Chapter 19. Mean and Median (For Ungrouped Data Only)

Exercise 19(A)

Solution 1:

The numbers given are $43, 51, 50, 57, 54$

The mean of the given numbers will be

$$= \frac{43+51+50+57+54}{5}$$

$$= \frac{255}{5}$$

$$= 51$$

Solution 2:

The first six natural numbers are $1, 2, 3, 4, 5, 6$

The mean of first six natural numbers

$$= \frac{1+2+3+4+5+6}{3}$$

$$= \frac{21}{3}$$

$$= 3.5$$

Solution 3:

The first ten odd natural numbers are $1, 3, 5, 7, 9, 11, 13, 15, 17, 19$

The mean of first ten odd numbers

$$= \frac{1+3+5+7+9+11+13+15+17+19}{10}$$

$$= \frac{100}{10}$$

$$= 10$$

Solution 4:

The all factors of 10 are $1, 2, 5, 10$

The mean of all factors of 10 are

$$= \frac{1+2+5+10}{4}$$

$$= \frac{18}{4}$$

$$= 4.5$$

Solution 5:

The given values are $x+3, x+5, x+7, x+9, x+11$

The mean of the values are

$$= \frac{x+3+x+5+x+7+x+9+x+11}{5}$$

$$= \frac{5x+35}{5}$$

$$= \frac{5(x+7)}{5}$$

$$= x+7$$

Solution 6:

(i)The given numbers are $9.8, 5.4, 3.7, 1.7, 1.8, 2.6, 2.8, 8.6, 10.5, 11.1$

$$\bar{x} = \frac{x1 + x2 + x3 + x4 + x5 + \ldots\ldots + xn}{n}$$

$$= \frac{9.8 + 5.4 + 3.7 + 1.7 + 1.8 + 2.6 + 2.8 + 8.6 + 10.5 + 11.1}{10}$$

$$= 5.8$$

(ii) The value of $\sum_{i=1}^{10}\left(x_i - \bar{x}\right)$

We know that

$$\sum_{i=1}^{n}\left(xi - \bar{x}\right) = \left(x1 - \bar{x}\right) + \left(x2 - \bar{x}\right) \ldots\ldots\ldots + \left(xn - \bar{x}\right) = 0$$

Here

$$\bar{x} = 5.8$$

Therefore

$$\sum_{i=1}^{10}\left(x_i - \bar{x}\right)$$

$$= (9.8-5.8)+(5.4-5.8)+(3.7-5.8)+(1.7-5.8)+(1.8-5.8)$$
$$+(2.6-5.8)+(2.8-5.8)+(8.6-5.8)+(10.5-5.8)+(11.1-5.8)$$
$$= 4-.4-2.1-4.1-4-3.2-3+2.8+4.7+5.3$$
$$= 0$$

Solution 7:

Given that the mean of 15 observations is 32

(i)resulting mean increased by 3

=32 + 3

=35

(ii)resulting mean decreased by 7

=32 - 7

= 25

(iii)resulting mean multiplied by 2

=32*2

=64

(iv)resulting mean divide by 0.5

$$= \frac{32}{.5}$$

$$= 64$$

(v)resulting mean increased by 60%

$$= 32 + \frac{60}{100} \times 32$$
$$= 32 + 19.2$$
$$= 51.2$$

(vi)resulting mean decreased by 20%

$$= 32 - \frac{20}{100} \times 32$$
$$= 32 - 6.4$$
$$= 25.6$$

Solution 8:

Given the mean of 5 numbers is 18

Total sum of 5 numbers

=18*5

=90

On excluding an observation, the mean of remaining 4 observation is 16

=16*4

=64

Therefore sum of remaining 4 observations

= total of 5 observations-total of 4 observations

= 90 - 64

= 26

Solution 9:

(i)Given that the mean of observations x, x + 2, x + 4, x + 6 and x + 8 is 11

Mean= $\frac{observations}{n}$

$$11 = \frac{x+x+2+x+4+x+6+x+8}{5}$$

$$11 = \frac{5x+20}{5}$$

$$x = \frac{35}{7}$$

$$x = 7$$

(ii)The mean of first three observations are

$$= \frac{x+x+2+x+4}{3}$$

$$= \frac{3x+6}{3}$$

$$= \frac{3*7+6}{3} \qquad [\text{since } x=7]$$

$$= \frac{21+6}{3}$$

$$= 9$$

Solution 10:

Given the mean of 100 observations is 40.

$$\frac{\sum x}{n} = \bar{x}$$

$$\Rightarrow \frac{\sum x}{n} = 40$$

$$\Rightarrow \quad x = 40*100$$

$$\Rightarrow \quad x = 4000$$

Incorrect value of x=4000

Correct value of x=Incorrect value of x-Incorrect observation + correct observation

=4000-83+53

=3970

Correct mean

$$= \frac{\text{correct value of } \sum x}{n}$$

$$= \frac{3970}{100}$$

$$= 39.7$$

Solution 11:

Given that the mean of 200 items was 50.

$$\text{Mean} = \frac{\sum x}{n}$$

$$\Rightarrow 50 = \frac{\sum x}{200}$$

$$\Rightarrow x = 10000$$

Incorrect value of $\sum x = 10000$

Correct value of

$$\sum x = 10000 - (92 + 8) + (192 + 88)$$

$$= 10000 - 100 + 280$$

$$= 10180$$

Correct mean

$$= \frac{\text{correct value of } \sum x}{n}$$

$$= \frac{10180}{200}$$

$$= 50.9$$

Solution 12:

Mean of 45 numbers = 18

$\Rightarrow$ Sum of 45 numbers $= 18 \times 45 = 810$

Mean of remaining (75 - 45) 30 numbers = 13

$\Rightarrow$ Sum of remaining 30 numbers $= 13 \times 30 = 390$

$\Rightarrow$ Sum of all the 75 numbers $= 810 + 390 = 1200$

$\Rightarrow$ Mean of all the 75 numbers $= \frac{1200}{75} = 16$

Solution 13:

Mean weight of 120 students = 52.75 kg

$\Rightarrow$ Sum of the weight of 120 students = $120 \times 52.75 = 6330$ kg

Mean weight of 50 students = 51 kg

$\Rightarrow$ Sum of the weight of 50 students = $50 \times 51 = 2550$ kg

$\Rightarrow$ Sum of the weight of remaining (120 - 50) 70 students

= Sum of the weight of 120 students - Sum of the weight of 50 students

= (6330 - 2550) kg

= 3780 kg

$\Rightarrow$ Mean weight of remaining 70 students = $\frac{3780}{70} = 54$ kg

Solution 14:

Let the number of boys and girls be x and y respectively.

Now,

Given, Mean marks of x boys in the examination = 70

$\Rightarrow$ Sum of marks of x boys in the examination = 70x

Given, Mean marks of y girls in the examination = 73

$\Rightarrow$ Sum of marks of y girls in the examination = 73y

Given, Mean marks of all students (x + y) in the examination = 71

$\Rightarrow$ Sum of marks of all students (x + y) students in the examination = 71(x + y)

Now, Sum of marks of all students (x + y) students in the examination

= Sum of marks of x boys in the examination

+ Sum of marks of y girls in the examination

$\Rightarrow 71(x+y) = 70x + 73y$

$\Rightarrow 71x + 71y = 70x + 73y$

$\Rightarrow x = 2y$

$\Rightarrow \frac{x}{y} = \frac{2}{1}$

$\Rightarrow x : y = 2 : 1$

Thus, the ratio of number of boys to the number of girls is 2 : 1.

Exercise 19(B)

Solution 1:

(i)Firstly arrange the numbers in ascending order

16, 16, 19, 25, 26, 28, 31, 32, 35

Now since

n=9(odd)

Therefore Median

$$= \left(\frac{n+1}{2}\right)^{th}$$

$$= \left(\frac{9+1}{2}\right)^{th}$$

$$= 5^{th}$$

Thus the median is 26

(ii)

Firstly arrange the numbers in ascending order

241, 243, 257, 258, 261, 271, 292, 299, 327, 347, 350

Now since n=11(Odd)

$$\text{Median} = \text{value of} \left(\frac{n+1}{2}\right)^{th} \text{term}$$

$$= 6^{th}\text{term}$$

$$= 271$$

Thus median is 271.

(iii) Firstly arrange the numbers in ascending order

$9, 14, 17, 21, 25, 34, 43, 50, 50, 63$

Now since n=10(even)

$$\text{Median} = \frac{1}{2}\left[\text{value of } \left(\frac{n}{2}\right)^{th} \text{ term} + \text{value of } \left(\frac{n}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}\left[\text{value of } \left(\frac{10}{2}\right)^{th} \text{ term} + \text{value of } \left(\frac{10}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}[25 + 34]$$

$$= \frac{1}{2}[59]$$

$$= 29.5$$

Thus the median is 29.5

(iv) Firstly arrange the numbers in ascending order

173,185,189,194,194,200,204,208,220,223

$$\text{Median} = \frac{1}{2}\left[\text{value of } \left(\frac{n}{2}\right)^{th} \text{ term} + \text{value of } \left(\frac{n}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}\left[\text{value of } \left(\frac{10}{2}\right)^{th} \text{ term} + \text{value of } \left(\frac{10}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}[200 + 194]$$

$$= \frac{1}{2}[394]$$

$$= 197$$

Thus the median is 197

Solution 2:

Given numbers are 34, 37, 53, 55, x, x+2, 77, 83, 89, 100

Here n = 10(even)

$$Median = \frac{1}{2}\left[value\ of \left(\frac{n}{2}\right)^{th} term + value\ of \left(\frac{n}{2}+1\right)^{th} term\right]$$

$$= \frac{1}{2}\left[value\ of \left(\frac{10}{2}\right)^{th} term + value\ of \left(\frac{10}{2}+1\right)^{th} term\right]$$

$$= \frac{1}{2}\left[value\ of\ (5)^{th} term + value\ of\ (5+1)^{th} term\right]$$

$$= \frac{1}{2}\left[value\ of\ (5)^{th} term + value\ of\ (6)^{th} term\right]$$

$$63 = \frac{1}{2}[x + x + 2]$$

$$\Rightarrow \frac{[2+2x]}{2} = 63$$

$$\Rightarrow x + 1 = 63$$

$$\Rightarrow x = 62$$

Solution 3:

For any given set of data, the median is the value of its middle term.

Here, total observations = n = 10 (even)

If n is even, we have

$$\text{Median} = \frac{1}{2}\left[\text{value of} \left(\frac{n}{2}\right)^{th} \text{ term + value of} \left(\frac{n}{2}+1\right)^{th} \text{ term}\right]$$

Thus, for n = 10, we have

$$\text{Median} = \frac{1}{2}\left[\text{value of} \left(\frac{10}{2}\right)^{th} \text{ term + value of} \left(\frac{10}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}\left[\text{value of } 5^{th} \text{ term + value of } 6^{th} \text{ term}\right]$$

Hence, if 7^{th} number is diminished by 8, there is no change in the median value.

Solution 4:

Here, total observations = n = 10 (even)

Thus, we have

$$\text{Median} = \frac{1}{2}\left[\text{value of }\left(\frac{10}{2}\right)^{th}\text{ term} + \text{value of }\left(\frac{10}{2}+1\right)^{th}\text{ term}\right]$$

$$= \frac{1}{2}\left[\text{value of }5^{th}\text{ term} + \text{value of }6^{th}\text{ term}\right]$$

According to given information, data in ascending order is as follows:

	1^{st} Term	2^{nd} Term	3^{rd} Term	4^{th} Term	5^{th} Term	6^{th} Term	7^{th} Term	8^{th} Term	9^{th} Term	10^{th} Term
Marks	Less than 30			35	40	48	66	More than 75		

$$\therefore \text{Median} = \frac{1}{2}(40+48) = \frac{88}{2} = 44$$

Hence, the median score of the whole group is 44.

Solution 5:

Total number of observations = 9 (odd)

Now, if n = odd

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th}\text{ term}$$

$$\Rightarrow \text{Median} = \left(\frac{9+1}{2}\right)^{th}\text{ term} = 5^{th}\text{ term} = x+5$$

Now, Median = 18 (given)

$\therefore x + 5 = 18$

$\Rightarrow x = 13$

Exercise 19(C)

Solution 1:

$$\text{Mean of the given data} = \frac{8 + 12 + 16 + 22 + 10 + 4}{6}$$

$$= \frac{72}{6} = 12$$

(i) Multiplied by 3

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$,
then mean of $ax_1, ax_2, ax_3, \ldots, ax_n$ is $a\bar{x}$.
Thus, when each of the given data is multiplied by 3,
the mean is also multiplied by 3.
Mean of the original data is 12.
Hence, the new mean = 12 x 3 = 36.

(ii) Divided by 2

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$,
then mean of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots, \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$.
Thus, when each of the given data is divided by 2,
the mean is also divided by 3.
Mean of the original data is 12.
Hence, the new mean $= \frac{12}{2} = 6$.

(iii) multiplied by 3 and then divided by 2

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$,

then mean of $\frac{a}{b}x_1, \frac{a}{b}x_2, \frac{a}{b}x_3, \ldots, \frac{a}{b}x_n$ is $\frac{a}{b}\bar{x}$.

Thus, when each of the given data is multiplied by $\frac{3}{2}$,

the mean is also multiplied by $\frac{3}{2}$.

Mean of the original data is 12.

Hence, the new mean $= \frac{3}{2} \times 12 = \frac{36}{2} = 18$

(iv) increased by 25%

New mean = Original mean + 25% of original mean

$\Rightarrow$ New mean = 12 + 25% of 12

$\Rightarrow$ New mean $= 12 + \frac{25}{100} \times 12$

$\Rightarrow$ New mean $= 12 + \frac{1}{4} \times 12$

$\Rightarrow$ New mean = 12 + 3

$\Rightarrow$ New mean = 15

(v) decreased by 40%

New mean = Original mean − 40% of original mean

$\Rightarrow$ New mean = 12 − 40% of 12

$\Rightarrow$ New mean $= 12 - \frac{40}{100} \times 12$

$\Rightarrow$ New mean $= 12 - \frac{2}{5} \times 12$

$\Rightarrow$ New mean $= 12 - 0.4 \times 12$

$\Rightarrow$ New mean = 12 − 4.8

$\Rightarrow$ New mean = 7.2

Solution 2:

$$\text{Mean of given data} = \frac{18 + 24 + 15 + 2x + 1 + 12}{5}$$

$$\Rightarrow \quad 21 = \frac{70 + 2x}{5}$$

$$\Rightarrow \quad 5 \times 21 = 70 + 2x$$

$$\Rightarrow \quad 105 = 70 + 2x$$

$$\Rightarrow \quad 2x = 105 - 70$$

$$\Rightarrow \quad 2x = 35$$

$$\Rightarrow \quad x = \frac{35}{2}$$

$$\Rightarrow \quad x = 17.5$$

Solution 3:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

$$\text{Mean of given data} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Given that mean of 6 numbers is 42.

That is,

$$\frac{x_1 + x_2 + x_3 + \ldots + x_6}{6} = 42$$

$$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_6 = 6 \times 42$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 252 - x_6 \ldots (1)$$

Also, given that the mean of 5 numbers is 45.

That is,

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 45$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 5 \times 45$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 225 \ldots (2)$$

From equations (1) and (2), we have,

$$x_1 + x_2 + x_3 + x_4 + x_5 = 252 - x_6 = x_1 + x_2 + x_3 + x_4 + x_5 = 225$$

$$252 - x_6 = 225$$

$$\Rightarrow x_6 = 252 - 225 = 27$$

Solution 4:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

Mean of given data $= \dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

Given that mean of 10 numbers is 24.

That is,

$$\frac{x_1 + x_2 + x_3 + \ldots + x_{10}}{10} = 24$$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} = 10 \times 24$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} = 240$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 240 + x_{11} \ldots (1)$

Also, given that mean of 11 numbers is 25.

That is,

$$\frac{x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11}}{11} = 25$$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 11 \times 25$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 275 \ldots (2)$

From equations (1) and (2), we have:

$x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 240 + x_{11} = 275$

$240 + x_{11} = 275$

$\Rightarrow x_{11} = 275 - 240 = 35$

Solution 5:

Consider the given data:

44, 47, 63, 65, x+13, 87, 93, 99, 110

Here the number of observations is 9, which is odd.

Thus, the median of the given data is $\left(\dfrac{n+1}{2}\right)^{th}$ observation.

From the given data, $\left(\dfrac{9+1}{2} = 5\right)^{th}$ observation is x + 13

Also, given that the median is 78.

Thus, we have

$x + 13 = 78$

$\Rightarrow x = 78 - 13$

$\Rightarrow x = 65$

Solution 6:

Consider the given data:

24, 27, 43, 48, x – 1, x + 3, 68, 73, 80, 90.

Here the number of observations is 10, which is even.

Thus, the median of given data is $\frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$.

From the given data, $\left(\frac{10}{2}=5\right)^{th}$ observation is x – 1

and $\left(\frac{10}{2}+1=6\right)^{th}$ observation is x + 3.

Also, given that the median is 58.

Thus, we have

$\frac{1}{2}[x-1+x+3] = 116$

$\Rightarrow 2x + 2 = 116$

$\Rightarrow 2x = 116 - 2$

$\Rightarrow 2x = 114$

$\Rightarrow x = \frac{114}{2}$

$\Rightarrow x = 57$

Solution 7:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Therefore,

$$\text{Mean of given data} = \frac{30 + 32 + 24 + 34 + 26 + 28 + 30 + 35 + 33 + 25}{10}$$

$$= \frac{297}{10}$$

$$= 29.7$$

(i)

Let us tabulate the observations and their deviations from the mean

Observations x_i	Devaiations $x_i - \bar{x}$
30	0.3
32	2.3
24	−5.7
34	4.3
26	−3.7
28	−1.7
30	0.3
35	5.3
33	3.3
25	−4.7
Total	0

From the table, it is clear that the sum of the deviations from

(ii)

Consider the given data:

30, 32, 24, 34, 26, 28, 30, 35, 33, 25

Let us rewrite the above data in ascending order.

24, 25, 26, 28, 30, 30, 32, 33, 34, 35

There are 10 observations, which is even.

$$\text{Therefore, median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2} + 1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} \text{term} + \left(\frac{10}{2} + 1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[(5)^{th} \text{term} + (5 + 1)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[5^{th} \text{term} + 6^{th} \text{term}\right]$$

$$= \frac{1}{2}[30 + 30]$$

$$= \frac{1}{2}[60]$$

$$= 30$$

Solution 8:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

$$\text{Mean } = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Therefore,

$$\text{Mean of given data} = \frac{35 + 48 + 92 + 76 + 64 + 52 + 51 + 63 + 71}{9}$$

$$= \frac{552}{9}$$

$$= 61.33$$

Let us rewrite the given data in ascending order:

Thus, we have

35, 48, 51, 52, 63, 64, 71, 76, 92

There are 9 observations, which is odd.

$$\text{Therefore, median } = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

$$\Rightarrow \text{ Median } = \left(\frac{9+1}{2}\right)^{th} \text{ observation}$$

$$\Rightarrow \text{ Median } = \left(\frac{10}{2}\right)^{th} \text{ observation}$$

$$\Rightarrow \text{ Median } = 5^{th} \text{observation}$$

$$\Rightarrow \text{ Median } = 63$$

If 51 is replaced by 66, the new set of data in ascending order is:

35, 48, 52, 63, 64, 66, 71, 76, 92

Since median $= 5^{th}$observation,

We have, new median $= 64$

Solution 9:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Therefore,

$$\text{Mean of given data} = \frac{x + x + 2 + x + 4 + x + 6 + x + 8}{5}$$

$$= \frac{5x + 20}{5}$$

$$= x + 4$$

Also, it's given that mean of the given data is 11.

$\Rightarrow x + 4 = 11$

$\Rightarrow x = 7$

$$\text{Hence the mean of the first three observations} = \frac{x + x + 2 + x + 4}{3}$$

$$= \frac{3x + 6}{3}$$

$$= x + 2$$

$$= 7 + 2$$

$$= 9$$

Solution 10:

Let us find the factors of 72:

$$
\begin{aligned}
72 &= 1 \times 72 \\
&= 2 \times 36 \\
&= 3 \times 24 \\
&= 4 \times 18 \\
&= 6 \times 12 \\
&= 8 \times 9 \\
&= 9 \times 8 \\
&= 12 \times 6 \\
&= 18 \times 4 \\
&= 24 \times 3 \\
&= 36 \times 2 \\
&= 72 \times 1
\end{aligned}
$$

Therefore, the data set is:

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

$$
\text{Mean of the above data set} = \frac{1+2+3+4+6+8+9+12+18+24+36+72}{12} = \frac{195}{12} = 16.25
$$

Since the number of observations is 12, which is even, median is given by

$$
\begin{aligned}
\text{Median} &= \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right] \\
&= \frac{1}{2}\left[\left(\frac{12}{2}\right)^{th} \text{term} + \left(\frac{12}{2}+1\right)^{th} \text{term}\right] \\
&= \frac{1}{2}\left[6^{th} \text{term} + 7^{th} \text{term}\right] \\
&= \frac{1}{2}[8+9] \\
&= \frac{1}{2} \times 17 \\
&= 8.5
\end{aligned}
$$