JEE MAIN 2025

Sample Paper - 5

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

1. There are three subjects in the question paper consisting of Physics (Q. no. 1 to 25), Chemistry (Q, no. 26 to 50), and Mathematics (Q. no. 51 to 75).

2. Each subject is divided into two sections. Section A consists of 20 multiple-choice questions & Section B consists of 5 numerical value-type questions.

3. There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero marks will be awarded for not attempted questions.

4. For Section B questions, 4 marks will be awarded for correct answers and zero for unattempted and incorrect answers.

5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.

6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

PHYSICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

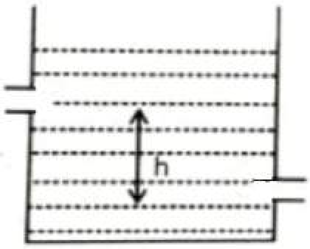
1. A wire frame in the shape of an equilateral triangle is hinged at one vertex so that it can swing freely in a vertical plane, with the plane of the triangle always remaining vertical. The side of the frame is $\frac{1}{\sqrt{3}}m$. The time period in seconds of small oscillations of the frame will be

A) $\frac{\pi}{\sqrt{2}}$ B) $\pi\sqrt{2}$ C) $\frac{\pi}{\sqrt{6}}$ D) $\frac{\pi}{\sqrt{5}}$

2. The dimensions of angular momentum are

A) $M^0L^{-1}T^0$ B) $M^0L^{-2}T^0$ C) M^1LT^{-2} D) $M^1L^2T^{-1}$

3. There are two identical small holes on the opposite sides of a tank containing liquid. The tank is open at the top. The difference in height between the two holes is h. As the liquid comes out of the two holes, the tank will experience a net horizontal force proportional to.

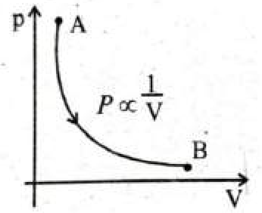


A) $\sqrt{h}$ B) h C) $h^{3/2}$ D) h^2

4. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a mass less spring of force constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

A) $2\pi\sqrt{m/K}$ B) $2\pi\sqrt{\frac{m(YA+KL)}{YAK}}$ C) $2\pi\sqrt{\frac{mYA}{KL}}$ D) $2\pi\sqrt{\frac{mL}{YA}}$

5. Internal energy of the gas as it expands according to the graph AB which is a recatangular hyperbola



A) Increasing continuously

B) decreasing continuously

C) Always constant

D) Initially Increasing then decreasing.

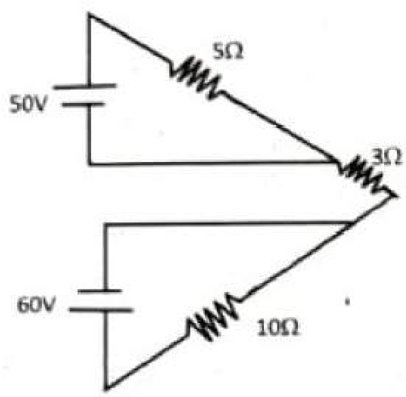
6. There are four concentric shells A,B,C and D of radii a,2a,3a and 4a respectively. Shells B & D are given charges +q & -q respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is___(take $\frac{1}{4\pi\varepsilon_0} = K$)

A) $\frac{Kq}{6a}$ B) $\frac{Kq}{2a}$ C) $\frac{Kq}{3a}$ D) $\frac{Kq}{4a}$

7. Two Identical discs initially at rest are in contact on a table. A third disc of same mass but of double radius strikes them symmetrically and itself comes to rest after impact. The co-efficient of restitution is:

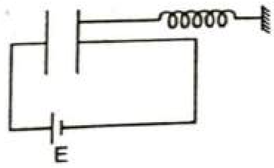
A) $\frac{9}{16}$ B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{1}{16}$

8. Find out the value of current through 3Ω resistance for the given circuit



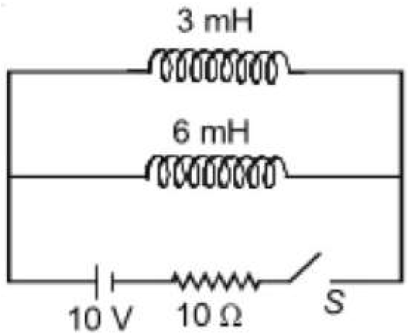
A) 10 amp B) 6 amp C) 4 amp D) zero

9. One plate of a capacitor is connected to a spring as shown in figure. Area of both the plates is A. In steady state separation between the plates is 0.8d (spring was unstretched and the distance between the plates was d when the capacitor was uncharged). The force constant of the spring is approximately



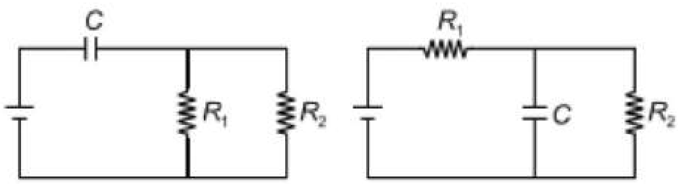
A) $\frac{4\varepsilon_0 AE^2}{d^3}$ B) $\frac{2\varepsilon_0 AE}{d^2}$ C) $\frac{6\varepsilon_0 E}{Ad^3}$ D) $\frac{\varepsilon_0 AE^3}{2d^3}$

10. The current through 3 mH inductor in steady state after closing switch S is



A) $\frac{1}{3}$ ampere B) $\frac{2}{3}$ ampere C) 1 ampere D) $\frac{3}{2}$ ampere

11. STATEMENT-1 : Time constants of the circuits shown in the figure are same.



AND

STATEMENT-2 : Instantaneous current through the capacitor branch is same at any instant for both the circuits, if batteries are inserted in the circuits at t=0.

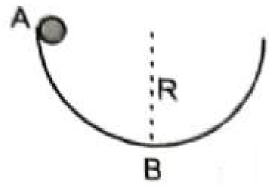
A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

C) Statement-1 is True, Statement-2 is False

D) Statement-1 is False, Statement-2 is True

12. A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R(>> r) as shown in figure. If the sphere starts from rest at the top point of the hemisphere. Find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.

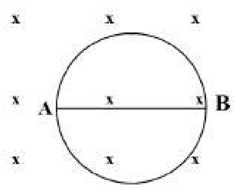


A) $\frac{10}{7}mg$ B) $\frac{17}{7}mg$ C) $\frac{5}{7}mg$ D) $\frac{7}{5}mg$

13. Refractive index of a prism is $\sqrt{\frac{7}{3}}$ and the angle of prism is 60^0. The minimum angle of incidence of a ray that will be transmitted through the prism is _______

A) 30^0 B) 40^0 C) 60^0 D) 90^0

14. The radius of the conducting loop shown in figure is R. Magnetic field is decreasing at a constant rate α. Resistance per unit length of the loop is ρ. Then current in wire AB is (AB is one of the diameters)

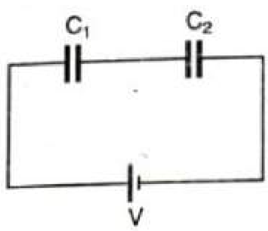


A) $\frac{R\alpha}{2\rho}$ *from A to B* B) $\frac{R\alpha}{2\rho}$ *from B to A* C) $\frac{2R\alpha}{\rho}$ *from A to B* D) zero

15. In YDSE, coherent monochromatic light having wavelength 600 nm has fallen on slits. First order bright fringe is at 4.84 mm from central maxima. Determine the wavelength for which the first order dark fringe will be observed at same location on screen ? Take $D = 3m$

A) 600 nm B) 1200 nm C) 300 nm D) 900 nm

16. Two capacitors having capacitance C_1 & C_2 are connected in series and a potential difference V is applied across them. Then:



V_1, V_2 & U_1, U_2 be the potentials drop & energy store in C_1 & C_2 respectively.

Match the entries in Column-I with Column-II :

Column-I		Column-II	
a)	$V_1 < V_2$	p)	$C_1 < C_2$
b)	$U_1 < U_2$	q)	$C_1 > C_2$
c)	V_1	r)	$C_1V/(C_1+C_2)$
d)	V_2	s)	$C_2V/(C_1+C_2)$

A) $(a \to q);(b \to q);(c \to s);(d \to r)$ B) $(a \to r);(b \to q);(c \to p);(d \to s)$

C) $(a \to q);(b \to p);(c \to r);(d \to s)$ D) $(a \to s);(b \to r);(c \to p);(d \to q)$

17. A changing electric field produces magnetic field. The direction of this magnetic field is

A) In the direction of electric field

B) In the direction opposite to the electric field

C) Perpendicular to the direction of electric field

D) Independent of the direction of electric field

18. A photon collides with a stationary hydrogen atom in ground state ineleastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of microsecond another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?

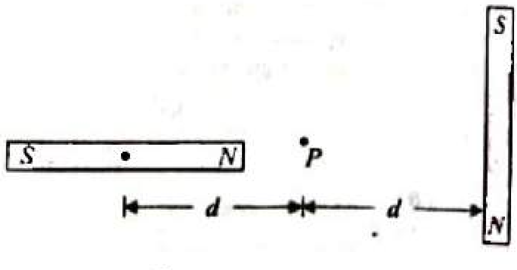
A) 2 photons of energy 10.2 eV

B) 2 photons of energy 1.4 eV

C) one photon of energy 0.2 eV and an electron of energy 1.4 eV

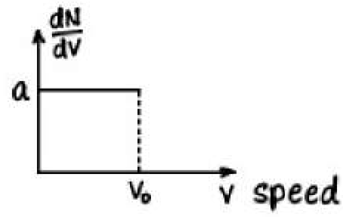
D) one photon of energy 10.2 eV and an electron of energy 1.4 eV

19. Two short bar magnets of magnetic moment M each are placed at a distance 2 d apart. The magnetic field. Midway between them at P is



A) $\frac{\mu_0}{4\pi}\frac{3M}{d^3}$ B) $\frac{\mu_0}{4\pi}\frac{M\sqrt{5}}{d^3}$ C) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$ D) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$

20. Graph shows a hypothetical speed distribution for a ample of N gas particles (for $V > V_0, \frac{dN}{dV} = 0$). If V_{ave} and V_{rms} are the average speed and rms speed of the gas molecules, then



A) $V_{ave} = \frac{V_0}{3}$ B) $V_{rms} = \frac{V_0}{\sqrt{2}}$ C) $V_{ave} : V_{rms} = 3 : \sqrt{2}$ D) $V_{ave} : V_{rms} = \sqrt{3} : 2$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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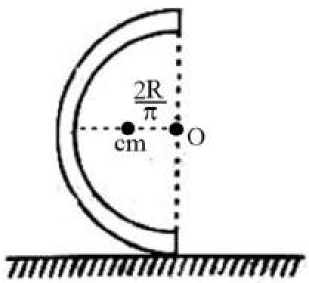
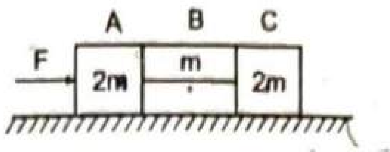
21. A half section of thin uniform pipe of mass m and radius r is released from rest. Pipe rolls without slipping. The change in PE of pipe when it has rolled through 90^0 is $\frac{nmgR}{\pi}$. Then the value of n is_

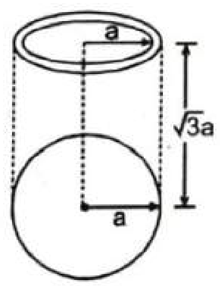


22. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C friction coefficient between B and C is μ.Minimum value of F to prevent block B from downward slipping is $\left(\frac{n}{2\mu}\right)mg$. Then the value of n is _____.



23. A uniform stick of length l and mass m lies on a smooth table. It rotates with angular velocity ω about an axis perpendicular to the table and though one end of the stick. The angular momentum of the stick about the end is $\frac{ml^2\omega}{n}$.Find the value of n_______.

24. A uniform ring of mass m is lying at a distance $\sqrt{3}a$ from centre of a sphere of mass M just over the sphere where a is the radius of ring as well as that of sphere. Then, gravitational force exerted is $\frac{\sqrt{3}GMm}{na^2}$ then the value of n is ____



25. Two tuning forks A & B when sounded together produces 4 beats/s. If B is loaded with wax then also beat frequency remains same. Frequency of A is 242 Hz, find frequency of B?

CHEMISTRY **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

26. Which of the following is not the name of an element with atomic number 104?

A) Nobelium B) Rutherfordium C) Kurchatovium D) Unnilqauadium

27. During halogen test, sodium fusion is boiled with $con.HNO_3$ to

A) remove unreacted sodium

B) decompose cyanide or sulphide of sodium

C) extract halogen from organic compound

D) Maintain the p^H of extract

28. In the ionization of $N_2\ to\ N_2^{\oplus}$, the electron is lost from a

A) $\sigma-orbital$ B) $\pi-orbial$ C) $\overset{*}{\sigma}-orbial$ D) $\overset{*}{\pi}-orbial$

29. A certain substance 'A' tetramerises in water to the extent of 80%. A solution of 2.5g of 'A' in 100g of water lowers the freezing point by 0.3^0C. The m.wt of 'A' is $\left(K_f = 1.86\,k.kg.mol^{-1}\right)$

A) 122 B) 31 C) 344 D) 62

30. Which technique among the following is most appropriate in separation of a mixture of 100 mg of

p-nitrophenol and picric acid?

A) Steam distillation B) Distillation under reduced pressure.

C) Sublimation D) Thin layer chromatography.

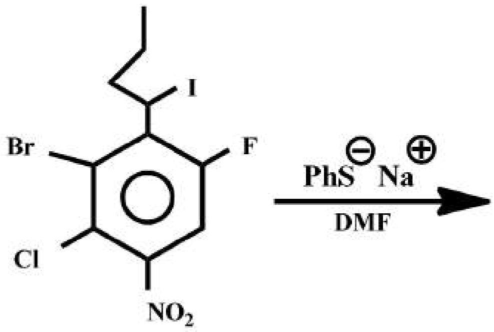
31. The equilibrium constants $K_{p_1}\ and\ K_{p_2}$ for the reactions $X \rightleftharpoons 2Y\ and\ Z \rightleftharpoons P+Q$ respectively are in the ratio of $1:9$. If the degree of dissociation of X and Z be equal then the ratio of total pressure at these equilibria is

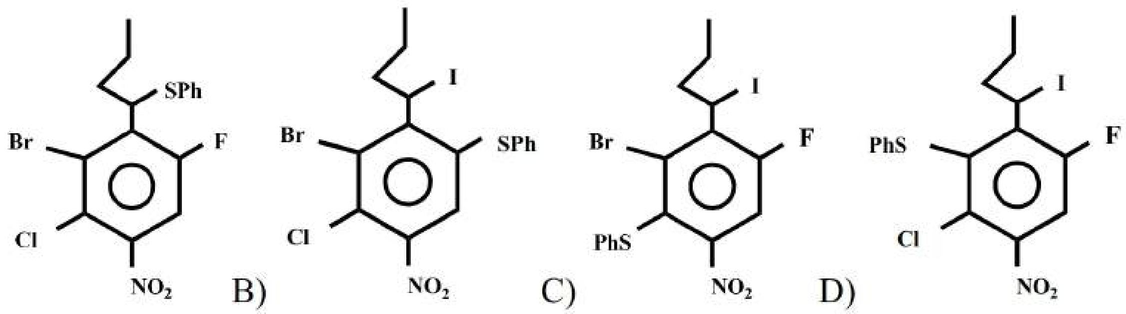
A) $1:36$ B) $1:1$ C) $1:3$ D) $1:9$

32. Transition metal complex with the highest value of crystal field splitting energy (Δ_0) will be

A) $\left[Cr(H_2O_6)\right]^{+3}$ B) $\left[Mo(H_2O_6)\right]^{+3}$ C) $\left[Fe(H_2O_6)\right]^{+3}$ D) $\left[Os(H_2O_6)\right]^{+3}$

33. The major product of the reaction is





A) B) C) D)

34. When an aldehyde is heated with Fehilings solution, a reddish brown precipitate is formed which is

A) CuO B) Cu C) Cu_2O D) $Cu-C\equiv C-Cu$

35. Which of the following cannot be prepared by using willamson's synthesis?

A) Methoxy benzene B) Benzyl p-nitro phenyl ether

C) t-butyl methyl ether D) Ditertiarybutyl ether

36. Among the following the metal with the highest melting point will be

A) Hg B) Ag C) Ga D) Cs

37. The element of group 15 which can form a strong bond with hydrogen is

A) Nitrogen B) Phosphorous C) Arsenic D) Antimony

38. Which of the following is not a mineral of fluorine?

A) Fluorspar B) Cryolite C) Fluoroapetite D) Carnallite

39. Consider the following statements:

According to Werner's theory

i) Secondary valencies are non-ionisable and directional

ii) Secondary valencies are satisfied by neutral molecules or negative ions .

iii) Ligands form coordinate bonds with metal ions or atoms.

Iv) The charge on the complex ion is always equal to the oxidation state of the metal atom.

A) all are correct B) (i), (ii) and (iii) are correct

C) (i), (ii) and (iv) are correct D) (iii) and (iv) are correct

40. Zinc and mercury do not show variable valency like other d-block elements because

A) They are soft

B) Their (n-1) d-shells are completely filled

C) They have only two electrons in the outermost shell

D) Their d-shells are incompletely filled.

41. Assertion (A): The purple colour of $KMnO_4$ is due to charge transfer transition

Reason (R): The intense colour, in most of the transition metal complexes is due to d – d transition.

A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true but R is false

D) A is false but R is true

42. Which of the following doesn't undergo Friedal-craft's reaction?

A) Xylene B) Nitrobenzene C) Cumene D) Toluene

43. Match the transition element ions given in Column I with the characteristic (s) of products given in Column II.

Column –I	**Column – II**
a) Cu^{2+}	P) Form amphoteric oxide
b) Zn^{2+}	Q) Diamagnetic and colourless compounds
c) Cr^{3+}	R) Coloured hydrated transition metal ion
d) Ni^{2+}	S) Paramagnetic

A) a – RS, b – PQ, c – PRS, d – QRS

B) a – PQ, b – RS, c – PRS, d – QRS

C) a – PRS, b – RS, c – PQ, d – QRS

D) a – RS, b – PQ, c – QRS, d – PRS

44. Oxidation state of iodine in $H_4IO_6^-$ is

A) +7 B) +5 C) +1 D) -1

45. Which of the following does not give a white precipitate when dilute hydrochloric acid is added?

A) Ag^+ B) Ba^{+2} C) Pb^{+2} D) Hg_2^{+2}

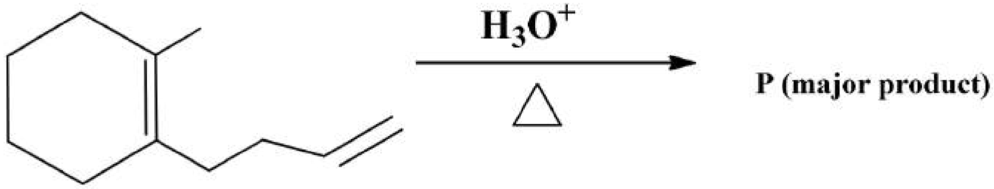
SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

46. The wave length of an electron and a neutron will become equal when the velocity of electron is x times the velocity of neutron. The value of x is (nearest integer)

$\left(Mass\ of\ electron = 9.1\times10^{-31}kg\right)$, $\left(Mass\ of\ neutron = 1.6\times10^{-27}kg\right)$

47. The major product in the following reaction is

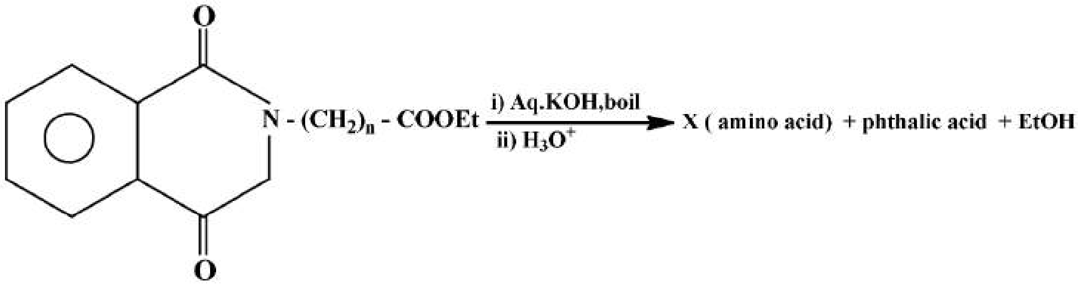


The degree of unsaturation of product P is _______

48. The volume of $0.15M\ H_2SO_4$ solution required to neutralise 15 ml of $0.5M\ KOH$ in presence of phenolphthalein indicator is ___ ml.

49. An known amine A with benzene sulphonyl chloride yields a derivative which dissolves in aq.KOH solution. Compound A on reaction with nitrous acid gives an alcohol B which responds to iodoform test. If hydrochloride of A contains 37.2% of chlorine by weight, the total number of carbon atoms present in compounds A and B is ___.

50.



If the molecular weight of tripeptide formed by X is 189, the value of n is ________

MATHEMATICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

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Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

51. If a denotes the number of permutation of n different things taken all at a time, b the number of permutation of n-2 different things taken 10 at a time and c, the number of permutations of n-12 different things taken all at a time such that $a = 182bc$, then value of n is:

A) 10 B) 12 C) 14 D) 18

52. STATEMENT – 1: $f(x) = |x[x]|$ is discontinuous at all Integers , where [.] denotes G.I.F

STATEMENT – 2: If a function is non-differentiable at a point then it may be continuous at that point

A) Statement 1 is true, statement 2 is true

B) Statement 1 is false, statement 2 is false

C) Statement 1 is true, statement 2 is false

D) Statement 1 is false, statement 2 is true

53. If a,b,c are real, then $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$ is decreasing in:

A) $\left(-\frac{2}{3}\left(a^2+b^2+c^2\right),0\right)$ B) $\left(0,\frac{2}{3}\left(a^2+b^2+c^2\right)\right)$ C) $\left(0,\frac{\left(a^2+b^2+c^2\right)}{3}\right)$ D) Never decreases

54. If p^{th}, q^{th}, r^{th}, terms of a G.P are the positive numbers a,b,c respectively then angle between the vectors $\log a^3\hat{i} + \log b^3\hat{j} + \log c^3\hat{k}$ *and* $(q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$ is:

A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$ C) 0 D) $\sin^{-1}\left(\frac{1}{\sqrt{p^2+q^2+r^2}}\right)$

55. Consider the data on X taking the values 0, 2, 4, 8, …. , 2^n with frequencies $^nC_0, ^nC_1, ^nC_2, ..., ^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ___.

A) 15 B) 8 C) 4 D) 6

56. Complete set of real values of 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all its roots real

A) $\left[\frac{3}{4}, \infty\right)$ B) $[1, \infty)$ C) $[2, \infty)$ D) $[0, \infty)$

57. The value of $\int_{-20\pi}^{20\pi} [\sin x + \cos x] dx$ is: (where $[.]$ denotes greatest integer function)

A) 10π B) -20π C) 20π D) -10π

58. Statement-1: The number of non-negative integral solutions of 2x + y + z = 21 is 132

Statement-2: For $n \in N, (n^2)!$ is divisible by $(n!)^n$.

A) Statement 1 is true, statement 2 is true

B) Statement 1 is false, statement 2 is false

C) Statement 1 is true, statement 2 is false

D) Statement 1 is false, statement 2 is true

59. Of all the functions that can be defined from the set $A:\{1,2,3,4\} \rightarrow B:\{5,6,7,8,9\}$ a mapping is randomly selected. The chance that the selected mapping is strictly monotonic is

A) $1/105$ B) $\frac{2}{125}$ C) $\frac{4}{4096}$ D) $\frac{5}{2048}$

60. Three distinct numbers a_1, a_2, a_3 are in increasing G.P $a_1^2 + a_2^2 + a_3^2 = 364$ and $a_1 + a_2 + a_3 = 26$, then the value of a_{10} if a_n is the n^{th} *term* of the given G.P is:

A) 2.3^9 B) 3^9 C) 2.3^{10} D) 3^{12}

61. Let $x=\left(5\sqrt{2}+7\right)^{19}$, then $x\{x\}$ $\left(\{x\}\,\textit{denotes the fractional part of } x\right)$ is equal to:

A) 2^{19} B) 3^{19} C) 0 D) 1

62. If $I_a=\int_0^{\pi/2}\frac{dx}{2\cos x+\sin x+a}$, then the value of I_a for

Column – I	Column – II
A) a = 1	P) $\ln\left(\frac{3}{2}\right)$
B) a = 3	Q) $\frac{1}{2}\log 3$
C) a = 2	R) $\frac{2}{\sqrt{11}}\left(\tan^{-1}\frac{3}{\sqrt{11}}-\tan^{-1}\frac{1}{\sqrt{11}}\right)$
D) a = 4	S) $\tan^{-1}\left(\frac{1}{3}\right)$

A) A – Q ; B – P ; C – S ; D – R B) A – Q ; B – S ; C – R ; D – P

C) A – Q ; B – S ; C – P ; D – R D) A – Q ; B – P ; C – Q ; D – S

63. A curve passing through $(2,3)$ and satisfying the differential equation

$\int_0^x ty(t)\,dt=x^2y(x),(x>0)$ is :

A) $x^2+y^2=13$ B) $y^2=\frac{9}{2}x$ C) $\frac{x^2}{8}+\frac{y^2}{18}=1$ D) $xy=6$

64. Let $\frac{\tan\left(\frac{\pi}{4}+\alpha\right)}{5}=\frac{\tan\left(\frac{\pi}{4}+\beta\right)}{3}=\frac{\tan\left(\frac{\pi}{4}+\gamma\right)}{2}$. Then $12\sin^2(\alpha-\beta)+15\sin^2(\beta-\gamma)-7\sin^2(\gamma-\alpha)$

is equal to

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) 1 D) 0

65. A 13 ft. ladder is leaning against a wall when its base starts to slide away. At the instant when the base is 12 ft. away from the wall, the base is moving away from the wall at the rate of 5 ft/sec. The rate at which the angle θ between the ladder and the ground is changing is

A) $-\frac{12}{13} rad/\sec$ B) $-1\ rad/\sec$ C) $-\frac{13}{12}\ rad/\sec$ D) $-\frac{10}{13}\ rad/\sec$

66. Two aero planes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

A) 0.06 B) 0.14 C) 0.312 D) 0.70

67. The area of the region bounded by the curve $y=\frac{16-x^2}{4}$ *and* $y=\sec^{-1}\left[-\sin^2 x\right]$

(where $[.]$ denotes greatest integer function is:)

A) $\frac{1}{3}(4-\pi)^{3/2}$ B) $8(4-\pi)^{3/2}$ C) $\frac{8}{3}(4-\pi)^{3/2}$ D) $\frac{8}{3}(4-\pi)^{1/2}$

68. If $f(x)=\frac{e^{[x]+|x|}-3}{[x]+|x|+1}$, then: (where $[.]$ represents greatest integer function)

A) $\lim_{x\to 0^+} f(x)=-2$ B) $\lim_{x\to 0^-} f(x)=0$ C) $\lim_{x\to 0^-} f(x)=2$ D) $\lim_{x\to 0} f(x)$ *exist*

69. Let $g(x)=\frac{1}{4}f\left(2x^2-1\right)+\frac{1}{2}f\left(1-x^2\right)\forall x\in R$, where $f''(x)>0\forall x\in R, g(x)$ *is* necessarily increasing in the interval

A) $\left(-\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}}\right)$

B) $\left(-\sqrt{\frac{2}{3}},0\right)\cup\left(\sqrt{\frac{2}{3}},\infty\right)$

C) $(-1,1)$

D) None of these

70. The largest integral value of x satisfying the inequality $\left(\tan^{-1}(x)\right)^2-4\left(\tan^{-1}(x)\right)+3>0$ is:

A) 0 B) 1 C) 2 D) 3

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

71. Let O be an interior point of ΔABC such that $\overrightarrow{OA}+2\overrightarrow{OB}+3\overrightarrow{OC}=\vec{0}$ then ratio of the area of ΔABC to the area of ΔAOC is _______.

72. Find number of integral values of k for which the line $3x+4y-k=0$, lies between the circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-12y+113=0$, without cutting a chord on either of circle.

73. Let $\vec{a}=3\hat{i}+2\hat{j}+4\hat{k}$, $\vec{b}=2\left(\hat{i}+\hat{k}\right)$ and $\vec{c}=4\hat{i}+2\hat{j}+3\hat{k}$. If the equation $x\vec{a}+y\vec{b}+z\vec{c}=\alpha\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$ has a non-trivial solution, then find the sum of all distinct possible values of α.

74. For each positive integer n, consider the point P with abscissa n on the curve $y^2-x^2=1$. If d_n represents the shortest distance from the point P to the line $y=x$ then $\underset{n\to\infty}{Lt}(n.d_n)$ has value $\frac{1}{K\sqrt{K}}$, then K is

75. Let $f(x)=\int_0^x 3^t\left(3^t-4\right)(x-t)\,dt\ (x\geq 0)$. If $x=a$ is the point where $f(x)$ attains it's local minimum value then value of 3^a is ;

KEY SHEET

PHYSICS

1	**D**	2	**D**	3	**B**	4	**B**	5	**C**
6	**A**	7	**A**	8	**D**	9	**A**	10	**B**
11	**C**	12	**B**	13	**A**	14	**D**	15	**B**
16	**A**	17	**C**	18	**D**	19	**B**	20	**D**
21	**2**	22	**5**	23	**3**	24	**8**	25	

CHEMISTRY

26	**A**	27	**B**	28	**A**	29	**D**	30	**D**
31	**A**	32	**D**	33	**A**	34	**C**	35	**D**
36	**B**	37	**A**	38	**D**	39	**B**	40	**B**
41	**B**	42	**B**	43	**A**	44	**A**	45	**B**
46	**1758**	47	**3**	48	**25**	49	**6**	50	**1**

MATHEMATICS

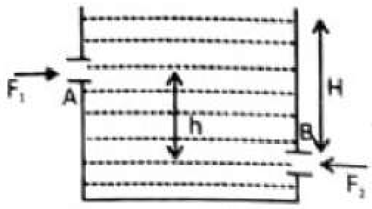
51	**C**	52	**D**	53	**A**	54	**A**	55	**D**
56	**A**	57	**B**	58	**A**	59	**B**	60	**A**
61	**D**	62	**C**	63	**D**	64	**D**	65	**B**
66	**C**	67	**C**	68	**A**	69	**B**	70	**B**
71	**3**	72	**30**	73	**7**	74	**2**	75	**7**

SOLUTIONS
PHYSICS

1. $T = 2\pi\sqrt{\frac{1}{mgd}}$

2. Angular momentum is defined by the equation L=MVR

3. Let height of liquid above second hole (2) be 'H'



$\therefore v_1 = \sqrt{2g(H-h)}$

Thus the force experienced by the tank at point

'A' is $F_1 = \frac{d\rho}{dt} = A\rho V_1^2$

Similarly $F_2 = A\rho V_2^2$

V_2 is the velocity of liquid coming out at second hole (2).

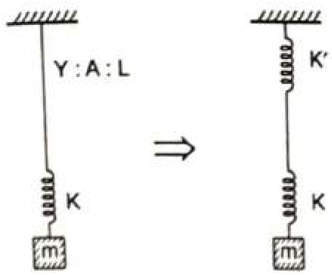
At point B $V_2 = \sqrt{2gH}$

$\therefore$ The net force on tank is, $\vec{F} = \vec{F_2} + \vec{F_1}$

$= A\rho\left(V_2^2 - V_1^2\right) = A\rho 2gh = A(2g)h$

$F_{net} \alpha\, h$

4.



$K' = \frac{YA}{L}$

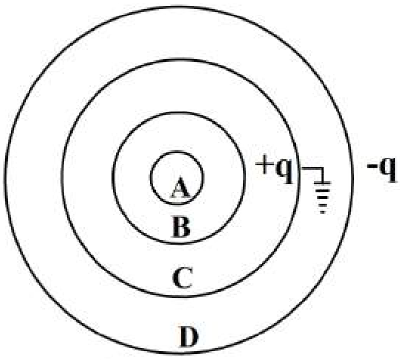
$\therefore K_{eq} = \frac{KK'}{K+K'} = \frac{\frac{KYA}{L}}{K+\frac{YA}{L}} = \frac{KYL}{KL+YA}$

$\therefore \qquad T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$= 2\pi\sqrt{\frac{m(KL+YA)}{KYA}}$

5. **The given graph represent isothermal process and for isothermal process internal energy is constant.**

6.



$$\frac{Kq}{3a}+\frac{KQ}{3a}-\frac{Kq}{4a}=0$$

$$Q={}^{-q}\!/\!_{4}$$

$$V_A=\frac{Kq}{2a}-\frac{K\,{}^{q}\!/\!_{4}}{3a}-\frac{Kq}{4a}=\frac{Kq}{6a}$$

$$V_A-V_C=\frac{Kq}{6a}$$

7. $\sin\theta=\frac{1}{3},\cos\theta=\frac{\sqrt{8}}{3}$

According to conservation of momentum $mu=2mv\cos\theta$

$$u=2v\frac{\sqrt{8}}{3}$$

$$e=\frac{v}{u\cos\theta}=\frac{9}{16}$$

8. As the branch of the circuit containing 3Ω resistor is open so no current flows through it.

9. In equilibrium electrostatic attraction between the plates = spring force

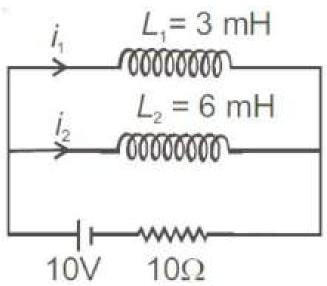
$$\therefore\quad \frac{q^2}{2\varepsilon_0 A}=kx$$

$$\therefore\quad \frac{(CE)^2}{2\varepsilon_0 A}=k\left(d-0.8d\right)$$

$$\therefore\quad \frac{\left(\frac{\varepsilon_0 A}{0.8d}\right)^2 E^2}{2\varepsilon_0 A}=0.2dk$$

$$\therefore\quad k=\frac{\varepsilon_0 AE^2}{0.256d^3}=\frac{4\varepsilon_0 AE^2}{d^3}$$

10. As magnetic flux is same through inductors, $L_1 i_1=L_2 i_2$



$$\Rightarrow\quad i_1=2i_2\quad .....(i)$$

Also, $i_1 + i_2 = \frac{\varepsilon}{R} = \frac{10\text{V}}{10\Omega} = 1\text{A}$ .....(ii)

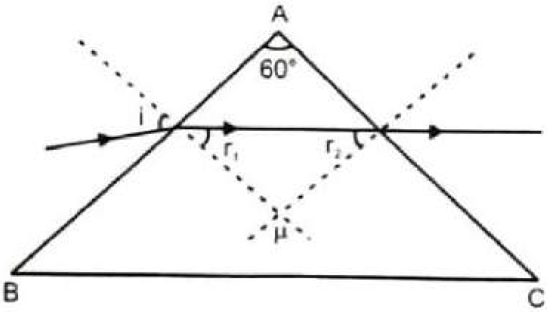
(i) and (ii)

$\Rightarrow \quad i_1 = \frac{2}{3}\text{A}$

11. Time constants of both the circuits $= \frac{R_1 R_2}{R_1 + R_2} \cdot C$ but that does not ensure same charge at any time on capacitors or same current in branches of the capacitor.

12. At the bottom, $N - mg = \frac{mv^2}{r}$

13. $r_2 < \theta_c; A - r_1 < \theta c$



$r_1 > A - \theta_c$

$Sin\, r_1 > \sin(A - \theta_c)$

$\frac{\sin i}{\mu} > \sin(A - \theta_c)$

$\sin i > \mu(\sin A \cos\theta_c - \cos A \sin\theta_c)$

$\sqrt{\frac{7}{3}}\left(\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{7}} - \sqrt{\frac{3}{7}}\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

$\sin i > \frac{1}{2}\ or\ i > 30^0$

14. According to Lenz's law e.m.fs of the same magnitude in the clockwise direction are induced in the two loops into which the figure is divided. So, current is induced in the clockwise direction in the outer boundary but no current in wire AB.

15. $\frac{\lambda_1 D}{d} = 4.84mm$

Let required wavelength is λ_2 the according to given information $\frac{\lambda_2 D}{2d} = 4.84mm$

$\Rightarrow \frac{\lambda_1}{\lambda_2 / 2} = 1 \Rightarrow \lambda_2 = 1200mm$

16. Let V_1 and V_2 be P.D. across C_1 and C_2. Then

$$C_1 V_1 = C_2 V_2 \Rightarrow V_2 = \frac{C_1 V_1}{C_2}$$

Also
$$V_1 + V_2 = V$$
$$V_1 + \frac{C_1 V_1}{C_2} = V$$

$$V_1\left(\frac{C_1+C_2}{C_2}\right)=V$$

$$V_1=\frac{C_2V}{(C_1+C_2)}$$

Similarly, $$V_2=\frac{C_1V}{C_1+C_2}$$

$$\therefore \quad \frac{V_1}{V_2}=\frac{C_2}{C_1} \Rightarrow V_1 < V_2 \text{ if } C_1 > C_2$$

Also, $U=\frac{q^2}{2C} \Rightarrow U_1 < U_2 \text{ if } C_1 > C_2$

17. The field due to current (either conventional or displacement) is normal to the direction of current.

18. The first photon will excite the hydrogen atom (in ground state) in first excited state $(E_2-E_1=10.2eV)$. Hence, during de-excitation a photon of 10.2eV will be released. The second photon of energy 15eV can ionize the atom. Hence the balance energy 1.4eV is retained by the electron.

19. The magnetic fields at P due to horizontal and vertical magnets are respectively $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$ towards right and $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ upwards.

Their resultant is $\frac{\mu_0}{4\pi}\frac{M}{d^3}\sqrt{2^2+1^2}=\frac{\mu_0}{4\pi}\frac{M\sqrt{5}}{d^3}$

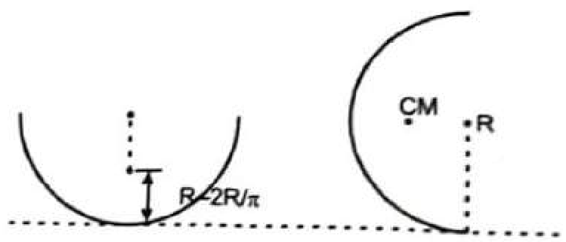
20.

$$dN = a\,dV \Rightarrow N = aV_0$$

$$V_{ave} = \frac{\int V\,dN}{N} = \frac{aV_0 \frac{V_0^2}{2}}{aV_0} = \frac{V_0}{2}$$

$$V_{rms}^2 = \frac{\int V^2 dN}{N} = \frac{\int V^2 a\,dV}{aV_0} = \frac{V_0^2}{3} \Rightarrow V_{rms} = \frac{V_0}{\sqrt{3}}$$

21.



$$\therefore \Delta PE_{max} = mgR - \left[mg\left(R-\frac{2R}{\pi}\right)\right]$$

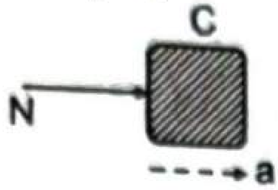
$$= mgR - mgR + mg\left(\frac{2R}{\pi}\right)$$

$$\Rightarrow \Delta PE_{max} = mg\left(\frac{2R}{\pi}\right)$$

22. Horizontal acceleration of the system is $a = \frac{F}{2m+m+2m} = \frac{F}{5m}$.

Let N be the normal reaction between B and C.

Free body diagram of C gives



Now B will not slide downwards if $\mu N \geq m_B g$

or $\quad (\mu)\left(\frac{2}{5}F\right) \geq mg$

or $\quad F \geq \frac{5}{2\mu} mg$

So, minimum value of F is $\frac{5}{2\mu} mg$

23. Angular momentum $L = I\omega = \frac{ml^2}{3}.\omega$

24. For all point out the sphere we can treat it as point mass at its centre so, effectively it will be the force between ring and point mass.

Gravitation field at $x = \sqrt{3}a$ on axis of ring is

$$E = \frac{Gmx}{\left(R^2+x^2\right)^{3/2}} = \frac{Gm\left(\sqrt{3}a\right)}{\left(a^2+3a^2\right)^{3/2}}$$

$$= \frac{\sqrt{3}Gm}{8a^2}$$

$$F = ME = \frac{\sqrt{3}GMm}{8a^2}$$

CHEMISTRY

26. CONCEPTUAL
27. CONCEPTUAL
28. M.O configuration of N_2 *is*

$$\sigma_{1s}^2 \overset{*}{\sigma}_{1s}^2 \sigma_{2s}^2 \overset{*}{\sigma}_{2s}^2 \left(\pi_{2p_x}^2 = \pi_{2p_y}^2\right) \sigma_{2p_z}^2$$

29. $\alpha = \dfrac{1-i}{1-1/n}$ *and* $\Delta T_f = ik_f m$

30. TLC is a technique used to isolate non-volatile mixtures.

31.

$$X \rightleftharpoons 2Y \qquad\qquad Z \rightleftharpoons P + Q$$

$$1 \quad 0 \qquad\qquad 1 \quad 0 \quad 0$$

$$1-\alpha \quad 2\alpha \qquad\qquad 1-\alpha \quad \alpha \quad \alpha$$

$$K_{P_1} = \frac{4\alpha^2 P_1}{1-\alpha^2} \qquad K_{P_2} = \frac{\alpha^2 P_2}{1-\alpha^2}$$

$$\frac{K_{P_1}}{K_{P_2}} = \frac{1}{9} = \frac{4P_1}{P_2} \Rightarrow \frac{P_1}{P_2} = \frac{1}{36}$$

32. 5d series member has more Δ_0 than 3d and 4d series.
33. -I is a good leaving group and also bonded to benzylic carbon.
34. Cu^{+2} *is reduced to* Cu^{+}
35.

$$CH_3 - \underset{CH_3}{\overset{CH_3}{C}} - Cl + CH_3 - \underset{CH_3}{\overset{CH_3}{C}} - O^- Na^+ \rightarrow CH_3 - \overset{CH_3}{C} - CH_2$$

$$(major)$$

36. CONCEPTUAL

37. CONCEPTUAL

38. CONCEPTUAL

39. The charge on the complex ion is not always equal to the oxidation state of the metal atom. It actually depends on the nature of ligands and oxidation state of the metal atom.

40. CONCEPTUAL

41. The purple colour of $KMnO_4$ is due to charge transfer transition

The intense colour, in most of the transition metal complexes is due to d – d transition

42. CONCEPTUAL

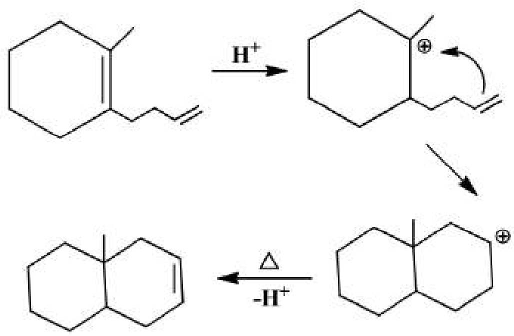
43. CONCEPTUAL

44. CONCEPTUAL

45. CONCEPTUAL

46. $\lambda_e = \frac{h}{m_e v_e}$ and $\lambda_n = \frac{h}{m_n v_n}$

47.



48.

$$\begin{array}{ccc} H_2SO_4 & & KOH \\ N_1V_1 & = & N_2V_2 \\ 0.3\times V_1 & = & 0.5\times 15 \\ V_1 & = & \frac{0.5\times 15}{0.3} = 25ml \end{array}$$

49. $\begin{matrix} CH_3 \\ CH_3 \end{matrix} \rangle CH - NH_2$, $\begin{matrix} CH_3 \\ CH_3 \end{matrix} \rangle CH - \overset{\oplus}{N}H_3\, Cl^{\ominus}$, $wt\%\ of\ Cl = 37.2\%$

A

50. $Tripeptide + 2H_2O \rightarrow 3\, moles\ of$ Amino $Acid$

$$\therefore \left(\begin{matrix} NH_2 \\ | \\ CH_2 \end{matrix} \right)_n - COOH \times 3 = 189 + 36$$

$\Rightarrow n = 1$

$X\ is\ \begin{matrix} NH_2 \\ | \\ CH_2 - COOH \end{matrix}$

MATHS

51. $a = n!$

$b = \frac{(n-2)!}{(n-12)!}$

$c = (n-12)!$

$n(n-1) = 182$

$= 14 \times 13$

$n = 14$

52. $f(x) = \left| x[x] \right|$

$f(x)$ is continuous at $x = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = f(0) = 0$

53. $f'(x) = \begin{vmatrix} 1 & 0 & 0 \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix} + \begin{vmatrix} x+a^2 & ab & ac \\ 0 & 1 & 0 \\ ac & bc & x+c^2 \end{vmatrix} + \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ 0 & 0 & 1 \end{vmatrix}$

$= (x+b^2)(x+c^2) - b^2c^2 + (x+a^2)(x+c^2) - a^2c^2 + (x+a^2)(x+b^2) - a^2b^2$

$= 3x^2 + 2x(a^2+b^2+c^2) < 0$

$f(x)$ is decreasing in

$x \in \left(\frac{-2}{3}(a^2+b^2+c^2), 0 \right)$

54. $a = A(R)^{p-1} \to \log a = \log A + (p-1)\log R$

$b = A.(R)^{q-1} \to \log b = \log A + (q-1)\log R$

$c = A.(R)^{r-1} \to \log c = \log A + (r-1)\log R$

Angle between two vectors

$\alpha.\beta = 3\sum (q-r)\log a$

$= 0$

So, angle between vectors = $\frac{\pi}{2}$

55.

$x_i(observation)$	0	2	2^2		2^n
$f_i(frequency)$	${}^nC_0$	${}^nC_1$	${}^nC_2$		${}^nC_n$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\frac{0\times^n C_0 + 2\times^n C_1 + 2^2\times^n C_2......2^n\times^n C_n}{^n C_0 +^n C_1 +^n C_1.......^n C_n} = \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$$\Rightarrow 3^n = 3^6$$

$$\Rightarrow n = 6$$

56. $a^2 - a\left(2x^2+1\right) + x^4 + x = 0$

$$a = \frac{2x^2 + 1 \pm (2x-1)}{2}$$

$a = x^2 + x$ $\quad\quad$ $a = x^2 - x + 1$

$a \ge -1/4$ $\quad\quad$ $a \ge 3/4$ $\quad\quad$ $(\because x \in R)$

57. $$[\sin x + \cos x] = \left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right] = \begin{cases} 1 & 0 \le x \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le x \le \frac{3\pi}{4} \\ -1 & \frac{3\pi}{4} < x < \pi \\ -2 & \pi < x \le \frac{3\pi}{2} \\ -1 & \frac{3\pi}{2} < x \le \frac{7\pi}{2} \\ 0 & \frac{7\pi}{4} < x \le 2\pi \end{cases}$$

$$I = 20\left[\int_0^{\pi/2} 1dx + \int_{\pi/2}^{3\pi/4} 0dx + \int_{3\pi/4}^{\pi} -1dx + \int_{\pi}^{3\pi/2} -2dx + \int_{3\pi/2}^{7\pi/4} -1dx + \int_{7\pi/4}^{2\pi} 0dx\right]$$

$$= 20\times\left[\frac{\pi}{2} - \pi + \frac{3\pi}{4} - 2\times\frac{3\pi}{2} + 2\pi - \frac{7\pi}{4} + \frac{3\pi}{2}\right] = 20\times -\pi = -20\pi$$

58. Statement I : Req. number of solutions

$$= \sum_{x=0}^{10} {}^{21-x+2-1}C_{2-1} = 132.$$

Statement II : Is true by the definition of n!.

59. $n(s) = 5^4$

$n(A) = 2.{}^5C_4 = 10$

$$P(A) = \frac{10}{625} = \frac{2}{125}$$

60. $\frac{a}{r} + a + ar = 26 \quad .....(i)$

$$\left(\frac{a}{r}\right)^2+a^2+a^2r^2=364 \qquad \ldots\text{(ii)}$$

$$a^2\left(\left(r+\frac{1}{r}\right)^2-1\right)=364$$

$$a\left(r+\frac{1}{r}\right)+a=26$$

$$\left(r+\frac{1}{r}\right)=\frac{26}{a}-1$$

$$a^2\left(\left(\frac{26}{a}-1\right)^2-1\right)=364$$

$$a^2\left[\left(\frac{26}{a}-1+1\right)\left(\frac{26}{a}-1-1\right)\right]=364$$

$$a^2\left[\frac{26}{a}\times 2\left(\frac{13-a}{a}\right)\right]=364$$

$$13-a=\frac{364}{52}$$

$$a=6$$

$$\therefore\ r=3$$

$$a_{10}=2.(3)^9$$

61. Let $f=\left(5\sqrt{2}-7\right)^{19}$

$x-f=an\,\text{integer}\Rightarrow[\text{x}]+\{x\}-f=an\,\text{integer}$

$\Rightarrow\{x\}-f=an\,\text{integer},but\ \text{-}1<+\{x\}-f<1\Rightarrow f$

So, $x\{x\}=x.f=1^{19}=1$

62. If $t=\tan\frac{x}{2}$, then $dx=\frac{2dt}{1+t^2}$

A) $a=1,\ I_1=\int_0^1\frac{dt}{2\left(1-t^2\right)+2t-\left(1+t^2\right)}$

$$I_1=-2\int_0^1\frac{dt}{t^2-2t-3}=2\int_0^1\frac{dt}{4-\left(t-1\right)^2}=\frac{1}{2}\log 3$$

Similarly, for others

63. $xy\left(x\right)=x^2y'\left(x\right)+2xy\left(x\right),\qquad xy\left(x\right)+x^2y'\left(x\right)=0$

$x\frac{dy}{dx}+y=0,\quad xy=c$

64. Taking first two $\dfrac{\tan\left(\frac{\pi}{4}+\alpha\right)}{\tan\left(\frac{\pi}{4}+\beta\right)}=\frac{5}{3}$

$\therefore \quad \dfrac{\cos(\alpha+\beta)}{\sin(\alpha-\beta)}=\dfrac{8}{2}=4$

$\therefore \quad \cos(\alpha+\beta)=4\sin(\alpha-\beta) \qquad \ldots.(\mathbf{i})$

Similarly

$\cos(\beta+\gamma)=5\sin(\beta-\gamma) \qquad \ldots.(\mathbf{ii})$

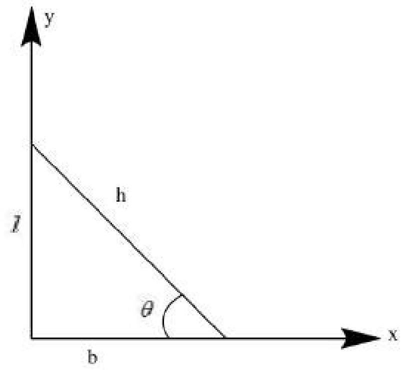
$3\cos(\gamma+\alpha)=-7\sin(\gamma-\alpha) \qquad \ldots.(\mathbf{iii})$

After marking $\sin^{-2}$ terms

$(\mathrm{i})+(\mathrm{ii})-(\mathrm{iii})$

Then $\quad 12\sin^2(\alpha-\beta)+15\sin^2(\beta-\gamma)-7\sin^2(\gamma-\alpha)=0$

65.



$\dfrac{db}{dt}=5\,ft/\sec$, when $b=12$ then $l=5$

$h^2=l^2+b^2$

$b=h\cos\theta$

$\dfrac{db}{dt}=-h\sin\theta\dfrac{d\theta}{dt}$

$5=-l\dfrac{d\theta}{dt}$

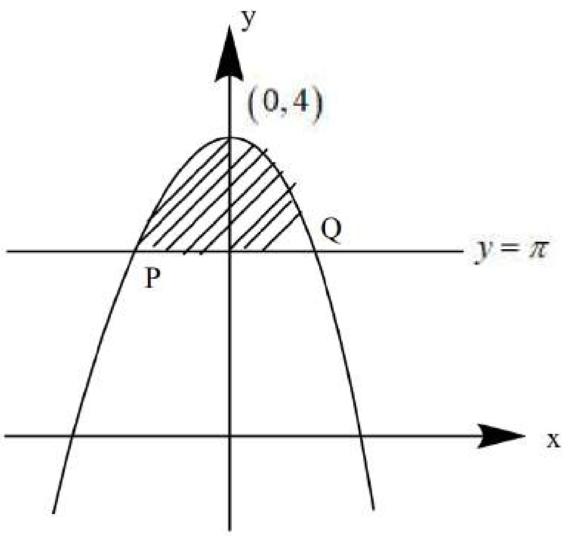
$\dfrac{d\theta}{dt}=-1\,rad/\sec$

66. Required probability $=P(I')P(II)+P(I')P(II')P(I')P(II)+\ldots\ldots$

$=0.7\times0.2+0.7\times0.8\times0.2+\ldots\ldots$

$=\dfrac{0.14}{1-0.56}=0.312$

67.



$y-\sec^{-1}\left[-\sin^2 x\right]=\pi$

$A=\int_{-a}^{a}\left(\frac{16-x^2}{4}-\pi\right)dx$ where $a=2(4-\pi)^{\frac{1}{2}}$

68. $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{e^{0+h}-3}{0+x+1}=-2$

$\lim_{x\to 0^-} f(x)=\lim_{h\to 0}\frac{e^{-1+h}-3}{-1-h+1}=\lim_{h\to 0}\frac{e^{-1+h}-3}{-h}=+\infty$

69. $f''(x)>0$

$\Rightarrow f'$ is inc.fn

To find : where g is nec.Inc

G is inc $\Rightarrow g'>0$

$\Rightarrow \frac{1}{4}.f'(2x^2-1)(4x)+\frac{1}{2}f'(1-x^2)(-2x)>0$

$\Rightarrow x\{f'(2x^2-1)-f'(1-x^2)\}>0$

Case I: $x>0\to(1)\ f'(2x^2-1)>f'(1-x^2)$

$\Rightarrow 2x^2-1>1-x^2$

$\Rightarrow x\in\left(-\infty,-\sqrt{\frac{2}{3}}\right)\cup\left(\sqrt{\frac{2}{3}},\infty\right)\to(2)$

$(1)\cap(2)\Rightarrow x\in\left(\sqrt{\frac{2}{3}},\infty\right)$ ....(3)

Case II: $x<0\to(3)\ f'(2x^2-1)<f'(1-x^2)$

$\Rightarrow 2x^2-1<1-x^2$

$\Rightarrow x\in\left(-\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}}\right)\to(4)$

$(3)\cap(4)\Rightarrow\in\left(-\sqrt{\frac{2}{3}},0\right)\to(6)$

$\therefore g\, is\, inc\, in\, x\in(5)\cup(6)$

$$\Rightarrow x \in \left(-\sqrt{\frac{2}{3}}, 0\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$$

70. $\tan^{-1}(x) = t$

$t^2 - 4t + 3 > 0$

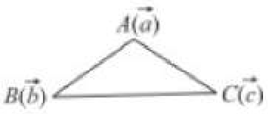
$t \in (-\infty, 1) \cup (3, \infty)$

$\tan^{-1}(x) \in \left(-\frac{\pi}{2}, 1\right)$

$x \in (-\infty, \tan 1)$

Largest integral $x = 1$

71.



$\vec{a} + 2\vec{b} + 3\vec{c} = 0$

Area of $\Delta ABC = \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

$\vec{a} \times \vec{b} = -\frac{3(\vec{a} \times \vec{c})}{2}$

$\vec{b} \times \vec{c} = -\frac{\vec{a} \times \vec{c}}{2}$

$\vec{c} \times \vec{a} = -\vec{a} \times \vec{c}$

Area of $\Delta ABC = \frac{1}{2} |-3(\vec{a} \times \vec{c})| = \frac{3}{2} |\vec{a} \times \vec{c}|$

$\Delta AOC = \frac{1}{2} |\vec{a} \times \vec{c}|$

$\frac{arc\ \ \Delta ABC}{arc\ \ \Delta AOC} = 3$

72. $C_1 \equiv (1,1), C_2 \equiv (9,6)$, $r_1 = 1, r_2 = 2$

$C_1M_1 \geq r_1$

$C_2M_2 \geq r_2$

$\left|\frac{3+4-k}{5}\right| \geq 1$

$|7-k| \geq 5$ $\qquad$ C_1 is below the line $3x + 4y - k = 0$

$k - 7 \geq 5$ $\qquad$ $7 - k < 0$

$k \geq 12$ $\qquad$ ...(i)

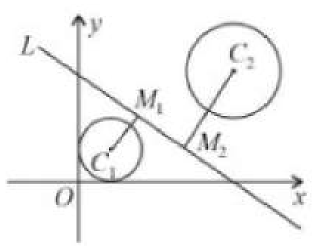
$|27 + 24 - k| \geq 10$ $\qquad$ C_2 lies above the line $3x + 4y - k = 0$

$51 - k \geq 10$ $\qquad$ $51 - k > 0$

$k \leq 41$ $\qquad$ ...(ii)

From (i) and (ii)

$k \in [12, 41]$ $\qquad$ Number of integral values = 30



73. Equating the components

$3x + 2y + 3z = \alpha x, 2x + 2z = \alpha y$ and $4x + 2y + 3z = \alpha z$

Hence,

$$(3-\alpha)x+2y+4z=0$$
$$2x-\alpha y+2z=0$$
$$4x+2y+(3-\alpha)z=0$$

For non-trivial solution $\begin{vmatrix} 3-\alpha & 2 & 4 \\ 2 & -\alpha & 2 \\ 4 & 2 & 3-\alpha \end{vmatrix}=0$

Hence $\alpha=-1 \ or \ 8 \Rightarrow sum=7$

74. Curve is rectangular hyperbola

$$P\left(n,\sqrt{n^2+1}\right) \Rightarrow d_n=\left|\frac{n-\sqrt{n^2+1}}{\sqrt{2}}\right|$$

$$\Rightarrow \lim_{n\to\infty} n.d_n=\frac{1}{2\sqrt{2}} \Rightarrow K=2$$

75. $f'(x)=0$

$$f'(x)=\int_0^x 3^t\left(3^t-4\right)dt+x.3^x\left(3^x-4\right)-3^x.x\left(3^x-4\right)$$

$$f'(x)=\frac{1}{2\ln 3}\left(3^{2x}-8.3^x+7\right)$$

$$f'(x)=\frac{1}{2\ln 3}\left(3^x-1\right)\left(3^x-7\right)$$

$x=\log_3 7$ is the point of minima

$$\Rightarrow 3^a=3^{\log_3 7}=7$$