CHAPTER 4

GEOMETRY

9RKSMH

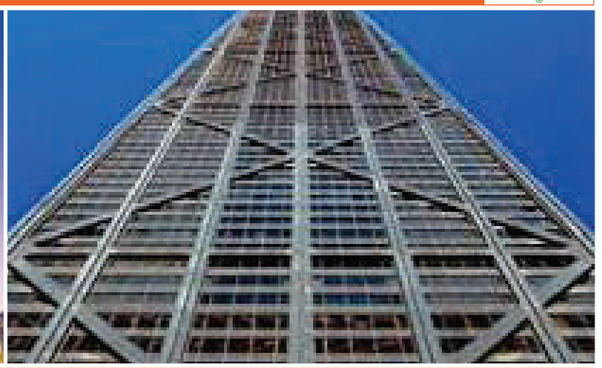
Learning Objectives

- To understand the formation of triangles and the basic elements of a triangle.
- To know the types of triangles and their properties.
- To draw parallel and perpendicular lines using a set square.

4.1 Introduction

We already studied the basic geometrical concepts such as angles and its types, drawing line segments, drawing and measuring angles in the first term. In this term, we will study triangles and their types, construction of parallel and perpendicular lines to a given line segment.

MATHEMATICS ALIVE – TRIANGLES IN REAL LIFE



The triangle is used in most types of construction work including bridges, buildings, cell-phone towers, aeroplane wings and pitched roofs. Its use in construction gives an object the quality of stiffness, resulting in rigid and strong structures.

Think about the situation:

A teacher distributes 2, 3, 4 and 5 sticks of equal lengths to four students and asks them to form a closed figure. Three students make the following figures.

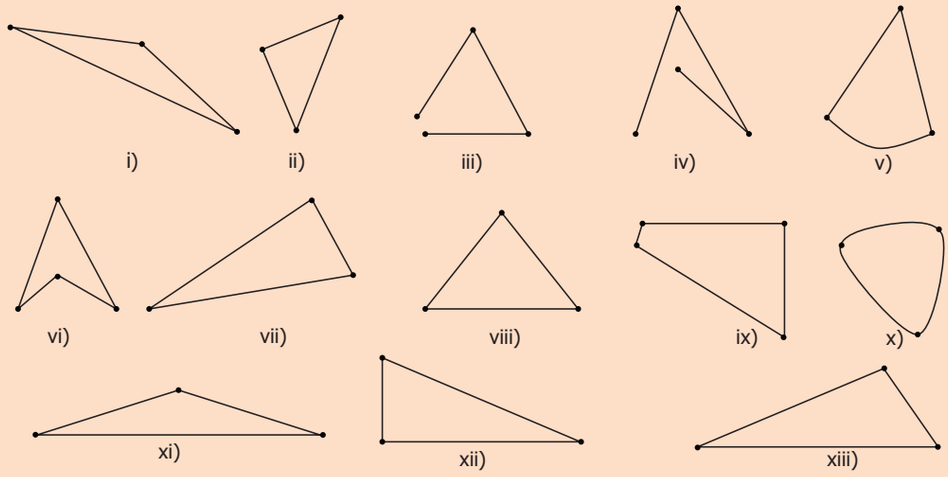
But one of the students who has 2 sticks with him creates the following figure.

He is not able to create a closed figure. Do you know why? Can you guess the least number of sticks required to form a closed figure? Three sticks. If you had formed a closed figure with three sticks, then what shape would you get? Is there any special name for it? Yes. Its ***triangle.***

A closed figure formed by three line segments is called a triangle.

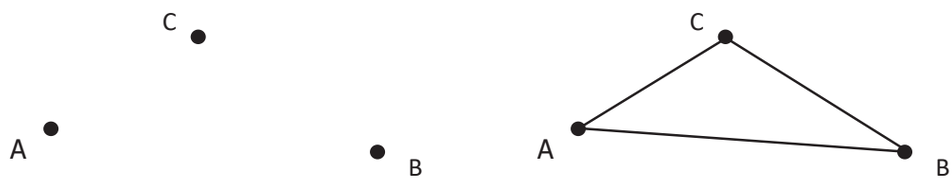
Activity

Classify the given shapes into triangles and non triangles.



4.2 Basic Elements of a Triangle

Mark 3 points A, B, C on a paper, such that they do not lie on a straight line. Join the line segments AB, BC and CA.

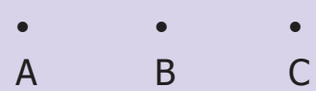


This forms a ***triangle ABC*** represented as ***∆ ABC or ∆BCA or ∆CAB.***

In ∆ ABC, the line segments **AB, BC** and **CA** are called the ***sides of the triangle*** and ***∠CAB, ∠ABC*** and ***∠BCA (∠A, ∠B & ∠C)*** are called the ***angles of the triangle.*** The point of intersection of two sides of the triangle is called the ***vertex.*** **A, B** and **C** are ***three vertices of Δ ABC.*** Hence, ***a triangle has 3 sides, 3 angles and 3 vertices.***

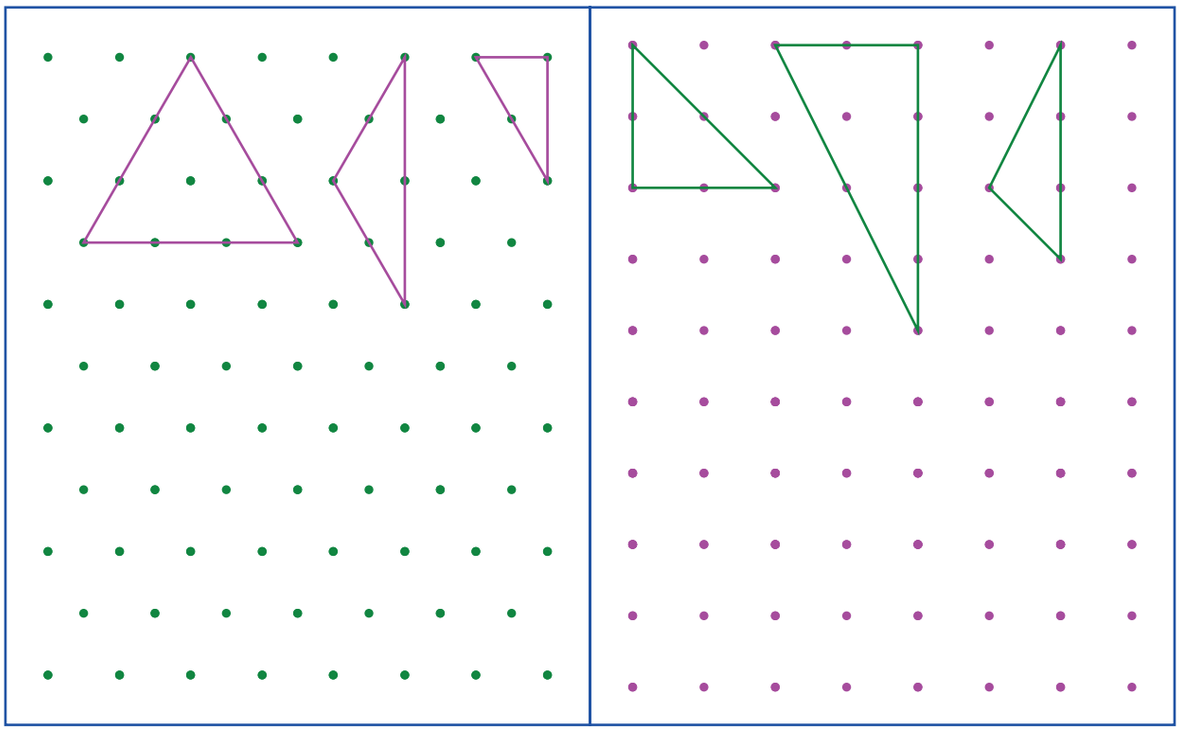
Think

Can a triangle be drawn using 3 points on a straight line?



4.3 Types and Properties of Triangles

Some triangles are drawn in the dotted sheet. Try to draw as many triangles as you can. Then, measure the sides and angles of all triangles and fill the table given below



S. No	Measure of angles	Sum of the measure of angles	Nature of angles	Measure of sides	Nature of Sides
1	60° , 60°, 60°	180°	Three angles are equal	3 cm, 3 cm, 3cm	Three sides are equal

From the table, we observe the following:

In a triangle,

- If the measure of all angles are different, then all sides are different.
- If the measure of two angles are equal, then two sides are equal.
- If the measure of three angles are equal, then three sides are equal and each angle measures 60°.
- Sum of three angles of a triangle is 180°.

Activity

Students are divided into groups and each group is given 3 sticks of length 9 units, 2 sticks of length 3 units, 2 sticks of length 2 units, 1 stick of length 5 units and 1 stick of length 4 units. Using the given sticks they are asked to form three triangles, find the length of the sides of each triangle and tabulate them.

Triangle	Length of side 1	Length of side 2	Length of side 3	All sides are equal / 2 sides are equal / 3 sides are different
1				
2				
3				

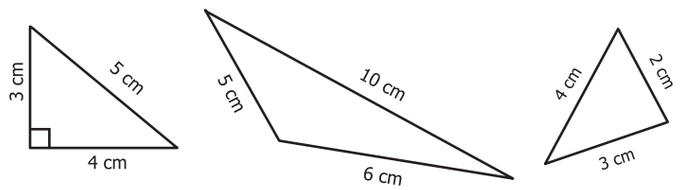
Read the table and answer the following questions.

1. Was each group able to form 3 triangles?
2. In each of the triangle formed, how many sides are equal?

4.3.1 Types of triangle based on its sides

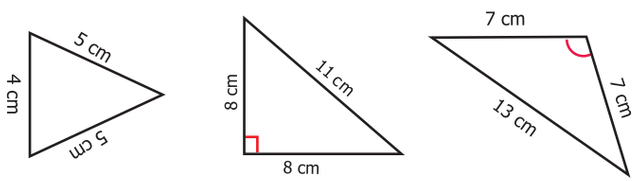
i) If three sides of a triangle are different in lengths, then it is called a ***Scalene Triangle***

Examples:



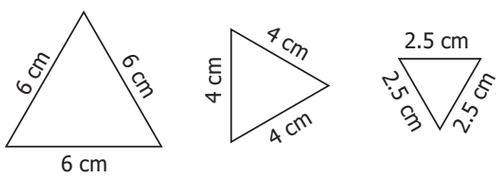
ii) If any two sides of a triangle are equal in length, then it is called an ***Isosceles Triangle***

Examples:



iii) If three sides of a triangle are equal in length, then it is called an ***Equilateral Triangle***

Examples:



Thus, based on the sides of triangles, we can classify triangles into 3 types.

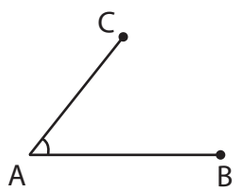
Try these

Complete the following table. In any triangle,

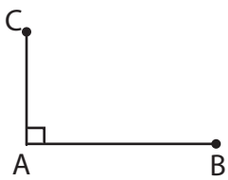
Sl. No	Side 1	Side 2	Side 3	Type of Triangle
1.	6cm	7cm	8cm	Scalene Triangle
2.	5cm	5cm	5cm	
3.	2.2cm	2.5cm	3.2cm	
4.	7cm	7cm	10cm	
5.	10cm	10cm	10cm	
6.	10cm	8cm	8cm	

4.3.2 Types of triangle based on its angles

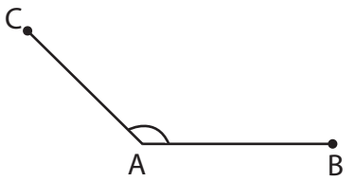
Write the given angles as acute, obtuse or right angle formed by two line segments AB and AC



∠A is _____

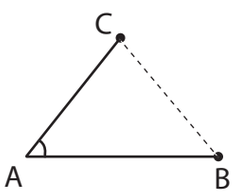


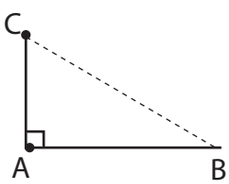
∠A is _____

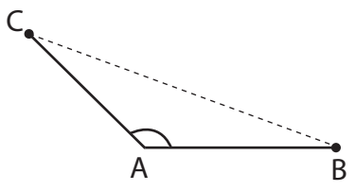


∠A is _____

Now, join the third side to form a triangle in each case and identify the kinds of angles and list them down.







∠A is _____ ∠A is _____ ∠A is _____

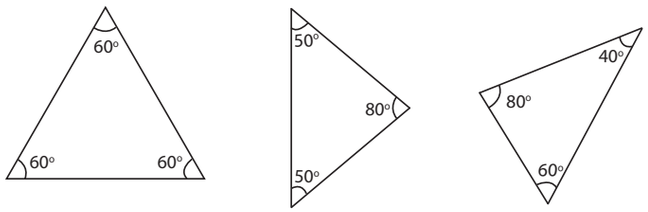
∠B is _____ ∠B is _____ ∠B is _____

∠C is _____ ∠C is _____ ∠C is _____

Now carefully look at these three triangles,

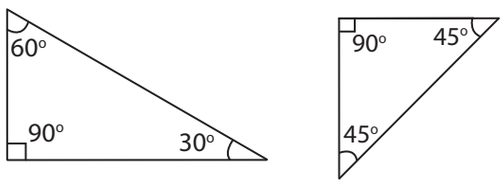
i) If three angles of a triangle are acute angles (between 0° and 90°), then it is called an ***Acute Angled Triangle.***

Examples:



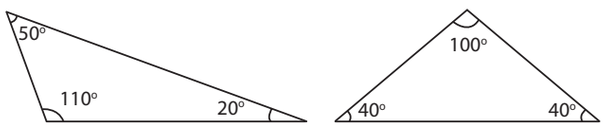
ii) If an angle of a triangle is a right angle (90°), then it is called a ***Right Angled Triangle.***

Examples:



iii) If an angle of a triangle is an obtuse angle (between 90° and 180°),then it is called an ***Obtuse Angled Triangle.***

Examples:



Thus, based on the angles of triangles, we can classify triangles into 3 types.

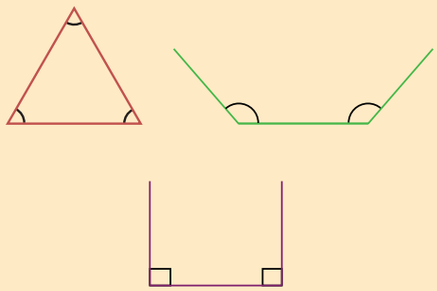
Try these

Complete the table

S.No.	∠ A	∠ B	∠ C	Sum of three angles	Can a △ABC be formed?	Type of Triangle
1	60°	60°	60°	180°	Yes	Acute angled triangle
2	50°	40°	90°			
3	60°	30°	90°			
4	95°	40°	35°			
5	110°	40°	30°			
6	150°	60°	70°			

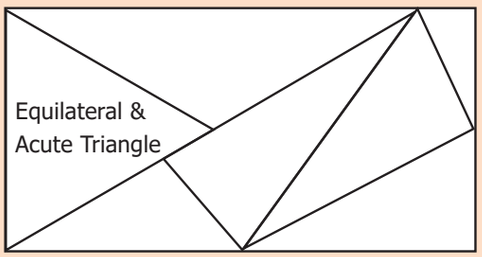
DO YOU KNOW?

A triangle can have three acute angles, but cannot have more than one right angle or an obtuse angle.



Activity

In the given figure, there are some triangles. Measure their sides and angles and name them in two ways. (One is done for you!)



4.3.3 Triangle Inequality property

Think about the situation:

Three students Kamala, Madhan and Sumathi are asked to form triangles with the given sticks of measure **6cm, 8cm, 5cm; 4cm, 10cm, 5cm** and **10cm, 6cm, 4cm** respectively. All of them try to form a triangle. While Kamala, the first girl is successful in forming a triangle, Madhan and Sumathi, next to Kamala are struggling. Why?

When they are trying to join the ends of the two smaller sticks, they find that the two smaller sticks coincide with the longer stick or shorter than the longer stick and they are unable to form triangles. From this, they understand that,

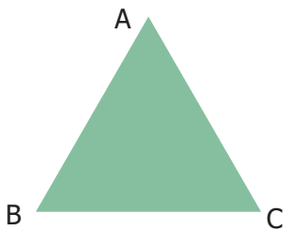
To form a triangle the sum of two smaller sides must be greater than the third side. Thus,

In a triangle, the sum of any two sides of a triangle is greater than the third side. This is known as Triangle Inequality property.

$AB + BC > CA$

$BC + CA > AB$

$CA + AB > BC$



Note

If three sides are equal in length, then definitely a triangle can be formed

DO YOU KNOW?

If any two sides of the triangle are given, then the length of the third side will lie between the difference and sum of the lengths of two given sides.

Example 1: Can a triangle be formed with 7 cm, 10 cm and 5 cm as its sides?

Solution: Instead of checking triangle inequality by all the sides in the triangle, check only with two smaller sides.

Sum of two smaller sides of the triangle = 5+7=12 cm > 10 cm, the third side.

It is greater than the third side.

So, a triangle can be formed with the given sides.

Example 2: Can a triangle be formed with 7 cm, 7 cm and 7 cm as its sides?

Solution: If three sides are equal, then definitely a triangle can be formed, as the triangle inequality is satisfied.

Example 3: Can a triangle be formed with 8 cm, 3 cm and 4 cm as its sides?

Solution: The sum of two smaller sides = 3+4=7 cm <8 cm, the third side.

It is less than the third side.

So, a triangle cannot be formed with the given sides.

Try these

Can a triangle be formed with the given sides? If yes, state the type of triangle formed.

S.No.	$\overline{AB}$	$\overline{BC}$	$\overline{CA}$	Can a ΔABC be formed?	Type of triangle
1	7 cm	10 cm	6 cm		
2	10 cm	8 cm	8 cm		
3	8.5 m	7.3 m	6.8 m		
4	4 cm	5 cm	12 cm		
5	15 m	20 m	20 m		
6	23 cm	20 cm	18 cm		
7	3.2 cm	1.5 cm	1.5 cm		

Example 4: Can a triangle be formed with the angles 80°, 30°, 40°?

Solution: The sum of three angles = 80°+ 30°+ 40° = 150° **(not equal to 180°)**

In a triangle, the sum of three angles is 180°.

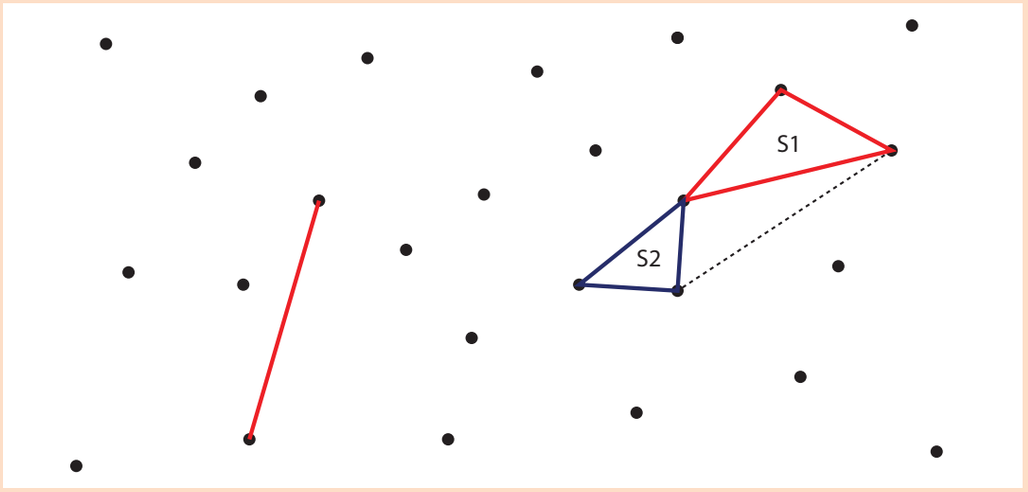
So, a triangle cannot be formed with the given angles.

Think

Can the difference between two larger sides be less than the third side?

Activity

A triangle game : In each turn a student must draw one line connecting two dots. A line should not cross other lines or touch other dots than the two that are connected to. If a student closes a triangle with his line then he gets a point. Once there are no more lines that can be drawn the game is over and the student who gains more points wins the game.



Think

In a right angled triangle, what measures can the other two angles have?

Exercise 4.1

1. Fill in the blanks:

a) Every triangle has at least _____ acute angles.

b) A triangle in which none of the sides equal is called a _____.

c) In an isosceles triangle _____angles are equal.

d) The sum of three angles of a triangle is _____.

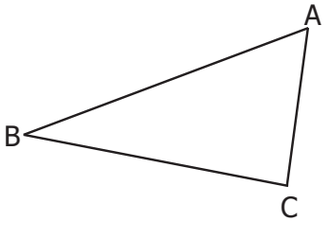
e) A right angled triangle with two equal sides is called _____.

2. Match the following:

(i)	No sides are equal	-	Isosceles triangle
(ii)	One right angle	-	Scalene triangle
(iii)	One obtuse angle	-	Right angled triangle
(iv)	Two sides of equal length	-	Equilateral triangle
(v)	All sides are equal	-	Obtuse angled triangle

9RUNP5

3. In ΔABC, name the

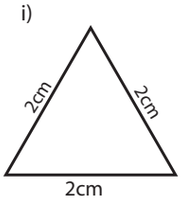


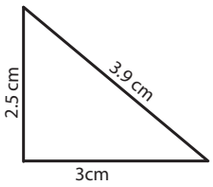
a) Three sides: _____ , _____ , _____

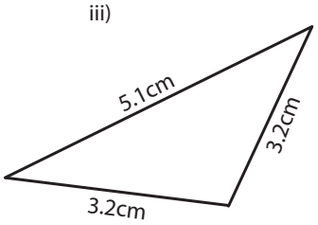
b) Three Angles: _____ , _____ , _____

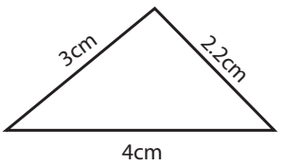
c) Three Vertices: _____ , _____ , _____

4. Classify the given triangles based on its sides as scalene, isosceles or equilateral.

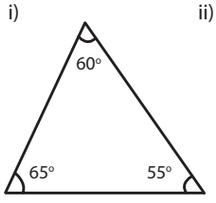
i) 

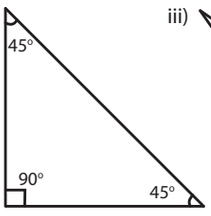
ii) 

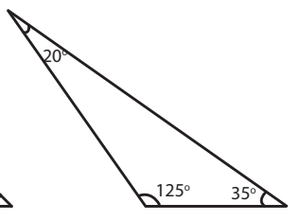
iii) 

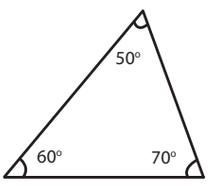
iv) 

5. Classify the given triangles based on its angles as acute angled, right angled or obtuse angled.

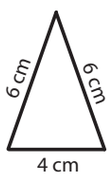
i) 

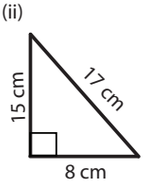
ii) 

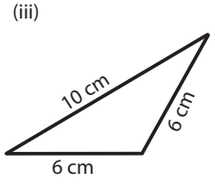
iii) 

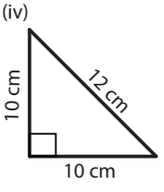
iv) 

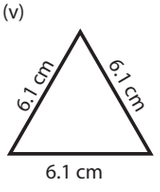
6. Classify the following triangles based on its sides and angles.

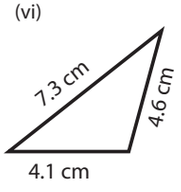
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) 

7. Can a triangle be formed with the following sides? If yes, name the type of triangle.

(i) 8 cm, 6 cm, 4 cm

(ii) 10 cm, 8 cm, 5 cm

(iii) 6.2 cm, 1.3 cm, 3.5 cm

(iv) 6 cm, 6 cm, 4 cm

(v) 3.5 cm, 3.5 cm, 3.5 cm

(vi) 9 cm, 4 cm, 5 cm

8. Can a triangle be formed with the following angles? If yes, name the type of triangle.

(i) 60°, 60°, 60°

(ii) 90°, 55°, 35°

(iii) 60°, 40°, 42°

(iv) 60°, 90°, 90°

(v) 70°, 60°, 50°

(vi) 100°, 50°, 30°

9. Two angles of the triangles are given. Find the third angle.

(i) 80°, 60°

(ii) 75°, 35°

(iii) 52°, 68°

(iv) 50°, 90°

(v) 120°, 30°

(vi) 55°, 85°

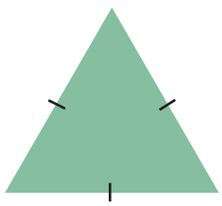
10. I am a closed figure with each of my three angles is 60°. Who am I?

11. Using the given information, write the type of triangle in the table given below

S.No.	∠1	∠2	∠3	Type of triangle based on angles	Type of triangle based on sides
i.	60°	40°	80°	Acute angled triangle.	Scalene Triangle
ii.	50°	50°	80°		
iii.	45°	45°	90°		
iv.	55°	45°	80°		
v.	75°	35°	70°		
vi.	60°	30°	90°		
vii.	25°	64°	91°		
viii.	120°	30°	30°		

Objective Type Questions

12. The given triangle is_____.



a) a right angled triangle b) an equilateral triangle

c) a scalene triangle d) an obtuse angled triangle

13. If all angles of a triangle are less than a right angle, then it is called _____.

a) an obtuse angled triangle b) a right angled triangle

c) an isosceles right angled triangle d) an acute angled triangle

14. If two sides of a triangle are 5 cm and 9 cm, then the third side is_____.

a) 5 cm b) 3 cm c) 4 cm d) 14 cm

15. The angles of a right angled triangle are

a) acute, acute, obtuse b) acute, right, right

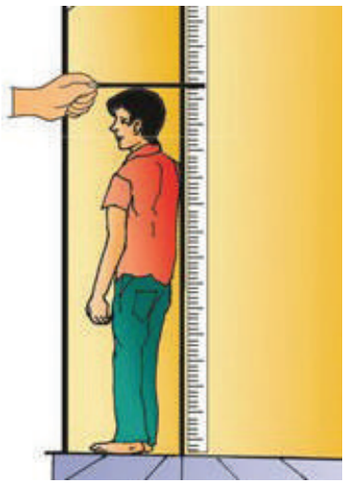
c) right, obtuse, acute d) acute, acute, right

16. An equilateral triangle is

a) an obtuse angled triangle b) a right angled triangle

c) an acute angled triangle d) a scalene triangle

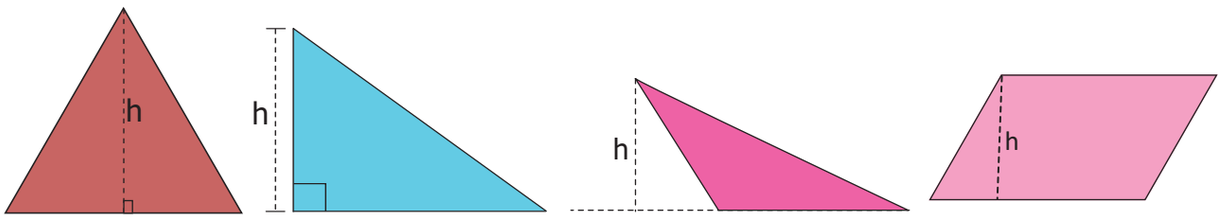
4.4 Construction of Perpendicular Lines



4.4.1 Introduction

Have you ever noticed that the wall and floor are always perpendicular to each other? So, to measure our heights, we make use of scale represented on the walls as shown in the figure.

In Geometry, to measure the height of figures, we use perpendicular lines .Using a set square, find the height of the given figures.



h=_____cm h=_____cm h=_____cm h=_____cm

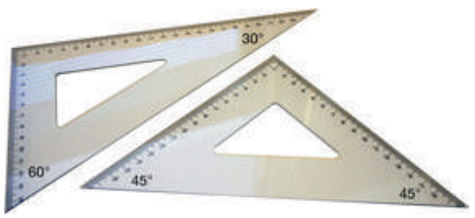
Let us learn to construct perpendicular lines by using set square.

4.4.2 Set Squares

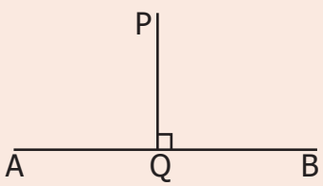
The set squares are two triangle shaped instruments in the Geometry Box. Each of them has a right angle. One set square has the angles 30°, 60°, 90° and the other set square has the angles 45°, 45°, 90°. The perpendicular edges are graduated in centimetres.

Set squares have several uses:

- To construct the specific angles 30°, 45°, 60°, 90°
- To draw parallel and perpendicular lines
- To measure the height of the shapes

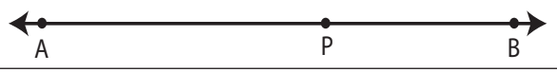
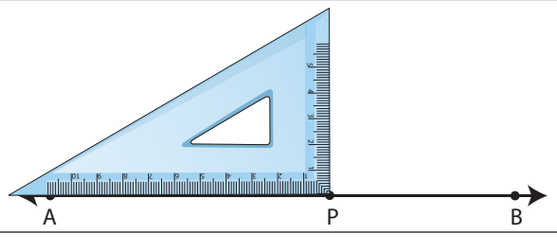
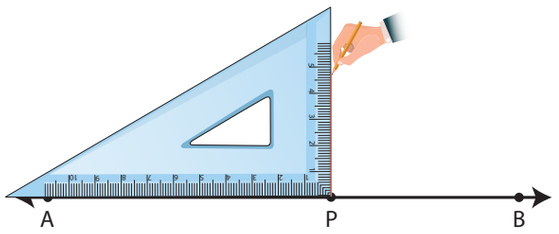
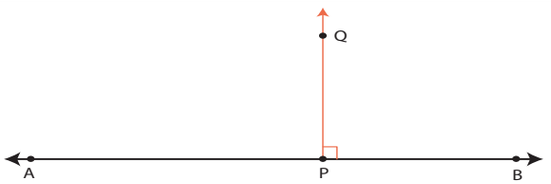


DO YOU KNOW?

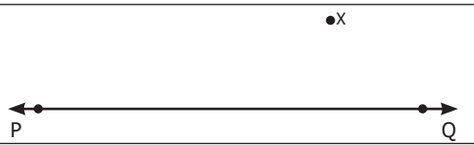
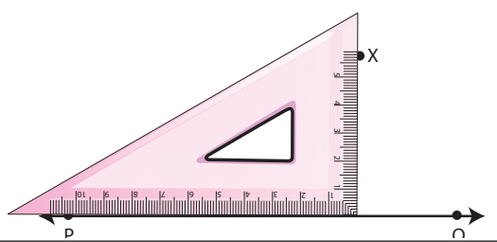
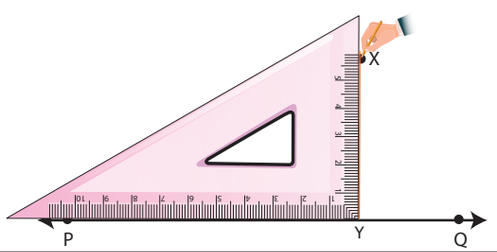
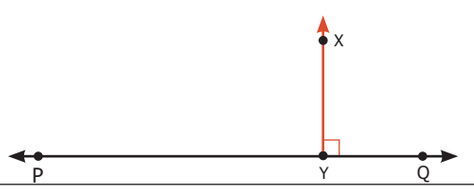


If the perpendicular from P meets AB at Q, the point Q is called the foot of the perpendicular from P to AB and the symbol "⊥" means "is perpendicular to". i.e., PQ ⊥ AB

Example 5: Construct a line perpendicular to the given line at a point on the line.

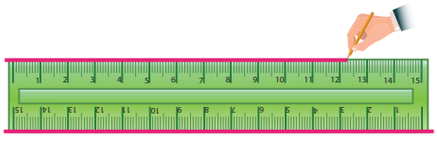
Step 1: Draw a line AB and take a point P anywhere on the line.	
Step 2: Place the set square on the line in such a way that the vertex which forms right angle coincides with P and one arm of the right angle coincides with the line AB.	
Step 3: Draw a line PQ through P along the other arm of the right angle of the set square.	
Step 4: The line PQ is perpendicular to the line AB at P. That is, PQ ⊥ AB and ∠APQ= ∠BPQ=90°.	

Example 6: Construct a line perpendicular to the given line through a point above it.

Step 1: Draw a line PQ. Take a point X anywhere above the line PQ.	
Step 2: Place one of the arms of the right angle of a set square along the line PQ and the other arm of its right angle touches the point X.	
Step 3: Draw a line through the point X meeting PQ at Y.	
Step 4: The line XY is perpendicular to the line PQ at Y. That is, XY ⊥ PQ	

4.5 Construction of Parallel Lines

Place a scale on a paper and draw lines along both the edges of the scale as shown.



Place the set square at two different points on ℓ_1 and find the distance between ℓ_1 and ℓ_2. Are they equal? Yes. Thus, the perpendicular distance between a set of parallel lines remains the same.

Note

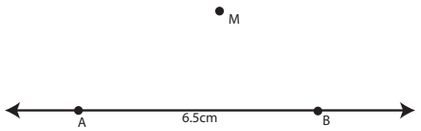
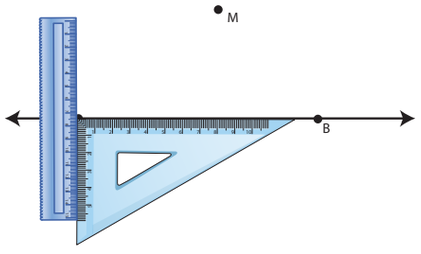
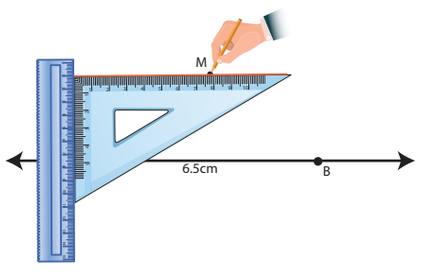
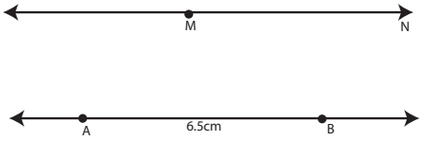
Parallel line segments need not be of equal length

Think

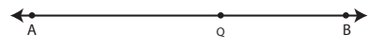
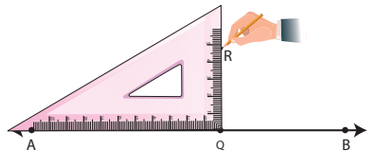
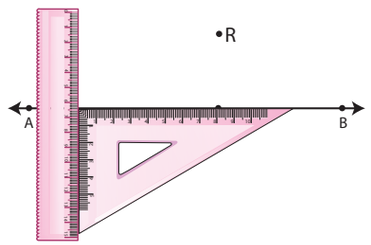
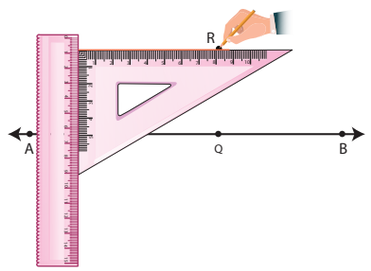
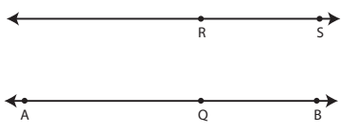
Identify the parallel lines in English alphabets (Capital Letters) and list the letters.

Examples: E W

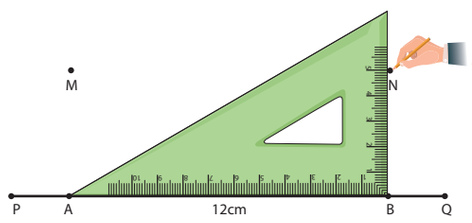
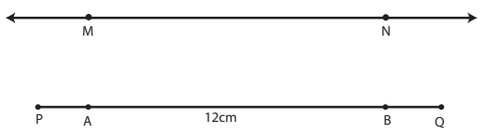
Example 7: Draw a line segment AB = 6.5 cm and mark a point M above it. Through M draw a line parallel to AB.

Step 1: Draw a line. Mark two points A and B on the line such that AB = 6.5 cm. Mark a point M anywhere above the line.	
Step 2: Place the set square below AB in such a way that one of the edges that form a right angle lies along AB. Place the scale along the other edge of the set square as shown in the figure.	
Step 3: Holding the scale firmly, Slide the set square along the edge of the scale until the other edge of the set square reaches the point M. Through M draw a line as shown.	
Step 4: The line MN is parallel to AB. That is, MN ∥ AB	

Example 8: Draw a line and mark a point R at a distance of 4.8 cm above the line. Through R draw a line parallel to the given line.

Step 1: Using a scale draw a line AB and mark a point Q on the line.	
Step 2: Place the set square in such a way that the vertex of the right angle coincides with Q and one of the edges of right angle lies along AB. Mark the point R such that QR = 4.8 cm.	
Step 3: Place the scale and the set square as shown in the figure.	
Step 4: Hold the scale firmly and slide the set square along the edge of the scale until the other edge touches the point R. Draw a line RS through R.	
Step 5: The line RS is parallel to AB. That is, RS ∥ AB.	

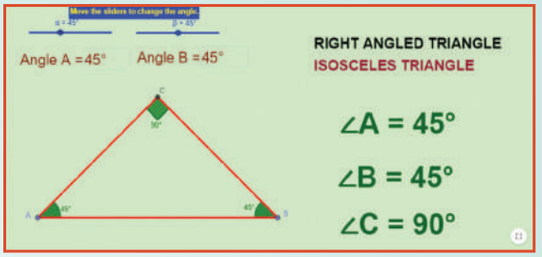
Example 9: Draw a line segment PQ = 12 cm. Mark two points M, N at a distance of 5 cm above the line segment PQ. Through M and N draw a line parallel to PQ.

Step 1: Using a scale, draw a line segment PQ = 12 cm. Mark two points A and B on the line segment.	
Step 2: Using the set square as shown, mark points M and N such that AM = BN = 5 cm.	
Step 3: Using the scale, join M and N. MN is parallel to PQ. That is, MN ∥ PQ.	

GEOMETRY

ICT CORNER

Expected Outcome



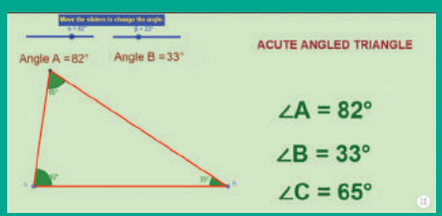
Step 1

Open the Browser and type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Geometry" will open. The work sheet contains three activities. 1. Types of triangles, 2. Perpendicular line construction and 3. Parallel line construction.

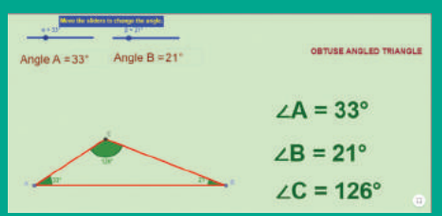
In the first activity move the sliders or enter the angle to change the Angles of the triangle and check what type of triangle is it and compare with the angles.

Step 2

In the second and third activity you can learn how to draw Perpendicular and parallel lines through a Video.



Step1



Step1

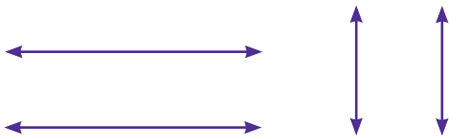
Browse in the link:

Geometry: https://ggbm.at/dPXHSSTF or Scan the QR Code.

B442_MAT_6_T2_EM

Exercise 4.2

1. Draw a line segment AB = 7 cm and mark a point P on it. Draw a line perpendicular to the given line segment at P.
2. Draw a line segment LM = 6.5 cm and take a point P not lying on it. Using a set square construct a line perpendicular to LM through P.
3. Find the distance between the given lines using a set square at two different points on each of the pairs of lines and check whether they are parallel.

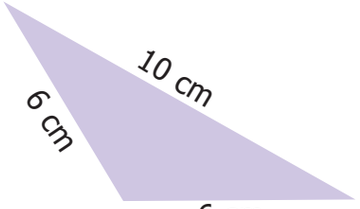


4. Draw a line segment measuring 7.8 cm. Mark a point B above it at a distance of 5 cm. Through B draw a line parallel to the given line segment.
5. Draw a line and mark a point R below it at a distance of 5.4 cm Through R draw a line parallel to the given line.

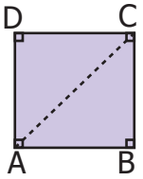
Exercise 4.3

Miscellaneous Practice Problems

1. What are the angles of an isosceles right angled triangle?
2. Which of the following correctly describes the given triangle?



   (a) It is a right isosceles triangle.
   (b) It is an acute isosceles triangle.
   (c) It is an obtuse isosceles triangle.
   (d) It is an obtuse scalene triangle.
3. Which of the following is not possible?
   (a) An obtuse isosceles triangle (b) An acute isosceles triangle
   (c) An obtuse equilateral triangle (d) An acute equilateral triangle
4. If one angle of an isosceles triangle is 124°, then find the other angles.
5. The diagram shows a square ABCD. If the line segment joins A and C, then mention the type of triangles so formed.



6. Draw a line segment AB of length 6 cm. At each end of this line segment AB, draw a line perpendicular to the line AB. Are these lines parallel?

Challenge Problems

7. Is a triangle possible with the angles 90°, 90° and 0°? Why?
8. Which of the following statements is true? Why?
   (a) Every equilateral triangle is an isosceles triangle.
   (b) Every isosceles triangle is an equilateral triangle.

9. If one angle of an isosceles triangle is 70°, then find the possibilities for the other two angles.

10. Which of the following can be the sides of an isosceles triangle?

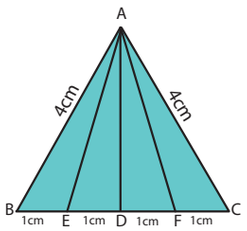
a) 6cm, 3cm, 3cm b) 5cm, 2cm, 2cm c) 6cm, 6cm, 7cm d) 4cm, 4cm, 8cm

11. Study the given figure and identify the following triangles.

(a) equilateral triangle (b) isosceles triangles

(c) scalene triangles (d) acute triangles

(e) obtuse triangles (f) right triangles



12. Two sides of the triangle are given in the table. Find the third side of the triangle.

Sl. No.	Side - 1	Side - 2	The length of the third side (any three measures)
i.	7 cm	4 cm	
ii.	8 cm	8 cm	
iii.	7.5 cm	3.5 cm	
iv.	10 cm	14 cm	

13. Complete the following table:

Types of Triangle / Its Angles	Acute angled triangle	Right angled triangle	Obtuse angled triangle
Any two angles	Always acute angles	i.	Always acute angles
Third angle	ii.	Right angle	iii.

Summary

- A closed figure formed by three line segments is called a triangle.
- A triangle has 3 sides, 3 angles and 3 vertices.
- Based on the sides of triangles, we can classify triangles into 3 types as scalene triangle, isosceles triangle and equilateral triangle.
- Based on the angles of triangles, we can classify triangles into 3 types as acute angled triangle, right angled triangle and obtuse angled triangle.
- In a triangle, the sum of any two sides is greater than the third side. This is known as Triangle Inequality property.
- Sum of three angles of a triangle is 180°.
- Parallel and Perperndicular lines can easily be drawn using set squares.
- The distance between a set of parallel lines always remains the same.