus the triangle formed by the two radii is called the segment of the circle.

(h) Area = Area of the sector – Area DOAB = $\frac{\theta}{360} \times pr^2 - \frac{1}{2} \times r^2 \sin q$



(i) Perimeter of segment = length of the arc + length of segment AB

$$= \frac{\theta}{360} \times 2pr + 2r \sin\left(\frac{\theta}{2}\right)$$

$$= \frac{\pi r\theta}{180} + 2r \sin\left(\frac{\theta}{2}\right)$$

(j) **Congruency:** Two circles can be congruent if and only if they have equal radii.

Properties

(a) The perpendicular from the centre of a circle to a chord bisects the chord. The converse is also true.

(b) The perpendicular bisectors of two chords of a circle intersect at its centre.

(c) There can be one and only one circle passing through three or more non-collinear points.

(d) If two circles intersect in two points then the line through the centres is the perpendicular bisector of the common chord.

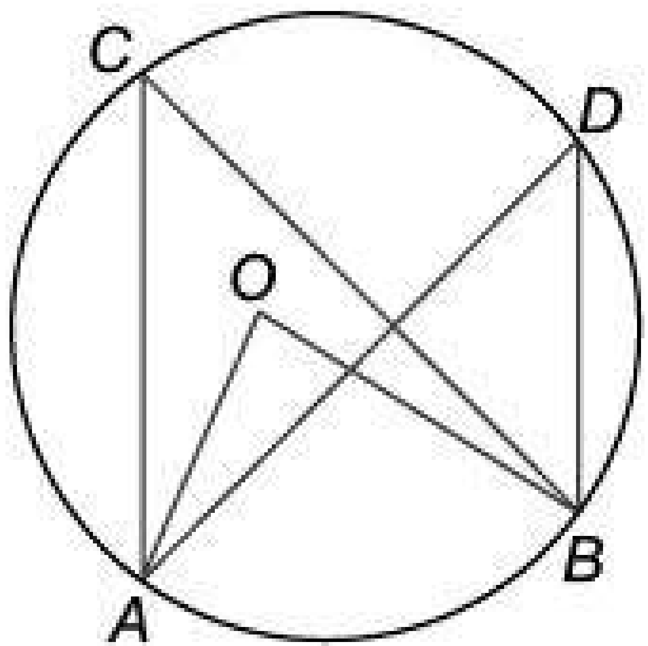
(e) If two chords of a circle are equal, then the centre of the circle lies on the angle bisector of the two chords.

(f) Equal chords of a circle or congruent circles are equidistant from the centre.

(g) Equidistant chords from the centre of a circle are equal to each other in terms of their length.

(h) The degree measure of an arc of a circle is twice the angle subtended by it at any point on the alternate segment of the circle. This can be clearly seen in the following figure:

With respect to the arc *AB*, –*AOB* = 2 –*ACB*.

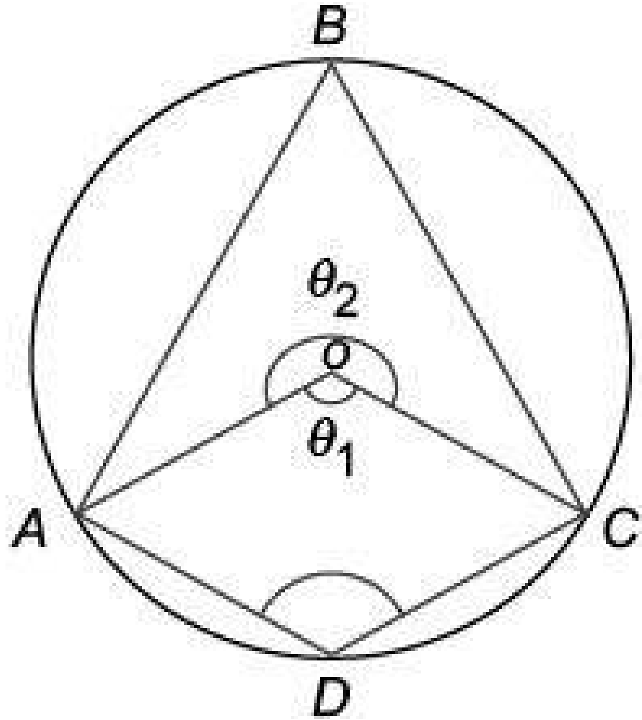


(i) Any two angles in the same segment are equal. Thus, –*ACB* = –*ADB*.

(j) The angle subtended by a semi-circle is a right angle. Conversely, the arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

(k) Any angle subtended by a minor arc in the alternate segment is acute, and any angle subtended by a major arc in the alternate segment is obtuse.

In the figure below



–*ABC* is acute and

–*ADC* = obtuse

Also q_1 = 2 –B

And q_2 = 2 –*D*

\$q_1 + q_2$ = 2(–*B* + –*D*)

= 360° = 2(–B + –D)

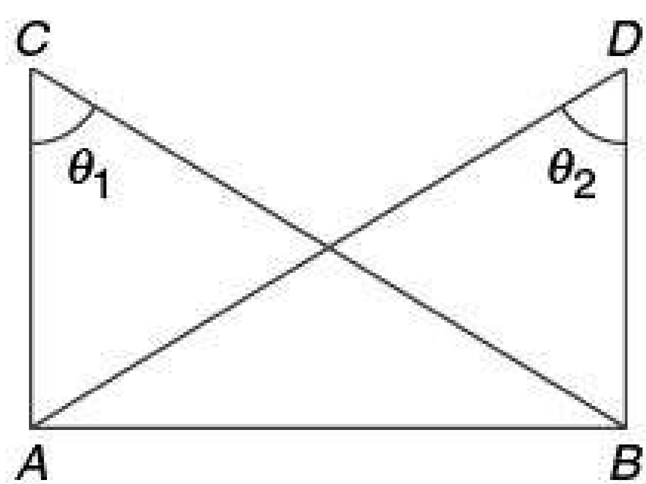
or –*B* + –*D* = 180°

or sum of opposite angles of a cyclic quadrilateral is 180°.

(l) If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, the four points are concyclic. Thus, in the following figure:

If,$q_1 = q_2$

Then *ABCD* are concyclic, that is, they lie on the same circle.



(m) Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres.) The converse is also true.

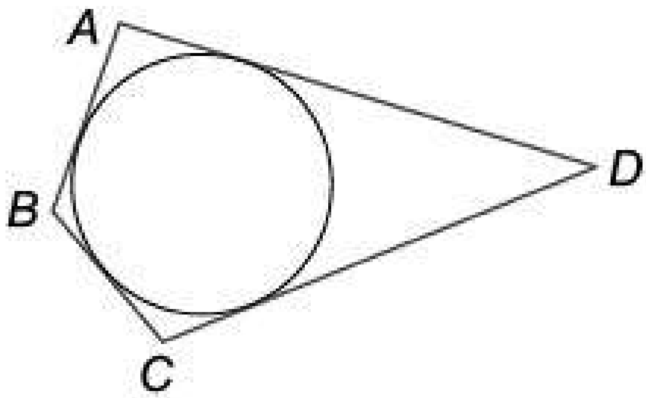
(n) If the sum of the opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Secant: A line that intersects a circle at two points.

Tangent: A line that touches a circle at exactly one point.

(o) If a circle touches all the four sides of a quadrilateral then the sum of the two opposite sides is equal to the sum of other two

$AB + DC = AD + BC$



(p) In two concentric circles, the chord of the larger circle that is tangent to the smaller circle is bisected at the point of contact.

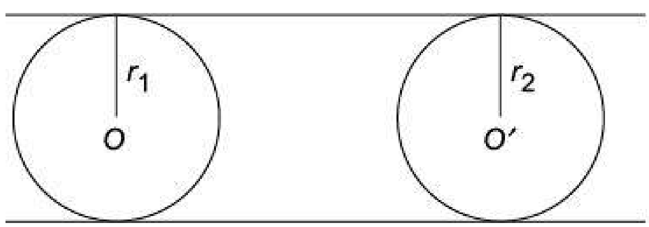
Tangents

- Length of direct common tangents is

  $= \sqrt{(\text{Distance between their centres})^2 - (r_1 - r_2)^2}$

  where r_1 and r_2 are the radii of the circles

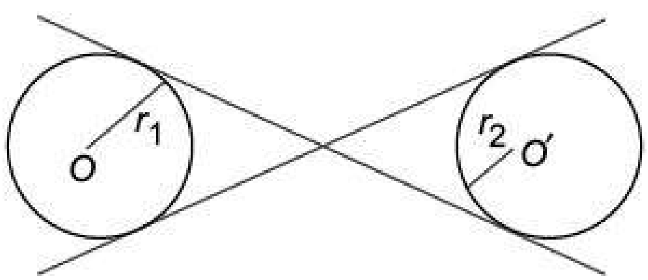
  $= \sqrt{(OO')^2 - (r_1 - r_2)^2}$



- Length of transverse common tangents is

  $= \sqrt{(\text{distance between their centres})^2 - (r_1 + r_2)^2}$

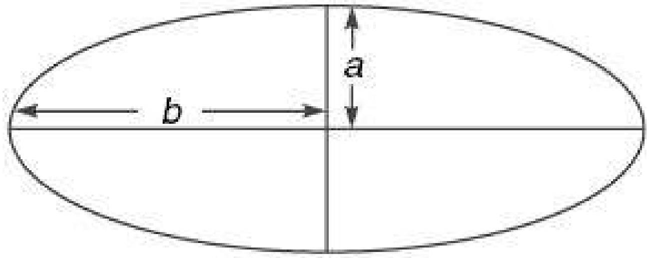
  $= \sqrt{(OO')^2 - (r_1 + r_2)^2}$



ELLIPSE

- Perimeter = $p(a + b)$

- Area = pab



STAR

Sum of angles of a star = $(2n - 8) \times p/2 = (n - 4)p$

PART II: MENSURATION

The following formulae hold true in the area of mensuration:

1. Cuboid

A cuboid is a three dimensional box. It is defined by the virtue of it's length l, breadth b and height h. It can be visualised as a room which has its length, breadth and height different from each other.

1. Total surface area of a cuboid = $2(lb + bh + lh)$
2. Volume of the cuboid = lbh

2. Cube of side s

A cube is a cuboid which has all its edges equal i.e. length = breadth = height = s.

1. Total surface area of a cube = $6s^2$.
2. Volume of the cube = s^3.

3. Prism

A prism is a solid which can have any polygon at both its ends. It's dimensions are defined by the dimensions of the polygon at it's ends and its height.

1. Lateral surface area of a right prism = Perimeter of base * height
2. Volume of a right prism = area of base* height
3. Whole surface of a right prism = Lateral surface of the prism + the area of the two plane ends.

4. Cylinder

A cylinder is a solid which has both its ends in the form of a circle. Its dimensions are defined in the form of the radius of the base (r) and the height h. A gas cylinder is a close approximation of a cylinder.

1. Curved surface of a right cylinder = $2prh$ where r is the radius of the base and h the height.
2. Whole surface of a right circular cylinder = $2prh + 2pr^2$
3. Volume of a right circular cylinder = pr^2h

5. Pyramid

A pyramid is a solid which can have any polygon as its base and its edges converge to a single apex. Its dimensions are defined by the dimensions of the polygon at its base and the length of its lateral edges which lead to the apex. The Egyptian pyramids are examples of pyramids.

1. Slant surface of a pyramid = 1/2 * Perimeter of the base* slant height
2. Whole surface of a pyramid = Slant surface + area of the base
3. Volume of a pyramid = $\frac{\text{area of the base*}}{3}$ height

6. Cone

A cone is a solid which has a circle at its base and a slanting lateral surface that converges at the apex. Its dimensions are defined by the radius of the base (*r*), the height (*h*) and the slant height (*l*). A structure similar to a cone is used in ice cream cones.

1. Curved surface of a cone = prl where l is the slant height
2. Whole surface of a cone = $prl + pr^2$
3. Volume of a cone = $\frac{\pi r^2 h}{3}$

7. Sphere

A sphere is a solid in the form of a ball with radius r.

1. Surface Area of a sphere = $4pr^2$
2. Volume of a sphere = $\frac{4}{3}pr^3$

8. Frustum of a pyramid

When a pyramid is cut the left over part is called the frustum of the pyramid.

1. Slant surface of the frustum of a pyramid = 1/2 * sum of perimeters of end * slant height.
2. Volume of the frustum of a pyramid = $\frac{k}{3}$ $[E_1 + (E_1 \Diamond E_2)^{1/2} + E_2]$ where k is the thickness and E_1, E_2 the areas of the ends.

9. Frustum of a cone

When a cone is cut the left over part is called the frustum of the cone.

1. Slant surface of the frustum of a cone = $p(r_1 + r_2)l$ where l is the slant height.
2. Volume of the frustum of a cone = $\frac{\pi}{3}$ $k(r_1^2 + r_1 r_2 + r_2^2)$

WORKED-OUT PROBLEMS

Problem 11.1 A right triangle with hypotenuse 10 inches and other two sides of variable length is rotated about its longest side thus giving rise to a solid. Find the maximum possible area of such a solid.

(a) (250/3)*p*in^2 (b) (160/3)*p*in^2

(c) 325/3*p*in^2 (d) None of these

Solution Most of the questions like this that are asked in the CAT will not have figures accompanying them. Drawing a figure takes time, so it is always better to strengthen our imagination. The beginners can start off by trying to imagine the figure first and trying to solve the problem. They can draw the figure only when they don't arrive at the right answer and then find out where exactly they went wrong. The key is to spend as much time with the problem as possible trying to understand it fully and analysing the different aspects of the same without investing too much time on it.

Let's now look into this problem. The key here lies in how quickly you are able to visualise the figure and are able to see that

(i) the triangle has to be an isosceles triangle,

(ii) the solid thus formed is actually a combination of two cones,

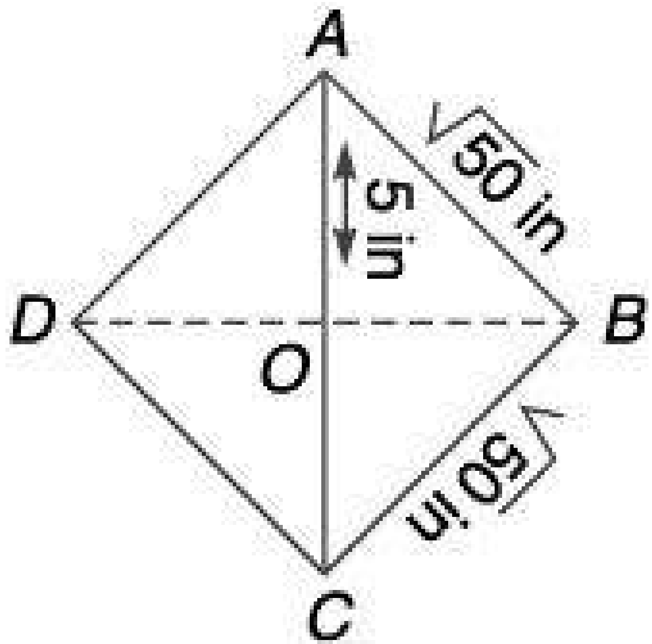
(iii) the radius of the base has to be the altitude of the triangle to the hypotenuse.

After you have visualised this comes the calculation aspect of the problem. This is one aspect where you can score over others.

In this question the figure would be somewhat like this (as shown alongside) with triangles *ABC* and *ADC* representing the cones and *AC* being the hypotenuse around which the triangle *ABC* revolves. Now that the area has to be maximum with *AC* as the hypotenuse, we must realise that *ADB* has to be an isosceles triangle, which automatically makes *BCD* an isosceles triangle too. The next step is to calculate the radius of the base, which is essentially the height of the triangle *ABC*. To find that, we have to first find *AB*. We know

$AC^2 = AB^2 + BC^2$

For triangle to be one with the greatest possible area *AB* must be equal to *BC* that is, $AB = BC = \sqrt{50}$, since $AO = 1/2AC = 5$ inches.



Now take the right angle triangle *ABO*, *BO* being the altitude of triangle *AOC*. By Pythagoras theorem, $AB^2 = AO^2 + BO^2$, so $BO^2 = 25$ inches

The next step is to find the area of the cone *ABD* and multiply it with two to get the area of the whole solid

Area of the cone $ADB = p/3\ (BO)^2 \times AO$

$= (1/3)\ (25) \times 5p = (125/3)p$

Therefore, area solid $ABCD = 2 \times (125/3)p = (250/3)p$

Problem 11.2 A right circular cylinder is to be made out of a metal sheet such that the sum of its height and radius does not exceed 9 cm can have an area of maximum.

(a) $54\ p\ \text{cm}^2$ (b) $108\ p\ \text{cm}^2$

(c) $81\ p\ \text{cm}^2$ (d) None of these

Solution Solving this question requires the knowledge of ratio and proportion also. To solve this question, one must know that for $a^2b^3c^4$ to have the maximum value when $(a + b + c)$ is constant, *a*, *b* and *c* must be in the ratio 1 : 2 : 3.

Now lets look at this problem. Volume of a cylinder $= pr^2h$.

If you analyse this formula closely, you will find that *r* and *h* are the only variable term. So for volume of the cylinder to be maximum, r^2h has to be maximum under the condition that $r + h = 9$. By the information given above, this is possible only when $r : h = 2 : 1$, that is, $r = 6$, $h = 3$. So,

Volume of the cone $= pr^2h$

$= p \times 6^2 \times 3$

$= \mathbf{108}p$

Problem 11.3 There are five concentric squares. If the area of the circle inside the smallest square is 77 square units and the distance between the corresponding corners of consecutive squares is 1.5 units, find the difference in the areas of the outermost and innermost squares.

Solution Here again the ability to visualise the diagram would be the key. Once you gain expertise in this aspect, you will be able to see that you will be able to see that the diameter of the circle is equal to the side of the innermost square that is

$pr^2 = 77$

or $\quad r = 3.5\sqrt{2}$

or $\quad 2r = 7\sqrt{2}$

Then the diagonal of the square is 14 sq units.

Which means the diagonal of the fifth square would be 14 + 12 units = 26.

Which means the side of the fifth square would be $13\sqrt{2}$.

Therefore, the area of the fifth square = 338 sq units

Area of the first square = 98 sq units

Hence, the difference would be 240 sq. units.

Problem 11.4 A spherical pear of radius 4 cm is to be divided into eight equal parts by cutting it in halves along the same axis. Find the surface area of each of the final piece.

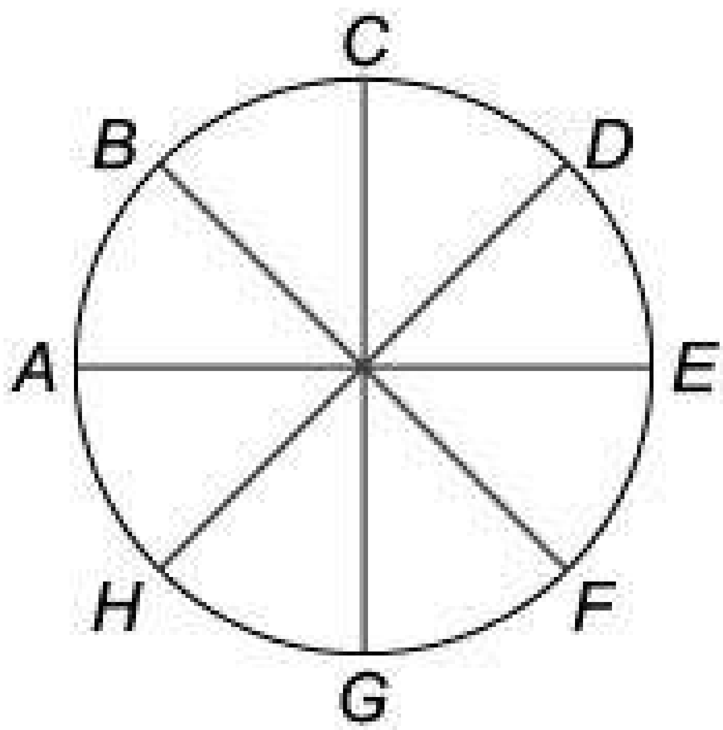


Fig. (a)

(a) 20 *p* (b) 25 *p*

(c) 24 *p* (d) 19 *p*

Solution The pear after being cut will have eight parts each of same volume and surface area. The figure will be somewhat like the above Figure (a) if seen from the top before cutting. After cutting it look something like the Figure (b).

Now the surface area of each piece = Area *ACBD* + 2 (Area *CODB*).

The darkened surface is nothing but the arc *AB* from side glance which means its surface area is one eighth the area of the sphere, that is, $1/8 \times 4pr^2 = (1/2)pr^2$.

Now *CODB* can be seen as a semicircle with radius 4 cm.

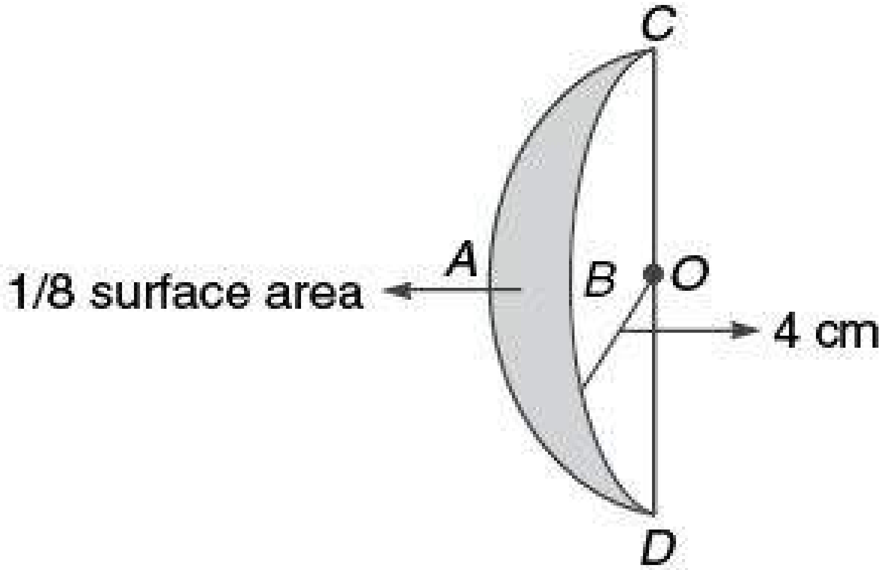


Fig. (b)

Therefore, 2 (Area *CODB*) = 2[(1/2)] $p\ r^2 = p\ r^2$

fi surface area of each piece = (1/2) $p\ r^2 + p\ r^2$

= (3/2) $p\ r^2$

= 24p

Problem 11.5 A solid metal sphere is melted and smaller spheres of equal radii are formed. 10% of the volume of the sphere is lost in the process. The smaller spheres have a radius, that is 1/9th the larger sphere. If 10 litres of paint were needed to paint the larger sphere, how many litres are needed to paint all the smaller spheres?

(a) 90 (b) 81

(c) 900 (d) 810

Solution Questions like this require, along with your knowledge of formulae, your ability to form equations. Stepwise, it will be something like this

Step 1: Assume values.

Step 2: Find out volume left.

Step 3: Find out the number of small spheres possible.

Step 4: Find out the total surface area of each small spheres as a ratio of the original sphere.

Step 5: Multiply it by 10.

Step 1: Let radius of the larger sphere be R and that of smaller ones be r.

Then, volume = $\frac{4}{3}pR^3$ and $\frac{4}{3}pr^3 = \frac{4}{3}p(R/9)^3$ respectively for the larger and smaller spheres.

Step 2: Volume lost due to melting = $\frac{4}{3}pR^3 \times \frac{10}{100} = \frac{4\pi R^3}{30}$

Volume left = $\frac{4}{3}pR^3 \times \frac{90}{100} = \frac{4\pi R^3 \times 0.9}{3}$

Step 3: Number of small spheres possible = Volume left/Volume of the smaller sphere

$$= \frac{\frac{4}{3}\pi R^3 \times 0.9}{\frac{4}{3}\pi \times (R/9)^3} = 9^3 \times 0.9$$

Step 4: Surface area of larger sphere = $4pR^2$

Surface area of smaller sphere = $4pr^2 = 4p\ (R/9)^2 = \frac{4\pi R^2}{81}$

Surface area of all smaller spheres = Number of small spheres × Surface area of smaller sphere

$= (9^3 \times .9) \times (4pR^2)/81$

$= 8.1 \times (4pR^2)$

Therefore, ratio of the surface area is $\frac{[8.1 \times (4\pi R^2)]}{4\pi R^2} = 8.1$

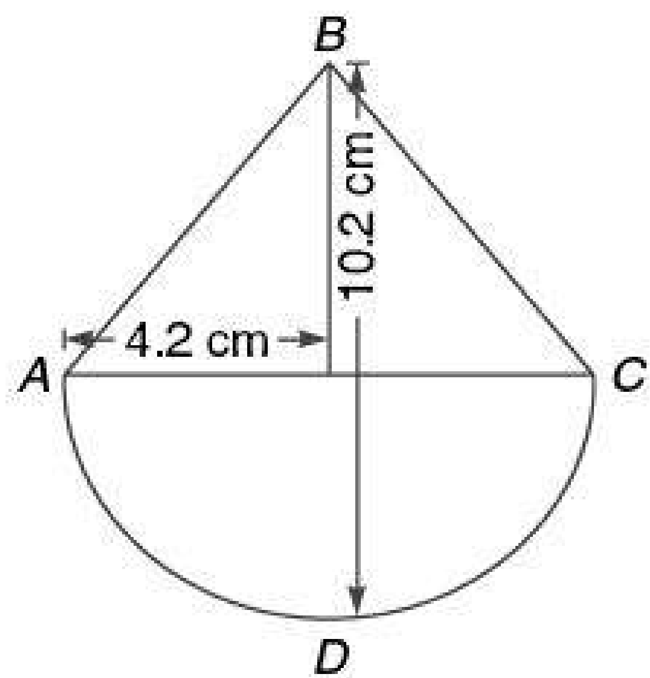
Step 5:

8.1 × number of litres = 8.1 × 10 = 81

Problem 11.6 A solid wooden toy in the shape of a right circular cone is mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

(a) 104 cm^3 (b) 162 cm^3

(c) 427 cm^3 (d) 266 cm^2



Solution Volume of the cone is given by –1/3 × pr^2h

Here, r = 4.2 cm; h = 10.2 – r = 6 cm

Therefore the volume of the cone = 1/3 $p \times (4.2)^2 \times 6$ cm

$= 110.88 \text{ cm}^3$

Volume of the hemisphere = $\frac{1}{2} \times \frac{4}{3}$ $pr^3 = 155.23$

Total volume = 110.88 + 155.232 = 266.112

Problem 11.7 A vessel is in the form of an inverted cone. Its depth is 8 cm and the diameter of its top, which is open, is 10 cm. It is filled with water up to the brim. When bullets, each of which is a sphere of radius 0.5 cm, are dropped into the vessel 1/4 of the water flows out. Find the number of bullets dropped in the vessel.

(a) 50 (b) 100 (c) 150 (d) 200

Solution In these type of questions it is just your calculation skills that is being tested. You just need to take care that while trying to be fast you don't end up making mistakes like taking the diameter to be the radius and so forth. The best way to avoid such mistakes is to proceed systematically. For example, in this problem we can proceed thus:

Volume of the cone = $\frac{1}{3}$ $pr^2h = \frac{200}{3} p \text{ cm}^3$

volme of all the lead shots = Volume of water that spilled out = $\frac{50}{3} p \text{ cm}^3$

Volume of each lead shot = $\frac{4}{3} pr^3 = \frac{\pi}{6}$ cm^3

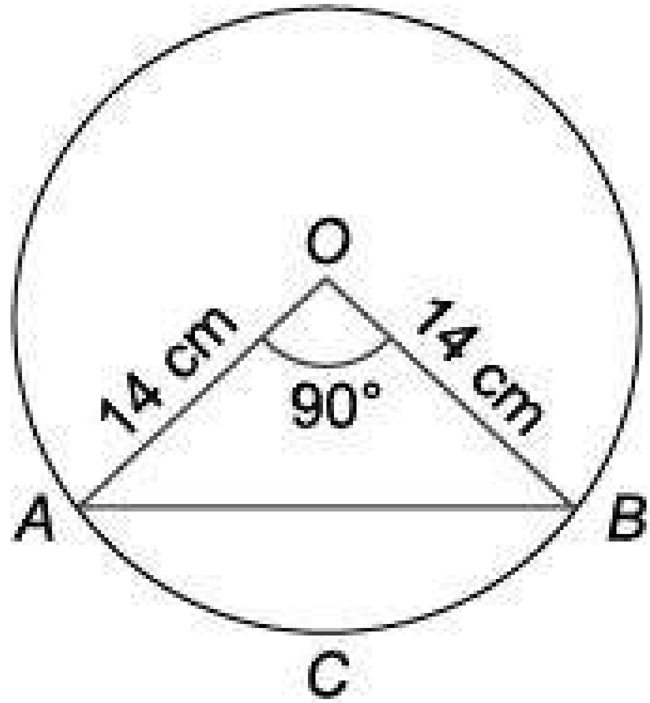
Number of lead shots = (Volume of water that spilled out)/(Volume of each lead shot)

$$= \frac{\frac{50}{3}\pi}{\frac{\pi}{6}} = \frac{50}{3} \times 6 = 100$$

Problem 11.8 *AB* is a chord of a circle of radius 14 cm. The chord subtends a right angle at the centre of the circle. Find the area of the minor segment.

(a) 98 sq cm (b) 56 sq cm

(c) 112 sq cm (d) None of these



Solution Area of the sector *ACBO* = $\frac{90\pi \times 14^2}{360}$

= 154 sq cm

Area of the triangle *AOB* = $\frac{14 \times 14}{2}$

= 98 sq cm

Area of the segment *ACB* = Area sector *ACBO* – Area of the triangle *AOB* = **56 sq cm**

Problem 11.9 A sphere of diameter 12.6 cm is melted and cast into a right circular cone of height 25.2 cm. Find the diameter of the base of the cone.

(a) 158.76 cm (b) 79.38 cm

(c) 39.64 cm (d) None of these

Solution In questions like this, do not go for complete calculations. As far as possible, try to cancel out values in the resulting equations.

Volume of the sphere = $\frac{4}{3} pr^3 = \frac{4}{3} p(6.3)^3$

Volume of the cone = $\frac{1}{3} pr^2h = \frac{\pi}{3} r^2(25.2)$

Now, volume of the cone = volume of the sphere

Therefore, *r* (radius of the cone) = 39.69 cm

Hence the diameter = **79.38 cm**

Problem 11.10 A chord *AB* of a circle of radius 5.25 cm makes an angle of 60° at the centre of the circle. Find the area of the major and minor segments.(Take *p* = 3.14)

(a) 168 cm^2 (b) 42 cm^2

(c) 84 cm^2 (d) None of these

Solution The moment you finish reading this question, it should occur to you that this has to be an equilateral triangle. Once you realise this, the question is reduced to just calculations.

Area of the minor sector = $\frac{60}{360}$ × *p* × 5.25^2

= 14.4375 cm^2

Area of the triangle = $\frac{\sqrt{3}}{4}$ × 5.25^2 = 11.93 cm^2

Area of the minor segment = Area of the minor sector – Area of the triangle = 2.5 cm^2

Area of the major segment = Area of the circle – Area of the minor segment.

= 86.54 cm^2 – 2.5 cm^2 = **84 cm^2**

Problem 11.11 A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

(a) 1 : 2 (b) 2 : 1

(c) 3 : 1 (d) None of these

Solution Questions of this type should be solved without the use of pen and paper. A good authority over formulae will make things easier.

Volume of the cone = $\frac{\pi r^2 h}{3}$ = Volume of a hemisphere = $\frac{2}{3}$ pr^3.

Height of a hemisphere = Radius of its base

So the question is effectively asking us to find out *h*/*r*

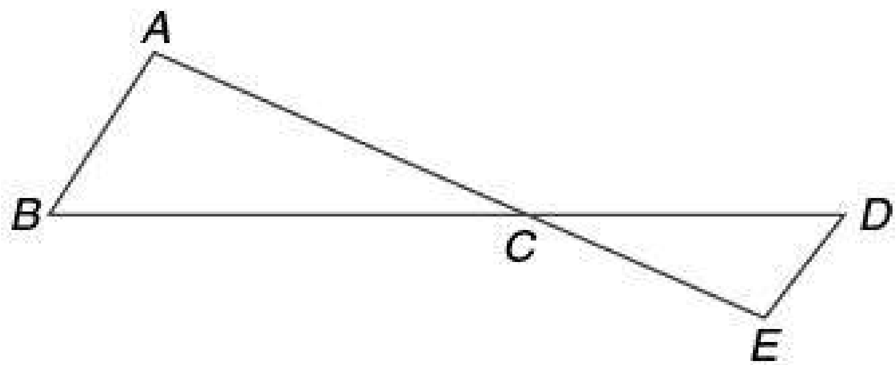
By the formula above we can easily see that *h*/*r* = **2/1**

LEVEL OF DIFFICULTY (I)

1. A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts the shadow 50 m long on the ground. Find the height of the tower.

(a) 100 m (b) 120 m
(c) 25 m (d) 200 m

2. In the figure, D*ABC* is similar to D*EDC*.



If we have *AB* = 4 cm,

ED = 3 cm, *CE* = 4.2 and

CD = 4.8 cm, find the value of *CA* and *CB*

(a) 6 cm, 6.4 cm (b) 4.8 cm, 6.4 cm
(c) 5.4 cm, 6.4 cm (d) 5.6 cm, 6.4 cm

3. The area of similar triangles, *ABC* and *DEF* are 144 cm^2 and 81 cm^2 respectively. If the longest side of larger D*ABC* be 36 cm, then the longest side of smaller D*DEF* is

(a) 20 cm (b) 26 cm
(c) 27 cm (d) 30 cm

4. Two isosceles D*s* have equal angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.

(a) 4/5 (b) 5/4
(c) 3/2 (d) 5/7

5. The areas of two similar Ds are respectively 9 cm^2 and 16 cm^2. Find the ratio of their corresponding sides.

(a) 3 : 4 (b) 4 : 3
(c) 2 : 3 (d) 4 : 5

6. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, find the distance between their tops.

(a) 12 cm (b) 14 cm

(c) 13 cm (d) 11 cm

7. The radius of a circle is 9 cm and length of one of its chords is 14 cm. Find the distance of the chord from the centre.

(a) 5.66 cm (b) 6.3 cm

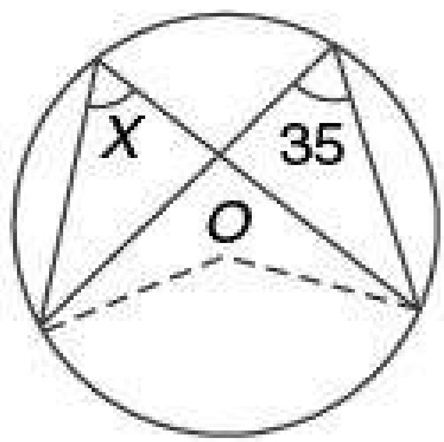
(c) 4 cm (d) 7 cm

8. Find the length of a chord that is at a distance of 12 cm from the centre of a circle of radius 13 cm.

(a) 9 cm (b) 8 cm

(c) 12 cm (d) cm

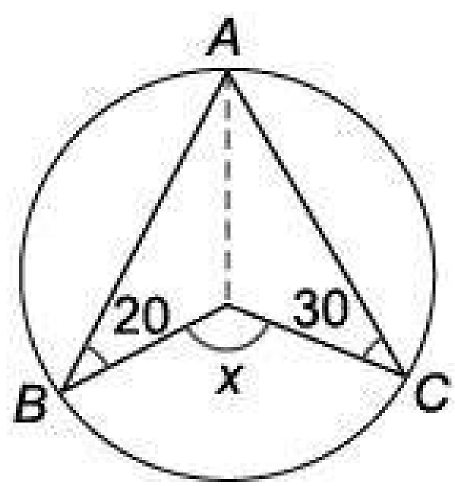
9. If *O* is the centre of circle, find –*x*



(a) 35° (b) 30°

(c) 39° (d) 40°

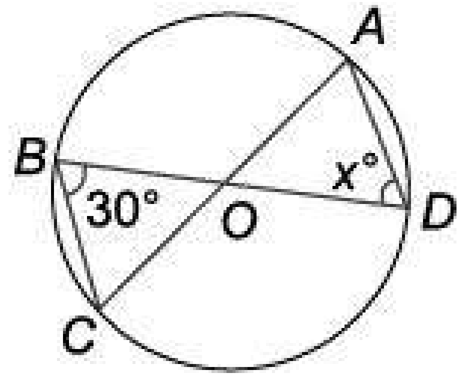
10. Find the value of –*x* in the given figure.



(a) 120° (b) 130°

(c) 110° (d) 100°

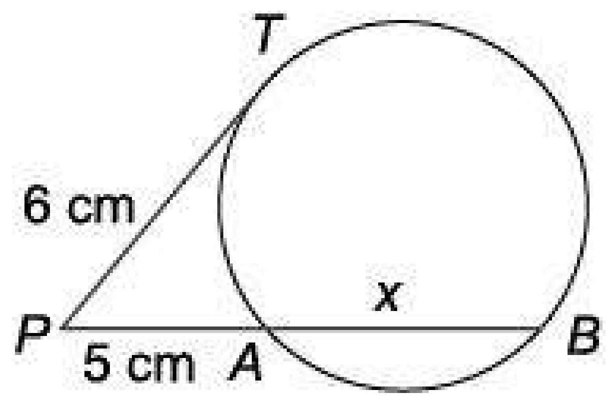
11. Find the value of *x* in the figure, if it is given that *AC* and *BD* are diameters of the circle.



(a) 60° (b) 45°

(c) 15° (d) 30°

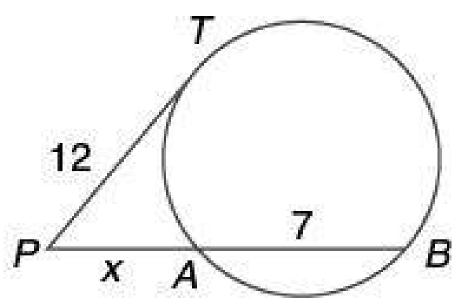
12. Find the value of x in the given figure.



(a) 2.2 cm (b) 1.6 cm

(c) 3 cm (d) 2.6 cm

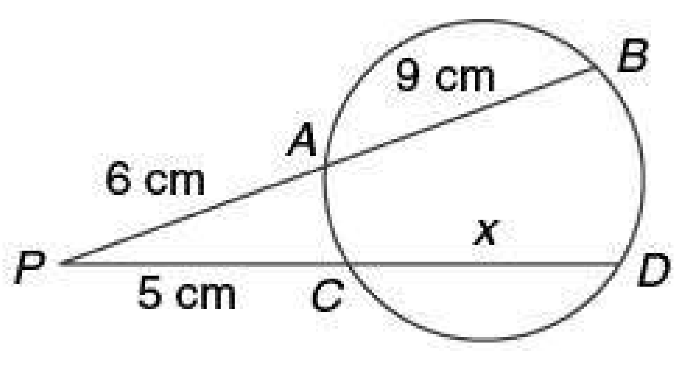
13. Find the value of x in the given figure.



(a) 16 cm (b) 9 cm

(c) 12 cm (d) 7 cm

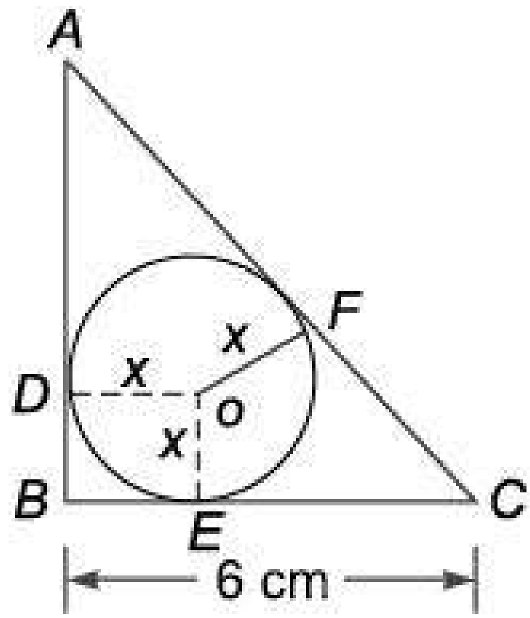
14. Find the value of x in the given figure.



(a) 13 cm (b) 12 cm

(c) 16 cm (d) 15 cm

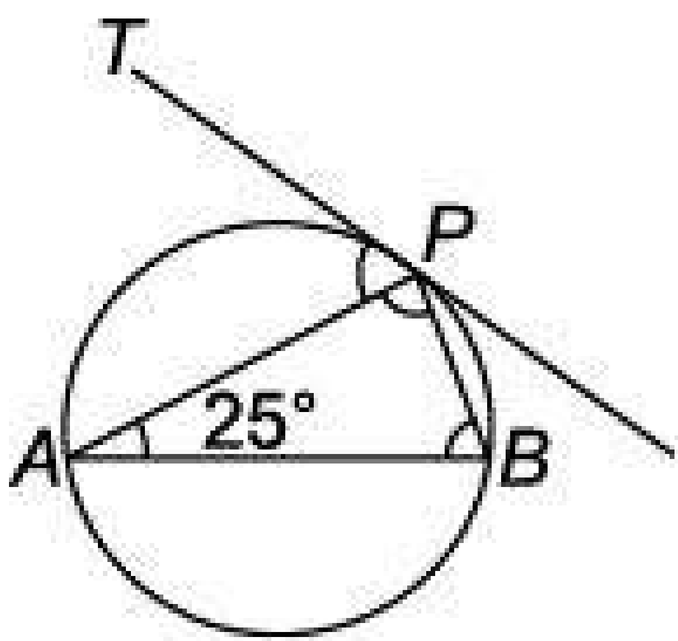
15. *ABC* is a right angled triangle with *BC* = 6 cm and *AB* = 8 cm. A circle with centre *O* and radius *x* has been inscribed in D*ABC*. What is the value of *x*.



(a) 2.4 cm (b) 2 cm

(c) 3.6 cm (d) 4 cm

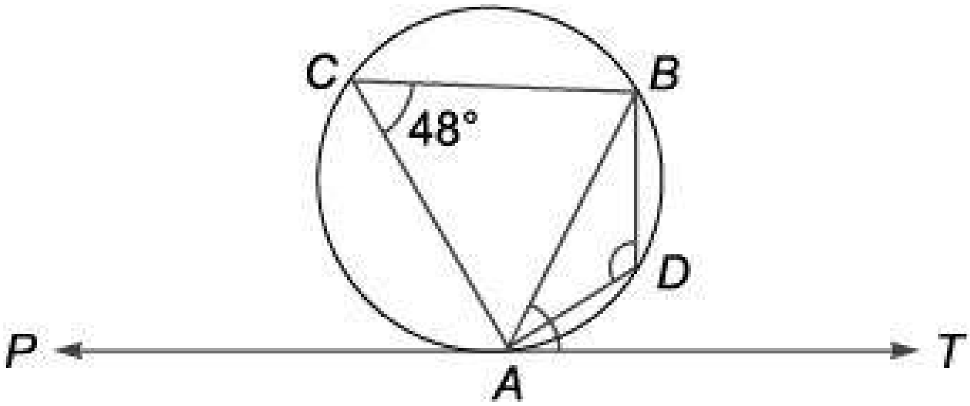
16. In the given figure *AB* is the diameter of the circle and –*PAB* = 25°. Find –*TPA*.



(a) 50° (b) 65°

(c) 70° (d) 45°

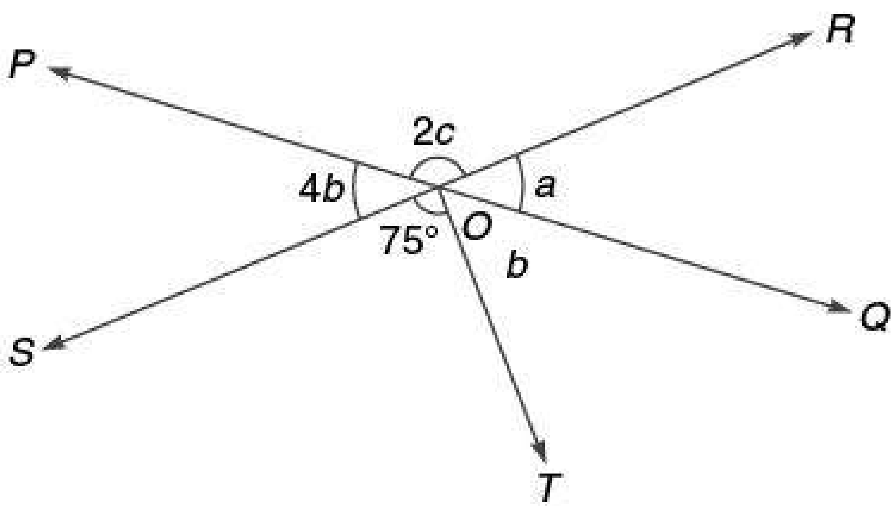
17. In the given figure find –*ADB*.



(a) 132° (b) 144°

(c) 48° (d) 96°

18. In the given two straight line, *PQ* and *RS* intersect each other at *O*. If –*SOT* = 75°, find the value of *a*, *b* and *c*.



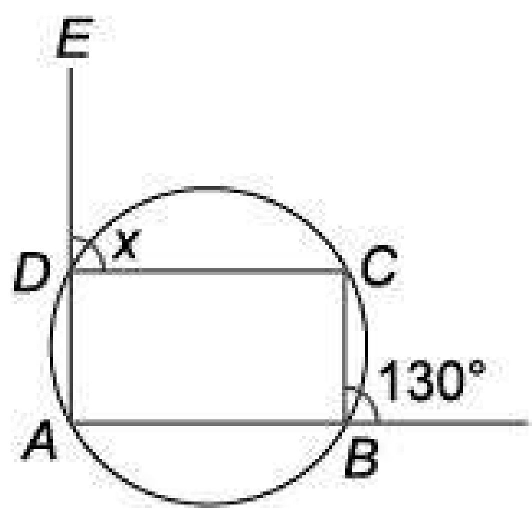
(a) $a = 84°$, $b = 21°$, $c = 48°$

(b) $a = 48°$, $b = 20°$, $c = 50°$

(c) $a = 72°$, $b = 24°$, $c = 54°$

(d) $a = 64°$, $b = 28°$, $c = 45°$

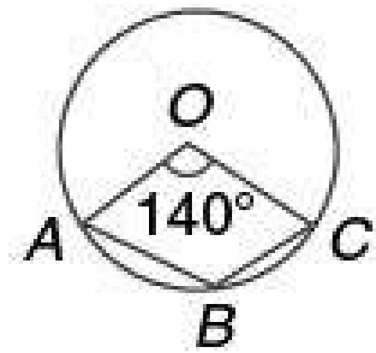
19. In the following figure *A*, *B*, *C* and *D* are the concyclic points. Find the value of *x*.



(a) 130° (b) 50°

(c) 60° (d) 30°

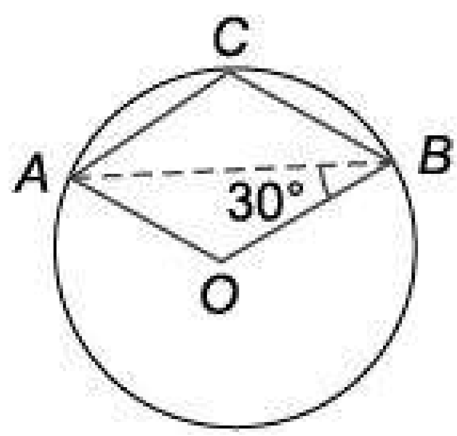
20. In the following figure, it is given that *O* is the centre of the circle and –*AOC* = 140°. Find –*ABC*.



(a) 110° (b) 120°

(c) 115° (d) 130°

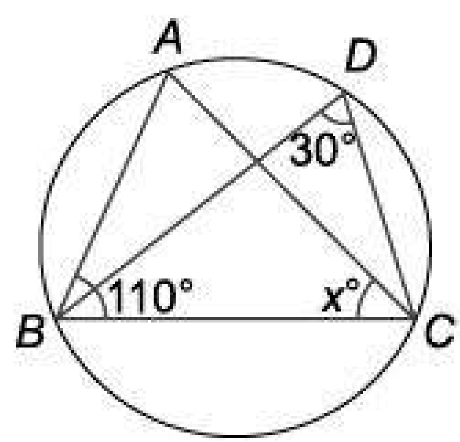
21. In the following figure, *O* is the centre of the circle and –*ABO* = 30°, find –*ACB*.



(a) 60° (b) 120°

(c) 75° (d) 90°

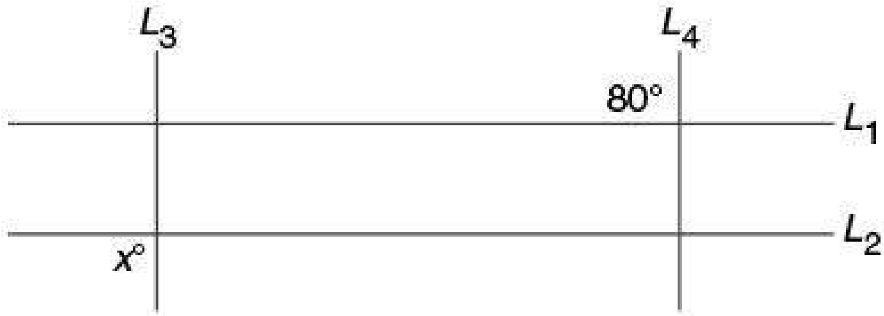
22. In the following figure, find the value of *x*



(a) 40° (b) 25°

(c) 30° (d) 45°

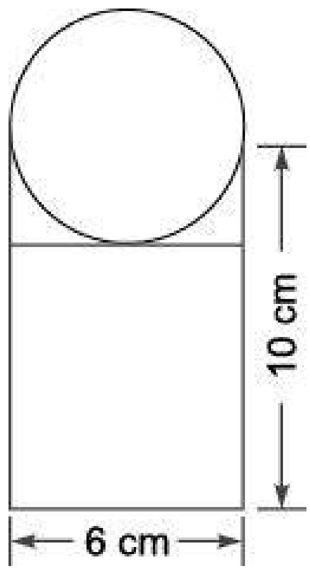
23. If $L_1 \parallel L_2$ in the figure below, what is the value of *x*.



(a) 80° (b) 100°

(c) 40° (d) Cannot be determined

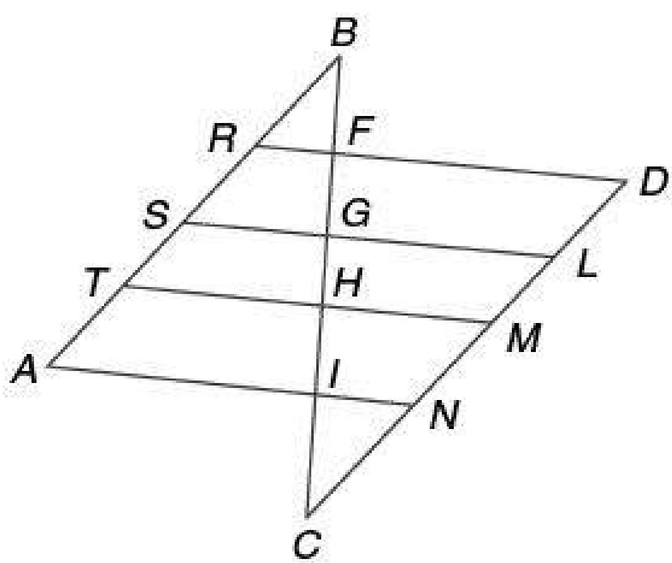
24. Find the perimeter of the given figure.



(a) (32 + 3*p*) cm (b) (36 + 6*p*) cm

(c) (46 + 3*p*) cm (d) (26 + 6*p*) cm

25. In the figure, *AB* is parallel to *CD* and *RD* ‖ *SL* ‖ *TM* ‖ *AN*, and *BR* : *RS* : *ST* : *TA* = 3 : 5 : 2 : 7. If it is known that *CN* = 1.333 *BR*. find the ratio of *BF* : *FG* : *GH* : *HI* : *IC*



(a) 3 : 7 : 2 : 5 : 4

(b) 3 : 5 : 2 : 7 : 4

(c) 4 : 7 : 2 : 5 : 3

(d) 4 : 5 : 2 : 7 : 3

LEVEL OF DIFFICULTY (II)

1. In a triangle *ABC*, point *D* is on side *AB* and point *E* is on side *AC*, such that *BCED* is a trapezium. *DE* : *BC* = 3 : 5. Calculate the ratio of the area of D*ADE* and the trapezium *BCED*.

   (a) 3 : 4 (b) 9 : 16

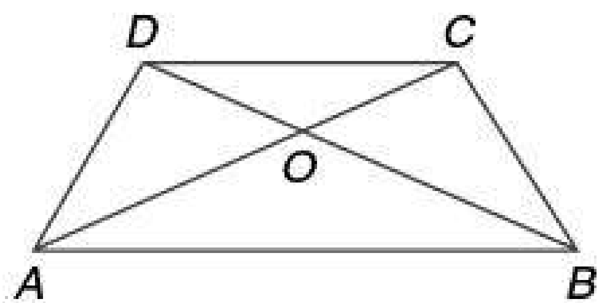
   (c) 3 : 5 (d) 9 : 25

2. *D*, *E*, *F* are the mid-points of the sides *BC*, *CA* and *AB* respectively of a D*ABC*. Determine the ratio of the area of triangles *DEF* and *ABC*.

   (a) 1 : 4 (b) 1 : 2

   (c) 2 : 3 (d) 4 : 5

3. In the adjoining figure, *ABCD* is a trapezium in which *AB*||*DC* and *AB* = 3 *DC*. Determine the ratio of the areas of (D*AOB* and D*COD)*.



   (a) 9 : 1 (b) 1 : 9

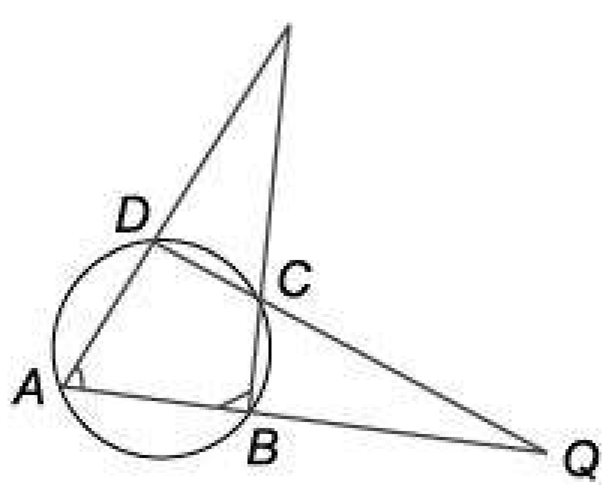
   (c) 3 : 1 (d) 1 : 3

4. A ladder reaches a window that is 8 m above the ground on one side of the street. Keeping its foot on the same point, the ladder is twined to the other side of the street to reach a window 12 m high. Find the width of the street if the ladder is 13 m.

   (a) 15.2 m (b) 14 m

   (c) 14.6 m (d) 12 m

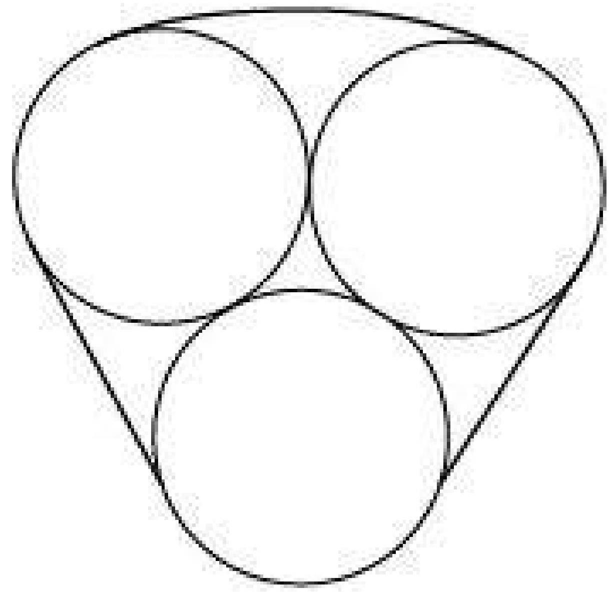
5. In the adjoining figure –*A* = 60° and –*ABC* = 80°, –*BQC*



(a)40° (b) 80°

(c)20° (d) 30°

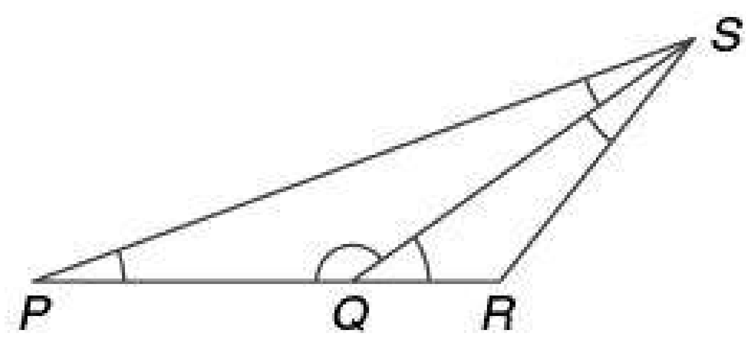
6. The diagram below represents three circular garbage cans, each of diameter 2 m. The three cans are touching as shown. Find, in metres, the perimeter of the rope encompassing the three cans.



(a) $2p + 6$ (b) $3p + 4$

(c) $4p + 6$ (d) $6p + 6$

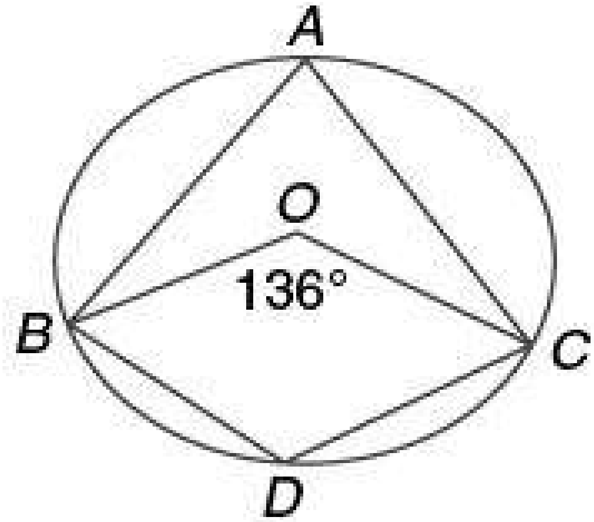
7. In the figure below, $PQ = QS$, $QR = RS$ and angle $SRQ = 100°$. How many degrees is angle QPS?



(a)20° (b) 40°

(c)15° (d) 30°

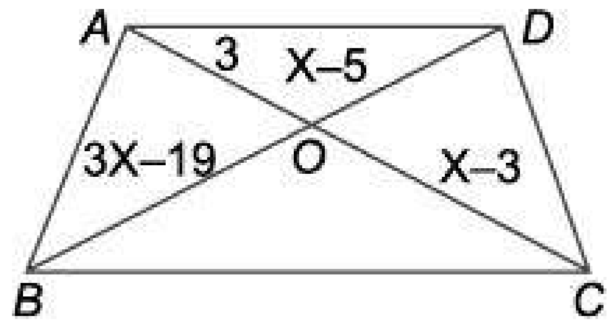
8. In the figure, $ABDC$ is a cyclic quadrilateral with O as centre of the circle. If $\angle BOC = 136°$, find $\angle BDC$.



(a) 110° (b) 112°

(c) 109° (d) 115°

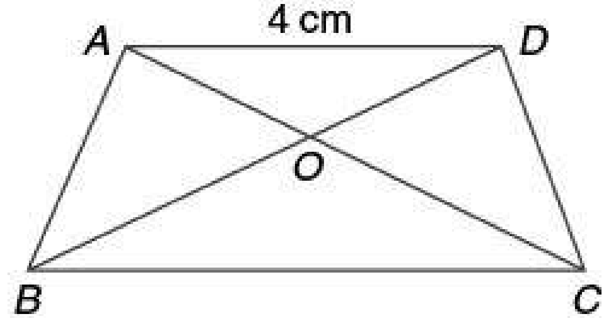
9. In the given figure, $AD \parallel BC$. Find the value of x.



(a) $x = 8, 9$ (b) $x = 7, 8$

(c) $x = 8, 10$ (d) $x = 7, 10$

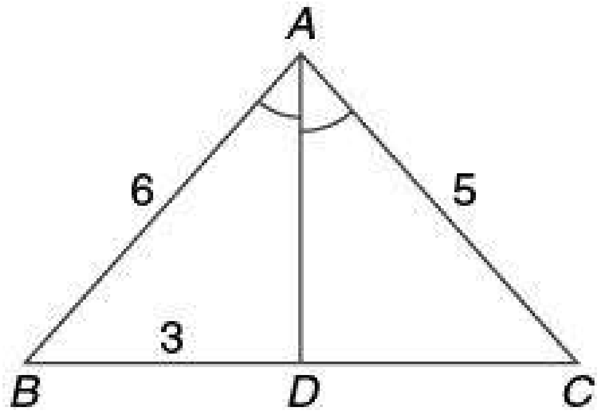
10. In the given figure $\frac{AO}{OC} = \frac{DO}{OB} = 1/2$ and $AB = 4$ cm. Find the value of BC in the above figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AD = 4$ cm. Find the value of BC.



(a) 7 cm (b) 8 cm

(c) 9 cm (d) 10 cm

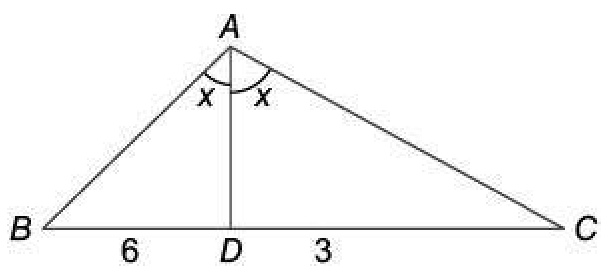
11. In the above figure, AD is the bisector of –BAC, AB = 6 cm, AC = 5 cm and BD = 3 cm. Find DC.



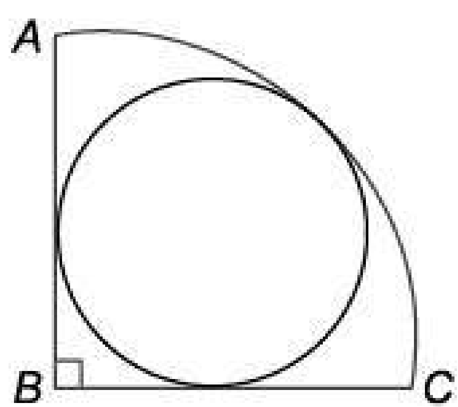
(a) 11.3 cm
(b) 2.5 cm
(c) 3.5 cm
(d) 4 cm

12. In a DABC, AD is the bisector of –BAC, AB = 8 cm, BD = 6 cm and DC = 3 cm. Find AC.

(a) 4 cm
(b) 6 cm
(c) 3 cm
(d) 5 cm



13. If ABC is a quarter circle and a circle is inscribed in it and if AB = 1 cm, find radius of smaller circle.



(a) $\sqrt{2} - 1$
(b) $(\sqrt{2} + 1)/2$
(c) $\sqrt{2} - 1/2$
(d) $1 - 2\sqrt{2}$

14. ABC is an equilateral triangle. Point D is on AC and point E is on BC, such that $AD = 2CD$ and CE

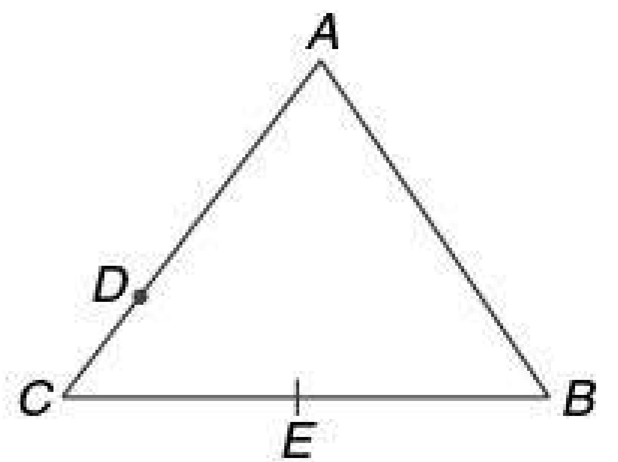
= *EB*. If we draw perpendiculars from *D* and *E* to other two sides and find the sum of the length of two perpendiculars for each set, that is, for *D* and *E* individually and denote them as per (*D*) and per (*E*) respectively, then which of the following option will be correct.

(a) Per (*D*) > per (*E*)

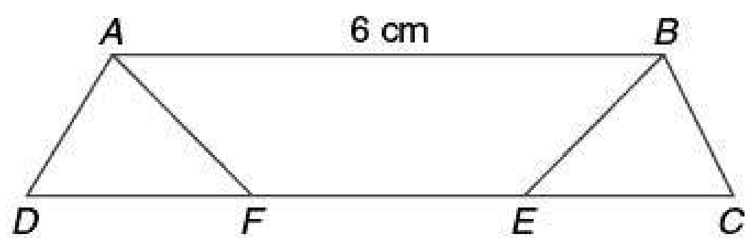
(b) Per (*D*) < per (*E*)

(c) Per (*D*) = per (*E*)

(d) None of these



15. *ABCD* is a trapezium in which *AB* is parallel to *DC*, *AD* = *BC*, *AB* = 6 cm, *AB* = *EF* and *DF* = *EC*. If two lines *AF* and *BE* are drawn so that area of *ABEF* is half of *ABCD*. Find *DF*/*CD*.



(a) 1/4

(b) 1/3

(c) 2/5

(d) 1/6

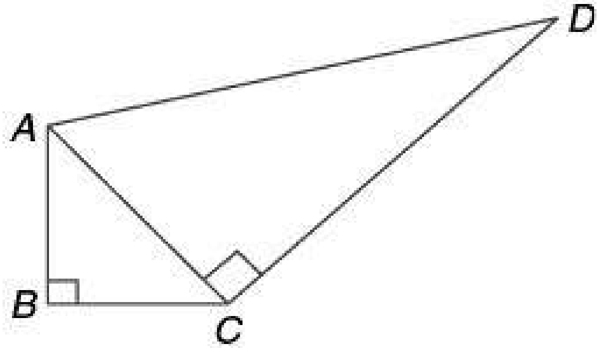
16. In the given figure, D*ABC* and D*ACD* are right angle triangles and *AB* = *x* cm, *BC* = *y* cm, *CD* = *z* cm and *x* ◊ *y* = *z* and *x*, *y* and *z* has minimum integral value. Find the area of *ABCD*

(a) 36 cm^2

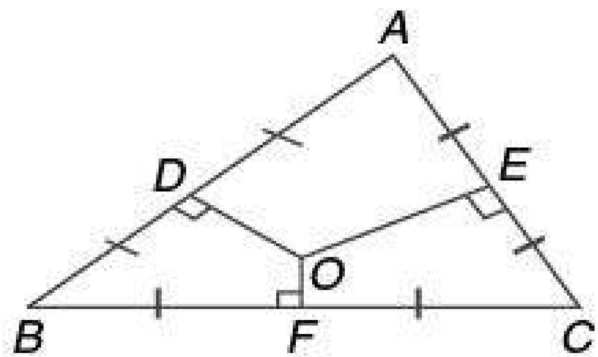
(b) 64 cm^2

(c) 24 cm^2

(d) 25 cm^2



17. *OD*, *OE* and *OF* are perpendicular bisectors to the three sides of the triangle. What is the relationship between *m –BAC* and *m –BOC*?



(a) *m–BAC* = 180 – *m–BOC*

(b) *m–BOC* = 90 + 1/2 *m–BAC*

(c) *m–BAC* = 90 + 1/2 *m–BOC*

(d) *m–BOC* = 2*m–BAC*

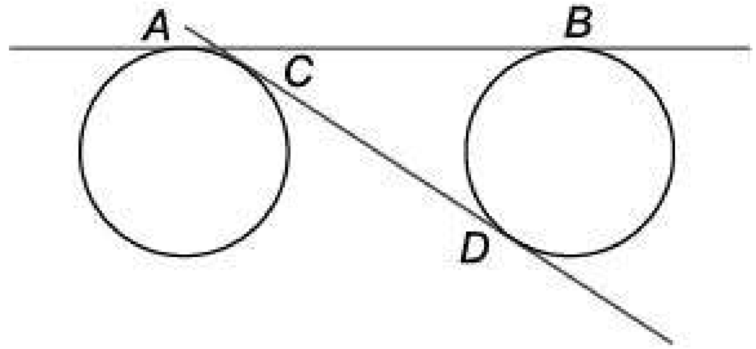
18. If two equal circles of radius 5 cm have two common tangent *AB* and *CD* which touch the circle on *A*, *C* and *B*, *D* respectively and if *CD* = 24 cm, find the length of *AB*.

(a) 27 cm

(b) 25 cm

(c) 26 cm

(d) 30 cm



19. If a circle is provided with a measure of 19° on centre, is it possible to divide the circle into 360 equal parts?

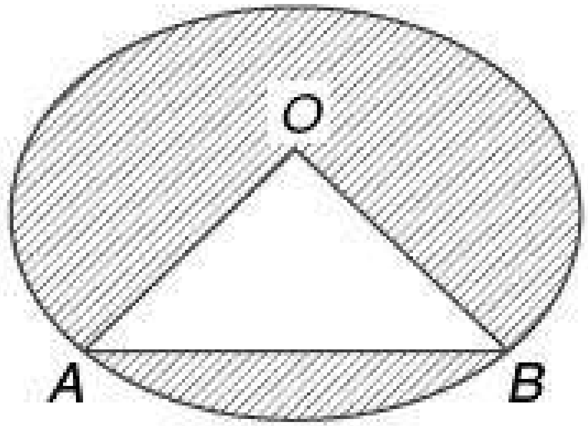
(a) Never

(b) Possible when one more measure of 20° is given

(c) Always

(d) Possible if one more measure of 21° is given

20. *O* is the centre of a circle of radius 5 cm. The chord *AB* subtends an angle of 60° at the centre. Find the area of the shaded portion (approximate value).



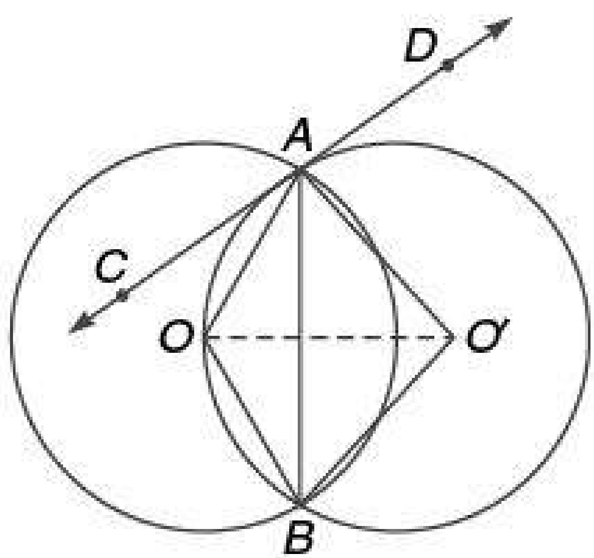
(a) 50 cm^2

(b) 62.78 cm^2

(c) 49.88 cm^2

(d) 67.67 cm^2

21. Two circles *C* (*O*, *r*) and *C* (*O*¢, *r*¢) intersect at two points *A* and *B* and *O* lies on *C* (*O*¢, *r*¢). A tangent *CD* is drawn to the circle *C* (*O*¢, *r*¢) at *A*. Then



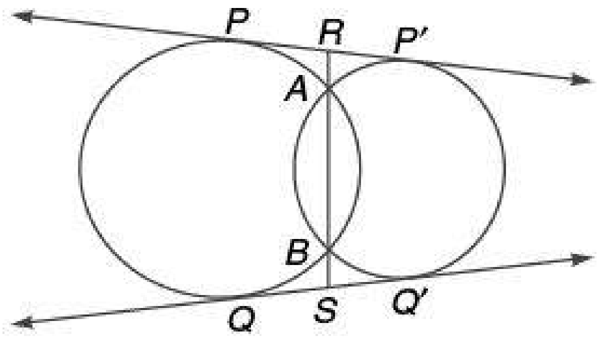
(a) –*OAC* = –*OAB*

(b) –*OAB* = –*AO*¢*O*

(c) –AO¢B = –AOB

(d) –OAC = –AOB

22. PP¢ and QQ¢ are two direct common tangents to two circles intersecting at points A and B. The common chord on produced intersects PP¢ in R and QQ¢ in S. Which of the following is true?



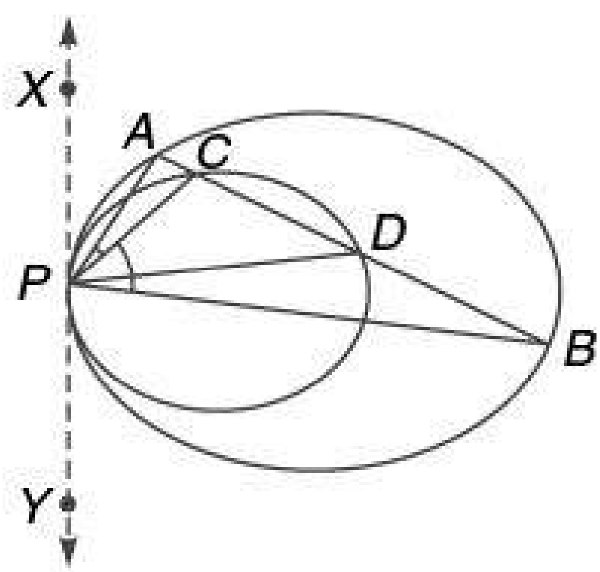
(a) $RA^2 + BS^2 = AB^2$

(b) RS^2 = PP¢^{2} + AB^2

(c) RS^2 = PP¢^{2} + QQ¢^{2}

(d) RS^2 = BS^2 + PP¢^{2}

23. Two circles touch internally at point P and a chord AB of the circle of larger radius intersects the other circle in C and D. Which of the following holds good?



(a) –CPA = –DPB

(b) 2 –CPA = –CPD

(c) –APX = –ADP

(d) –BPY = –CPD + –CPA

24. In a trapezium $ABCD$, $AB \parallel CD$ and $AD = BC$. If P is point of intersection of diagonals AC and BD,

then all of the following is wrong except

(a) $PA \lozenge PB = PC \lozenge PD$

(b) $PA \lozenge PC = PB \lozenge PD$

(c) $PA \lozenge AB = PD \lozenge DC$

(d) $PA \lozenge PD = AB \lozenge DC$

25. All of the following is true except:

(a) The points of intersection of direct common tan gents and indirect common tangents of two circles divide the line segment joining the two centres respectively externally and internally in the ratio of their radii.

(b) In a cyclic quadrilateral $ABCD$, if the diagonal CA bisects the angle C, then diagonal BD is parallel to the tangent at A to the circle through A, B, C, D.

(c) If TA, TB are tangent segments to a circle $C(O, r)$ from an external point T and OT intersects the circle in P, then AP bisects the angle TAB.

(d) If in a right triangle ABC, BD is the perpendicular on the hypotenuse AC, then

(i) $AC \lozenge AD = AB^2$ and

(ii) $AC \lozenge AD = BC^2$

LEVEL OF DIFFICULTY (I)

1. In a right angled triangle, find the hypotenuse if base and perpendicular are respectively 36015 cm and 48020 cm.

   (a) 69125 cm (b) 60025 cm

   (c) 391025 cm (d) 60125 cm

2. The perimeter of an equilateral triangle is $72\sqrt{3}$ cm. Find its height.

   (a) 63 metres (b) 24 metres

   (c) 18 metres (d) 36 metres

3. The inner circumference of a circular track is 440 cm. The track is 14 cm wide. Find the diameter of the outer circle of the track.

   (a) 84 cm (b) 168 cm

   (c) 336 cm (d) 77 cm

4. A race track is in the form of a ring whose inner and outer circumference are 352 metre and 396 metre respectively. Find the width of the track.

   (a) 7 metres (b) 14 metres

   (c) $14p$ metres (d) $7p$ metres

5. The outer circumference of a circular track is 220 metre. The track is 7 metre wide everywhere. Calculate the cost of levelling the track at the rate of 50 paise per square metre.

   (a) ` 1556.5 (b) ` 3113

   (c) ` 593 (d) ` 693

6. Find the area of a quadrant of a circle whose circumference is 44 cm

   (a) 77 cm^2 (b) 38.5 cm^2

   (c) 19.25 cm^2 (d) $19.25p$ cm^2

7. A pit 7.5 metre long, 6 metre wide and 1.5 metre deep is dug in a field. Find the volume of soil removed in cubic metres.

   (a) 135 m^3 (b) 101.25 m^3

   (c) 50.625 m^3 (d) 67.5 m^3

8. Find the length of the longest pole that can be placed in an indoor stadium 24 metre long, 18 metre wide and 16 metre high.

   (a) 30 metres (b) 25 metres

   (c) 34 metres (d) $\sqrt{580}$ metres

9. The length, breadth and height of a room are in the ratio of 3 : 2 : 1. If its volume be 1296 m^3, find its breadth

(a) 18 metres | (b) 18 metres
(c) 16 metres | (d) 12 metres

10. The volume of a cube is 216 cm^3. Part of this cube is then melted to form a cylinder of length 8 cm. Find the volume of the cylinder.

(a) 342 cm^3 | (b) 216 cm^3
(c) 36 cm^3 | (e) Data inadequate

11. The whole surface of a rectangular block is 8788 square cm. If length, breadth and height are in the ratio of 4 : 3 : 2, find length.

(a) 26 cm | (b) 52 cm
(c) 104 cm | (d) 13 cm

12. Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the side of the resulting cube.

(a) 11 cm | (b) 12 cm
(c) 13 cm | (d) 24 cm

13. Find curved and total surface area of a conical flask of radius 6 cm and height 8 cm.

(a) 60p, 96p | (b) 20p, 96p
(c) 60p, 48p | (d) 30p, 48p

14. The volume of a right circular cone is 100p cm^3 and its height is 12 cm. Find its curved surface area.

(a) 130 p cm^2 | (b) 65 p cm^2
(c) 204 p cm^2 | (d) 65 cm^2

15. The diameters of two cones are equal. If their slant height be in the ratio 5 : 7, find the ratio of their curved surface areas.

(a) 25 : 7 | (b) 25 : 49
(c) 5 : 49 | (d) 5 : 7

16. The curved surface area of a cone is 2376 square cm and its slant height is 18 cm. Find the diameter.

(a) 6 cm | (b) 18 cm
(c) 84 cm | (d) 12 cm

17. The ratio of radii of a cylinder to a that of a cone is 1 : 2. If their heights are equal, find the ratio of their volumes?

(a) 1 : 3 | (b) 2 : 3
(c) 3 : 4 | (d) 3 : 1

18. A silver wire when bent in the form of a square, encloses an area of 484 cm^2. Now if the same wire is bent to form a circle, the area of enclosed by it would be

(a) 308 cm^2 (b) 196 cm^2

(c) 616 cm^2 (d) 88 cm^2

19. The circumference of a circle exceeds its diameter by 16.8 cm. Find the circumference of the circle.

(a) 12.32 cm (b) 49.28 cm

(c) 58.64 cm (d) 24.64 cm

20. A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel?

(a) 70 cm (b) 135 cm

(c) 17.5 cm (d) 35 cm

21. The volume of a right circular cone is $100p$ cm^3 and its height is 12 cm. Find its slant height.

(a) 13 cm (b) 16 cm

(c) 9 cm (d) 26 cm

22. The short and the long hands of a clock are 4 cm and 6 cm long respectively. What will be sum of distances travelled by their tips in 4 days? (Take $p = 3.14$)

(a) 954.56 cm (b) 3818.24 cm

(c) 2909.12 cm (d) 2703.56 cm

23. The surface areas of two spheres are in the ratio of 1 : 4. Find the ratio of their volumes.

(a) 1 : 2 (b) 1 : 8

(c) 1 : 4 (d) 2 : 1

24. The outer and inner diameters of a spherical shell are 10 cm and 9 cm respectively. Find the volume of the metal contained in the shell. (Use $p = 22/7$)

(a) 6956 cm^3 (b) 141.95 cm^3

(c) 283.9 cm^3 (d) 478.3 cm^3

25. The radii of two spheres are in the ratio of 1 : 2. Find the ratio of their surface areas.

(a) 1 : 3 (b) 2 : 3

(c) 1 : 4 (d) 3 : 4

26. A sphere of radius r has the same volume as that of a cone with a circular base of radius r. Find the height of cone.

(a) $2r$ (b) $r/3$

(c) $4r$ (d) $(2/3)r$

27. Find the number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 12 m long, 5 m high and 0.25 m thick, while the sand and cement mixture occupies 5% of the total

volume of wall.

(a) 6080
(b) 3040
(c) 1520
(d) 12160

28. A road that is 7 m wide surrounds a circular path whose circumference is 352 m. What will be the area of the road?

(a) 2618 cm^2
(b) 654.5 cm^2
(c) 1309 cm^2
(d) 5236 cm^2

29. In a shower, 10 cm of rain falls. What will be the volume of water that falls on 1 hectare area of ground?

(a) 500 m^3
(b) 650 m^3
(c) 1000 m^3
(d) 750 m^3

30. Seven equal cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

(a) 750 cm^2
(b) 1500 cm^2
(c) 2250 cm^2
(d) 700 cm^2

31. In a swimming pool measuring 90 m by 40 m, 150 men take a dip. If the average displacement of water by a man is 8 cubic metres, what will be rise in water level?

(a) 30 cm
(b) 50 cm
(c) 20 cm
(d) 33.33 cm

32. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and height is 24 m?

(a) 55 m
(b) 330 m
(c) 220 m
(d) 110 m

33. Two cones have their heights in the ratio 1 : 2 and the diameters of their bases are in the ratio 2 : 1. What will be the ratio of their volumes?

(a) 4 : 1
(b) 2 : 1
(c) 3 : 2
(d) 1 : 1

34. A conical tent is to accommodate 10 persons. Each person must have 6 m^2 space to sit and 30 m^3 of air to breath. What will be the height of the cone?

(a) 37.5 m
(b) 150 m
(c) 75 m
(d) None of these

35. A closed wooden box measures externally 10 cm long, 8 cm broad and 6 cm high. Thickness of wood is 0.5 cm. Find the volume of wood used.

(a) 230 cubic cm
(b) 165 cubic cm

(c) 330 cubic cm (d) 300 cubic cm.

36. A cuboid of dimension 24 cm × 9 cm × 8 cm is melted and smaller cubes are of side 3 cm is formed. Find how many such cubes can be formed.

(a) 27 (b) 64

(c) 54 (d) 32

37. Three cubes each of volume of 216 m^3 are joined end to end. Find the surface area of the resulting figure.

(a) 504 m^2 (b) 216 m^2

(c) 432 m^2 (d) 480 m^2

38. A hollow spherical shell is made of a metal of density 4.9 g/cm^3. If its internal and external radii are 10 cm and 12 cm respectively, find the weight of the shell.

(Take *p* = 3.1416)

(a) 5016 gm (b) 1416.8 gm

(c) 14942.28 gm (d) 5667.1 gm

39. The largest cone is formed at the base of a cube of side measuring 7 cm. Find the ratio of volume of cone to cube.

(a) 20 : 21 (b) 22 : 21

(c) 21 : 22 (d) 42 : 11

40. A spherical cannon ball, 28 cm in diameter, is melted and cast into a right circular conical mould the base of which is 35 cm in diameter. Find the height of the cone correct up to two places of decimals.

(a) 8.96 cm (b) 35.84 cm

(c) 5.97 cm (d) 17.92 cm

41. Find the area of the circle circumscribed about a square each side of which is 10 cm.

(a) 314.28 cm^3 (b) 157.14 cm^3

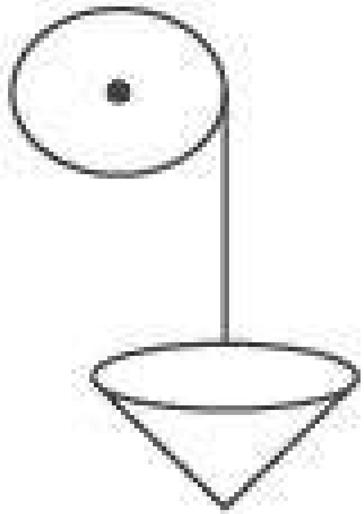
(c) 150.38 cm^3 (d) 78.57 cm^3

42. Find the radius of the circle inscribed in a triangle whose sides are 8 cm, 15 cm and 17 cm.

(a) 4 cm (b) 5 cm

(c) 3 cm (d) $2\sqrt{2}$ cm

43. In the given diagram a rope is wound round the outside of a circular drum whose diameter is 70 cm and a bucket is tied to the other end of the rope. Find the number of revolutions made by the drum if the bucket is raised by 11 m.



(a) 10 (b) 2.5

(c) 5 (d) 5.5

44. A cube whose edge is 20 cm long has circle on each of its faces painted black. What is the total area of the unpainted surface of the cube if the circles are of the largest area possible?

(a) 85.71 cm^2 (b) 257.14 cm^2

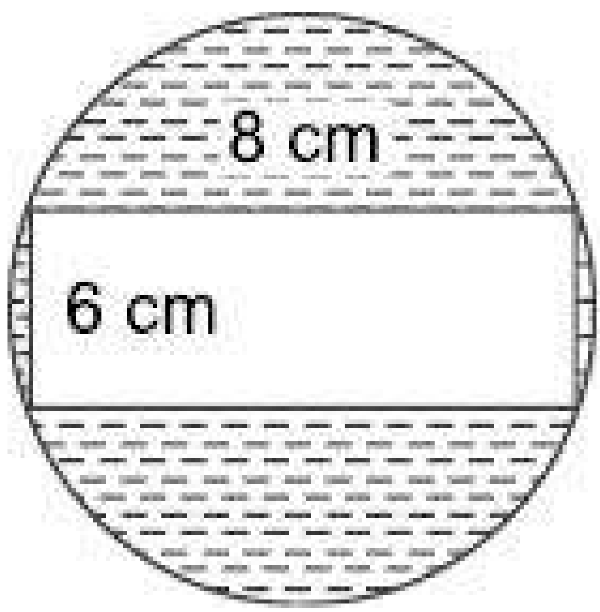
(c) 514.28 cm^2 (d) 331.33 cm^2

45. The areas of three adjacent faces of a cuboid are x, y, z. If the volume is V, then V^2 will be equal to

(a) xy/z (b) yz/x^2

(c) x^2y^2/z^2 (d) xyz

46. In the adjacent figure, find the area of the shaded region. (Use = 22/7)

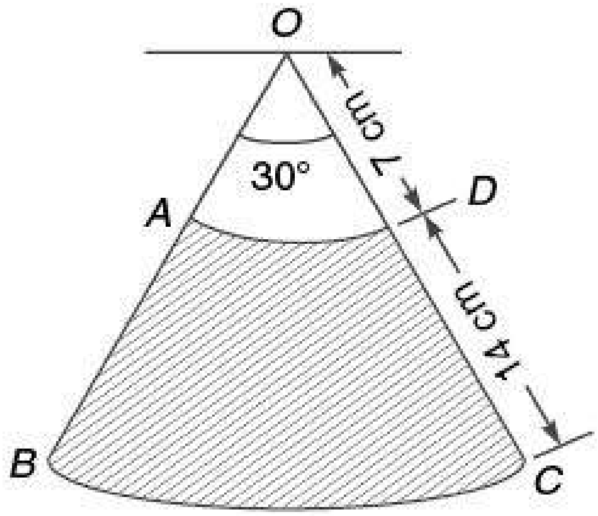


(a) 15.28 cm^2 (b) 61.14 cm^2

(c) 30.57 cm^2 (d) 40.76 cm^2

47. The diagram represents the area swept by the wiper of a car. With the dimensions given in the

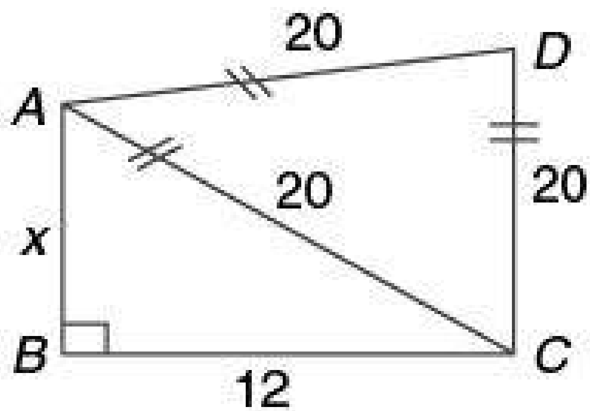
figure, calculate the shaded area swept by the wiper.



(a) 102.67 cm (b) 205.34 cm

(c) 51.33 cm (d) 208.16 cm

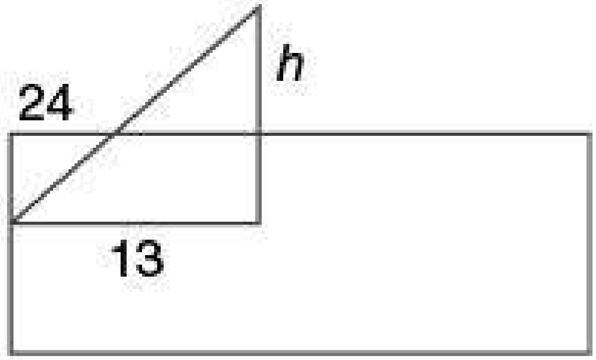
48. Find the area of the quadrilateral *ABCD*. (Given, $\sqrt{3}$ = 1.73)



(a) 452 sq units (b) 269 sq units

(c) 134.5 sq units (d) 1445 g cm

49. The base of a pyramid is a rectangle of sides 18 m × 26 m and its slant height to the shorter side of the base is 24 m. Find its volume.



(a) $156\sqrt{407}$ (b) $78\sqrt{407}$

(c) $312\sqrt{407}$

(d) $234\sqrt{407}$

50. A wire is looped in the form of a circle of radius 28 cm. It is bent again into a square form. What will be the length of the diagonal of the largest square possible thus?

(a) 44 cm

(b) $44\sqrt{2}$

(c) $176/2\sqrt{2}$

(d) $88\sqrt{2}$

LEVEL OF DIFFICULTY (II)

1. The perimeter of a sector of a circle of radius 5.7 m is 27.2 m. Find the area of the sector.

   (a) 90.06 cm^2 (b) 135.09 cm^2

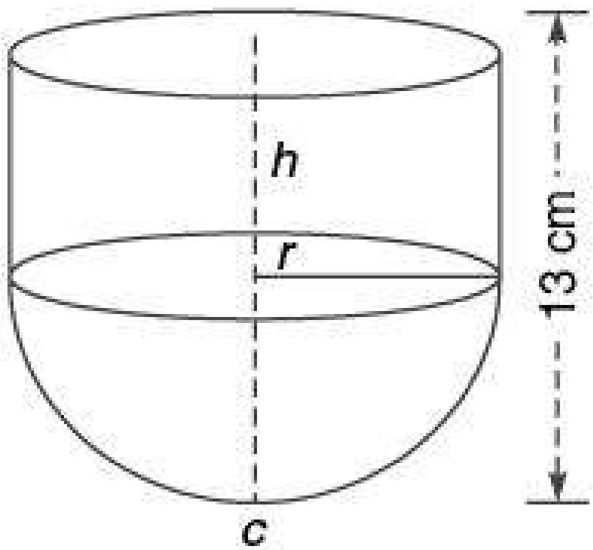
   (c) 45 cm^2 (d) None of these

2. The dimensions of a field are 20 m by 9 m. A pit 10 m long, 4.5 m wide and 3 m deep is dug in one corner of the field and the earth removed has been evenly spread over the remaining area of the field. What will be the rise in the height of field as a result of this operation?

   (a) 1 m (b) 2 m

   (c) 3 m (d) 4 m

3. A vessel is in the form of a hollow cylinder mounted on a hemispherical bowl. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find the capacity of the vessel. (Take p = 22/7).



   (a) 321.33 cm (b) 1642.67 cm^3

   (c) 1232 cm^3 (d) 1632.33 cm^3

4. The sides of a triangle are 21, 20 and 13 cm. Find the area of the larger triangle into which the given triangle is divided by the perpendicular upon the longest side from the opposite vertex.

   (a) 72 cm^2 (b) 96 cm^2

   (c) 168 cm^2 (d) 144 cm^2

5. A circular tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.

   (a) 3894 (b) 973.5

   (c) 1947 m (d) 1800 m

6. A steel sphere of radius 4 cm is drawn into a wire of diameter 4 mm. Find the length of wire.

(a) 10,665 mm (b) 42,660 mm

(c) 21,333 mm (d) 14,220 mm

7. A cylinder and a cone having equal diameter of their bases are placed in the Qutab Minar one on the other, with the cylinder placed in the bottom. If their curved surface area are in the ratio of 8 : 5, find the ratio of their heights. Assume the height of the cylinder to be equal to the radius of Qutab Minar. (Assume Qutab Minar to be having same radius throughout).

(a) 1 : 4 (b) 3 : 4

(c) 4 : 3 (d) 2 : 3

8. If the curved surface area of a cone is thrice that of another cone and slant height of the second cone is thrice that of the first, find the ratio of the area of their base.

(a) 81 : 1 (b) 9 : 1

(c) 3 : 1 (d) 27 : 1

9. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.

(a) 2 cm (b) 3 cm

(c) 1 cm (d) 3.5 cm

10. A hollow sphere of external and internal diameters 6 cm and 4 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone

(a) 4.75 cm (b) 9.5 cm

(c) 19 cm (d) 38 cm

11. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

(a) 7 : 9 (b) 49 : 81

(c) 9 : 7 (d) 27 : 23

12. If V be the volume of a cuboid of dimension x, y, z and A is its surface, then A/V will be equal to

(a) $x^2y^2z^2$ (b) $1/2\ (1/xy + 1/xz + 1/yz)$

(c) $\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ (d) $1/xyz$

13. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 a.m. and 9 : 35 a.m.

(a) 183.3 cm^2 (b) 366.6 cm^2

(c) 244.4 cm^2 (d) 188.39 cm^2

14. Two circles touch internally. The sum of their areas is $116p$ cm^2 and distance between their centres is 6 cm. Find the radii of the circles.

(a) 10 cm, 4 cm (b) 11 cm, 4 cm

(c) 9 cm, 5 cm | (d) 10 cm, 5 cm

15. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of conical part is 12 cm.

(a) 1440 cm^2 (b) 385 cm^2

(c) 1580 cm^2 (d) 770 cm^2

16. A solid wooden toy is in the form of a cone mounted on a hemisphere. If the radii of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of wood used in the toy.

(a) 343.72 cm^3 (b) 266.11 cm^3

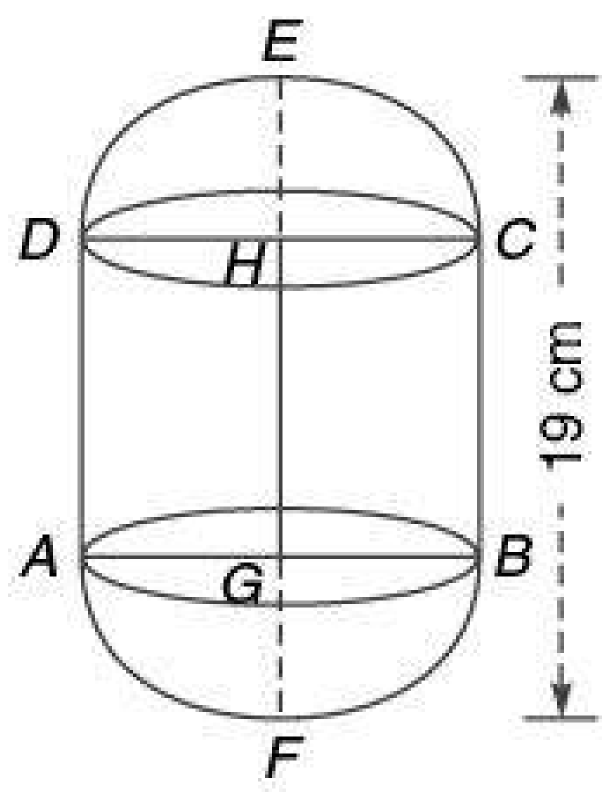
(c) 532.22 cm^3 (d) 133.55 cm^3

17. A cylindrical container whose diameter is 12 cm and height is 15 cm, is filled with ice cream. The whole ice-cream is distributed to 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of its base, find the diameter of the ice-cream cone.

(a) 6 cm (b) 12 cm

(c) 3 cm (d) 18 cm

18. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the total surface area of the solid.(Use = 22/7).



(a) 398.75 cm^2 (b) 418 cm^2

(c) 444 cm^2 (d) 412 cm^2

19. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

(a) 2 : 1 : 3 (b) 2.5 : 1 : 3

(c) 1 : 2 : 3 (d) 1.5 : 2 : 3

20. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost of painting 1 cm^2 of the surface is ` 0.05. Find the total cost of painting the vessel all over.

(Take $p = 22/7$)

(a) ` 97.65 (b) ` 86.4

(c) ` 184 (d) ` 96.28

21. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3.5 cm and the heights of conical and cylindrical portion are respectively 6 cm and 10 cm. Find the volume of the solid.

(Use $p = 3.14$)

(a) 117 cm^2 (b) 234 cm^2

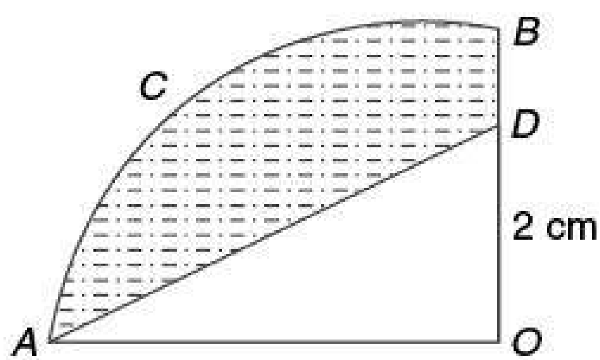
(c) 58.5 cm^2 (d) None of these

22. In the adjoining figure, *AOBCA* represents a quadrant of a circle of radius 3.5 cm with centre *O*. Calculate the area of the shaded portion.

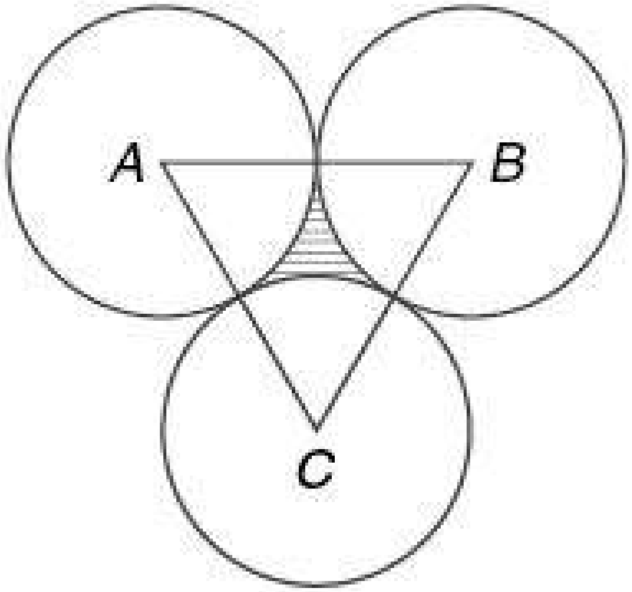
(Use $p = 22/7$)

(a) 35 cm^2 (b) 7.875 cm^2

(c) 9.625 cm^2 (d) 6.125 cm^2



23. Find the area of the shaded region if the radius of each of the circles is 1 cm.



(a) $2 - \frac{\pi}{3}$

(b) $\sqrt{3}$ – *p*

(c) $\sqrt{3} - \frac{\pi}{2}$

(d) $\sqrt{3}$ – *p*/4

24. A right elliptical cylinder full of petrol has its widest elliptical side 2.4 m and the shortest 1.6 m. Its height is 7 m. Find the time required to empty half the tank through a hose of diameter 4 cm if the rate of flow of petrol is 120 m/min

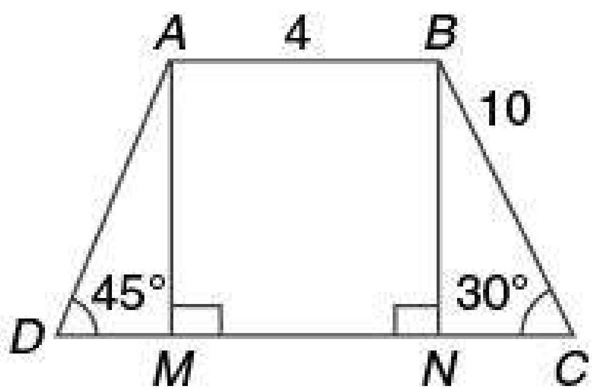
(a) 60 min

(b) 90 min

(c) 75 min

(d) 70 min

25. Find the area of the trapezium *ABCD*.



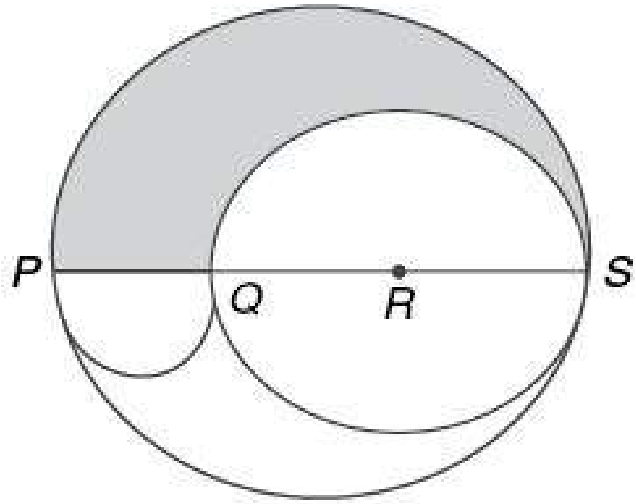
(a) $5/2(13 + 2\sqrt{3})$

(b) $\frac{5\sqrt{3}(13+5\sqrt{3})}{2}$

(c) $13(13 + 2\sqrt{3})$

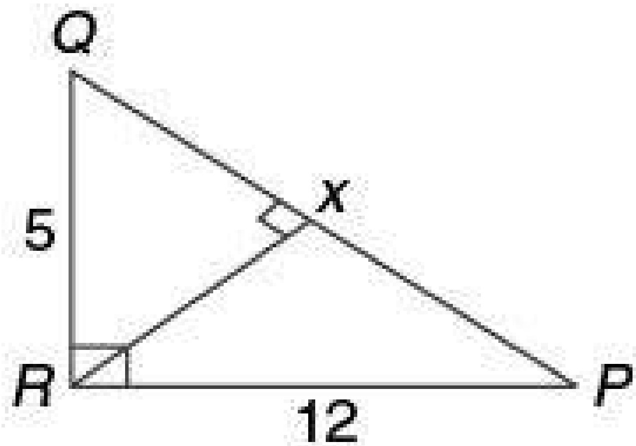
(d) None of these

26. *PQRS* is the diameter of a circle of radius 6 cm. The lengths *PQ*, *QR* and *RS* are equal. Semi-circles are drawn with *PQ* and *QS* as diameters as shown in the figure alongside. Find the ratio of the area of the shaded region to that of the unshaded region.



(a) 1 : 2
(b) 25 : 121
(c) 5 : 18
(d) 5 : 13

27. In the right angle triangle *PQR*, find *Rr*.

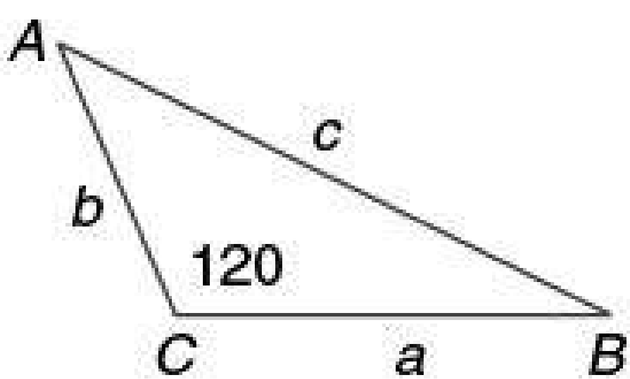


(a) 13/60
(b) 13/45
(c) 60/13
(d) 23/29

28. The radius of a right circular cylinder is increased by 50%. Find the percentage increase in volume

(a) 120%
(b) 75%
(c) 150%
(d) 125%

29. Two persons start walking on a road that diverge at an angle of 120°. If they walk at the rate of 3 km/h and 2 km/h repectively. Find the distance between them after 4 hours.



(a) $4\sqrt{19}$ km

(b) 5 km

(c) 7 km

(d) $8\sqrt{19}$ km

30. Water flows out at the rate of 10 m/min from a cylindrical pipe of diameter 5 mm. Find the time taken to fill a conical tank whose diameter at the surface is 40 cm and depth 24 cm.

(a) 50 min

(b) 102.4 min

(c) 51.2 min

(d) 25.6 min

31. The section of a solid right circular cone by a plane containing vertex and perpendicular to base is an equilateral triangle of side 12 cm. Find the volume of the cone.

(a) 72 cc

(b) 144 cc

(c) $72\sqrt{2}\,p$ cc

(d) $72\sqrt{3}\,p$ cc

32. Iron weighs 8 times the weight of oak. Find the diameter of an iron ball whose weight is equal to that of a ball of oak 18 cm in diameter.

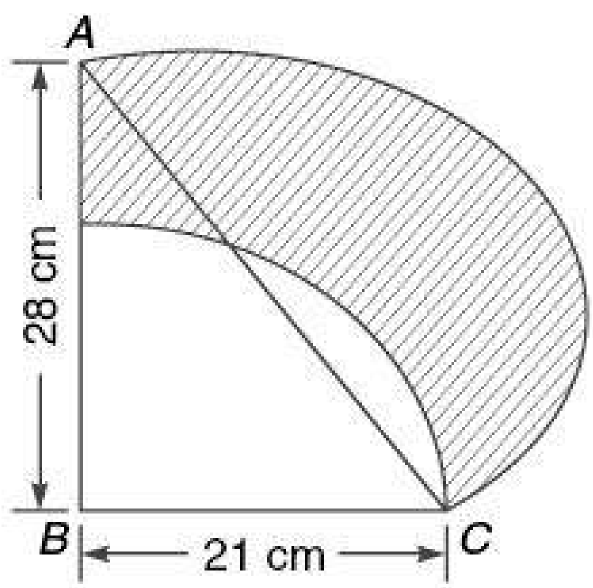
(a) 4.5 cm

(b) 9 cm

(c) 12 cm

(d) 15 cm

33. In the figure, *ABC* is a right angled triangle with –*B* = 90°, *BC* = 21 cm and *AB* = 28 cm. With *AC* as diameter of a semicircle and with *BC* as radius, a quarter circle is drawn. Find the area of the shaded portion correct to two decimal places



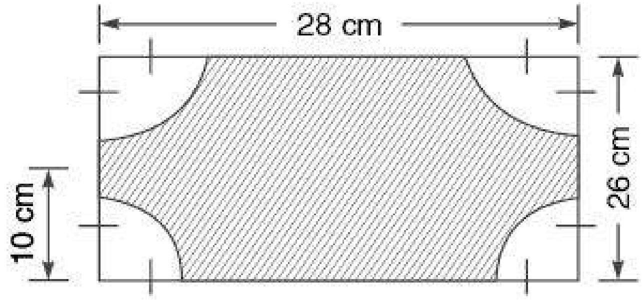
(a) 428.75 cm^2

(b) 857.50 cm^2

(c) 214.37 cm^2

(d) 371.56

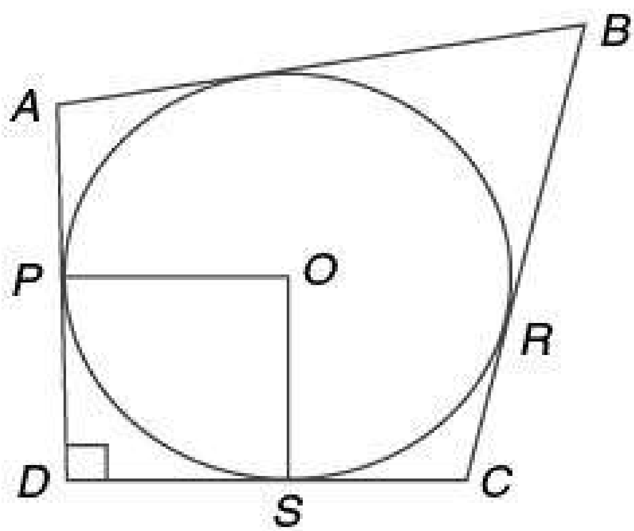
34. Find the perimeter and area of the shaded portion of the adjoining diagram:



(a) 90.8 cm, 414 cm^2 (b) 181.6 cm, 423.7 cm^2

(c) 90.8 cm, 827.4 cm^2 (d) 181.6 cm, 827.4 cm^2

35. In the adjoining figure, a circle is inscribed in the quadrilateral *ABCD*. Given that *BC* = 38 cm, *AB* = 27 cm and *DC* = 25 cm, and that *AD* is perpendicular to *DC*. Find the maximum limit of the radius and the area of the circle.

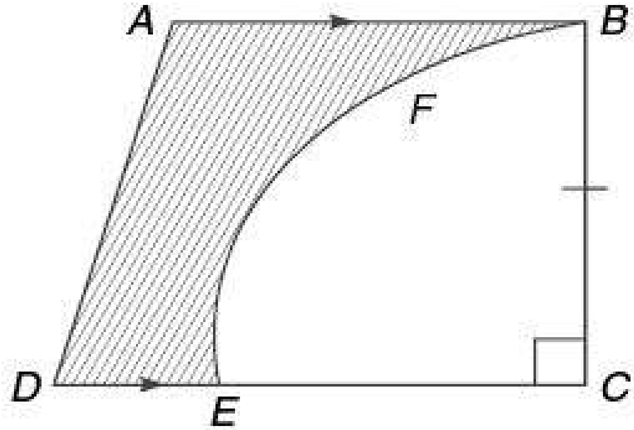


(a) 10 cm; 226 cm^2 (b) 14 cm; 616 cm^2

(c) 14 cm; 216 cm^2 (d) 28 cm; 616 cm^2

36. From a piece of cardboard, in the shape of a trapezium *ABCD* and *AB* || *DC* and –*BCD* = 90°, a quarter circle (*BFEC*) with *C* as its centre is removed. Given *AB* = *BC* = 3.5 cm and *DE* = 2 cm, calculate the area of the remaining piece of the cardboard.

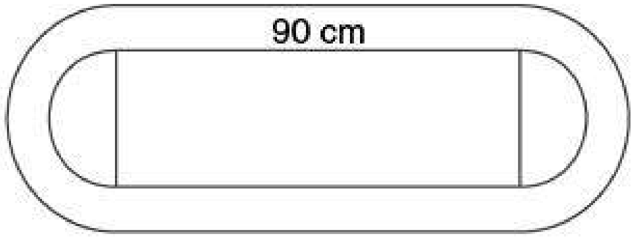
(Take *p* = 22/7)



(a) 3.325 cm^2 (b) 3.125 cm^2

(c) 6.125 cm^2 (d) 12.25 cm^2

37. The inside perimeter of a practice running track with semi-circular ends and straight parallel sides is 312 m. The length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



(a) 5166 m^2 (b) 5802.57 m^2

(c) 636.57 m^2 (d) 1273.14 m^2

38. Find the area of the triangle inscribed in a circle circumscribed by a square made by joining the mid-points of the adjacent sides of a square of side *a*.

(a) $3a^2/16$ (b) $\dfrac{3\sqrt{3}\,a^2}{16}$

(c) $3/4\ a^2(p - 1/2)$ (d) $\dfrac{3\sqrt{3}\,a^2}{32}$

39. Two goats are tethered to diagonally opposite vertices of a field formed by joining the mid-points of the adjacent sides of another square field of side 20÷2. What is the total grazing area of the two goats?

(a) $100p$ m^2 (b) $50(\sqrt{2} - 1)p$ m^2

(c) $100p(3 - 2\sqrt{2})$ m^2 (d) $200p(2 - \sqrt{2})$ m^2

40. The area of the circle circumscribing three circles of unit radius touching each other is

(a) $(p/3)(2 + \sqrt{3})^2$ (b) $6p(2 + \sqrt{3})^2$

(c) $3p(2 + \sqrt{3})^2$ (d) $\left(\dfrac{\pi}{6}\right)(2 + \sqrt{3})^2$

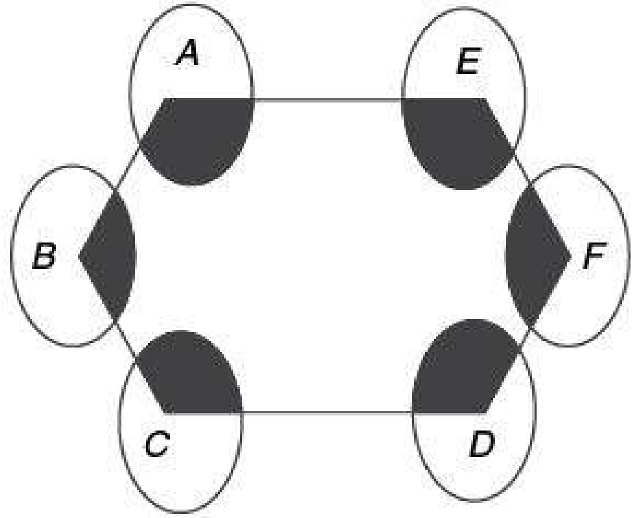
41. Find the ratio of the diameter of the circles inscribed in and circumscribing an equilateral triangle to its height.

(a) 1 : 2 : 1 (b) 1 : 2 : 3

(c) 1 : 3 : 4 (d) 3 : 2 : 1

42. Find the sum of the areas of the shaded sectors given that *ABCDFE* is any hexagon and all the circles are of same radius *r* with different vertices of the hexagon as their centres as shown in the

figure.



(a) pr^2

(b) $2pr^2$

(c) $5pr^2/4$

(d) $3pr^2/2$

43. Circles are drawn with four vertices as the centre and radius equal to the side of a square. If the square is formed by joining the mid-points of another square of side $2\sqrt{6}$, find the area common to all the four circles.

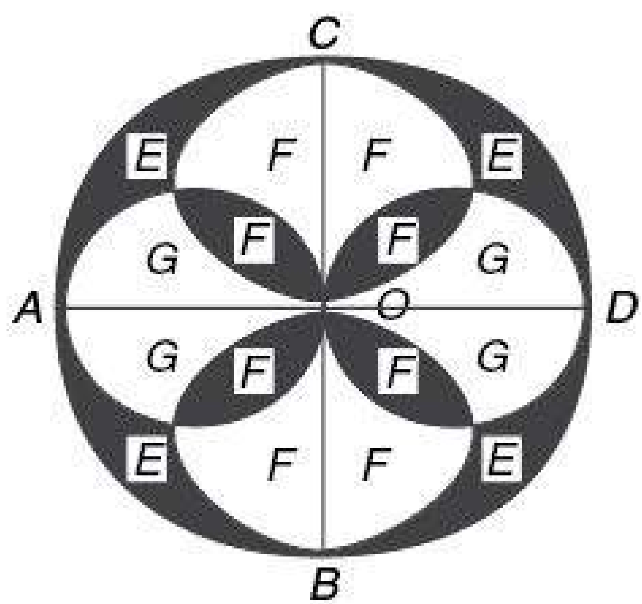
(a) $[(\sqrt{3}-1)/2)]\ 6p$

(b) $4p - 3\sqrt{3}$

(c) $1/2\ (p - 3\sqrt{3})$

(d) $4p - 12(\sqrt{3}-1)$

44. *ABDC* is a circle and circles are drawn with *AO*, *CO*, *DO* and *OB* as diameters. Areas *E* and *F* are shaded. *E*/*F* is equal to



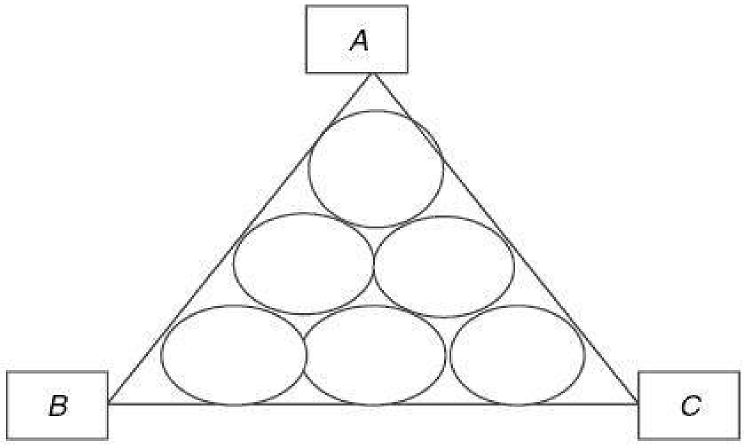
(a) 1

(b) 1/2

(c) 1/ 2

(d) $p/4$

45. The diagram shows six equal circles inscribed in equilateral triangle *ABC*. The circles touch externally among themselves and also touch the sides of the triangle. If the radius of each circle is *R*, area of the triangle is



(a) $(6 + p\sqrt{3})\ R^1$ (b) $9\ R^2$

(c) $R^2\ (12 + 7\sqrt{3})$ (d) $R^2\ (9 + 6\sqrt{3})$

ANSWER KEY

GEOMETRY

Level of Difficulty (I)

1. (a)	2. (d)	3. (c)	4. (a)
5. (a)	6. (c)	7. (a)	8. (d)
9. (a)	10. (d)	11. (d)	12. (a)
13. (b)	14. (a)	15. (b)	16. (b)
17. (a)	18. (a)	19. (b)	20. (a)
21. (b)	22. (a)	23. (d)	24. (d)
25. (b)			

Level of Difficulty (II)

1. (b)	2. (a)	3. (a)	4. (a)
5. (c)	6. (a)	7. (a)	8. (b)
9. (a)	10. (b)	11. (b)	12. (a)
13. (a)	14. (c)	15. (b)	16. (a)
17. (d)	18. (c)	19. (c)	20. (d)
21. (a)	22. (b)	23. (a)	24. (b)
25. (d)			

MENSURATION

Level of Difficulty (I)

1. (b)	2. (d)	3. (b)	4. (a)

5. (d)	6. (b)	7. (d)	8. (c)
9. (d)	10. (d)	11. (b)	12. (b)
13. (a)	14. (b)	15. (d)	16. (c)
17. (c)	18. (c)	19. (d)	20. (d)
21. (a)	22. (b)	23. (b)	24. (b)
25. (c)	26. (c)	27. (a)	28. (a)
29. (c)	30. (a)	31. (d)	32. (d)
33. (b)	34. (d)	35. (b)	36. (b)
37. (a)	38. (c)	39. (d)	40. (b)
41. (b)	42. (c)	43. (c)	44. (c)
45. (d)	46. (c)	47. (a)	48. (b)
49. (a)	50. (b)		

Level of Difficulty (II)

1. (d)	2. (d)	3. (b)	4. (b)
5. (c)	6. (c)	7. (b)	8. (a)
9. (c)	10. (d)	11. (a)	12. (c)
13. (a)	14. (a)	15. (d)	16. (b)
17. (a)	18. (b)	19. (c)	20. (d)
21. (d)	22. (d)	23. (c)	24. (d)
25. (d)	26. (d)	27. (c)	28. (d)
29. (a)	30. (c)	31. (d)	32. (b)
33. (a)	34. (a)	35. (d)	36. (c)
37. (c)	38. (d)	39. (a)	40. (a)
41. (b)	42. (b)	43. (d)	44. (a)
45. (c)			

Solutions and Shortcuts

GEOMETRY

Level of Difficulty (I)

1. (a)

   When the length of stick = 20 m, then length of shadow = 10 m i.e. in this case length = 2 × shadow with the same angle of inclination of the sun, the length of tower that casts a shadow of 50 m fi 2 × 50 m = 100 m

   i.e. height of tower = 100 m

2. (d)

   D*ABC* ~ D*EDC*

Then $\dfrac{AC}{EC} = \dfrac{BC}{DC} = \dfrac{AB}{ED}$

Then $\dfrac{AC}{4.2} = \dfrac{4}{3} = AC = 5.6$ cm and

$\dfrac{BC}{4.8} = \dfrac{4}{3} = BC = 6.4$ cm

3. (c)

For similar triangles fi (Ratio of sides)^2 = Ratio of areas

Then as per question = $\left(\dfrac{36}{x}\right)^2 = \dfrac{144}{81}$

{Let the longest side of DDEF = x}

fi $\dfrac{36}{x} = \dfrac{12}{9} = x = 27$ cm

4. (a)

(Ratio of corresponding sides)^2 = Ratio of area of similar triangle

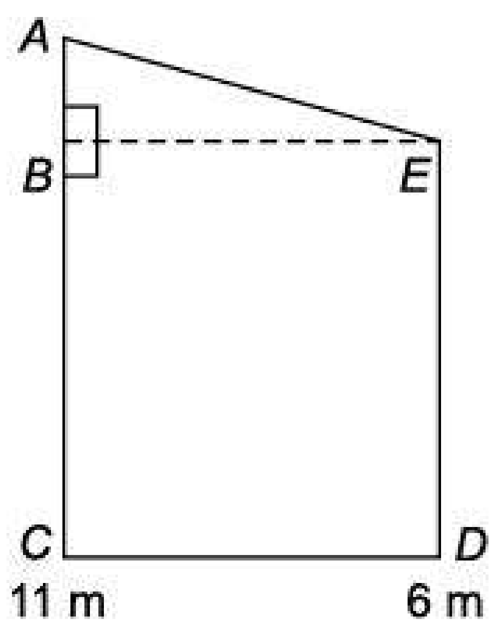
\ Ratio of corresponding sides in this question

$= \sqrt{\dfrac{16}{25}} = \dfrac{4}{5}$

5. (a)

Ratio of corresponding sides = $\sqrt{\dfrac{9}{16}} = \dfrac{3}{4}$

6. (c)



$BC = ED = 6$ m

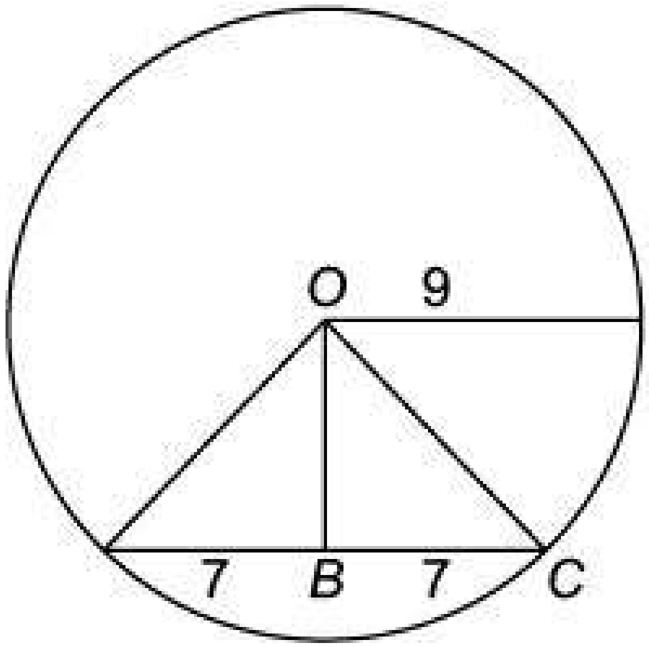
So $AB = AC - BC = 11 - 6 = 5$ m

$CD = BE = 12$ m

Then by Pythagoras theorem:

$AE^2 = AB^2 + BE^2$ fi $AE = 13$ m

7. (a)



In the DOBC; $BC = 7$ cm and $OC = 9$ cm, then using Pythagoras theorem.

$OB^2 = OC^2 - BC^2$

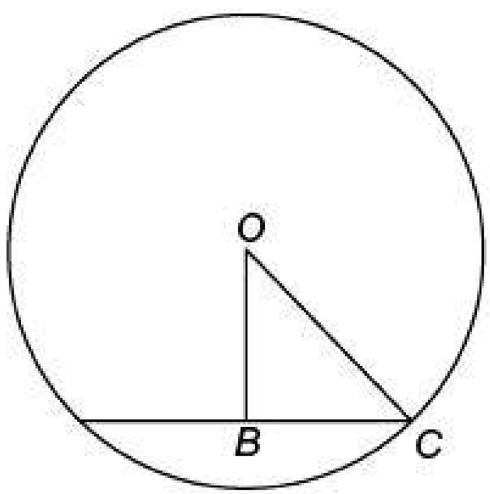
$OB = \sqrt{32} = 5.66$ cm (approx)

8. (d)

In the DOBC, $OB = 12$ cm, OC = radius = 13 cm. Then using Pythagoras theorem;

$BC^2 = OC^2 - OB^2 = 25$; $BC = 5$ cm

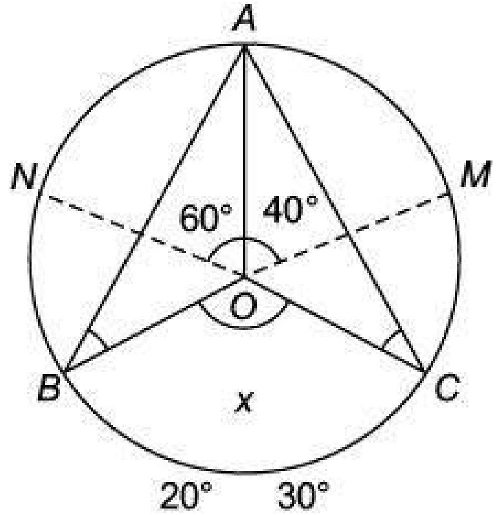
Length of the chord = $2 \times BC = 2 \times 5 = 10$ cm



9. (a)

–x = 35°; because angles subtended by an arc, anywhere on the circumference are equal.

10. (d)



–*AOM* = 2–*ABM* and

–*AON* = 2–*ACN*

because angle subtended by an arc at the centre of the circle is twice the angle subtended by it on the circumference on the same segment.

–*AON* = 60° and –*AOM* = 40°

–*X* = –*AON* + –*AOM*

(◊.◊ vertically opposite angles).

–*X* = 100°

11. (d)

D*BCO* and D*AOD* are similar (◊.◊ *AC* and *BD* one diameters). Thus

–*ADO* = –*CBD* (alternate opposite angles)

\–*X* = 30°

12. (a)

By the rule of tangents, we know:

$6^2 = (5 + n)5$ fi $36 = 25 + 5x$ fi $11 = 5x$ fi $x = 2.2$ cm

13. (b)

By the rule of tangents, we get

$12^2 = (x + 7)x$ fi $144 = x^2 + 7x$

fi $x^2 + 7x – 144 = 0$ fi $x^2 + 16x – 9x – 144 = 0$

fi $x(x + 16) – 9(x + 16)$ fi $x = 9$ or –16

–16 can’t be the length, hence this value is discarded, thus, $x = 9$

14. (a)

By the rule of chords, cutting externally, we get

fi $(9 + 6)6 = (5 + x)5$ fi $90 = 25 + 5x$ fi $5x = 65$ fi $x = 13$ cm

15. (b)

Inradius = area/semi perimeter

= 24/12 = 2 cm

16. (b)

–*APB* = 90° (angle in a semicircle = 90°)

–*PBA* = 180 – (90 + 25) = 65°

–*TPA* = –*PBA*(the angle that a chord makes with the tangent, is subtended by the chord on the circumference in the alternate segment).

= 65°

17. (a)

ADBC is a cyclic quadrilateral as all its four vertices are on the circumference of the circle. Also, the opposite angles of the cyclic quadrilateral are supplementary.

Therefore, –*ADB* = 180 – 48° = 132°

18. (a)

–*POS* = –*QOR* (vertically opposite angles)

So $a = 4b$

–*SOT* + –*TOQ* + –*QOR* = 180°

(sum of angles on a line = 180°)

$4b + 2c = 180°$

$84 + 2c = 180°$

fi$2c = 96°$ fi $c = 48°$

So $a = 84°$, $b = 21°$, $c = 48°$

19. (b)

–*ABC* = 180° – 130° = 50°

(\ sum of angles on a line = 180°)

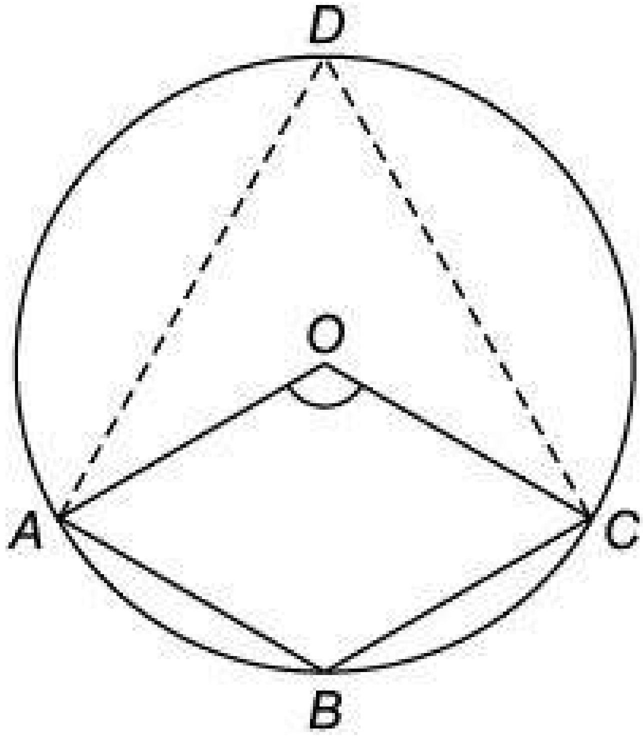
–*ADC* = 180° – –*ABC* = 130°

(◊.◊ opposite angles of a cyclic quadrilateral are supplementary).

–*x* = 180° – 130° = 50°

(\ sum of angles on a line = 180°)

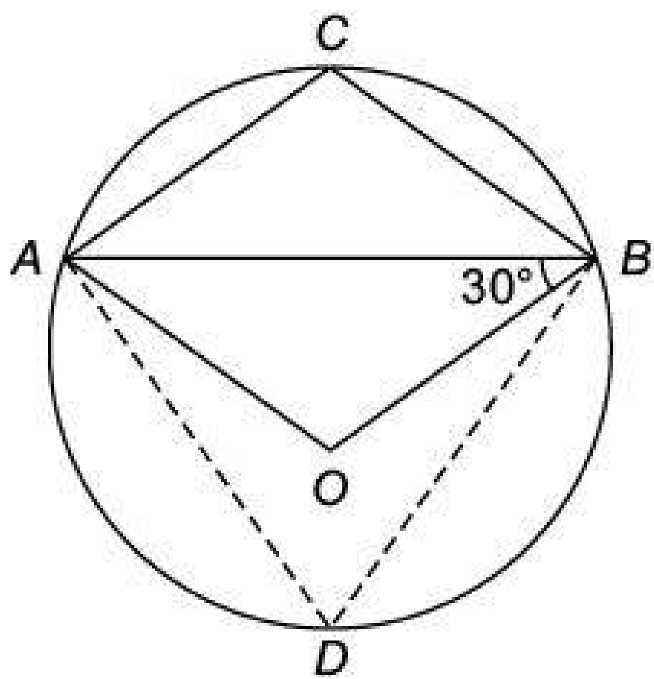
20. (a)



–ADC = $\frac{140}{2}$ = 70° (because the angle subtended by an arc on the circumference is half of what it subtends at the centre). $ABCD$ one cyclic quadrilateral

So –ABC = 180° – 70° = 110° (because opposite angles of a cyclic quadrilateral are supplementary).

21. (b)



OB = OA = radius of the circle

–AOB = 180 – (30 + 30)

{Sum of angles of a triangle = 180°)

fi 120°

Then –ADB = $\frac{120}{2}$ = 60°; because the angle subtended by a chord at the centre is twice of what it can subtend at the circumference. Again, $ABCD$ is a cyclic quadrilateral;

So –ACB = 180° – 60° = 120° (because opposite angles of cyclic quadrilateral are supplementary).

22. (a)

–BAC = 30° (◊.◊ angles subtended by an arc anywhere on the circumference in the same segment are equal).

In DBAC; –x = 180° – (110° + 30°) = 40°

(◊.◊ sum of angles of a triangle = 180°)

23. (d)

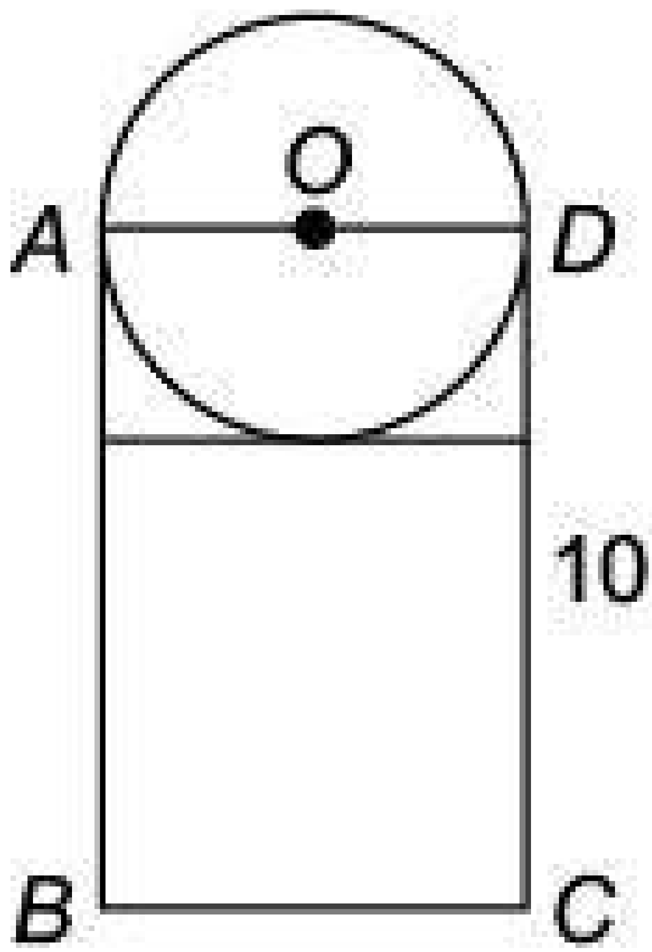
As L_4 and L_3 are not parallel lines, so there can't be any relation between 80° and x°.

Hence the answer cannot be determined.

24. (d)

Perimeter of the figure = 10 + 10 + 6 + 3p

= 26 + 3p



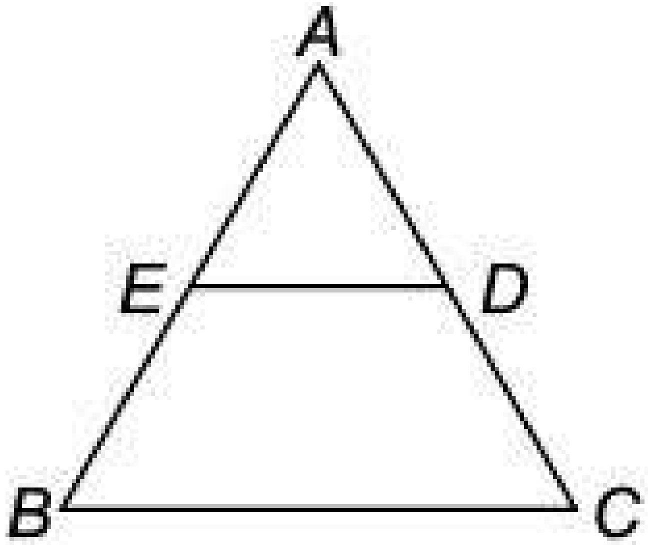
25. (b)CN : BR = 4 : 3

So the required answer is 3 : 5 : 2 : 7 : 4

Option (b) is correct.

Level of Difficulty (II)

1. (b)



D*ADE* is similar to D*ABC* (AAA property)

ED : *BC* = 3 : 5

Area of D*ADE* : Area of D*ABC* = 9 : 25

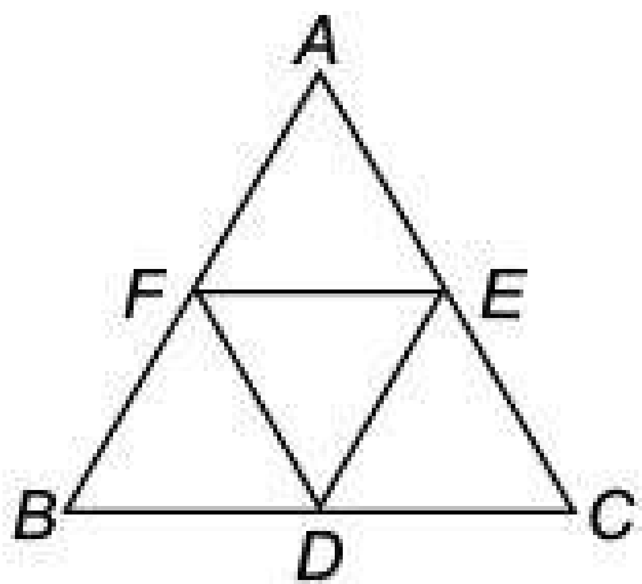
Area of trapezium = area of *ABC* – Area of *ADE*

= 25 – 9 = 16

Thus,

Area of D*ADE* : Area of trapezium *EDBC* = 9 : 16

2. (a)



The area of a triangle formed by joining the mid-points of the sides of another triangle is always $1/4^{th}$ of the area of the bigger triangle.

So, the ratio is = 1 : 4

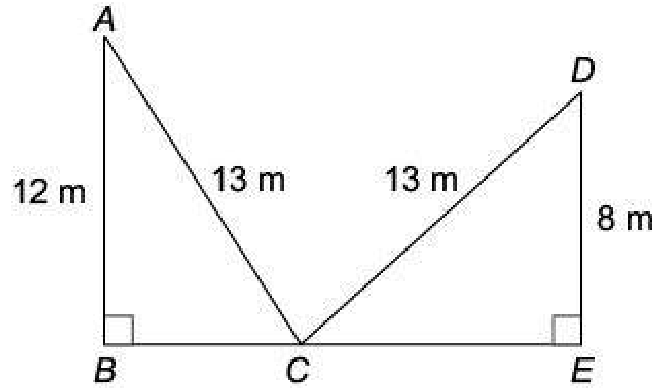
3. (a)

D*DOC* and D*AOB* are similar (by AAA property)

AB : *DC* = 3 : 1

So area of *AOB* : Area of *DOC* = $(3 : 1)^2$ fi 9 : 1

4. (a)



In ΔABC; $BC = \sqrt{13^2 - 12^2} = 5$

In ΔCDE; $CE = \sqrt{13^2 - 8^2} = \sqrt{105} = 10.2$

Width of street = $BC + CE = 5 + 10.2 = 15.2$ m

5. (c)

$ABCD$ is a cyclic quadrilateral. Therefore

$\angle DCB = 180 - \angle A = 180 - 60 = 120°$

$\angle ABC = 80°$; therefore $\angle BCQ = 180° - 120° = 60°$

And $\angle CBQ = 180° - 80° = 100°$

(because, sum of angles on a line=180°)

Then in ΔBCQ; $\angle Q = 180 - (100 + 60) = 20$

($\because$ sum of angles of triangle = 180°)

6. (a)

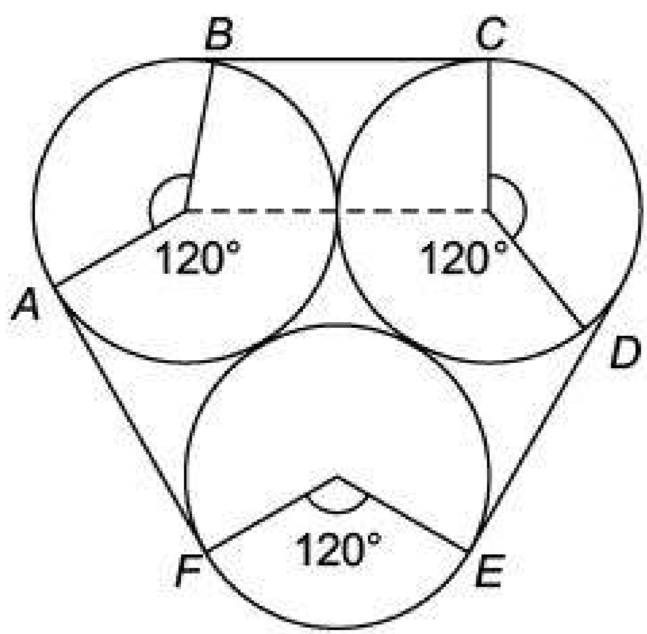
$\angle AOB = \angle CO'D = \angle FO''E = 120°$

Distance between 2 centres = 2 m

$\therefore BC = DE = FA = 2$ m

Perimeter of the figure = $BC + DE + FA$ + circumference of sectors AOB, $CO'B$ and $FO''E$.

But three equal sectors of 120° = 1 full circle of same radius.



Therefore, perimeter of surface

= 2*pr* + *BC* + *DE* + *FA* = (2*p* + 6)*m*

7. (a)

In D*QRS*; *QR* = *RS*, therefore –*RQS* = –*RSQ* (because angles opposite to equal sides are equal).

Thus –*RQS* + –*RSQ* = 180° – 100° = 80°

\ –*RQS* = –*RSQ* = 40°

–*PQS* = 180° – 40° = 140°

(sum of angles on a line = 180°)

Then again –*QRS* = –*QSP*

(◊.◊ angles opposite to equal sides are equal)

Thus –*QPS* + –*QSP* = 180° – 140° = 40°

And –*QPS* = –*QSP* = 20°

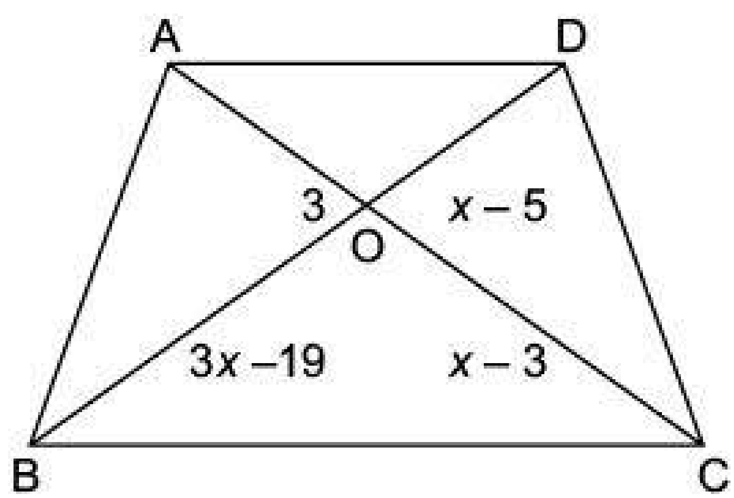
8. (b)

–*BOC* = 136°

–*BAC* = $\frac{136}{2}$ = 68° (because angle subtended by an arc anywhere on the circumference is half of the angle it subtends at the centre).

–*BDC* = 180° – 68° = 112° (\ *ABCD* is a cyclic quadrilateral and its opposite angles are supplementary)

9. (a)



D*ADO* and D*BOC* are similar (AAA property)

Then $\frac{3}{3x-19} = \frac{x-5}{x-3}$

fi$3x - 9 = 3x^2 - 15x - 19x + 95$

fi$3x^2 - 37x + 104 = 0$

On solving this quadratic equation, we get *x* = 8 or 9.

10. (b)

D*AOD* and D*BOC* are similar (AAA property)

$\frac{AO}{OC} = \frac{1}{2}$; therefore $\frac{AD}{BC} = \frac{1}{2}$ fi $\frac{4}{BC} = \frac{1}{2}$

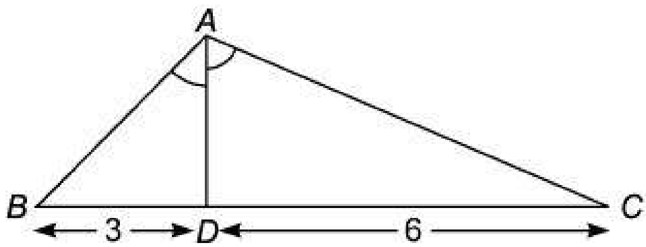
fiBC = 8 cm

11. (b)

In the given figure, DABD is similar to D ADC

Then $\frac{AB}{BD} = \frac{AC}{DC}$ fi $\frac{6}{3} = \frac{5}{DC}$ fi DE = 2.5 cm

12. (a)



As AD bisects –CAB, so DABD is similar to D ADC

Then $\frac{AB}{DB} = \frac{AC}{DC}$ fi $\frac{8}{6} = \frac{AC}{3}$ fi AC = 4 cm

13. (a)

Area of the quarter circle = $\frac{\pi r^2}{4}$ fi 0.25*p*. Going by options, we have to see that the area of the inserted circle is less than the area of the quarter circle.

Option (b) $\frac{(\sqrt{2}+1)}{2}$ fi Area = (1.5 + $\sqrt{2}$)*p*

2.9*p* > 0.25*p*. Hence discarded.

Option (c) $\sqrt{2} - \frac{1}{2}$ fi Area = $p\left(2 + \frac{1}{4} - \sqrt{2}\right)$ fi 0.75*p* > 0.25*p*. Hence discarded.

Option (d) 1 – $\sqrt[2]{2}$ fi Area = *p* (1 + 8 – $\sqrt[4]{2}$ fi 0.85*p* > 0.25*p*. Hence discarded.

Option (a) $\sqrt{2}$ – 1 = Area *p*(2 + 1 – $\sqrt[2]{2}$) fi 0.20*p*

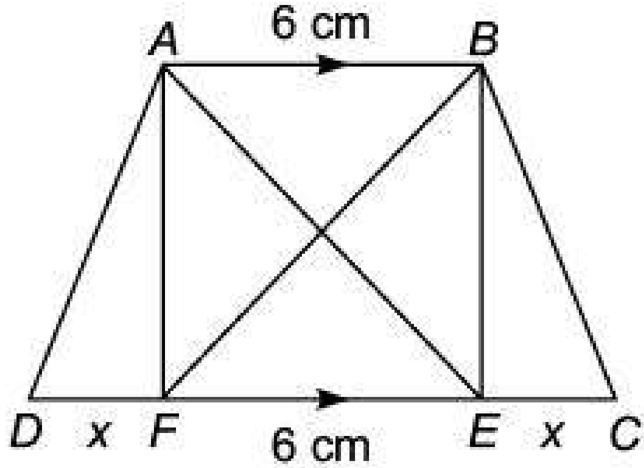
< 0.25*p*

Hence this option is correct.

14. (c)

Sum of length of all perpendiculars drawn on the sides of any equilateral triangle is constant. Perpendicular (D) = Perpendicular (E)

15. (b)



In the above question:

$FE = AB = 6$ cm

DADF @ DBEC; so $DF = EC$

Let $DF = EC = x$

Solving through options; e.g. option (b) 1/3; $x = 6$

Then by Pythagoras triplet $AF = 8$

Area of $ABEF = 8 \times 6 = 48$ cm^2

Area of DAFD + DBEC = $2 \times \frac{1}{2} \times 6 \times 8$ fi 48 cm^2

\ Area of $ABCD = 48 + 48 = 96$ cm^2. Hence the condition is proved.

16. (a)

Find the indivisible area of DABC and DACD

Add them then;

$\frac{1}{2}AB \times BC + \frac{1}{2}AC \times CD$ fi $\frac{1}{2}xy + \frac{1}{2}z\sqrt{x^2+y^2}$

fi replace x, y, z by the lowest integral values.

Here as we are talking about right angles, so we have to take the smallest Pythagorean triplet, which in this case would be = 3, 4, 5

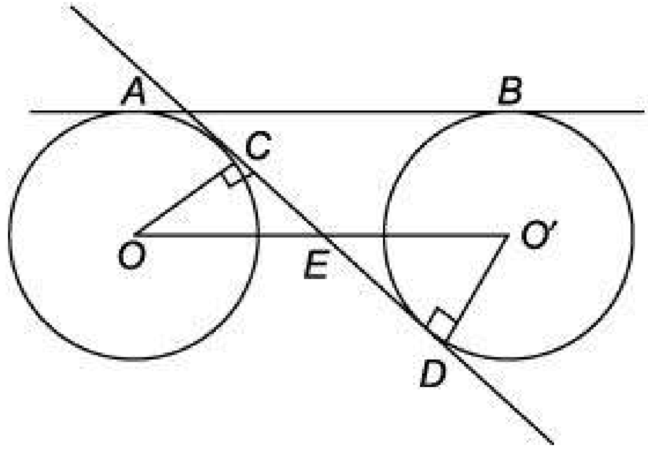
Answer fi 36 cm^2

17. (d)

As the point 'O' is formed by the ^ bisects to the three sides of the D, so point 'O' is the circumcenter. This means that virtually, points A, B and C are on the circumference of the circle.

Thus m–BOC = 2m–BAC (\angle subtended by an arc at the centre of the circle is twice the angle subtended at the circumference).

18. (c)



$OC = O'D = 5$ cm (radius)

$CD = 24$ cm

ΔCOE and $\Delta EO'D$ are similar therefore $OE = O'E$ and $CE = ED = 12$ cm

In ΔCOD; $OE^2 = CE^2 + OC^2$

$= 12^2 + 5^2 = 169$

$OE = 13$

$\therefore OO' = OE + EO' = 13 + 13 = 26$ cm

19. (c)

It will always be possible to divide a circle into 360 equal parts, because the sum of angles that can be subtended at the centre of the circle = 360°

20. (d)

Area of the shaded portion = Area of circle – Area of triangle

$\Rightarrow$ Area of circle = $\pi r^2 \Rightarrow \frac{22}{7} \times 5 \times 5 \Rightarrow \frac{22 \times 25}{7}$ cm^2

$= 78.50$ cm^2

Area of triangle $\Rightarrow \frac{1}{2} r^2 \sin\theta \Rightarrow \frac{1}{2} \times 25 \times \sin\theta$

$\Rightarrow \frac{25\sqrt{3}}{4} \Rightarrow 6.25 \times 1.732 \Rightarrow 10.8$

$\therefore$ Area of shaded portion = $78.50 - 10.8 = 67.7$ cm^2

21. (a)

$OB = OA$ – radius of circle

$\Rightarrow \angle CAO = \angle OBA$

(angles in alternate segments are equal).

Now, if $\quad \angle CAO = \angle OBA$

$\therefore \quad \angle CAO = \angle OAB$

$\therefore$ option (a) is correct.

22. (b)

23. (a)

Angle *XPA* = angle *ABP* = *x*

Angle *CPX* = angle *CDP* = *x* + *y*

Anlge *CDP* is exterior angle of triangle *PDB*

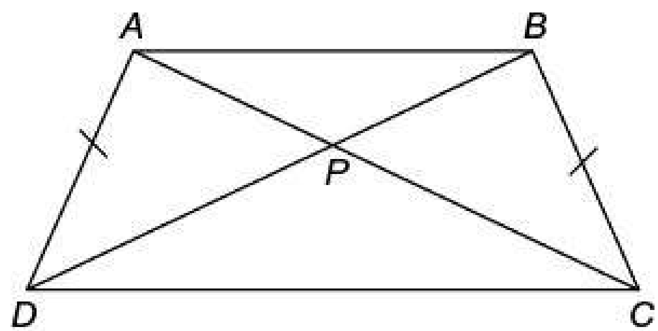
So angle *CDP* = *DBP* + *DPB*

X + *y* = *x* + *DPB*

DPB = *y*

So angle *CPA* = *DPB*

24. (b)



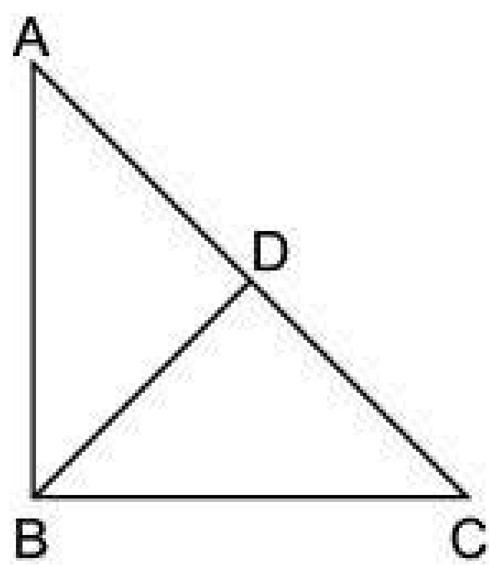
D*APD* ~ D*CPB*

\ $\frac{PA}{PB} = \frac{PD}{PC}$

i.e. *PA* ◊ *PC* = *PB* ◊ *PD*.

\ option (b)

25. (d)



AC ◊ $AD = AB^2$

AC ◊ $AD = BC^2$

D*ABC N* D*ADB*

\ $\frac{AC}{AB} = \frac{AB}{AD}$

(Corresponding sides of similar triangle are proportional)

$AC ◊ AD = AB^2$(i)

Also, DABC N DBDC fi $\frac{AC}{BC} = \frac{BC}{CD}$

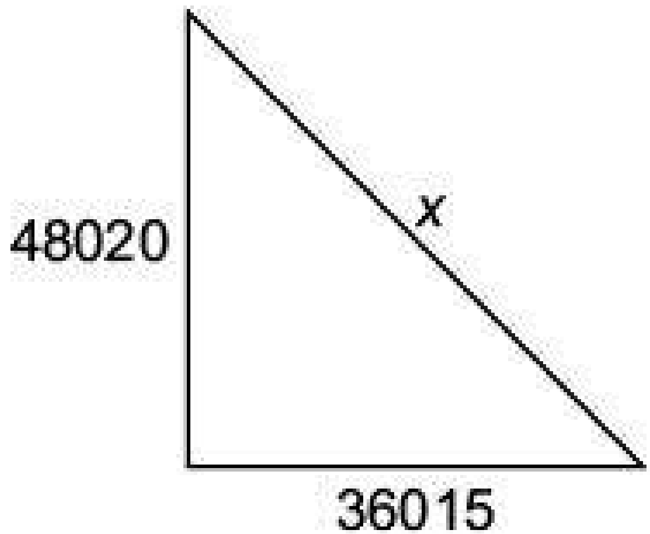
$AC ◊ CD = BC^2$ (ii)

Both conditions of option (d) are found, therefore (d) is the answer.

Mensuration

Level of Difficulty (I)

1. (b)



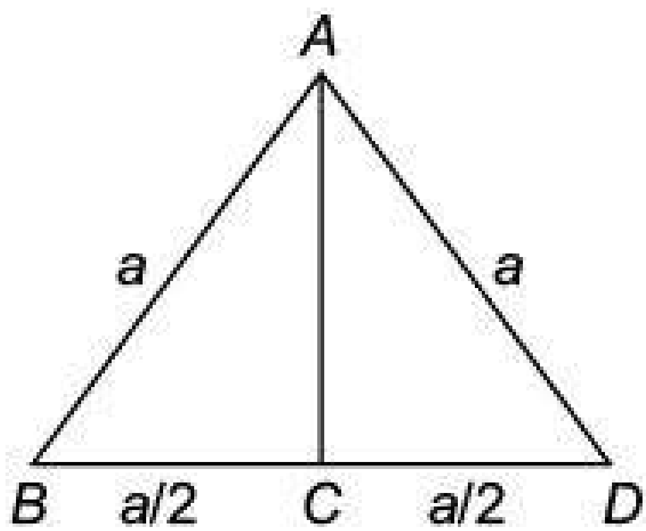
Let hypotenuse = x cm

Then, by Pythagoras theorem:

$x^2 = (48020)^2 + (36015)^2$

x fi 60025 cm

2. (d)



Let one side of the D be = a

Perimeter of equilateral triangle = 3a

\3$a = 72\sqrt{3} = a = 24\sqrt{3}$ cm

Height = AC; by Pythagoras theorem

$AC^2 = a^2 - \left(\frac{a}{2}\right)^2$

AC = 36 cm

3. (b)

Let inner radius = A; then $2pr = 440$ \ $p = 70$

Radius of outer circle = 70 + 14 = 84 cm

\ Outer diameter = 2 × Radius = 2 × 84 = 168

4. (a)

Let inner radius = r and outer radius = R

Width = $R - r = \frac{396}{2\pi} - \frac{352}{2\pi}$

fi$(R - r) = \frac{44}{2\pi}$ = 7 meters

5. (d)

Let outer radius = R; then inner radius = $r = R - 7$

$2pR$ = 220 fi 35m;

r = 35 – 7 = 28 m

Area of torch = $pR^2 - pr^2$ fi $p(R^2 - r^2)$ = 1386 m^2

Cost of traveling it = 1386 × $\frac{1}{2}$ = ` 693

6. (b)

Circumference of circle = $2pr$ = 44

= r = 7 cm

Area of a quadrant = $\frac{\pi r^2}{4}$ = 38.5 cm^2

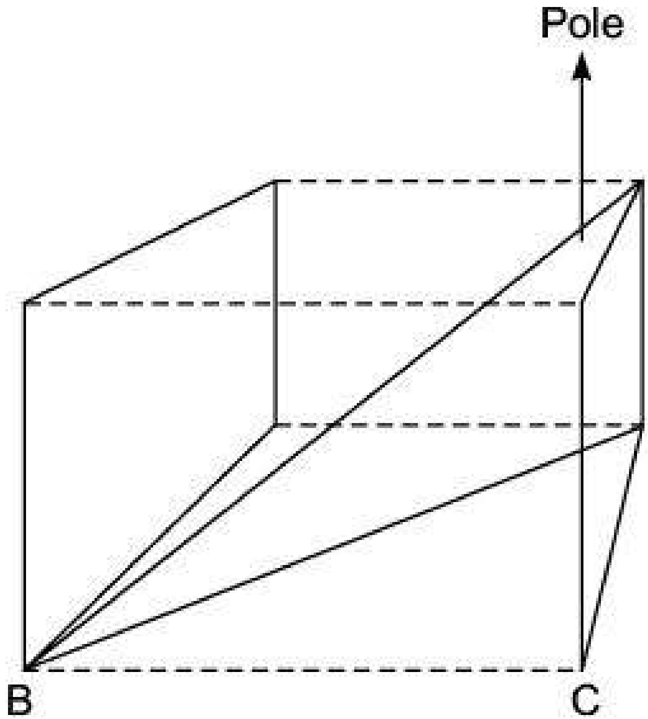
7. (d)

Volume of soil removed = $l \times b \times h$

= 7.5 × 6 × 1.5 = 67.5 m^3

8. (c)

The longest pole can be placed diagonally (3-dimensional)



$BC = \sqrt{18^2 + 24^2} = 30$

$AC = \sqrt{30^2 + 16^2} = 34$ m

9. (d)

Let the common ratio be = x

Then; length = $3x$, breadth = $2x$ and height = x

Then; as per question $3x$ ◊ $2x$ ◊ $x = 1296$ fi $6x^3 = 1296$

fi $x = 6$ m

Breadth = $2x = 12$ m

10. (d)

Data is inadequate as it's not mentioned that what part of the cube is melted to form cylinder.

11. (b)

Let the common ratio be = x

Then, length = $4x$, breadth = $3x$ and height = $2x$

As per question;

$2(4x$ ◊ $3x + 3x$ ◊ $2x + 2x$ ◊ $4x) = 8788$

$2(12x^2 + 6x^2 + 8x^2) = 8788$ fi $52x^2 = 8788$

fi $x = 13$

Length = $4x = 52$ cm

12. (b)

The total volume will remain the same, let the side of the resulting cube be = a. Then,

$6^3 + 8^3 + 10^3 = a^3$ fi $a = \sqrt[3]{1728} = 12$ cm

13. (a)

Slant length = $l = \sqrt{6^2 + 8^2} = 10$ cm

Then curved surface area = $prl = p \times 6 \times 10$ fi $60p$

And total surface area = $prl + pr^2$ fi $p((6 \times 10) + 6^2) = 96p$

14. (b)

Volume of a cone = $\frac{\pi r^2 h}{3}$

Then;$100p = \frac{\pi r^2 \cdot 12}{3}$ fi $r = 5$ cm

Curved surface area = prl

$l = \sqrt{h^2+r^2}$ fi $\sqrt{12^2+5^2} = 13$

then,$prl = p \times 13 \times 5 = 65p$ cm^2

15. (d)

Let the radius of the two cones be = x cm

Let slant height of 1st cone = 5 cm and

Slant height of 2nd cone = 7 cm

Then ratio of covered surface area = $\frac{\pi \times 5}{\pi \times 7} = 5 : 7$

16. (c)

Radius = $\frac{\pi rl}{\pi l} = \frac{2376}{3.14 \times 18} = 42$ cm

Diameter = 2 × Radius = 2 × 42 = 84 cm

17. (c)

Let the radius of cylinder = 1(r)

Then the radius of cone be = 2(R)

Then as per question = $\dfrac{\pi r^2 h}{\frac{\pi R^2 h}{3}}$ fi $\frac{3\pi r^2 h}{\pi R^2 h}$

fi $\frac{3r^2}{R^2}$ fi 3 : 4

18. (c)

The perimeter would remain the same in any case.

Let one side of a square be = a cm

Then $a^2 = 484$ fi $a = 22$ cm \ perimeter = $4a = 88$ cm

Let the radius of the circle be = r cm

Then $2pr = 88$ fi $r = 14$ cm

Then area = $pr^2 = 616$ cm^2

19. (d)

Let the radius of the circle be = p

Then $\quad 2pr - 2r = 16.8$ fi $r = 3.92$ cm

Then $\quad 2pr = 24.6$ cm

20. (d)

Let the radius of the wheel be = p

Then $5000 \times 2pr = 1100000$ cm fi $r = 35$ cm

21. (a)

Let the slant height be = l

Let radius = r

Then $v = \dfrac{\pi r^2 h}{3}$ fi $r = \sqrt{\dfrac{3v}{\pi h}}$ fi $\sqrt{\dfrac{3\times 100\pi}{\pi\times 12}}$ = 5 cm

$l = \sqrt{h^2+\pi^2} = \sqrt{12^2+5^2} = 13$ cm

22. (b)

In 4 days, the short hand covers its circumference

$4 \times 2 = 8$ times long hand covers its circumference

$4 \times 24 = 96$ times

Then they will cover a total distance of:-

$(2 \times p \times 4)8 + (2 \times p \times 6)96$ fi 3818.24 cm

23. (b)

Let the radius of the smaller sphere = r

Then, the radius of the bigger sphere = R

Let the surface area of the smaller sphere = 1

Then, the surface area of the bigger sphere = 4

Then, as per question

fi $\dfrac{4\pi r^2}{4\pi R^2} = \dfrac{1}{4}$ fi $\dfrac{r}{R} = \dfrac{1}{2}$ fi $R = 2r$

Ratio of their volumes

$= \dfrac{4\pi r^3}{3} \times \dfrac{3}{4\pi(2r)^3}$ fi 1: 8

24. (b)

Inner radius(p) = $\dfrac{9}{2}$ = 4.5 cm

Outer radius (R) = $\dfrac{10}{2}$ = 5 cm

Volume of metal contained in the shell = $\dfrac{4\pi R^3 - 4\pi r^3}{3}$

fi $\dfrac{4\pi}{3}(R^3 - r^3)$

fi 141.9 cm^3

25. (c)

Let smaller radius (r) = 1

Then bigger radius (R) = 2

Then, as per question

fi $\frac{4\pi r^2}{4\pi R^2} = \left(\frac{r}{R}\right)^2$ fi $\left(\frac{1}{2}\right)^2 = 1 : 4$

26. (c)

As per question fi $\frac{4\pi r^3}{3} = \frac{\pi r^2 h}{3}$ fi $h = 4r$

27. (a)

Volume of wall = 1200 × 500 × 25 = 15000000 cm^3

Volume of cement = 5% of 15000000 = 750000 cm^3

Remaining volume = 15000000 – 750000

= 14250000 cm^3

Volume of a brick = 25 × 12.5 × 7.5 = 2343.75 cm^3

Number of bricks used = $\frac{14250000}{2343.75}$ = 6080

28. (a)

Let the inner radius = r

Then 2pr = 352 m. Then r = 56

Then outer radius = r + 7 = 63 = R

Now,$pR^2 - pr^2$ = Area of road

fi$p(R^2 - r^2)$ = 2618 m^2

29. (c)

1 hectare = 10000 m^2

Height = 10 cm = $\frac{1}{10}$ m

Volume = 10000 × $\frac{1}{10}$ = 1000 m^3

30. (a)

Total surface area of 7 cubes fi 7 × 6a^2 = 1050

But on joining end to end, 12 sides will be covered.

So there area = 12 × a^2 fi 12 × 25 = 300

So the surface area of the resulting figure = 1050 – 300 = 750

31. (d)

Let the rise in height be = h

Then, as per the question, the volume of water should be equal in both the cases.

Now, $90 \times 40 \times h = 150 \times 8$

$h = \frac{150\times8}{90\times40} = \frac{1}{3}$ m $= \frac{100}{3}$ cm

= 33.33 cm

32. (d)

Slant height (l) = $\sqrt{7^2+24^2}$ = 25 m

Area of cloth required = covered surface area of cone = $prl = \frac{22}{7} \times 7 \times 25 = 550$ m^2

Amount of cloth required = $\frac{550}{5}$ = 110 m

33. (b)

If the ratio of their diameters = 2 : 1, then the ratio of their radii will also be = 2 : 1

Let the radii of the broader cone = 2 and height be = 1

Then the radii of the smaller cone = 1 and height be = 2

Ratio of volumes = $\frac{\pi 2^2 \cdot 1}{3} \prod \frac{\pi 1^2 \cdot 2}{3}$

$\frac{4\pi}{3} \times \frac{3}{2\pi}$ fi 2 : 1

34. (d)

Area of base = $6 \times 10 = 60$ m^2

Volume of tent = $30 \times 10 = 300$ m^3

Let the radius be = r, height = h, slant height = l

$pr^2 = 60$ fi $r = \sqrt{\frac{60}{\pi}}$

$300 = \frac{\pi r^2 h}{3}$ fi $900 = p \Diamond \frac{60}{\pi} \Diamond h$ fi $h = 15$ m

35. (b)

Volume of wood used = External volume – Outer Volume

fi $(10 \times 8 \times 6) - (10 - 1) \times (8 - 1) \times (6 - 1)$

fi $480 - (9 \times 7 \times 5) = 165$ cm^2

36. (b)

Total volume in both the cones will be equal. Let the number of smaller cubes = x

$x \Diamond 3^3 = 24 \times 9 \times 8$ fi $x = \frac{24\times72}{27} = 64$

37. (a)

Let one side of the cube = a

Then $a^3 = 216$ fi $a = 6$ m

Area of the resultant figure

= Area of all 3 cubes – Area of covered figure

fi $216 \times 3 - (4 \times a^2)$ fi $648 - 144$ fi $504\ m^2$

38. (c)

Volume of metal used = $\frac{4\pi R^3}{3} - \frac{4\pi r^3}{3}$

$= \frac{4\pi}{3}(12^3 - 10^3)$

$= 3047.89\ cm^3$

Weight = volume × densityfi 4.9×3047.89

fi 14942.28 gm

39. (d)

Volume of cube = $7^3 = 343\ cm^3$

Radius of cone = $\frac{7}{2}$ = 3.5 cm

Height of cone = 7

Ratio of volumes = $\frac{\frac{\pi r^2 h}{3}}{343} = \frac{22 \times 3.5 \times 3.5 \times 7}{7 \times 3 \times 343}$

fi 11:42

40. (b)

The volume in both the cases will be equal. Let the height of cone be = h

$4 \times \frac{22}{7} \times (14)^3 \times \frac{1}{3} = \frac{22}{7} \times \left(\frac{35}{2}\right)^2 \times \frac{h}{3}$

fi$4(14)^3 = h\left(\frac{35}{2}\right)^2 = h$

$= \frac{4 \times 14 \times 14 \times 14 \times 2 \times 2}{35 \times 35}$

$= h = 35.84$ cm

41. (b)

Diameter of circle = diagonal of square

$= \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$

◊.◊ Radius = $\frac{10\sqrt{2}}{2} = 5\sqrt{2}$

Area of circle = pr^2 fi $50p = 50 \times 3.14 = 157.14\ cm^3$

42. (c)

Area of triangle = rS; where r = inradius

$S = \dfrac{15+8+17}{2}$ = 20 cm

D = $\sqrt{S(S-a)(S-b)(S-c)}$

fi D $\sqrt{20(20-15)(20-8)(20-17)}$

D = $\sqrt{20\times5\times12\times3}$ = 60 cm^2

$r = \dfrac{\Delta}{S} = \dfrac{60}{20}$ = 3 cm

43. (c)

Circumference of the circular face of the cylinder = $2pr$

fi $2 \times \dfrac{22}{7} \times \dfrac{35}{100}$ = 2.2 m

Number of revolutions required to lift the bucket by 11 m = $\dfrac{11}{2.2}$ = 5

44. (c) Surface area of the cube = $6a^2 = 6 \times (20)^2$

= 2400

Area of 6 circles of radius 10 cm = $6pr^2$

= $6 \times p \times 100$

= 1885.71

Remaining area = 2400 – 1884 = 514.28

45. (d)

$x \lozenge y \lozenge z = lb \times bh \times lh = (lbh)^2$

(V) Volume of a cuboid = lbh

So $V^2 = (lbh)^2 = xyz$

46. (c)

Diameter of the circle = diagonal of rectangle

= $\sqrt{8^2+6^2}$ = 10 cm

Radius = $\dfrac{10}{2}$ = 5 cm

Area of shaded portion = $pr^2 - lb$

= $3.14 \times 5^2 - 8 \times 6$

= 30.57 cm^2

47. (a)

Larger Radius (R) = 14 + 7 = 21 cm

Smaller Radius (r) = 7 cm

Area of shaded portion $pR^2 \frac{\theta}{360} - \frac{\pi r^2 \theta}{360}$

fi $\frac{\pi\theta}{360}(21^2 - 7^2)$ fi 102.67 cm

48. (b)

Area of quadrilateral = Area of right angled triangle + Area of equilateral triangle x = $\sqrt{20^2 - 12^2}$ = 16

Area of quadrilateral = $\left(\frac{1}{2} \times 16 \times 12\right)$ + $\frac{\sqrt{3}}{4}$ × 20 × 20

= 269 units^2

49. (a)

Height = $\sqrt{24^2 - 13^2}$ = $\sqrt{407}$

Volume = $\frac{\text{Area of base} \times \text{height}}{3}$ fi $\frac{18 \times 26 \times \sqrt{407}}{3}$

fi $156\sqrt{407}$

50. (b)

The perimeter would remain the same in both cases. Circumference of circle = $2pr$ = 2 × $\frac{22}{7}$ × 28 = 176 cm

Perimeter of square = 176

Greatest side possible = $\frac{176}{4}$ = 44 cm

Length of diagonal = $\sqrt{44^2 + 44^2}$ = 62.216

= $\frac{88}{2}$ ◊ $\sqrt{2}$ = 44 $\sqrt{2}$

Level of Difficulty (II)

1. (d)

Let the angle subtended by the sector at the centre be = q

Then,

5.7 + 5.7 + (2p) × 5.7 × $\frac{\theta}{360}$ = 27.2

11.4 + $\frac{11.4 \times 3.14 \times \theta}{360}$ = 27.2

fi $\frac{\theta}{360}$ = 0.44

Area of the sector = $pr^2\frac{\theta}{360}$ fi $(22/7) \times (5.7)^2 \times 0.44$

= 44.92 approx.

2. (a)

Volume of mud dug out = $10 \times 4.5 \times 3 = 135$ m^3

Let the remaining ground rise by = h m

Then$\{(20 \times 9) - (10 \times 4.5)\}h = 135$

$135h = 135$ fi $h = 1$ m

3. (b)

Height of the cylinder = 13 – 7 = 6 cm

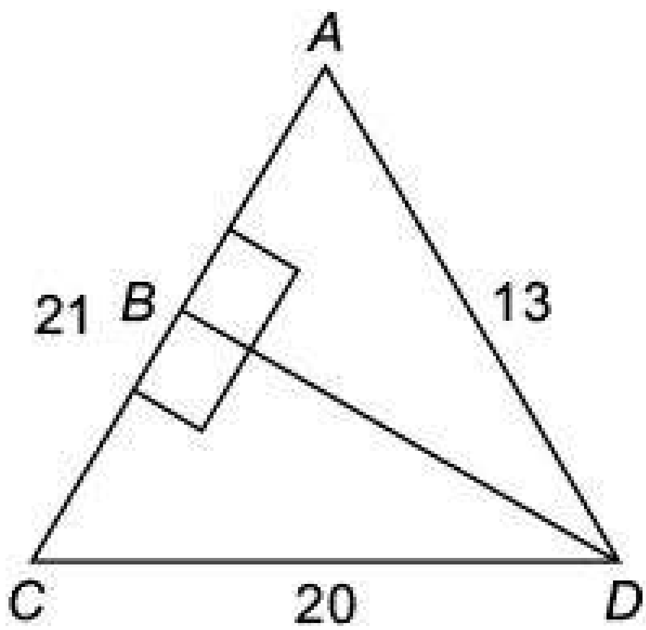
Radius of the cylinder and the hemisphere = 7 cm

Volume of the vessel = volume of cylinder + volume of hemisphere

fi $pr^2h + \frac{4\pi r^3}{3\times 2}$ fi $3.14 \times (7)^2 \times 6 + \frac{4\times 3.14\times (7)^3}{3\times 2}$

fi 1642.6 cm^2

4. (b)



Let the original triangle be = ACD

Longest side = AC = 21cm

In the right angled DABD, by Pythagorean triplets, we get $AB = 5$ and $BD = 12$

Then, $BC = 21 - 5 = 16$

By Pythagoras theorem,

$BD^2 = CD^2 - BC^2$ fi $BD = 12$ cm

Thus, on assumption is correct.

Area of the larger DBDC = $\frac{1}{2} \times 16 \times 12$ fi 96 cm^2

5. (c)

Radius = $\frac{105}{2}$ = 52.5 cm

Area of the entire canvas, used for the tent

= Area of cylinder + centre of cone

= 2*prh* + *prl*

= *pr*(2*h* + *l*) = 3.14 × 52.5 $(2\sqrt{53^2 - 52.5^2} + 53)$

= 5 × *l* (because area of canvas = *l* × *b* also)

= *l* fi 1947 m

6. (c)

The volume in both the cases would be the same.

Therefore = $\frac{4\pi r^3}{3}$ = pr^2h

$\frac{4\times3.14\times(4\times10)^3}{3}$ = 3.14 × 2^2 × *h*

fi*h* = $\frac{64000}{3}$ = 21333.33 mm

7. (b)

As the cylinder and cone have equal diameters. So they have equal area. Let cone's height be h_2 and as per question, cylinder's height be h_1.

$\frac{2\pi r h_1}{\pi r\sqrt{h_2^2 + r^2}} = \frac{8}{5}$.

On solving we get the desired ratio as 3 : 4

8. (a)

Let the slant height of 1st cone = *L*

Then the slant height of 2nd cone = 3*L*

Let the radius of 1st cone = r_1

And let the radius of 2nd cone = r_2

Then,$pr_1L = 3 \times pr_2 \times 3L$

fi$pr_1L = 9pr_2L$ fi $r_1 = 9r_2$

Ratio of area of the base

$\frac{\pi r_1^2}{\pi r_2^2}$ fi $\left(\frac{r_1}{r_2}\right)^2 = \left(\frac{9}{1}\right)^2$ fi 81 : 1

9. (c)

Let the internal radius of the cylinder = *r*

Then, the volume of sphere = Volume of sphere cylinder fi $\dfrac{4\pi \cdot 6^3}{3}$ = $ph(5^2 - r^2)$

fi $\dfrac{864\pi}{3}$ = $32p(25 - r^2)$

fi$r^2 = 16 = r = 4$ cm

So thickness of the cylinder = 5 – 4 = 1 cm

10. (d)

The volume in both the cases would be the same.

Let the height of the cone = h

Then, external radius = 6 cm

Internal radius = 4 cm

fi $\dfrac{4\pi(6^3 - 4^3)}{3} = \dfrac{\pi \cdot 4^2 \cdot h}{3}$

fi$h = \dfrac{6^3 - 4^3}{4}$ fi $h = \dfrac{216-64}{4}$ = 38 cm

11. (a)

Let arc side of the cube be = a units

Total surface area of 3 cubes = $3 \times 6a^2$

$= 18a^2$

Total surface area of cuboid = $18a^2 - 4a^2 = 14a^2$

Ratio = $\dfrac{14a^2}{18a^2}$ = 7 : 9

12. (c)

$A = 2(xy + yz + zx)$

$V = xyz$

$A/V = \dfrac{2(xy+yz+zx)}{xyz} = \dfrac{2}{z} + \dfrac{2}{x} + \dfrac{2}{y}$

fi $2\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)$

13. (a)

The entire dial of the clock = 360°

Every 5 minutes = $\dfrac{360}{12}$ = 30°

So 35 minutes = 30 × 17 = 210

Area = $pr^2 \times \dfrac{8}{360} = 3.14 \times 100 \times \dfrac{210}{360}$

fi 183.3 cm^2

14. (a)

Let the radius of the bigger circle = R

Let the radius of the smaller circle = r

Then as per question; $R - r = 6$

Solving through options; only option (a) satisfies this condition.

15. (d)

Radius of cylinder, hemisphere and cone = 5 cm

Height of cylinder = 13 cm

Height of cone = 12 cm

Surface area of toy = $2prh + \frac{4\pi r^2}{2} + prL$

$L = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13$

Then fi $(2 \times 3.14 \times 5 \times 13) + (2 \times 3.14 \times 25) + (3.14 \times 5 \times 13)$ fi 770 cm^2

16. (b)

Height of cone = 10.2 – 4.2 = 6 cm

Volume of wood = $\frac{\pi r^2 h}{3} + \frac{4\pi r^5}{3 \times 2}$

fi $\frac{3.14 \times (4.2)^2 \times 6}{3} + \frac{4 \times 3.14 \times (4.2)^3}{3 \times 2}$

fi 266 cm^3

17. (a)

Volume of ice cream = pr^2h

$= 3.14 \times (6)^2 \times 15$

= 1695.6 cm^2

Volume of 1 cone = $\frac{\pi r^2 \times 24}{3}$

Then fi $10 \times 3.14 \times r^2 \times 8 = 1695.6$

fi$r$ = 3 cm (approx.)

Sodiameter = $2 \times r$ = 6 cm

18. (b)

Radius of cylinder and hemispheres = $\frac{7}{2}$ = 3.5 cm

Height of cylinder = 19 – (3.5 × 2) = 12 cm

Total surface area of solid = $2prh + 4pr^2$

fi $2 \times 3.14 \times 3.5 \times 12 + 4 \times 3.14 \times (3.5)^2$

fi 418 cm^2

19. (c)

As they stand on the same base so their radius is also same.

Then; volume of cone = $\frac{\pi r^2 h}{3}$

Volume of hemisphere = $\frac{2\pi r^2}{3}$

Volume of cylinder = pr^2h

Ratio = $\frac{\pi r^2 h}{3}$: $\frac{2\pi r^3}{3}$: pr^2h

fi $\frac{h}{3}$: $\frac{2r}{3}$: h

fi$h : 2r : 3h$

Radius of a hemisphere = Its height

So $h : 2h : 3h$ fi $1 : 2 : 3$

20. (d)

Total surface to be painted

= external surface area + internal surface + surface area of ring area

fi $2pR^2 + 2pr^2 = 2p(R^2 + r^2) + 2p(R^2 - r^2)$

Cost of painting

fi $2 \times 3.14 \times \left\{\left(\frac{25}{2}\right)^2 + \left(\frac{24}{2}\right)^2 + \left(\frac{25}{2}\right)^2 - \left(\frac{24}{2}\right)^2\right\}$

$\times\ 0.05$

fi $2 \times 6.28 \times (12.5)^2 \times 0.05$ = ` 96.28

21. (e)

Radius = $\frac{3.5}{2}$ = 1.75 cm

Volume of solid = $pr^2h + \frac{\pi r^2 h}{3} + \frac{2\pi r^3}{3}$

fi $pr^2\left(h + \frac{h}{3} + \frac{r}{3}\right)$

fi $3.14 \times (1.75)^2 \times \left(10 + \frac{6}{3} + \frac{1.75}{3}\right)$

fi 121 cm^3

22. (d)

Area of shaded portion = Area of quadrant – Area of triangle

fi $\frac{\pi r^2}{4} - \frac{1}{2} \times 3.5 \times 2 = \frac{3.14 \times (3.5)^2}{4} - 3.5$

fi 6.1 cm^2

23. (c)

ABC is an equilateral triangle with sides = 2 cm

Area of shaded portion = Area of equilateral triangle –Area of 3 quadrant

fi i.e. $\frac{\sqrt{3}}{4}a^2 - 3\left(\pi r^2 \frac{\theta}{360}\right)$; q = 60° (\ *A*, *B*, *C* is an equilateral triangle)

fi $\frac{\sqrt{3}}{4} \times 2^2 - 3\left(3.14 \times 1 \times \frac{60}{360}\right)$

fi $\sqrt{3} - \frac{3.14}{2} = \sqrt{3} - \frac{\pi}{2}$

24. (d)

Volume of the elliptical cylinder

$= p \times \frac{2.4}{2} \times \frac{1.6}{2} \times 7$

$= 3.14 \times 1.2 \times 0.8 \times 7$ fi 9 m^3

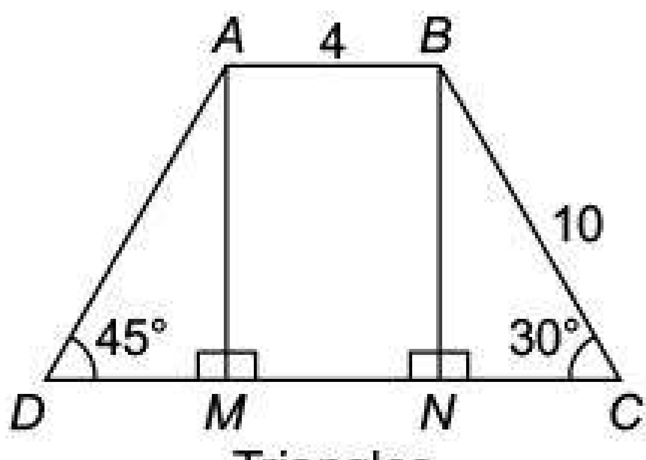
Amount of water emptied per minute

$= 120 \times 3.14 \times \left(\frac{2}{100}\right)^2$

Time required to empty half the tank

$= \frac{4.5}{120 \times 3.14 \times (0.02)^2} = 70$ min

25. (e)



Triangles

AB and *DC* are the parallel sides

Height = *AM* = *BN*

AB = *MN* = 4

D*BNC* and D*AMD* are right angled triangles

In DBNC fi sin 30 = $\frac{BN}{10}$ fi BN = 5

Using Pythagoras theorem $NC = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

In DADM; AM = 5; tan 45 = $\frac{AM}{DM}$ = 1 = $\frac{5}{DM}$

fiDM = 5

Area of trapezium fi $\frac{1}{2}$ (Sum of 11 sides) × height

fi $\frac{1}{2}(4 + 4 + 5\sqrt{3} + 5) \times 5 = \frac{5(13+5\sqrt{3})}{2}$(Answer)

26. (d)

$PQ = QR = RS = \frac{12}{3}$ = 4 cm

Area of unshaded region fi $\frac{\pi 6^2}{2} + \frac{\pi 4^2}{2}$

fi 18p + 8p fi 26p

Area of shaded region fi $\frac{\pi 6^2}{2} - \frac{\pi 4^2}{2}$

fi 18p – 8p = 10p

Ratio = $\frac{10\pi}{26\pi}$ fi $\frac{5}{13}$ fi 5 : 13

27. (c)

$QP = \sqrt{5^2 + 12^2}$ = 13

Area of the triangle = $\frac{1}{2} \times b \times h = 30$

fi As Rx is a ^ drawn to the hypotenuse

So $Rx = \frac{2 \times \text{Area}}{\text{Hypotenuse}} = \frac{60}{13}$

28. (d)

Let initial area = p

Then volume = pr^2h

New radius = $p + \frac{\pi}{2}$ fi $\frac{3\pi}{2}$

New volume = $p - \frac{9}{4}r^2h$

Increased volume = $\frac{5}{4}pr^2h$

Percent increase = $\frac{5\pi r^2 h}{4\pi r^2 h}$ × 100 = 125%

29. Distance after 4 hours = $AB = C$

$a = 3 \times 4 = 12$; $b = 2 \times 4 = 8$

and $\frac{a+b+c}{2}$ fi $\frac{12+8+C}{2}$ fi $\left(\frac{10+C}{2}\right)$

Area = $\sqrt{S(S-a)(S-b)(S-c)}$

Area = $\frac{1}{2}ab \sin 120°$

Area fi $48 \times \frac{\sqrt{3}}{2} = 24\sqrt{3}$

As per question:

$$24\sqrt{3} = \sqrt{\left(10+\frac{c}{2}\right)\left(\frac{c}{2}-2\right)\left(2+\frac{c}{2}\right)\left(10-\frac{c}{2}\right)}$$

On solving, we get $c = 4\sqrt{19}$ km

30. (c)

Volume of the cone = $\frac{\pi r^2 h}{3}$ fi $\frac{3.14 \times 20 \times 20 \times 24}{3}$

fi 10048 cm^3

Diameter of pipe = 5 m

Volume of water flowing out of the pipe per minute

fi $10 \times (2.5)^2 \times 3.14$ fi 196.25 cm^3

Time taken to fill the tank = $\frac{10048}{196.25}$ = 51.2 mins

31. (d)

One side of the equilateral triangle = diameter of cone.

Therefore radius of cone = $\frac{12}{2} = 6$

Height of cone = Height of equilateral triangle be

◊.◊ Height of cone = $\frac{\sqrt{3}a}{2} = 6\sqrt{3}$

Volume of cone = $\frac{\pi r^2 h}{3}$

fi $\frac{\pi \times 6^2 \times 6\sqrt{3}}{3} = 72\sqrt{3}\pi$ cm^3

32. (b)

Let the radius of iron ball = r_1

Let the radius of oak ball = r_0

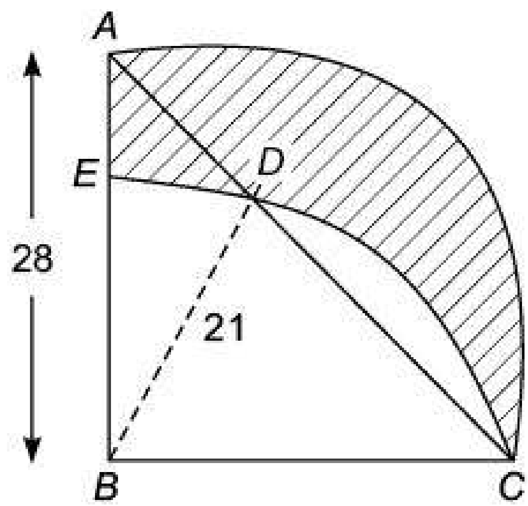
Then, as iron weights 8 times more than Oak

\ $\frac{4\pi r_0^3}{3} = \frac{8 \times 4\pi r_1^3}{3} = \frac{r_0}{r_1} = 2$ fi $r_0 = 2r_1$

So diameter of iron = $\frac{1}{2}$ diameter of oak

fi $\frac{1}{2} \times 18 = 9$ cm

33. (a)



Area of shaded portion = Area of *ADC* – Area of sector *DC* + Area of D*ADB* – sector *BED*

fi Area of *ADC* = *p* × (17.5)^2 × $\frac{1}{2}$ = 481 cm^2

$\frac{\angle DBC}{\angle ABC} = \frac{21}{28}$ fi –*DBC* = 67.5 and –*DBA* = 22.5

fi Area of sector *DC* = $\left(\pi \times 21^2 \times \frac{67.5}{360}\right)$

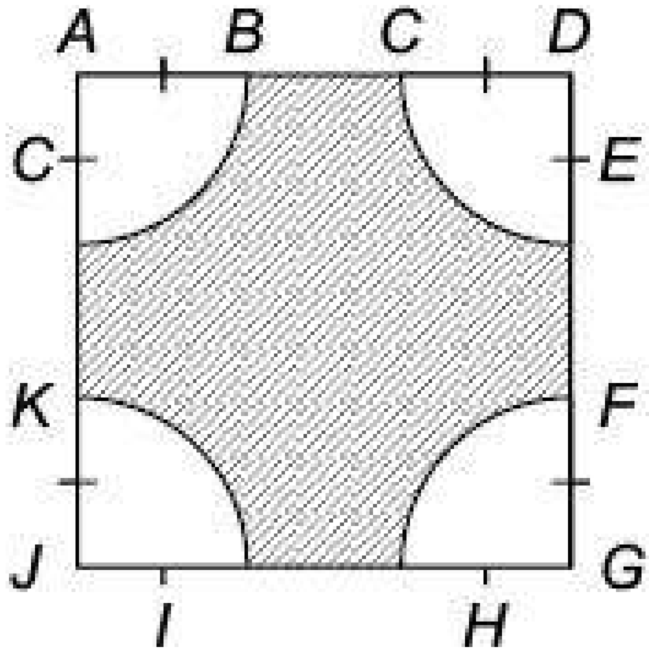
$-\left(\frac{1}{2} \times 21^2 \times Sin 67.5\right)$ = 56 cm^2

fi Area of *ADE* = $\left(\frac{1}{2} \times 28 \times 21\right)$

$-\left(204 + \frac{1}{2} \times 21^2 \times \sin 22.5\right)$ = 5.6 cm^2

Thus area of shaded portion = 480 – 56 + 5.6 = 429 cm^2

34. (a)



KJ = radius of semicircles = 10 cm

4 Quadrants of equal radius = 1 circle of that radius

Area of shaded portion fi Area of rectangle – Area of circle

fi $(28 \times 26) - (3.14 \times 10^2)$ fi 414 cm^2

$BC = 28 - (10 + 10) = 8$ and $EF = 26 - (10 + 10) = 6$

Perimeter of shaded portion = 28 cm + $2pr$

Answer fi 414 cm^2 = Area and

Perimeter = 90.8

35. (d)

Go through the option

Only option (d) is correct.

36. (c)

Area of remaining cardboard = Area of trapezium

– Area of quadrant

fi Area of trapezium = $\frac{1}{2}$(sum of parallel sides)

× height

$= \frac{1}{2} \times (AB + DC) \times BC$

fi $\frac{1}{2} \times (3.5 + 5.5) \times 3.5$

$= 4.5 \times 3.5 = 15.75$ cm^2

Area of quadrant = $\frac{\pi r^2}{4}$ fi $\frac{3.14 \times 3.5 \times 3.5}{4}$ = 9.6

fi Area of remaining cardboard = 15.7 – 9.6 = 6.1 cm^2

37. (c)

Circumference of the 2 semicircles = 21 – (90 + 90)

= 132

2 semicircles = 1 circle with equal radius

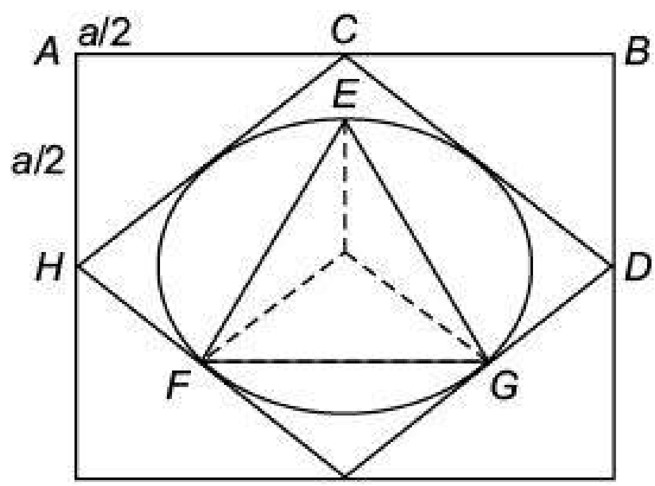
So 2*pr* = 132 fi 2*r* = $\frac{132}{3.14}$ fi 42 m diameter

Area of track = Area within external border – Area within internal border

fi $p(23^2 - 21^2) + 90 \times 46 - 90 \times 4^2$

fi 88p + 360 fi 636.3 m^2

38. (d)



AB = side of the outermost triangle = *a*

AC = *CB* = *a*/2

$HC = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}$

Diameter of circle = $\frac{a}{\sqrt{2}}$; radius = $\frac{a}{2\sqrt{2}}$

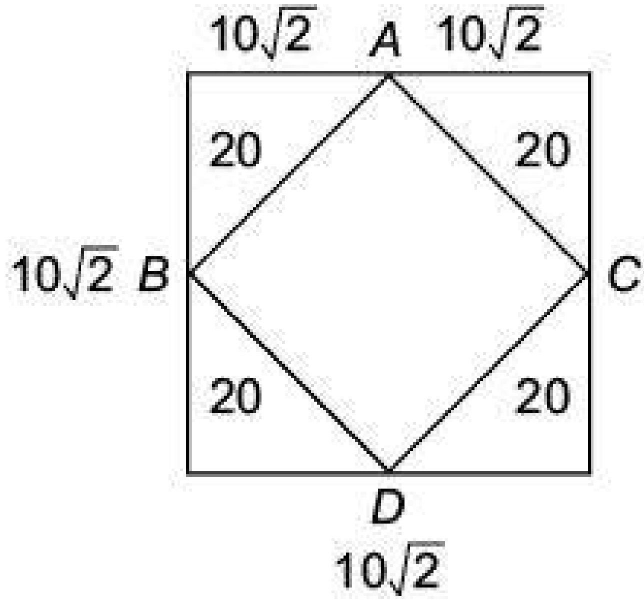
O is the centre of the circle. Then –*EOF* = 120°

Then Area of D*EOF* = $\frac{1}{2}$ *EO* ◊ *OF* ◊ sin 120°

fi $\frac{1}{2} \times \frac{a^2}{8} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}a^2}{32}$

Then area of D*EFG* = $\frac{3\sqrt{3}a^2}{32}$

39. (a)

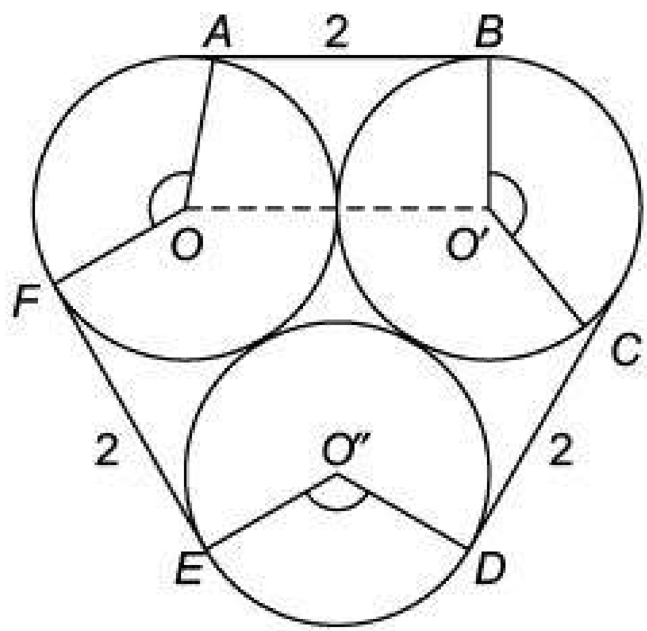


The length of rope of goat = $10\sqrt{2}$ m

Then the two goats will graze an area = Area of a semicircle with area $10\sqrt{2}$ m.

So total area grazed = $\frac{\pi r^2}{2}$ fi 100p m^2

40. (a)



In this figure, the sides $AB = CD = FE$ = distance between 2 radii = 2 cm

–*AOP* = –*BO*¢*C* = –*EO* ≤ *D* = 120°

So perimeter of the bigger triangle = (2 + 2 + 2 + 2 × 3.14 × 1) (because 3 sectors of circles of 120° = 1 full circle of same radius)

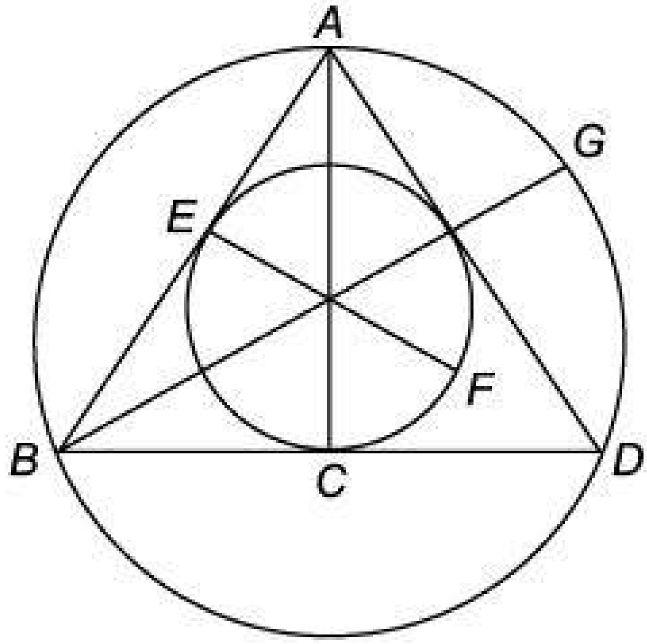
Let the radius of longer circle be = R

Then 2 × p × R = 12.28 fi $R = \frac{12.28}{2 \times 3.14} = 2$

Area = AR^2 fi 3.14 × 4 fi 13 cm^2 (approximately)

Option (a) is closest to the answer.

41. (b)



Let arc side of equilateral triangle = a

Then height = $\frac{a\sqrt{3}}{2}$

Area = $\frac{\sqrt{3}}{2}a^2$; $S = \frac{a+a+a}{2} = \frac{3a}{2}$

Diameter of inner circle = $\frac{2 \times \text{Area}}{5}$

$= \frac{\sqrt{3}}{2}a^2 \times \frac{2}{3a} = \frac{a}{\sqrt{3}}$

Diameter of outer circle = $\frac{a^3}{2 \times \text{Area}} = a^3 \times \frac{a}{\sqrt{3}a^2}$

fi $\frac{2a}{\sqrt{3}}$

Ratio = $\frac{a}{\sqrt{3}} : \frac{2a}{\sqrt{3}} : \frac{a\sqrt{3}}{2}$ fi Ratio = 1 : 2 : 3

42. (b)

Sum of interior angles of a hexagon = 120°

6 sectors with same radius o = 2 full circles of same radius

So area of shaded region fi $2pr^2$

43. (d)

44. (a)

$AO = CO = DO = OB$ = radius of bigger circle = r(let)

Then area of $(G + F) = \frac{\pi r^2}{2}$

Area of $2(G + F) = pr^2$. Also area of $2G + F + E = pr^2$

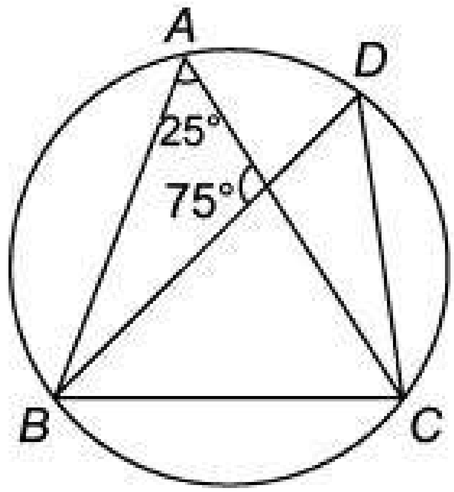
i.e. $2G + F + F = 2G + F + E$ fi $F = E$

So the ratio of areas E and $F = 1 : 1$

45. (c)

Extra Practice Exercise on Geometry and Mensuration

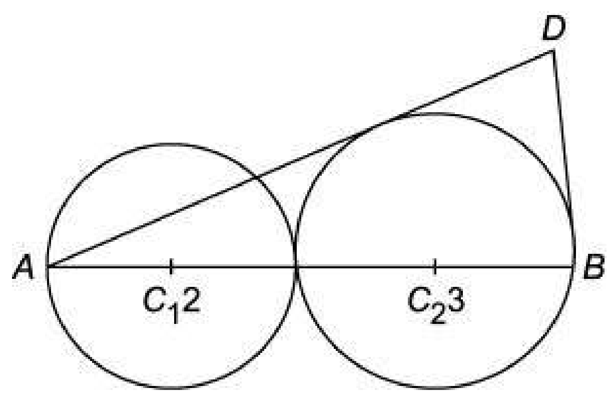
1. In the figure given what is the measure of –*ACD*



(a) 75° (b) 80°

(c) 90° (d) 105°

2. Two circles C_1 and C_2 of radius 2 and 3 respectively touch each other as shown in the figure. If *AD* and *BD* are tangents then the length of *BD* is



(a) $3\sqrt{6}$ (b) $5\sqrt{6}$

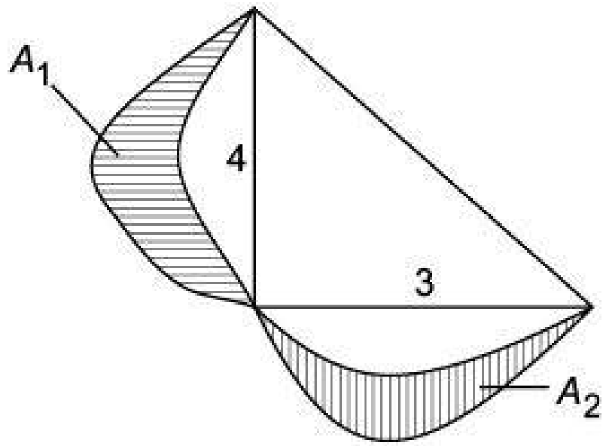
(c) $(7\sqrt{6})/3$ (d) 6

3. If the sides of a triangle measure 13, 14, 15 cm respectively, what is the height of the triangle for the base side 14.

(a) 10 (b) 12

(c) 14 (d) 13

4. A right angled triangle is drawn on a plane such that sides adjacent to right angle are 3 cm and 4 cm. Now three semi-circles are drawn taking all three sides of the triangle as diameters respectively (as shown in the figure). What is the area of the shaded regions $A_1 + A_2$



(a) 3*p* (b) 4*p*

(c) 5*p* (d) None of these

5. A lateral side of an isosceles triangle is 15 cm and the altitude is 8 cm. What is the radius of the circumscribed circle

(a) 9.625 (b) 9.375

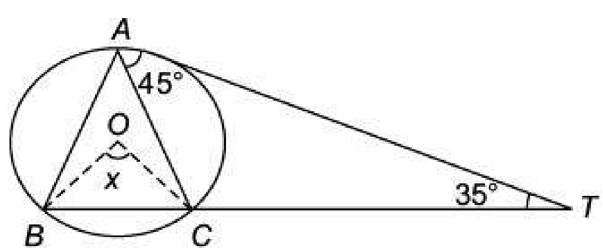
(c) 9.5 (d) 9.125

6. Let *a*, *b*, *c* be the length of the sides of triangle *ABC*. Given $(a + b + c)(b + c - a) = abc$. Then the value of *a* will lie in between

(a) –1 and 1 (b) 0 and 4

(c) 0 and 1 (d) 0 and

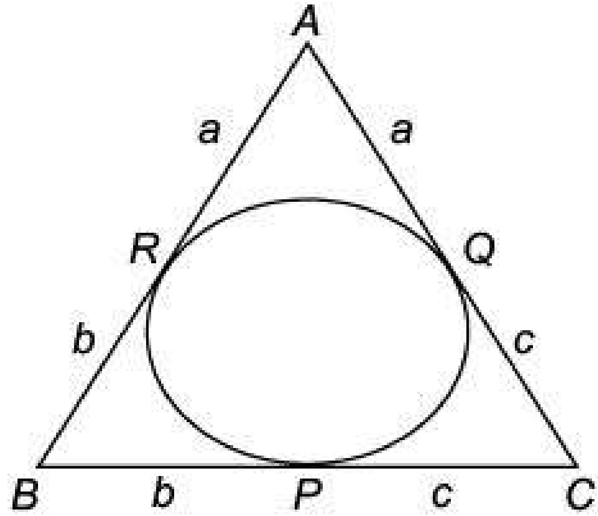
7. In the figure given below (not drawn to scale). *A*, *B* and *C* are three points on a circle with centre *O*. The chord *BC* is extended to point *T* such that *AT* becomes a tangent to the circle at point *A*. If –*CTA* = 35° and –*CAT* = 45° calculate *x*° (–*BOC*)



(a) 100° (b) 90°

(c) 110° (d) 65°

8. In the given figure



$AB = 20$

$BC = 15$

$CA = 19$

Calculate *a*, *b*, *c*

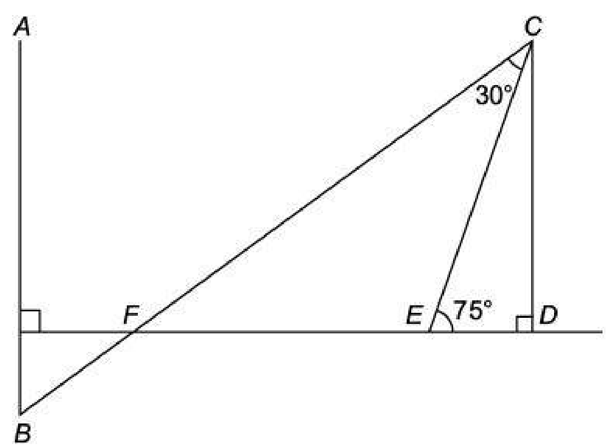
(a) $a = 12$, $b = 8$, $c = 7$

(b) $a = 8$, $b = 12$, $c = 7$

(c) $a = 9$, $b = 10$, $c = 15$

(d) $a = 10$, $b = 15$, $c = 9$

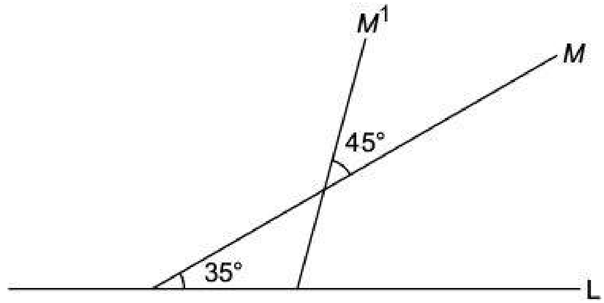
9. In the figure given below, *AB* is perpendicular to *ED*. –*CED* = 75° and –*ECF* = 30°. What is the measure of –*ABC*?



(a) 60° (b) 45°

(c) 55° (d) 30°

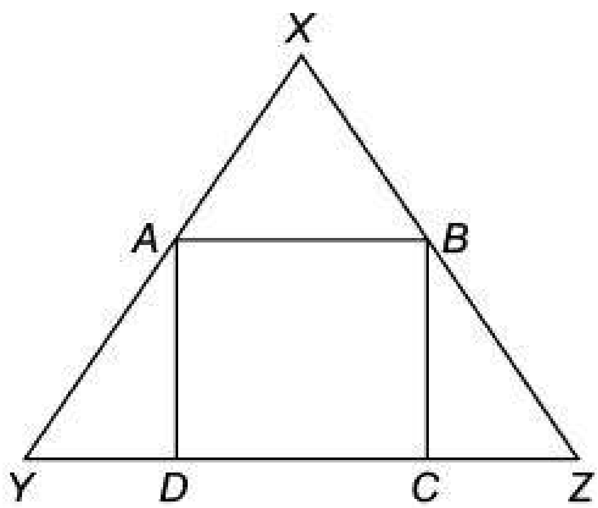
10. The angle between lines *L* and *M* measures 35° degrees. If line *M* is rotated 45° degrees counter clockwise about point *P* to line M^1 what is the angle in degrees between lines *L* and M^1



(a) 90° (b) 80°

(c) 75° (d) 60°

11. In the figure given below, *XYZ* is a right angled triangle in which –*Y* = 45° and –*X* = 90°. *ABCD* is a square inscribed in it whose area is 64 cm^2. What is the area of triangle *XYZ*?



(a) 100 (b) 64

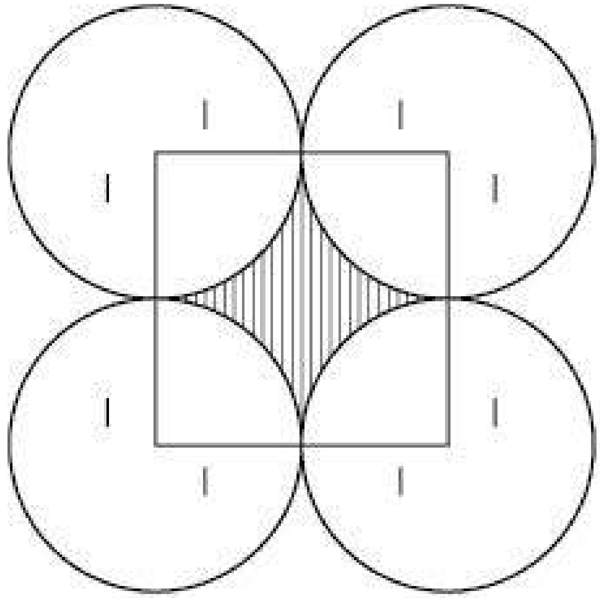
(c) 144 (d) 81

12. The area of circle circumscribed about a regular hexagon is 144*p*. What is the area of hexagon?

(a) $300\sqrt{3}$ (b) $216\sqrt{3}$

(c) 256 (d) 225

13. Find the area of the shaded portion



(a) $4 - p$ (b) $6 - p$

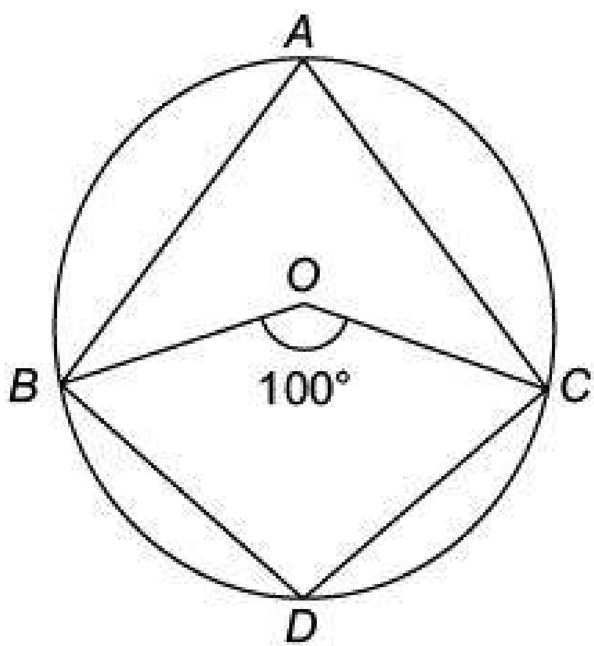
(c) $5 - p$ (d) p

14. The numerical value of the product of the three sides (which are integers when measured in cm) of a right angled triangle having a perimeter of 56 cm is 4200. Find the length of the hypotenuse.

(a) 24 (b) 25

(c) 15 (d) 30

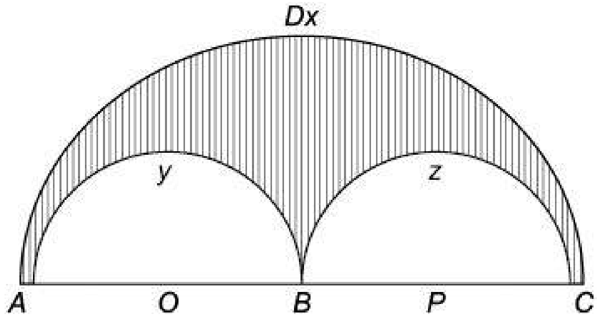
15. In the figure *ABDC* is a cyclic quadrilateral with *O* as centre of the circle. Find –*BDC*.



(a) 105° (b) 120°

(c) 130° (d) 95°

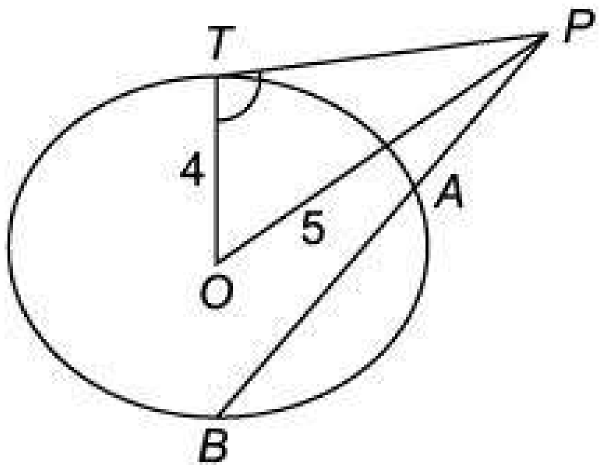
16. *B*, *O*, *P* are centres of semicircles *AXC*, *AYB* & *BZC* respectively. *AC* = 12 cm. Find the area of the shaded region.



(a) 9p (b) 18p

(c) 20p (d) 25p

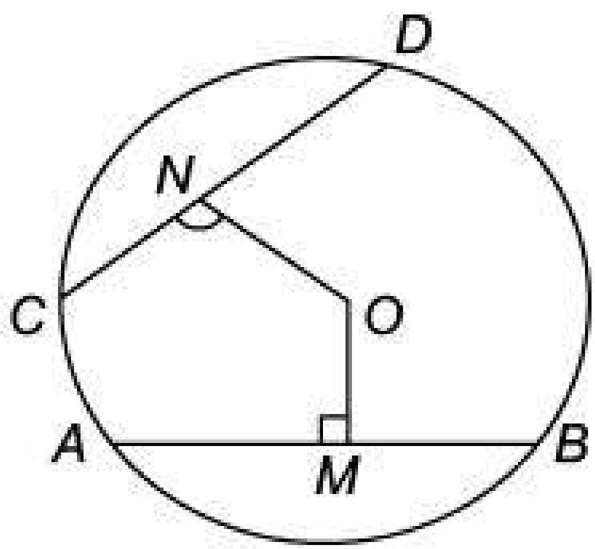
17. O is the centre of the circle. $OP = 5$ and $OT = 4$, and $AB = 8$. The line PT is a tangent to the circle. Find PB



(a) 9 cm (b) 10 cm

(c) 7 cm (d) 8 cm

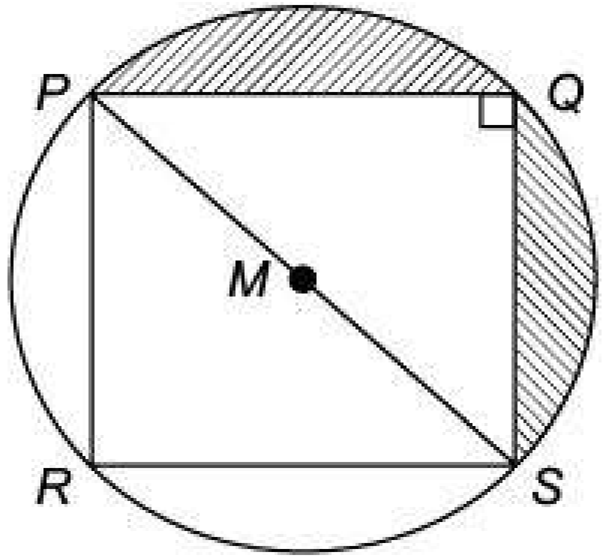
18. In the figure given below, $AB = 16$, $CD = 12$ and $OM = 6$. Calculate ON.



(a) 8 (b) 10

(c) 12 (d) 14

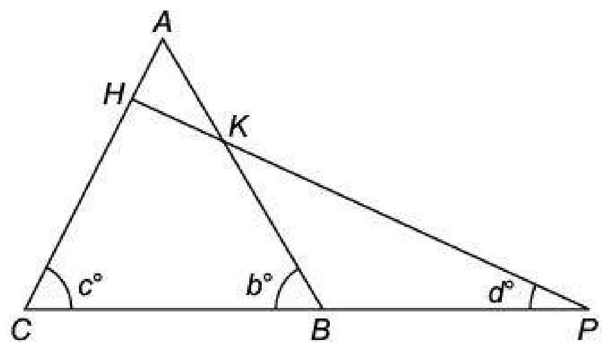
19. In the figure, M is the centre of the circle. 1(QS) = 10÷2, 1(PR) = 1(RS) and PR is parallel to QS. Find the area of the shaded region.



(a) $90p - 90$ (b) $50p - 100$

(c) $150p - 150$ (d) $125p - 125$

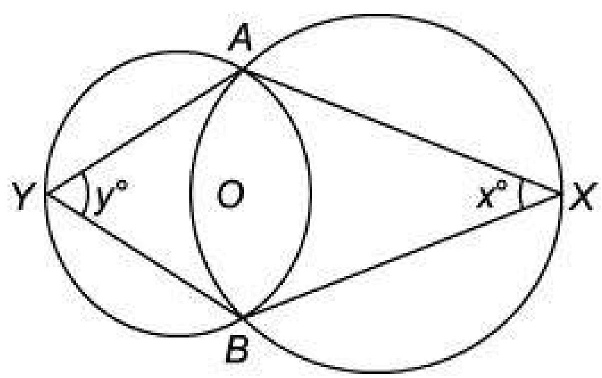
20. In the given figure *PBC* and *PKH* are straight lines. If $AH = AK$, $b = 70°$, $c = 40°$, the value of d is



(a) 20° (b) 25°

(c) 15° (d) 35°

21. In the given figure, circle *AXB* passes through '*O*' the centre of circle *AYB*. *AX* and *BX* and *AY* and *BY* are tangents to the circles *AYB* and *AXB* respectively. The value of $y°$ is

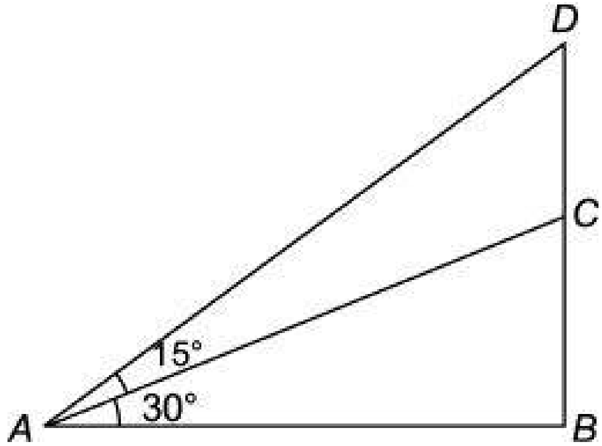


(a) $180° - x°$, (b) $180° - 2x°$

(c) ½ $(90° - x°)$ (d) $90° - (x°/2)$

22. In the figure, $AB = x$

Calculate the area of triangle *ADC* ($–B = 90°$)



(a) ½ x^2 sin 30° (b) ½ x^2 cos 30°

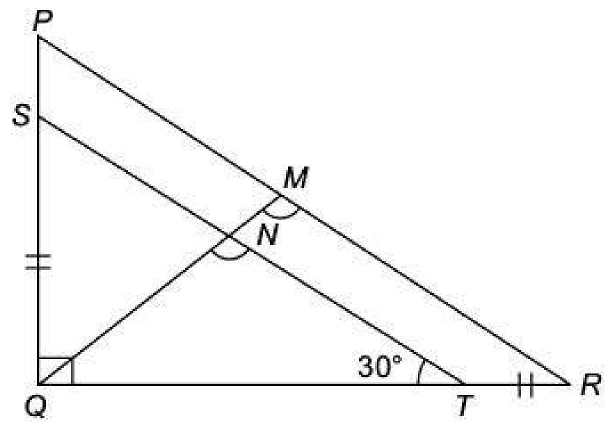
(c) ½ x^2 tan 30° (d) ½ x^2(tan 45° – tan 30°)

23. In the given figure *SQ* = *TR* = *a*, *QT* = *b*, *QM* ^ *PR*, *ST* is parallel to *PR*.

m – *STQ* = 30°

m – *SQT* = 90°

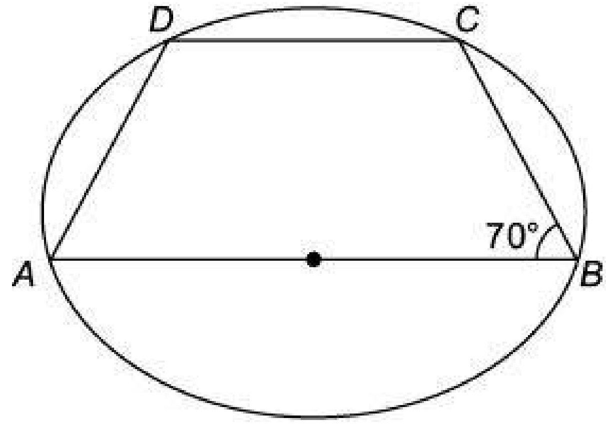
Find *QM*.



(a) (a + b)/2 (b) 2(a + b)

(c) (2a + b)/2 (d) (a + 2b)/2

24. In the figure given, *AB* is a diameter of the circle and *C* and *D* are on the circumference such that –*CAD* = 40°. Find the measure of the –*ACD*



(a) 40°

(b) 50°

(c) 60°

(d) None of these

25. Six solid hemispherical balls have to be arranged one upon the other vertically. Find the minimum total surface area of the cylinder in which the hemispherical balls can be arranged, if the radii of each hemispherical ball is 7 cm.

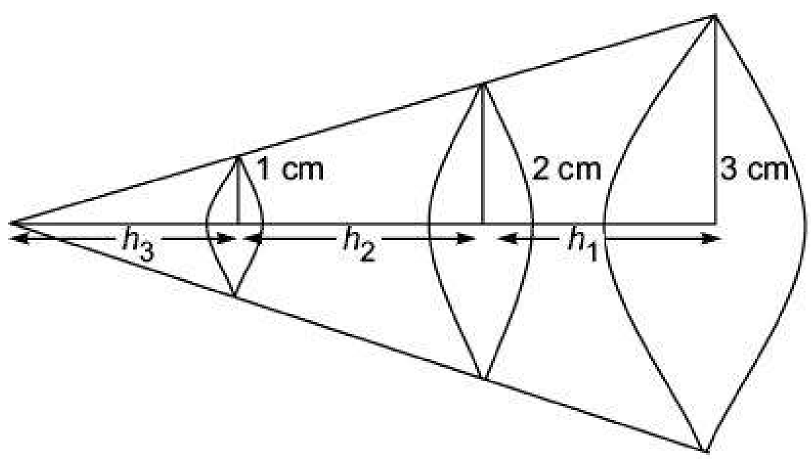
(a) 2056

(b) 2156

(c) 1232

(d) None of these

Questions 26 and 27: In the following figure, there is a cone which is being cut and extracted in three segments having heights h_1, h_2 and h_3 and the radius of their bases 1 cm, 2 cm and 3 cm respectively, then



26. The ratio of the volumes of the smallest segment to that of the largest segment is

(a) 1 : 27

(b) 27 : 1

(c) 1 : 19

(d) None of these

27. The ratio of the curved surface area of the second largest segment to that of the full cone is:

(a) 2 : 9

(b) 4 : 9

(c) cannot be determined

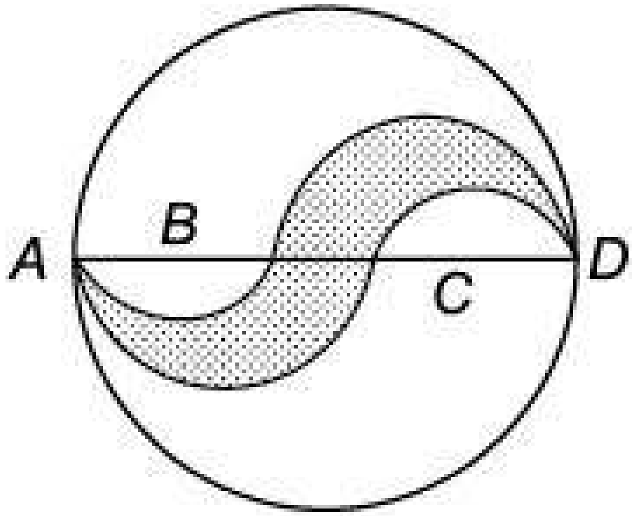
(d) None of these

28. On a semicircle with diameter *AD*, Chord *BC* is parallel to the diameter. Further each of the chords *AB* and *CD* has Length 2 cm while *AD* has length 8 cm. Find the length of *BC*.

(a) 7.5 cm (b) 7 cm

(c) 7.75 cm (d) Cannot be determined

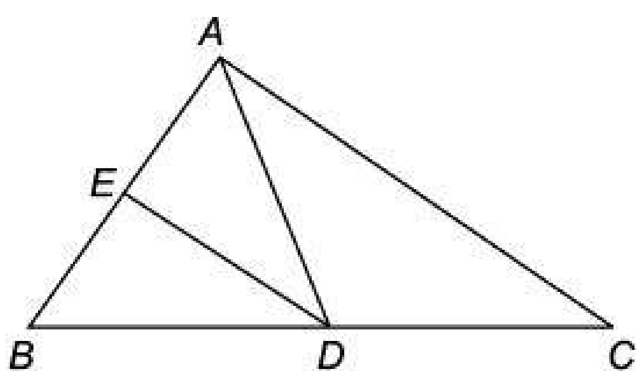
29. In the given figure, *B* and *C* are points on the diameter *AD* of the circle such that *AB* = *BC* = *CD*. Then find the ratio of area of the shaded portion to that of the whole circle.



(a) 1 : 3 (b) 2 : 3

(c) 1 : 2 (d) None of these

30. In the given figure, *ABC* is a triangle in which *AD* and *DE* are medians to *BC* and *AB* respectively, the ratio of the area of D*BED* to that of D*ABC* is



(a) 1 : 4 (b) 1 : 16

(c) Data inadequate (d) None of these

31. Two identical circles intersect so that their centres, and the points at which they intersect, from a square of side 1 cm. The area in square cm of the portion that is common to the two circles is

(a) $p/4$ (b) $p/_{2} - 1$

(c) $p/5$ (d) ÷2 – 1

32. If the height of a cone is trebled and its base diameter is doubled, then the ratio of the volume of the resultant cone to that of the original cone is

(a) 9 : 1 (b) 9 : 2

(c) 12 : 1 (d) 6 : 1

33. If two cylinders of equal volume have their heights in the ratio 2 : 3, then the ratio of their radii is

(a) $\sqrt{3} : \sqrt{2}$ (b) 2 : 3

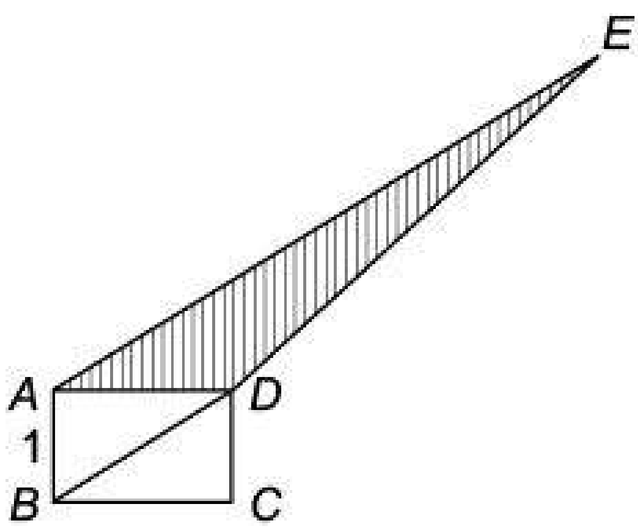
(c) $\sqrt{5} : \sqrt{3}$ (d) $\sqrt{6} : \sqrt{3}$

34. Through three given non-collinear points, how many circles can pass.

(a) 2 (b) 3

(c) Both 1 and 2 (d) None of these

35. The area of the rectangle *ABCD* is 2 and *BD* = *DE*. Find the area of the shaded region



(a) $\sqrt{5}$ (b) $2\sqrt{5}$

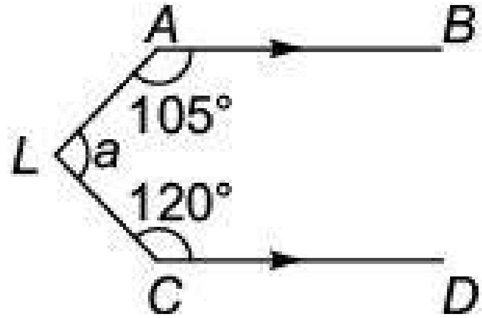
(c) $\sqrt{(5/2)}$ (d) 1

36. In a right-angled triangle, the square of the hypotenuse is equal to twice the product of the other two sides. The acute angles of the triangle are

(a) 30° and 30° (b) 30° and 60°

(c) 15° and 75° (d) 45° and 45°

37. Find –*ALC* if *AB* || *CD*



(a)75 (b) 135

(c)110 (d) 145

38. If the sides of a triangle are in the ratio of ½ : $^1/_3$: ¼ , the perimeter is 52 cm, then the length of the smallest side is

(a) 12 cm (b) 11 cm

(c) 8 cm (d) None of these

39. The number of distinct triangles with integral valued sides and perimeter as 14 is

(a) 2 (b) 3

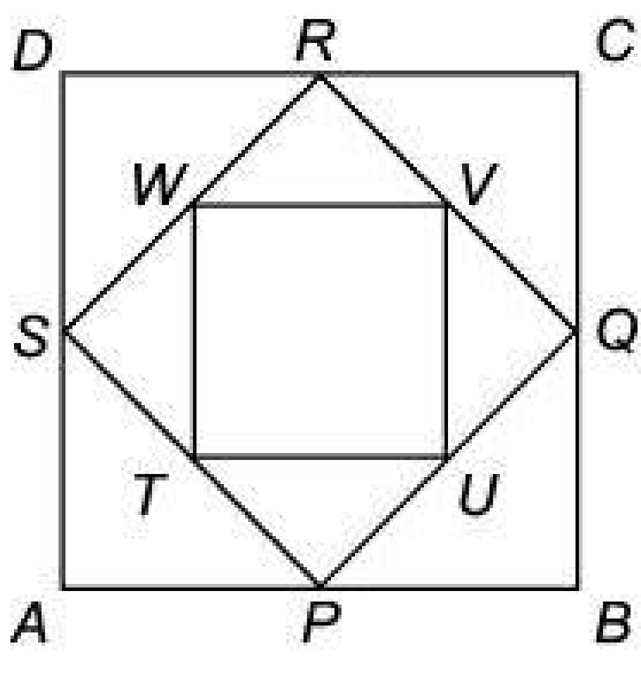
(c) 4 (d) 5

40. A polygon has 65 diagonals. Then, what is the number of sides of the same polygon?

(a) 11 (b) 12

(c) 14 (d) None of these

41. *PQRS* is a square drawn inside square *ABCD* of side 2*x* units by joining the midpoints of the sides *AB*, *BC*, *CD*, *DA*. The square *TUVW* is drawn inside *PQRS*, where *T*, *U*, *V*, *W* are the midpoints of *SP*, *PQ*, *QR* and ` If the process is repeated an infinite number of times the sum of the areas of all the squares will be equal to:



(a) $8x^2$ (b) $6x^2$

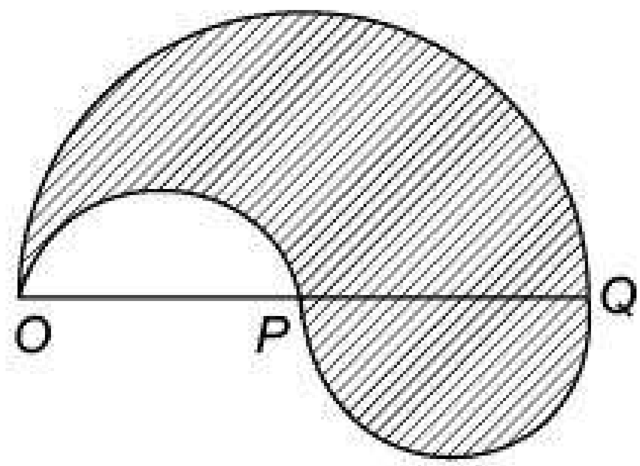
(c) $16x^2$ (d) $6x^2/2$

42. Suppose the same thing is done with an equilateral triangle of side x, wherein the mid points of the sides are connected to each other to form a second triangle and the mid points of the sides of the second triangle are connected to form a third triangle and so on an infinite number of times—then the sum of the areas of all such equilateral triangles would be:

(a) $3x^2$ (b) $6x^2$

(c) $12x^2$ (d) None of these

43. If in the figure given below $OP = PQ = 28$ cm and OQ, PQ and OP are all joined by semicircles, then the perimeter of the figure (shaded area) is equal to



(a) 352 cm (b) 264 cm

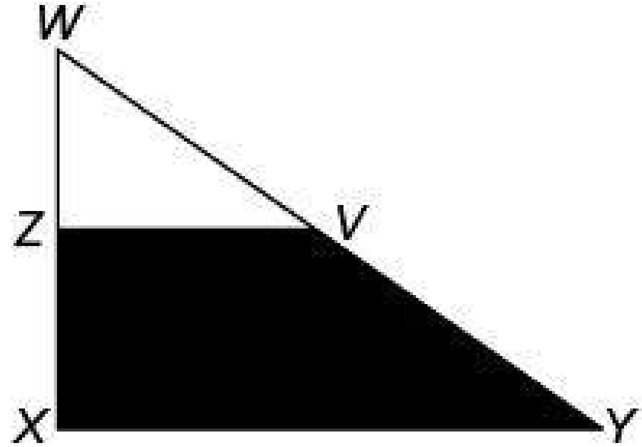
(c) 176 cm (d) 88 cm

44. For the question above, what is the shaded area?

(a) 1352 sq. cm (b) 1264 sq. cm

(c) 1232 sq. cm (d) 1188 sq. cm

45. What is the area of the shaded portion? It is given that $ZV \| XY$, $WZ = ZX$, $ZV = 2a$ and $ZX = 2b$.



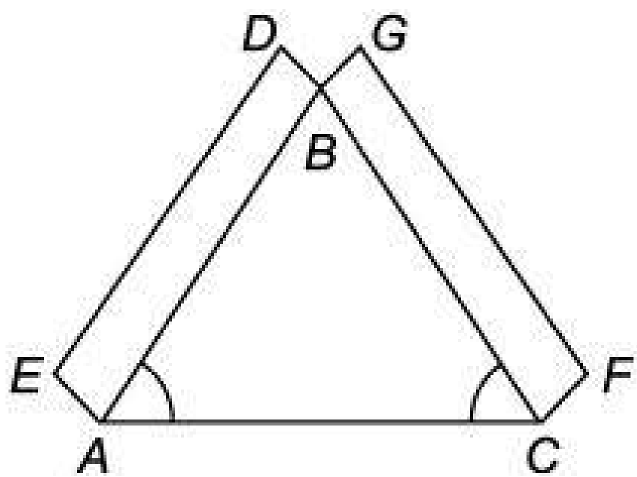
(a) $\frac{4ab}{2}$

(b) $\frac{8ab}{3}$

(c) $6ab$

(d) $3ab$

46. In the given figure there is an isosceles triangle *ABC* with angle *A* = angle *C* = 50° *ABDE* and *BCFG* are two rectangles drawn on the sides *AB* and *BC* respectively, such that *BD* = *BG* = *AE* = *CF*.



Find the value of the angle *DBG*.

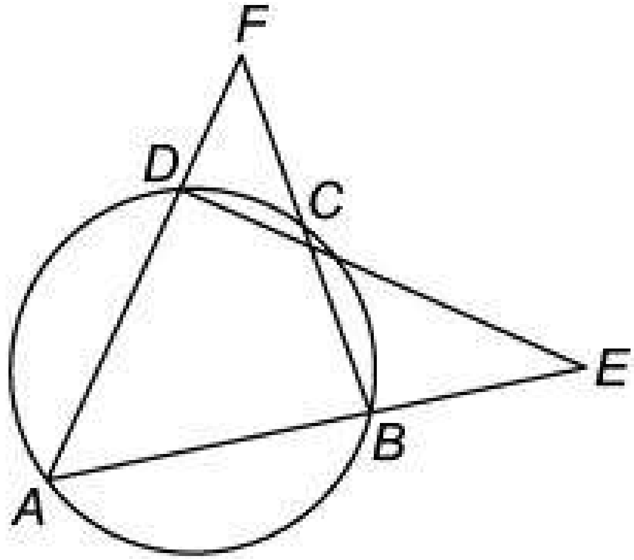
(a)80°

(b) 120°

(c) 100°

(d) 140°

47. In the figure, *ABE*, *DCE*, *BCF* and *ADF* are straight lines. *E* = 50°, *F* = 56°, find –*A*.



(a)47°

(b) 37°

(c)40°

(d) 42°

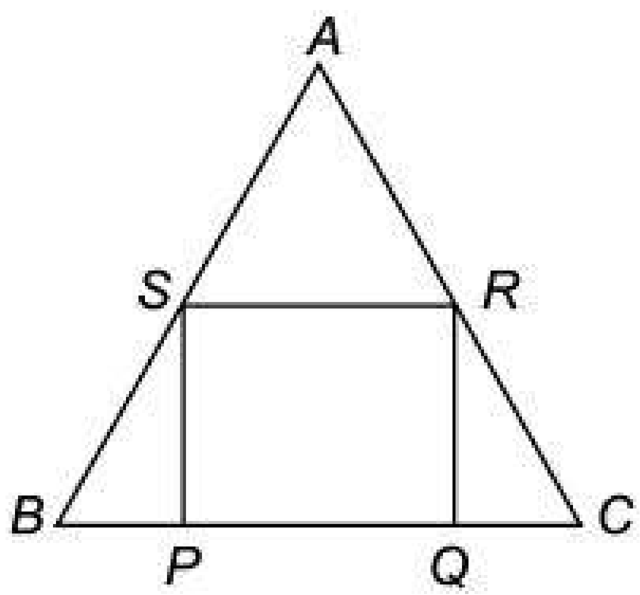
48. *ABC* is an equilateral triangle. *PQRS* is a square inscribed in it. Therefore

(a) $AR^2 = RC^2$

(b) $2AR^2 = RC^2$

(c) $3AR^2 = 4RC^2$

(d) $4AR^2 = 3RC^2$



49. Consider the five points comprising the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

(a) 4

(b) 6

(c) 8

(d) 10

50. In a triangle *ABC*, the internal bisector of the angle *A* meets *BC* at *D*. If $AB = 4$, $AC = 3$ and $–A = 60°$. Then, the length of *AD* is:

(a) $2\sqrt{3}$ (b) $(12\sqrt{3})/7$

(c) $(15\sqrt{3})/8$ (d) $(6\sqrt{3})/7$

Directions for Questions 51 and 52: Answer the questions based on the following information.

A rectangle *PRSU*, is divided into two smaller rectangles *PQTU* and *QRST* by the line *QT*. *PQ* = 40 cm. *QR* = 20 cm, and *RS* = 40cm. Points *A*, *B*, *F* are within rectangle *PQTU*, and points *C*, *D*, *E* are within the rectangle *QRST*. The closest pair of points among the pairs (*A*, *C*), (*A*, *D*), (*A*, *E*), (*F*, *C*), (*F*, *D*), (*F*, *E*), (*B*, *C*), (*B*, *D*), (*B*, *E*) are $40\sqrt{3}$ cm apart.

51. Which of the following statements is necessarily true?

(a) The closest pair of points among the six given points cannot be (*F*, *C*).

(b) Distance between *A* and *B* is greater than that between *F* and *C*.

(c) The closest pair of points among the six given points is (*C*, *D*), (*D*, *E*) or (*C*, *E*).

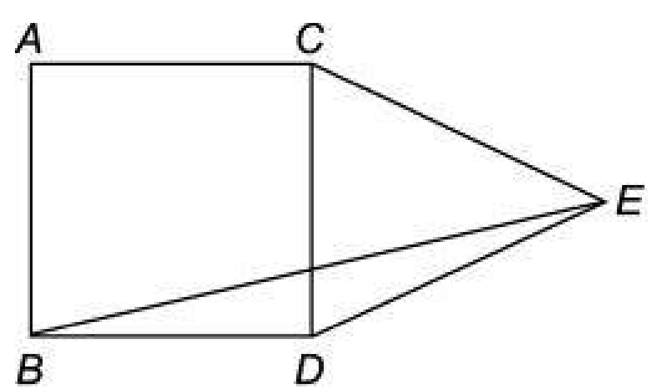
(d) None of the above.

52. *AB* > *AF* > *BF*; *CD* > *DE* > *CE*; and *BF* = $24\sqrt{5}$ cm. Which is the closest pair of points among all the six given points?

(a) *B*, *F* (b) *C*, *D*

(c) *A*, *B* (d) None of these

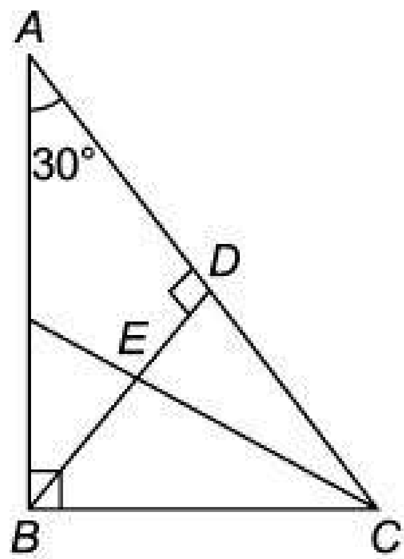
53. If *ABCD* is a square and *CDE* is an equilateral triangle, what is the measure of –*DEB*?



(a) 15° (b) 30°

(c) 20° (d) 45°

54. *AB* ^ *BC*, *BD* ^ *AC* and *CE* bisects –*C*, –*A* = 30°. Then, what is –*CED*?



(a)30° (b) 60°

(c)45° (d) 65°

55. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then, the ratio of the shorter side to the longer side is:

(a) 1/2 (b) 2/3

(c) 1/4 (d) 3/4

ANSWER KEY

1. (b)	2. (c)	3. (b)	4. (d)
5. (b)	6. (b)	7. (c)	8. (a)
9. (b)	10. (b)	11. (c)	12. (b)
13. (a)	14. (b)	15. (c)	16. (a)
17. (a)	18. (a)	19. (b)	20. (c)
21. d	22. (d)	23. (a)	24. (d)
25. (b)	26. (c)	27. (a)	28. (b)
29. (a)	30. (a)	31. (b)	32. (c)
33. (a)	34. (d)	35. (d)	36. (d)
37. (b)	38. (a)	39. (c)	40. (d)
41. (a)	42. (d)	43. (c)	44. (c)
45. (c)	46. (a)	47. (b)	48. (d)
49. (c)	50. (b)	51. (d)	52. (d)

53. (a)	54. (b)	55. (d)