Vector Algebra

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. The value of the expression $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2$ is

   (a) $\vec{a}.\vec{b}$ (b) $|\vec{a}|.|\vec{b}|$ (c) $|\vec{a}|^2|\vec{b}|^2$ (d) $(\vec{a}.\vec{b})$

2. The area of a triangle formed by vertices O, A, B where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is *[CBSE 2020 (65/2/1)]*

   (a) $3\sqrt{5}$ sq. units (b) $5\sqrt{5}$ sq. units (c) $6\sqrt{5}$ sq. units (d) 4 sq. units

3. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$ then $\vec{a}.\vec{b} \geq 0$ only when *[CBSE 2023 (65/4/1)]*

   (a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$ (c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

4. If $\vec{a}$ is any non-zero vector, then $(\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$ is equal to

   (a) $\vec{a}.\vec{b}$ (b) $\vec{a}$ (c) 0 (d) none of these

5. The vector of the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is *[NCERT Exemplar]*

   (a) $\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ (c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

6. The position vector of the point which divides the joining of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 : 1 is *[NCERT Exemplar]*

   (a) $\frac{3\vec{a} - 2\vec{b}}{2}$ (b) $\frac{7\vec{a} - 8\vec{b}}{4}$ (c) $\frac{3\vec{a}}{4}$ (d) $\frac{5\vec{a}}{4}$

7. The direction cosines of vector $\overrightarrow{BA}$, where coordinates of A and B are (1, 2, –1) and (3, 4, 0) respectively, are *[CBSE 2023 (65/3/2)]*

   (a) $-2, -2, -1$ (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (c) 2, 2, 1 (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

8. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4 respectively and $\vec{a}.\vec{b} = 2\sqrt{3}$ is *[NCERT Exemplar]*

   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

9. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is *[CBSE 2023 (65/1/1)]*

   (a) 3 (b) -3 (c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

10. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is [*NCERT Exemplar*]

(*a*) $\frac{2}{3}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{5}{2}$ (*d*) $\frac{2}{5}$

11. The vector from origin to the points *A* and *B* are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, respectively then the area of triangle *OAB* is [*NCERT Exemplar*]

(*a*) 340 (*b*) $\sqrt{25}$ (*c*) $\sqrt{229}$ (*d*) $\frac{1}{2}\sqrt{229}$

12. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to [*NCERT Exemplar*]

(*a*) $\vec{a}^{\,2}$ (*b*) $3\vec{a}^{\,2}$ (*c*) $4\vec{a}^{\,2}$ (*d*) $2\vec{a}^{\,2}$

13. If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then value of $|\vec{a} \times \vec{b}|$ is [*NCERT Exemplar*]

(*a*) 5 (*b*) 10 (*c*) 14 (*d*) 16

14. The projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on the vector $4\hat{i} - 4\hat{j} + 7\hat{k}$ is

(*a*) $\frac{5\sqrt{6}}{10}$ (*b*) $\frac{19}{9}$ (*c*) $\frac{9}{19}$ (*d*) $\frac{\sqrt{6}}{19}$

15. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is [*NCERT Exemplar*]

(*a*) 1 (*b*) 3 (*c*) $-\frac{3}{2}$ (*d*) None of these

16. Projection vector of $\vec{a}$ on $\vec{b}$ is [*NCERT Exemplar*]

(*a*) $\left(\frac{|\vec{a} \cdot \vec{b}|}{|\vec{b}|^2}\right)\vec{b}$ (*b*) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ (*c*) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$ (*d*) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\hat{b}$

17. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then the range of $|\lambda\vec{a}|$ is [*NCERT Exemplar*]

(*a*) [0, 8] (*b*) [– 12, 8] (*c*) [0, 12] (*d*) [8, 12]

18. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is [*NCERT Exemplar*]

(*a*) one (*b*) two (*c*) three (*d*) infinite

19. The position vector of the point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio 1 : 2 is [*NCERT Exemplar*]

(*a*) $\frac{3\vec{a} + 2\vec{b}}{3}$ (*b*) $\vec{a}$ (*c*) $\frac{5\vec{a} - \vec{b}}{3}$ (*d*) $\frac{4\vec{a} + \vec{b}}{3}$

20. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is [*CBSE 2023 (65/3/2)*]

(*a*) $\frac{\pi}{2}$ (*b*) π (*c*) $\frac{\pi}{4}$ (*d*) 0

21. In ΔABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If *D* is mid-point of *BC*, then vector $\overrightarrow{AD}$ is equal to [*CBSE 2023 (65/3/2)*]

(*a*) $4\hat{i} + 6\hat{k}$ (*b*) $2\hat{i} - 2\hat{j} + 2\hat{k}$ (*c*) $\hat{i} - \hat{j} + \hat{k}$ (*d*) $2\hat{i} + 3\hat{k}$

22. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if [*CBSE 2023 (65/5/1)*]

(*a*) $a_1b_1 - a_2b_2 + a_3b_3 = 0$ (*b*) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(*c*) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (*d*) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

23. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is [CBSE 2023 (65/5/1)]

(*a*) 1 (*b*) 5 (*c*) 7 (*d*) 12

24. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is [CBSE 2023 (65/2/1)]

(*a*) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (*b*) $-2\hat{i} - 3\hat{j} + 6\hat{k}$ (*c*) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (*d*) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

25. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is [CBSE 2023 (65/2/1)]

(*a*) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (*b*) $4\hat{i} + \hat{j} - 2\hat{k}$ (*c*) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (*d*) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

26. Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is [CBSE 2023 (65/2/1)]

(*a*) 0 (*b*) 12 (*c*) $\frac{12}{\sqrt{13}}$ (*d*) $\frac{-12}{\sqrt{13}}$

27. The value of $(\hat{i} \times \hat{j}).\hat{j} + (\hat{j} \times \hat{i}).\hat{k}$ is [CBSE 2023 (65/1/1)]

(*a*) 2 (*b*) 0 (*c*) 1 (*d*) – 1

28. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals [CBSE 2023 (65/1/1)]

(*a*) $\sqrt{14}$ (*b*) 3 (*c*) $\sqrt{12}$ (*d*) $\sqrt{17}$

Answers

1. (*c*)	2. (*a*)	3. (*b*)	4. (*b*)	5. (*c*)	6. (*d*)	7. (*b*)
8. (*b*)	9. (*a*)	10. (*a*)	11. (*d*)	12. (*d*)	13. (*d*)	14. (*b*)
15. (*c*)	16. (*a*)	17. (*c*)	18. (*b*)	19. (*d*)	20. (*a*)	21. (*d*)
22. (*b*)	23. (*c*)	24. (*d*)	25. (*b*)	26. (*a*)	27. (*d*)	28. (*b*)

Solutions of Selected Multiple Choice Questions

1. $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2$

$= \sin^2\theta|\vec{a}|^2|\vec{b}|^2 + |\vec{a}|^2|\vec{b}|^2\cos^2\theta = |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = |\vec{a}|^2|\vec{b}|^2$

$\therefore$ Option (*c*) is correct.

2. We have,

$$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix} = 8\hat{i} - 10\hat{j} + 4\hat{k}$$

$\therefore |\overrightarrow{OA} \times \overrightarrow{OB}| = \sqrt{(8)^2 + (-10)^2 + (4)^2} = \sqrt{64 + 100 + 16} = \sqrt{180} = 6\sqrt{5}$

$\therefore$ Area of $\Delta\, OAB = \frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}| = \frac{1}{2} \times 6\sqrt{5} = 3\sqrt{5}$ sq. units.

$\therefore$ Option (*a*) is correct.

3. $\because\ \vec{a}.\vec{b} \geq 0$

$|\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \cos\theta \geq 0 \qquad [\because |\vec{a}| \geq 0, |\vec{b}| \geq 0]$

$\Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$

$\therefore$ Option (*b*) is correct.

4. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\therefore (\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \vec{a}$

$\therefore$ Option (*b*) is correct.

7. We have,

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = (\hat{i} + 2\hat{j} - \hat{k}) - (3\hat{i} + 4\hat{j} + 0\hat{k})$$
$$= -2\hat{i} - 2\hat{j} - \hat{k}$$

$\Rightarrow \quad |\overrightarrow{BA}| = \sqrt{(-2)^2 + (-2)^2 + (-1)^2} = \sqrt{4+4+1} = 3$

direction cosines of $\overrightarrow{BA}$ are

$$\frac{-2}{3}, \frac{-2}{3}, \frac{-1}{3}$$

$\therefore$ Option (*b*) is correct.

8. Here, $|\vec{a}| = \sqrt{3}, |\vec{b}| = 4$ and $\vec{a}.\vec{b} = 2\sqrt{3}$

We know that, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta \quad \Rightarrow \quad 2\sqrt{3} = \sqrt{3}.4.\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$

$\therefore$ Option (*b*) is correct.

9. Given two vectors are $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other.

$\therefore \quad 2 \times (-4) + p \times (-6) + 1 \times 26 = 0$

$\Rightarrow \quad -8 - 6p + 26 = 0 \Rightarrow 18 = 6p \Rightarrow p = 3$

$\therefore$ Option (*a*) is correct.

12. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k} \quad \Rightarrow \quad |\vec{a}|^2 = x^2 + y^2 + z^2$

$$\vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(-z) + \hat{k}(-y) = z\hat{j} - y\hat{k}$$

$\therefore \quad (\vec{a} \times \hat{i})^2 = (z\hat{j} - y\hat{k}).(z\hat{j} - y\hat{k}) = y^2 + z^2$

Similarly, $\quad (\vec{a} \times \hat{j})^2 = x^2 + z^2$ and $(\vec{a} \times \hat{k})^2 = x^2 + y^2$

$\therefore \quad (\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = y^2 + z^2 + x^2 + z^2 + x^2 + y^2 = 2(x^2 + y^2 + z^2) = 2|\vec{a}|^2 = 2\vec{a}^2$

$\therefore$ Option (*d*) is correct.

15. We have, $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}|^2 = 1, |\vec{b}|^2 = 1, |\vec{c}|^2 = 1$

$\because \quad (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \quad |\vec{a}|^2 + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + |\vec{b}|^2 + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + |\vec{c}|^2 = 0$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0 \quad [\because \vec{a}.\vec{b} = \vec{b}.\vec{a}, \vec{b}.\vec{c} = \vec{c}.\vec{b}$ and $\vec{c}.\vec{a} = \vec{a}.\vec{c}]$

$\Rightarrow \quad 1 + 1 + 1 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow \quad \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$

$\therefore$ Option (*c*) is correct.

20. Given projection of $\vec{a}$ on $\vec{b}$ is zero.

$$i.e.., \ \vec{a} \cdot \frac{\vec{b}}{|\vec{b}|} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow ab\cos\theta = 0$$

$\Rightarrow \quad \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} \Rightarrow \vec{a} \perp \vec{b}$

$\therefore$ Angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$

$\therefore$ Option (*a*) is correct.

21. Given, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$

$\therefore \quad \overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$ (Since D is the mid point of BC)

$\overrightarrow{AD} = \frac{1}{2}\{\hat{i} + \hat{j} + 2\hat{k} + 3\hat{i} - \hat{j} + 4\hat{k}\}$

$\Rightarrow \quad \overrightarrow{AD} = \frac{1}{2}\{4\hat{i} + 6\hat{k}\} = 2\hat{i} + 3\hat{k}$

$\Rightarrow \quad \overrightarrow{AD} = 2\hat{i} + 3\hat{k}$

$\therefore$ Option (*d*) is correct.

22. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{a}$ and $\vec{b}$ are collinear if

$\vec{a} = \lambda\vec{b} \quad \Rightarrow \quad a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$\Rightarrow \quad a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \lambda b_1\hat{i} + \lambda b_2\hat{j} + \lambda b_3\hat{k}$

Comparing, we get

$a_1 = \lambda b_1 \quad \Rightarrow \quad \frac{a_1}{b_1} = \lambda$

Also $a_2 = \lambda b_2 \quad \Rightarrow \quad \frac{a_2}{b_2} = \lambda$

and $a_3 = \lambda b_3 \quad \Rightarrow \quad \frac{a_3}{b_3} = \lambda$

$\Rightarrow \quad \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

$\therefore$ Option (*b*) is correct.

23. $\vec{a} = 6\hat{i} - 2\hat{j} + 3\hat{k}$

$|\vec{a}| = \sqrt{6^2 + (-2)^2 + 3^2} = \sqrt{36 + 4 + 9} = \sqrt{49} = 7$

$\therefore$ Option (*c*) is correct.

24. We have, $\overrightarrow{PQ} = (4-2)\hat{i} + (4-1)\hat{j} + (-7+1)\hat{k}$

$\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$\therefore$ Unit vector along $\overrightarrow{PQ} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{4 + 9 + 36}} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$

$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$

$\therefore$ Option (*d*) is correct.

25. Let $\vec{c}$ be the position vector of mid point of AB and the position vector of A is $2\hat{i} + 3\hat{j} - 4\hat{k}$

A$(2\hat{i} + 3\hat{j} - 4\hat{k})$ $\quad$ $\vec{c}(3\hat{i} + 2\hat{j} - 3\hat{k})$ $\quad$ B$(x\hat{i} + y\hat{j} + z\hat{k})$

$\therefore \quad 3 = \frac{2+x}{2}, \frac{3+y}{2} = 2 \text{ and } \frac{z-4}{2} = -3$

$\Rightarrow \quad 2 + x = 6, 3 + y = 4 \text{ and } z - 4 = -6$

$\Rightarrow \quad x = 6 - 2, y = 4 - 3 \text{ and } z = -6 + 4$

$\Rightarrow \quad x = 4, y = 1 \text{ and } z = -2$

$\therefore$ Position vector of $B = 4\hat{i} + \hat{j} - 2\hat{k}$

$\therefore$ Option (*b*) is correct.

26. Let $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 2\hat{j}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \vec{a}.\frac{\vec{b}}{|\vec{b}|} = (2\hat{i} + 3\hat{j}).\frac{(3\hat{i} - 2\hat{j})}{\sqrt{9+4}}$

$$= \frac{6-6}{\sqrt{13}} = 0$$

$\therefore$ Option (*a*) is correct.

27. $(\hat{i} \times \hat{j}).\hat{j} + (\hat{j} \times \hat{i}).\hat{k}$

$= \hat{k}.\hat{j} + (-\hat{k}).\hat{k} = 0 + (-1) = -1$

$\therefore$ Option (*d*) is correct.

28. Given $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$

$\Rightarrow \quad 2\hat{i} - 2\hat{j} + 2\hat{k} + \vec{b} = \hat{i}$

$\Rightarrow \quad \vec{b} = \hat{i} - 2\hat{i} + 2\hat{j} - 2\hat{k} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$\Rightarrow \quad |\vec{b}| = \sqrt{(-1)^2 + (2)^2 + (-2)^2} = \sqrt{1+4+4} = 3$

$\Rightarrow \quad |\vec{b}| = 3$

$\therefore$ Option (*b*) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A) :** Direction cosines of vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ are $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$.

Reason (R) : If vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ then its direction ratios are $\frac{a}{|\vec{r}|}, \frac{b}{|\vec{r}|}, \frac{c}{|\vec{r}|}$, where $|\vec{r}| = \sqrt{a^2 + b^2 + c^2}$.

2. **Assertion (A) :** If $(\vec{a}-\vec{b}).(\vec{a}+\vec{b})=0$, then $\vec{a}$ and $\vec{b}$ are perpendicular.

Reason (R) : The projection of $\hat{i}+3\hat{j}+\hat{k}$ on $2\hat{i}-3\hat{j}+6\hat{k}$ is $-\frac{1}{7}$.

3. **Assertion (A) :** If $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}, \vec{b}=2\hat{i}+3\hat{j}+3\hat{k}$ then $\vec{a}\,.\,\vec{b}=9$.

Reason (R) : If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ then its magnitude $|\vec{a}|=\sqrt{a_1^2+a_2^2+a_3^2}$.

4. **Assertion (A) :** Cosine of the angle between the two vectors $2\hat{i}+2\hat{j}-\hat{k}$ and $6\hat{i}-3\hat{j}+2\hat{k}$ is $\frac{16}{21}$.

Reason (R) : Cosine of the angle between two vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$.

5. **Assertion (A) :** If $|\vec{a}\times\vec{b}|=1$ and $|\vec{a}\,.\,\vec{b}|=\sqrt{3}$ then the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$.

Reason (R) : $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$ and $|\vec{a}\,.\,\vec{b}|=|\vec{a}||\vec{b}|\cos\theta$.

Answers

1. (*a*) **2.** (*d*) **3.** (*b*) **4.** (*d*) **5.** (*a*)

Solutions of Assertion-Reason Questions

1. We have, $\vec{a}=\hat{i}+\hat{j}-2\hat{k}$

$$\therefore\ |\vec{a}|=\sqrt{(1)^2+(1)^2+(-2)^2}=\sqrt{6}$$

$\because$ Its direction cosines are $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$.

Clearly, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Hence, option (*a*) is correct.

2. $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=0 \Rightarrow |\vec{a}|^2-|\vec{b}|^2=0 \Rightarrow |\vec{a}|=|\vec{b}|$

If $\vec{a}=\hat{i}, \vec{a}=-\hat{i}$ then $|\vec{a}|=1=|\vec{b}|$ but $\vec{a}$ is not perpendicular to $\vec{b}$.

So A is false statement.

$$\text{The projection of } \hat{i}+3\hat{j}+\hat{k} \text{ on } 2\hat{i}-3\hat{j}+6\hat{k} = \frac{(\hat{i}+3\hat{j}+\hat{k}).(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{(2)^2+(-3)^2+(6)^2}}$$

$$=\frac{2-9+6}{7}=\frac{-1}{7}$$

R is true statement.

Hence, option (*d*) is correct.

3. We have, $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+3\hat{k}$

$$\therefore\quad \vec{a}\,.\,\vec{b}=(3\hat{i}-\hat{j}+2\hat{k})\,.\,(2\hat{i}+3\hat{j}+3\hat{k})$$

$$=3\times2-1\times3+2\times3=9$$

Clearly, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Hence, option (*b*) is correct.

4. We have, $\cos\theta = \dfrac{(2\hat{i}+2\hat{j}-\hat{k}).(6\hat{i}-3\hat{j}+2\hat{k})}{\sqrt{(2)^2+(2)^2+(-1)^2}.\sqrt{(6)^2+(-3)^2+(2)^2}}$

$$= \frac{2\times 6+2\times(-3)-1\times 2}{3\times 7} = \frac{4}{21}$$

Clearly, Assertion (A) is false and Reason (R) is true.

Hence, option (*d*) is correct.

5. We have,

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta \qquad ...(i)$$

and $$|\vec{a}.\vec{b}| = |\vec{a}||\vec{b}|\cos\theta \qquad ...(ii)$$

From $\dfrac{(i)}{(ii)}$, we have

$$\frac{|\vec{a}\times\vec{b}|}{|\vec{a}.\vec{b}|} = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{1}{\sqrt{3}} = \tan\theta$$

$$\Rightarrow \quad \tan\theta = \tan\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$$

Clearly, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Hence, option (*a*) is correct.

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.**

Solar panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6, 8, 4), P_2 (21, 8, 4), P_3 (21, 16, 10) and P_4 (6,16,10).

(i) Find the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4 – PV of P_1 where PV stands for position vector.

(ii) (a) Find the magnitudes of the vectors $\vec{A}$ and $\vec{B}$.

(b) Find the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof.

Sol. Given points are P_1 (6, 8, 4), P_2 (21, 8, 4), P_3 (21, 16, 10) and P_4 (6, 16, 10).

(i) We have, $\vec{A} = PV \text{ of } P_2 - PV \text{ of } P_1 = (21\hat{i} + 8\hat{j} + 4\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k})$

$\vec{A} = 15\hat{i} + 0\hat{j} + 0\hat{k}$

$\therefore$ Components of $\vec{A}$ are 15, 0, 0.

and $\vec{B} = PV \text{ of } P_4 - PV \text{ of } P_1 = (6\hat{i} + 16\hat{j} + 10\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k})$

$= 0\hat{i} + 8\hat{j} + 6\hat{k}$

$\therefore$ Components of $\vec{B}$ are 0, 8, 6.

(ii) (a) We have,

$$|\vec{A}| = \sqrt{(15)^2 + (0)^2 + (0)^2} = 15 \text{ units}$$

$$|\vec{B}| = \sqrt{(0)^2 + (8)^2 + (6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ units}$$

(b) We have,

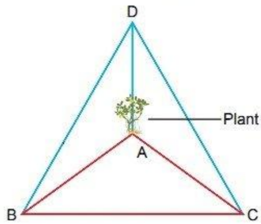
$$\vec{N} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix}$$

$$= \hat{i}(0 - 0) - \hat{j}(90 - 0) + \hat{k}(120 - 0) = 0\hat{i} - 90\hat{j} + 120\hat{k}$$

Its components are 0, –90, 120.

2. **Read the following passage and answer the following questions.**

Raghav purchased an air plant plant holder which is in shape of tetrahedron. Let A, B, C, D be the co-ordinates of the air plant holder where A = (1, 2 , 3), B = (3, 2, 1), C = (2, 1 , 2), D = (3, 4 , 3).



(i) Find the vector $\overrightarrow{AB}$.

(ii) Find the vector $\overrightarrow{CD}$.

(iii) (a) Find the unit vector along $\overrightarrow{BC}$ vector.

OR

(iii) (b) Find the area (ΔBCD).

Sol. $\because A \equiv (1, 2, 3), B \equiv (3, 2, 1), C \equiv (2, 1, 2), D \equiv (3, 4, 3)$

(i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (3-1)\hat{i} + (2-2)\hat{j} + (1-3)\hat{k} = 2\hat{i} - 2\hat{k}$

(ii) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = (3-2)\hat{i} + (4-1)\hat{j} + (3-2)\hat{k} = \hat{i} + 3\hat{j} + \hat{k}$

(iii) (a) $\because \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (2-3)\hat{i} + (1-2)\hat{j} + (2-1)\hat{k} = -\hat{i} - \hat{j} + \hat{k}$

$$\because \widehat{BC} = \frac{\overrightarrow{BC}}{|\overrightarrow{BC}|} = \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{(-1)^2 + (-1)^2 + 1^2}} = \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

$$= -\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

OR

(iii) (b) $\because \overrightarrow{BC} = -\hat{i} - \hat{j} + \hat{k}$

$$\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB} = (3-3)\hat{i} + (4-2)\hat{j} + (3-1)\hat{k}$$

$$= 2\hat{j} + 2\hat{k}$$

$\because \overrightarrow{BD}$ and $\overrightarrow{BD}$ are adjacent sides of ΔBCD.

$$= \overrightarrow{BC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 0 & 2 & 2 \end{vmatrix} = \hat{i}(-2-2) - \hat{j}(-2-0) + k(-2+0) = -4\hat{i} + 2\hat{j} - 2\hat{k}$$

$\therefore$ Area of $\Delta BCD = \frac{1}{2}|\overrightarrow{BC} \times \overrightarrow{BD}|$

$$= \frac{1}{2}\sqrt{(-4)^2 + 2^2 + (-2)^2} = \frac{1}{2} \times \sqrt{16+4+4}$$

$$= \frac{1}{2}\sqrt{24} = \frac{1}{2} \times 2\sqrt{6} = \sqrt{6} \text{ sq. units}$$

CONCEPTUAL QUESTIONS

1. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units. [*CBSE (AI) 2008*]

Sol. The unit vector in the direction of the given vector $\vec{a}$ is

$$\hat{a} = \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j}) = \frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$$

Therefore, the vector having magnitude equal to 7 and in the direction of $\vec{a}$ is

$$7\hat{a} = 7\left(\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right) = \frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j}$$

2. Write the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. [*CBSE Central 2016*]

Sol. Number of vectors of unit length perpendicular to both vectors = 2, namely $\pm(\vec{a} \times \vec{b})$.

3. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vector. [*CBSE Delhi 2009*]

Sol. Since $\vec{a} \parallel \vec{b}$, therefore $\vec{a} = \lambda\vec{b} \Rightarrow 3\hat{i} + 2\hat{j} + 9\hat{k} = \lambda(\hat{i} + p\hat{j} + 3\hat{k})$

$\Rightarrow \lambda = 3, 2 = \lambda p, 9 = 3\lambda$ or $\lambda = 3, p = \frac{2}{3}$ [By comparing the coefficients]

4. What is the cosine of the angle, which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with y-axis? [*CBSE Delhi 2010*]

Sol. We will consider $\vec{a} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$

Unit vector in the direction of $\vec{a}$ is

$$\hat{a} = \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{\sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2}}$$

$$= \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{\sqrt{4}} = \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{2}$$

$$= \frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$$

The cosine of the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with *y*-axis is $\left(\frac{1}{2}\right)$.

5. If $|\vec{a}| = 4, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 6\sqrt{3}$, then the value of $|\vec{a} \times \vec{b}|$. **[*CBSE East 2016*]**

Sol. We have,

$$\vec{a} \cdot \vec{b} = 6\sqrt{3} \quad \Rightarrow \quad |\vec{a}|.|\vec{b}|\cos\theta = 6\sqrt{3}$$

$$\Rightarrow \quad 4 \times 3 \cos\theta = 6\sqrt{3} \quad \Rightarrow \quad \cos\theta = \frac{6\sqrt{3}}{4 \times 3} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{6}$$

Now, $|\vec{a} \times \vec{b}| = |\vec{a}|.|\vec{b}|\sin\theta = 4 \times 3 \sin\frac{\pi}{6} = 4 \times 3 \times \frac{1}{2} = 6$

6. Write the value of the area of the parallelogram determined by the vectors $2\hat{i}$ and $3\hat{j}$. **[*CBSE (F) 2012*]**

Sol. Required area of parallelogram $= |2\hat{i} \times 3\hat{j}| = 6\,|\hat{i} \times \hat{j}| = 6\,|\hat{k}| = 6$ sq units.

[**Note:** Area of parallelogram whose sides are represented by $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$]

7. Find the scalar components of the vector $\overrightarrow{AB}$ with initial point $A(2, 1)$ and terminal point $B(-5, 7)$. **[*CBSE (AI) 2012*]**

Sol. Let $\overrightarrow{AB} = (-5 - 2)\hat{i} + (7 - 1)\hat{j} = -7\hat{i} + 6\hat{j}$

Hence, scalar components are –7, 6.

[**Note:** If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then *x, y, z* are called scalar components and $x\hat{i}, y\hat{j}, z\hat{k}$ are called vector components.]

8. For what value of '*a*' the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear? **[*CBSE Delhi 2011*]**

Sol. $\because$ $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

$$\therefore \quad \frac{2}{a} = \frac{-3}{6} = \frac{4}{-8} \quad \Rightarrow \quad a = \frac{2 \times 6}{-3} \quad \text{or} \quad a = \frac{2 \times (-8)}{4} \quad \Rightarrow a = -4$$

[**Note:** If $\vec{a}$ and $\vec{b}$ are collinear vectors then the respective components of $\vec{a}$ and $\vec{b}$ are proportional.]

9. Write the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$. **[*CBSE Delhi 2011*]**

Sol. Direction cosines of vector $-2\hat{i} + \hat{j} - 5\hat{k}$ are

$$\frac{-2}{\sqrt{(-2)^2 + 1^2 + (-5)^2}}, \quad \frac{1}{\sqrt{(-2)^2 + 1^2 + (-5)^2}}, \quad \frac{-5}{\sqrt{(-2)^2 + 1^2 + (-5)^2}} = \frac{-2}{\sqrt{30}}, \quad \frac{1}{\sqrt{30}}, \quad \frac{-5}{\sqrt{30}}$$

Note: If *l, m, n* are direction cosine of $a\hat{i} + b\hat{j} + c\hat{k}$ then

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \quad n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

10. If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \frac{1}{2}, |\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$, then find $|\vec{a} . \vec{b}|$.

[CBSE South 2016]

Sol. We have, $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $||\vec{a}||\vec{b}|\sin\theta\,\hat{n}| = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $|\vec{a}||\vec{b}|\sin\theta = \frac{1}{\sqrt{3}}$ [$\because$ θ is angle between $\vec{a}$ and $\vec{b}$]

$\Rightarrow$ $\frac{1}{2} \times \frac{4}{\sqrt{3}} . \sin\theta = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $\sin\theta = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin\theta = \frac{1}{2}$ $\Rightarrow$ $\theta = 30°$

Now, $|\vec{a} . \vec{b}| = |\vec{a}||\vec{b}|\cos\theta = \frac{1}{2} \times \frac{4}{\sqrt{3}} . \cos 30° = \frac{2}{\sqrt{3}} . \frac{\sqrt{3}}{2} = 1$

11. Give an example of vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$. *[CBSE Sample Paper 2018]*

Sol. Let $\vec{a} = x\hat{i} + y\hat{j}$; $\vec{b} = y\hat{i} + x\hat{j}$

$|\vec{a}| = \sqrt{x^2 + y^2}$, $|\vec{b}| = \sqrt{y^2 + x^2}$ Hence, $\vec{a} \neq \vec{b}$ but $|\vec{a}| = |\vec{b}|$

12. Find the unit vector in the direction of sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{j} + \hat{k}$.

[NCERT Exemplar]

Sol. Let $\vec{c}$ denotes the sum of $\vec{a}$ and $\vec{b}$.

We have, $\vec{c} = \vec{a} + \vec{b} = 2\hat{i} - \hat{j} + \hat{k} + 2\hat{j} + \hat{k} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ Unit vector in the direction of $\vec{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9}} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}$

Very Short Answer Questions

1. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors. *[CBSE 2020 (65/3/1)]*

Sol. We have,

$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$\Rightarrow$ $|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$ (Squaring both sides)

$\Rightarrow$ $|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

(Here θ is the angle between vectors $\vec{a}$ and $\vec{b}$)

$\Rightarrow$ $4|\vec{a}||\vec{b}|\cos\theta = 0$

$\Rightarrow$ $\cos\theta = 0 = \cos\frac{\pi}{2}$ $\Rightarrow$ $\theta = \frac{\pi}{2}$

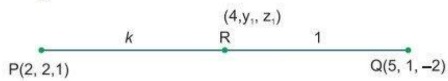
$\Rightarrow$ $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

2. The x-coordinate of a point on the line joining the point $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate. *[CBSE (AI) 2017]*

Sol. Let required point be $R(4, y_1, z_1)$ which divides PQ in ratio $k : 1$.

By section formula

$4 = \frac{5k + 2}{k + 1}$ $\Rightarrow$ $4k + 4 = 5k + 2$



$\Rightarrow \quad k = 2$

$\therefore \quad z_1 = \dfrac{2 \times (-2) + 1 \times 1}{2 + 1}$

$= \dfrac{-4 + 1}{3} = \dfrac{-3}{3} = -1$

3. Find 'λ' when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units. [*CBSE Delhi 2012*]

Sol. We know that projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|} \Rightarrow 4 = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$ ...(*i*)

Now, $\vec{a}.\vec{b} = 2\lambda + 6 + 12 = 2\lambda + 18$ also $|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{4 + 36 + 9} = 7$

Putting in (*i*), we get

$$4 = \frac{2\lambda + 18}{7} \quad \Rightarrow \quad 2\lambda = 28 - 18 \quad \Rightarrow \quad \lambda = \frac{10}{2} = 5$$

4. Find all the ·vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.

[*CBSE 2023 (65/1/1)*]

Sol. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ be the given vectors

$\therefore$ Its unit vector $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{(1)^2 + (1)^2 + (1)^2}} = \pm\dfrac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

Therefore, vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$ is given by

$$3\sqrt{3} \times \left[\pm\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})\right] = \pm 3(\hat{i} + \hat{j} + \hat{k}).$$

5. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}).(\vec{r} \times \hat{k}) - 12$. [*CBSE 2023 (65/3/2)*]

Sol. Given $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$\therefore \quad \vec{r} \times \vec{j} = (3\hat{i} - 2\hat{j} + 6\hat{k}) \times \hat{j} = 3\hat{i} \times \hat{j} - 2\hat{j} \times \hat{j} + 6\hat{k} \times \hat{j}$

$= 3\hat{k} - 2 \times 0 + 6 \times -\hat{i} = -6\hat{i} + 3\hat{k}$

and, $\quad \vec{r} \times \vec{k} = (3\hat{i} - 2\hat{j} + 6\hat{k}) \times \hat{k}$

$= -3\hat{j} - 2\hat{i} = -2\hat{i} - 3\hat{j}$

$\Rightarrow \quad (\vec{r} \times \hat{j}).(\vec{r} \times \hat{k}) = (-6\hat{i} + 3\hat{k}).(-2\hat{i} - 3\hat{j}) = 12$

$\therefore \quad (\vec{r} \times \hat{j}).(\vec{r} \times \hat{k}) - 12 = 12 - 12 = 0$

6. *X* and *Y* are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point *Z* which divides the line segment *XY* in the ratio 2 : 1 externally.

[*CBSE 2019 (65/4/1)*]

Sol. We have $\overrightarrow{OX} = 3\vec{a} + \vec{b}, \overrightarrow{OY} = \vec{a} - 3\vec{b}, \overrightarrow{OZ} = ?$

$$\overrightarrow{OZ} = \frac{2(\vec{a} - 3\vec{b}) - 1(3\vec{a} + \vec{b})}{2 - 1}$$

$$= \frac{-\vec{a} - 7\vec{b}}{1} = -\vec{a} - 7\vec{b}$$

$\therefore \quad \overrightarrow{OZ} = -\vec{a} - 7\vec{b}$

7. Position vectors of the points *A*, *B* and *C* as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$. *[CBSE 2023 (65/1/1)]*

Sol. Given point *A*, *B* and *C* such that

$$\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$$

Let given position vectors of the points *A*, *B* and *C* are respectively

$$\vec{a} = \vec{a}_1\,\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{b} = b_1\,\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ and } \vec{c} = c_1\,\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\because \quad \frac{\overrightarrow{AC}}{\overrightarrow{AB}} = \frac{5}{4}$$

$\therefore$ Point *C* divides *AB* externally in the ratio $5:4$ using section formula, we have

$$c_1 = \frac{5b_1 - 4a_1}{5-4} = 5b_1 - 4a_1\,;\ \ c_2 = \frac{5b_2 - 4a_2}{5-4} = 5b_2 - 4a_2$$

and $c_3 = \dfrac{5b_3 - 4a_3}{5-4} = 5b_3 - 4a_3$

$$\therefore \quad \vec{c} = c_1\,\hat{i} + c_2\hat{j} + c_3\hat{k} = (5b_1 - 4a_1)\,\hat{i} + (5b_2 - 4a_2)\hat{j} + (5b_3 - 4a_3)\hat{k}$$

$$= 5(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) - 4(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$

$$\vec{c} = 5\vec{b} - 4\vec{a}$$

8. If $|\vec{a}| = a$, then find the value of the following : *[CBSE Bhubneshwar 2015]*

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$$

Sol. Let $\vec{a}$ makes angle α, β, γ with *x*, *y* and *z* axis.

$\therefore\ |\vec{a} \times \hat{i}| = |\vec{a}|.1.\sin\alpha = a\sin\alpha$ similarly $|\vec{a} \times \hat{j}| = a\sin\beta$ and $|\vec{a} \times \hat{k}| = a\sin\gamma$

$$\therefore\ |\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = a^2\sin^2\alpha + a^2\sin\beta + a^2\sin^2\gamma = a^2[\sin^2\alpha + \sin^2\beta + \sin^2\gamma]$$

$$= a^2[1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma]$$

$$= a^2[3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)]$$

$$= a^2(3-1) \qquad \begin{bmatrix} \because l^2 + m^2 + n^2 = 1 \\ \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \end{bmatrix}$$

$$= 2a^2$$

9. The vectors $\vec{a} = 3\hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. If $|\vec{a}| = |\vec{b}|$, then find the value of *y*. *[CBSE Bhubneshwar 2015]*

Sol. $\because$ $\vec{a}$ and $\vec{b}$ are mutually perpendicular.

$\therefore \quad \vec{a}\,.\,\vec{b} = 0 \Rightarrow (3\hat{i} + x\hat{j}).(2\hat{i} + \hat{j} + y\hat{k}) = 0 \quad \Rightarrow \quad 6 + x + 0.\,y = 0$

$\Rightarrow \quad 6 + x = 0 \quad \Rightarrow \quad x = -6$

Again, $|\vec{a}| = |\vec{b}|$

$\Rightarrow \quad \sqrt{3^2 + x^2} = \sqrt{2^2 + 1 + y^2} \quad \Rightarrow \quad \sqrt{9 + 36} = \sqrt{5 + y^2}\ \ [\because x = -6]$

$\Rightarrow \quad \sqrt{45} = \sqrt{5 + y^2} \quad \Rightarrow \quad y^2 = 45 - 5$

$\Rightarrow \quad y = \pm\sqrt{40} = \pm 2\sqrt{10}$

10. Find the value of $\vec{a}.\vec{b}$ if $|\vec{a}| = 10, |\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 16$. [*CBSE Guwahati 2015*]

Sol. $\because\ |\vec{a} \times \vec{b}| = 16 \quad \Rightarrow \quad |\vec{a}|\,|\vec{b}| \sin\theta = 16$

$\Rightarrow \quad 10 \times 2 \sin\theta = 16 \qquad \Rightarrow \sin\theta = \dfrac{16}{20} = \dfrac{4}{5}$

$\Rightarrow \quad \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \dfrac{16}{25}} = \pm\dfrac{3}{5}$

$\therefore \quad \vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta = \pm 10 \times 2 \times \dfrac{3}{5} = \pm 12$

11. Find the vector of magnitude 6, which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$. [*NCERT Exemplar*]

Sol. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} - \hat{j} + 3\hat{k}$

So, any vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 4 & -1 & 3 \end{vmatrix} = \hat{i}(-3+2) - \hat{j}(6-8) + \hat{k}(-2+4) = -\hat{i} + 2\hat{j} + 2\hat{k} = \vec{r} \text{ [say]}$$

A vector of magnitude 6 in the direction of $\vec{r}$

$$= \frac{\vec{r}}{|\vec{r}|}.6 = \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}}.6 = \frac{-6}{3}\hat{i} + \frac{12}{3}\hat{j} + \frac{12}{3}\hat{k} = -2\hat{i} + 4\hat{j} + 4\hat{k}$$

12. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other. [*CBSE 2019 (65/4/1)*]

Sol. We have $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$.

Then, $\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$\because \quad (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = -8 + 3 + 5 = 0$

$\Rightarrow \quad (\vec{a} + \vec{b}) \perp (\vec{a} - \vec{b})$

13. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $(\vec{a} \times \vec{b})^2 = \vec{a}^2\, \vec{b}^2 - (\vec{a}.\vec{b})^2$ [*CBSE 2019 (65/5/3)*]

Sol. $\because \quad (\vec{a} \times \vec{b}) = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$ and $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$

RHS $= \vec{a}^2\, \vec{b}^2 - (\vec{a}.\vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta$

$= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2\theta) = |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta$

$= (\vec{a} \times \vec{b})^2 = \text{LHS}$ Hence proved.

14. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = \dfrac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$. [*CBSE 2023 (65/5/1)*]

Sol. $|\vec{a}| = 3,\ |\vec{b}| = \dfrac{2}{3}$ and $|\vec{a} \times \vec{b}| = 1$

$\Rightarrow \quad |\vec{a}|\,|\vec{b}| \sin\theta = 1$

$\Rightarrow \quad \sin\theta = \dfrac{1}{|\vec{a}||\vec{b}|} = \dfrac{1}{3 \times \dfrac{2}{3}} = \dfrac{1}{2} = \sin\dfrac{\pi}{6}$

$\Rightarrow \quad \theta = \dfrac{\pi}{6}$

15. Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. *[CBSE 2023 (65/5/1)]*

Sol. Adjacent sides of parallelogram are

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k},\ \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$\therefore$ Area of parallelogram $= |\vec{a} \times \vec{b}|$

$$= \sqrt{(20)^2 + 5^2 + (-5)^2} = \sqrt{400+25+25}$$

$$= \sqrt{450} = \sqrt{9 \times 25 \times 2} = 15\sqrt{2} \text{ sq.units}$$

16. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$. *[CBSE 2023 (65/2/1)]*

Sol. Given $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that

$$\vec{a}.\vec{b} = \vec{a}.\vec{c}$$

$\Rightarrow \quad \vec{a}.\vec{b} - \vec{a}.\vec{c} = 0 \quad \Rightarrow \vec{a}.(\vec{b} - \vec{c}) = 0$

$\Rightarrow \quad \vec{a}$ is perpendicular to $(\vec{b} - \vec{c})$.

$\therefore \quad$ Angle between $\vec{a}$ and $(\vec{b} - \vec{c})$ is 90° or, $\frac{\pi}{2}$

17. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. *[CBSE 2021-22 (Term-2) (65/3/2)]*

Sol.

$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

projection of $(\vec{b}+\vec{c})$ on $\vec{a}$ $= \dfrac{\vec{a}.(\vec{b}+\vec{c})}{|\vec{a}|}$

$= \dfrac{(2\hat{i} - 2\hat{j} + \hat{k}).(3\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{2^2 + (-2)^2 + 1^2}}$

$= \dfrac{(2\times3) + (-2\times1) + (1\times2)}{\sqrt{9}}$

$= \dfrac{6 - 2 + 2}{\sqrt{9}}$

$= \dfrac{6}{3}$

$= 2$ units

Answer: Projection of $(\vec{b}+\vec{c})$ on $\vec{a}$ is 2 units.

[Topper's Answer 2022]

Short Answer Questions

1. If $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$.

[CBSE 2021-22 (Term-2) (65/3/2)]

Sol.

$|\vec{a}| = 3$

$|\vec{b}| = 5$

$|\vec{c}| = 4$

$\vec{a} + \vec{b} + \vec{c} = 0.$

$(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = \vec{0}.\vec{0}$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\dfrac{(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)}{2}$

$= -\dfrac{(3^2 + 5^2 + 4^2)}{2}$

$= -\left(\dfrac{50}{2}\right) = -25$

Answer: $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -25$

[Topper's Answer 2022]

2. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-zero verctors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}, \vec{c} \neq 2\vec{b}$.

[CBSE 2021-22 (Term-2) (65/1/1)]

Sol. Consider $(\vec{a} - 2\vec{d}) \times (2\vec{b} - \vec{c})$ ½

$= \vec{a} \times 2\vec{b} - \vec{a} \times \vec{c} - 4\vec{d} \times \vec{b} + 2\vec{d} \times \vec{c}$ 1

$= 0$ 1

$\therefore (\vec{a} - 2\vec{d}) \parallel (2\vec{b} - \vec{c})$ ½

[CBSE Marking Scheme 2022]

3. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

[CBSE 2019 (65/2/1)]

Sol. We have, $\vec{b} + \vec{c} = (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \vec{b} + \vec{c} = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$\therefore$ Unit vector of $\vec{b} + \vec{c} = \dfrac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|} = \dfrac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 36 + 4}}$

Now, $\vec{a}\cdot\dfrac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|}=1 \quad \Rightarrow \quad (\hat{i}+\hat{j}+\hat{k})\cdot\dfrac{((2+\lambda)\hat{i}+6\hat{j}-2\hat{k})}{\sqrt{(2+\lambda)^2+40}}=1$

$\Rightarrow \quad 2+\lambda+6-2=\sqrt{(2+\lambda)^2+40}$

$\Rightarrow \quad \lambda+6=\sqrt{(2+\lambda)^2+40}$

Squaring both sides, we have

$$\lambda^2+12\lambda+36=(2+\lambda)^2+40=4+\lambda^2+4\lambda+40$$

$\Rightarrow \quad 8\lambda=44-36=8 \quad \Rightarrow \quad \lambda=\dfrac{8}{8}=1 \quad \Rightarrow \quad \lambda=1$

By putting the value of $\lambda=1$, unit vector of $\vec{b}+\vec{c}=\dfrac{3}{7}\hat{i}+\dfrac{6}{7}\hat{j}-\dfrac{2}{7}\hat{k}$.

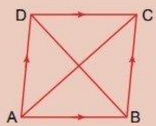
4. The two adjacent sides of a parallelogram are represented by $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also. [*CBSE 2021-22 (Term-2) (65/1/1)*]

Sol. Let *ABCD* be a parallelogram with

$$\overrightarrow{AB}=\overrightarrow{DC}=2\hat{i}-4\hat{j}-5\hat{k}$$

and $\overrightarrow{BC}=\overrightarrow{AD}=2\hat{i}+2\hat{j}+3\hat{k}$

$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=4\hat{i}-2\hat{j}-2\hat{k}$$



and $\overrightarrow{BD}=6\hat{j}+8\hat{k}$ 1

$\therefore \quad |\overrightarrow{AC}|=2\sqrt{6}$ and $|\overrightarrow{BD}|=10$

$\therefore$ Required unit vector $\hat{d}_1$ and $\hat{d}_2$ are

$$\hat{d}_1=\frac{2}{\sqrt{6}}\hat{i}-\frac{1}{\sqrt{6}}\hat{j}-\frac{\hat{k}}{\sqrt{6}} \text{ and } \hat{d}_2=\frac{3}{5}\hat{j}+\frac{4}{5}\hat{k}$$ 1

Now,

$$\text{Area of } \| ABCD=\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$$

$$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -2 & -2\\ 0 & 6 & 8\end{vmatrix}$$

$$=\frac{1}{2}|-4\hat{i}-32\hat{j}+24\hat{k}|$$

$$=\frac{1}{2}\sqrt{1616}=2\sqrt{101}$$ 1

[*CBSE Marking Scheme 2022*]

5. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=1, |\vec{b}|=2$ and $|\vec{c}|=3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$ and $\vec{b},\vec{c}$ are perpendicular to each other, then find $|3\vec{a}-2\vec{b}+2\vec{c}|$. [*CBSE 2019 (65/3/1)*]

Sol. Given projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$.

$$\Rightarrow \quad \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$$

$\therefore \quad \vec{b}.\vec{a} = \vec{c}.\vec{a}$ ...(i)

Also given $\vec{b} \perp \vec{c} \Rightarrow \vec{b}.\vec{c} = 0$ ...(ii)

Now, $|3\vec{a} - 2\vec{b} + 2\vec{c}|^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\vec{a}.\vec{b} - 8\vec{b}.\vec{c} + 12\vec{a}.\vec{c}$

$= 9\times(1)^2 + 4\times(2)^2 + 4\times(3)^2 - 12\vec{a}.\vec{b} - 8\times 0 + 12\vec{a}.\vec{b}$

$= 9 + 4\times 4 + 4\times 9 = 9 + 16 + 36 = 61$

$\Rightarrow \quad |3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61}$

6. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

[*CBSE Delhi 2017*]

Sol. Let $|\vec{a}| = |\vec{b}| = |\vec{c}| = x$ (say)

Since $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors.

Therefore, $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0 = \vec{b}.\vec{a} = \vec{c}.\vec{b} = \vec{a}.\vec{c}$

Now, $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c})$

$= \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c}$

$= x^2 + 0 + 0 + 0 + x^2 + 0 + 0 + 0 + x^2 = 3x^2$

$\Rightarrow \quad |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}x$

Let θ_1, θ_2 and θ_3 be the angles made by $(\vec{a} + \vec{b} + \vec{c})$ with $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

$\therefore \quad \cos\theta_1 = \dfrac{\vec{a}.(\vec{a} + \vec{b} + \vec{c})}{|\vec{a}|.|\vec{a} + \vec{b} + \vec{c}|} = \dfrac{\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c}}{x.\sqrt{3}x} = \dfrac{x^2 + 0 + 0}{\sqrt{3}x^2} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta_1 = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ similarly $\theta_2 = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ and $\theta_3 = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

i.e., $(\vec{a} + \vec{b} + \vec{c})$ is equally inclined with $\vec{a}, \vec{b}$ and $\vec{c}$.

7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

[*CBSE Delhi 2008, 2013*]

Sol. Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then,

$$\vec{a}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ c_1 & c_2 & c_3 \end{vmatrix} = (c_3 - c_2)\hat{i} + (c_1 - c_3)\hat{j} + (c_2 - c_1)\hat{k}$$

$\because \quad \vec{a}\times\vec{c} = \vec{b}$

$\Rightarrow \quad (c_3 - c_2)\hat{i} + (c_1 - c_3)\hat{j} + (c_2 - c_1)\hat{k} = \hat{j} - \hat{k}$

$\Rightarrow \quad c_3 - c_2 = 0, c_1 - c_3 = 1$ and $c_2 - c_1 = -1$ ...(i)

Also, $\vec{a}.\vec{c} = (\hat{i} + \hat{j} + \hat{k}).(c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$

$\Rightarrow \quad \vec{a}.\vec{c} = c_1 + c_2 + c_3$

$\Rightarrow \quad c_1 + c_2 + c_3 = 3 \quad [\because \vec{a}.\vec{c} = 3]$ ...(ii)

$\Rightarrow \quad c_1 + c_2 + c_1 - 1 = 3 \quad [\because c_1 - c_3 = 1]$ ... (iii)

$\Rightarrow \quad 2c_1 + c_2 = 4$

On solving $c_1 - c_2 = 1$ and $2c_1 + c_2 = 4$, we get

$$3c_1 = 5 \Rightarrow c_1 = \frac{5}{3}$$

$\therefore \quad c_2 = (c_1 - 1) = \left(\frac{5}{3} - 1\right) = \frac{2}{3}$ and $c_3 = c_2 = \frac{2}{3}$

Hence, $\vec{c} = \left(\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$.

8. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$ then show that the angle between $\vec{a}$ and $\vec{b}$ is $60°$.

[*CBSE Delhi 2008, 2014*]

Sol. $\vec{a} + \vec{b} + \vec{c} = 0 \quad \Rightarrow \quad (\vec{a} + \vec{b})^2 = (-\vec{c})^2$

$\Rightarrow \quad (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = \vec{c}.\vec{c}$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{c}|^2 \quad \Rightarrow \quad 9 + 25 + 2\vec{a}.\vec{b} = 49$

$\Rightarrow \quad 2\vec{a}.\vec{b} = 49 - 25 - 9$

$\Rightarrow \quad 2|\vec{a}||\vec{b}|\cos\theta = 15 \quad \Rightarrow \quad 30\cos\theta = 15$

$\Rightarrow \quad \cos\theta = \frac{1}{2} = \cos 60° \quad \Rightarrow \quad \theta = 60°$

9. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c} = 18$. **[*CBSE (AI) 2012*]**

Sol. Given, $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}, \vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

Vector $\vec{p}$ is perpendicular to both $\vec{a}$ and $\vec{b}$ *i.e.*, $\vec{p}$ is parallel to vector $\vec{a} \times \vec{b}$.

$$\therefore \quad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = \hat{i}\begin{vmatrix} 4 & 2 \\ -2 & 7 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 4 \\ 3 & -2 \end{vmatrix} = 32\hat{i} - \hat{j} - 14\hat{k}$$

Since $\vec{p}$ is parallel to $\vec{a} \times \vec{b} \Rightarrow \vec{p} = \mu(32\hat{i} - \hat{j} - 14\hat{k})$

Also, $\vec{p}.\vec{c} = 18 \quad \Rightarrow \quad \mu(32\hat{i} - \hat{j} - 14\hat{k}).(2\hat{i} - \hat{j} + 4\hat{k}) = 18$

$\Rightarrow \quad \mu(64 + 1 - 56) = 18 \Rightarrow 9\mu = 18$ or $\mu = 2$

$\therefore \quad \vec{p} = 2(32\hat{i} - \hat{j} - 14\hat{k}) = 64\hat{i} - 2\hat{j} - 28\hat{k}$

10. The magnitude of the vector product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ. **[*CBSE (F) 2013*]**

Sol. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}; \ \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}; \ \vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

From question

$$\left|\vec{a} \times \frac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|}\right| = \sqrt{2} \quad \Rightarrow \quad \left|\frac{\vec{a} \times (\vec{b} + \vec{c})}{|\vec{b} + \vec{c}|}\right| = \sqrt{2} \qquad ...(i)$$

$\vec{b} + \vec{c} = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$\therefore \quad |\vec{b} + \vec{c}| = \sqrt{(2 + \lambda)^2 + 6^2 + (-2)^2} = \sqrt{4 + \lambda^2 + 4\lambda + 36 + 4} = \sqrt{\lambda^2 + 4\lambda + 44}$

$$\Rightarrow \quad \vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 + \lambda & 6 & -2 \end{vmatrix} = (-2 - 6)\hat{i} - (-2 - 2 - \lambda)\hat{j} + (6 - 2 - \lambda)\hat{k} = -8\hat{i} + (4 + \lambda)\hat{j} + (4 - \lambda)\hat{k}$$

Putting it in (*i*), we get

$$\left|\frac{-8\hat{i}+(4+\lambda)\hat{j}+(4-\lambda)\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}\right|=\sqrt{2} \quad\Rightarrow\quad \frac{\sqrt{(-8)^2+(4+\lambda)^2+(4-\lambda)^2}}{\sqrt{\lambda^2+4\lambda+44}}=\sqrt{2}$$

Squaring both sides, we get

$$\frac{64+16+\lambda^2+8\lambda+16+\lambda^2-8\lambda}{\lambda^2+4\lambda+44}=2 \quad\Rightarrow\quad \frac{96+2\lambda^2}{\lambda^2+4\lambda+44}=2$$

$\Rightarrow \quad 8\lambda = 8 \qquad \Rightarrow \qquad \lambda = 1$

11. Show that the points *A, B, C* with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle. **[*CBSE (AI) 2017*]**

Sol. Given, position vector of $A = 2\hat{i}-\hat{j}+\hat{k}$

position vector of $B = \hat{i}-3\hat{j}-5\hat{k}$

position vector of $C = 3\hat{i}-4\hat{j}-4\hat{k}$

$\Rightarrow \quad \overrightarrow{AB} = -\hat{i}-2\hat{j}-6\hat{k}$; $\quad \overrightarrow{AC}=\hat{i}-3\hat{j}-5\hat{k}$ and $\overrightarrow{BC}=2\hat{i}-\hat{j}+\hat{k}$

Now, $|\overrightarrow{AB}|^2 = \overrightarrow{AB}\cdot\overrightarrow{AB} = 1+4+36=41$; $|\overrightarrow{AC}|^2 = 1+9+25=35$; $|\overrightarrow{BC}|^2 = 4+1+1=6$

$\because \quad |\overrightarrow{AB}|^2 = |\overrightarrow{AC}|^2 + |\overrightarrow{BC}|^2 \Rightarrow$ *A, B, C* are the vertices of right triangle.

Now, $\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -2 & -6\\ 1 & -3 & -5\end{vmatrix} = \hat{i}(10-18)-\hat{j}(5+6)+\hat{k}(3+2) = -8\hat{i}-11\hat{j}+5\hat{k}$

$\therefore \quad |\overrightarrow{AB}\times\overrightarrow{AC}| = \sqrt{(-8)^2+(-11)^2+5^2} = \sqrt{64+121+25}=\sqrt{210}$

$\therefore$ Area $(\Delta ABC) = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \frac{\sqrt{210}}{2}$ sq. units

Alternate method to find area:

Area of $\Delta ABC = \frac{1}{2}\times|\overrightarrow{BC}|\times|\overrightarrow{AC}| = \frac{1}{2}\times\sqrt{35}\times\sqrt{6} = \frac{\sqrt{210}}{2}$ sq. units

12. Find a unit vector perpendicular to the plane of triangle *ABC*, where the coordinates of its vertices are *A*(3, –1, 2), *B*(1, –1, –3) and *C*(4, –3, 1). **[*CBSE Bhubaneshwar 2015*]**

Sol. Here, $\overrightarrow{AB} = (1-3)\hat{i}+(-1+1)\hat{j}+(-3-2)\hat{k}$

$= -2\hat{i}+0.\hat{j}-5\hat{k}$

And $\overrightarrow{AC} = (4-3)\hat{i}+(-3+1)\hat{j}+(1-2)\hat{k}$

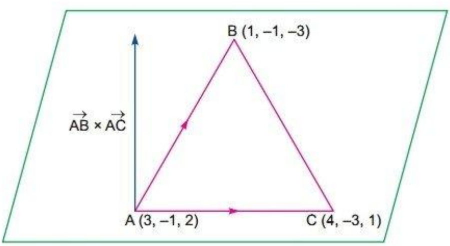
$=\hat{i}-2\hat{j}-\hat{k}$

$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 0 & -5\\ 1 & -2 & -1\end{vmatrix}$

$=(0-10)\hat{i}-(2+5)\hat{j}+(4-0)\hat{k}$

$=-10\hat{i}-7\hat{j}+4\hat{k}$

Since, $\overrightarrow{AB}\times\overrightarrow{AC}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$.



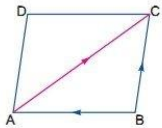
$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC}$ is perpendicular to the plane of triangle ABC.

$\therefore$ Required vector $= \dfrac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|}$

$$= \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{(-10)^2 + (-7)^2 + 4^2}} = \frac{1}{\sqrt{165}}(-10\hat{i} - 7\hat{j} + 4\hat{k})$$

$$= \frac{-10}{\sqrt{165}}\hat{i} - \frac{7}{\sqrt{165}}\hat{j} + \frac{4}{\sqrt{165}}\hat{k}$$

13. Find the area of a parallelogram $ABCD$ whose side AB and the diagonal AC are given by the vectors $3\hat{i} + \hat{j} + 4\hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively. **[CBSE (F) 2017]**



Sol. Here, $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -\overrightarrow{AB} + \overrightarrow{AC}$

$$= -3\hat{i} - \hat{j} - 4\hat{k} + 4\hat{i} + 5\hat{k} = \hat{i} - \hat{j} + \hat{k}$$

$\therefore \quad \overrightarrow{AD} = \overrightarrow{BC} = \hat{i} - \hat{j} + \hat{k}$

$\therefore$ Area of parallelogram $= |\overrightarrow{AB} \times \overrightarrow{AD}|$

$$= \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} \right|$$

$$= |(1+4)\hat{i} - (3-4)\hat{j} + (-3-1)\hat{k}|$$

$$= |5\hat{i} + \hat{j} - 4\hat{k}|$$

$$= \sqrt{5^2 + 1^2 + (-4)^2} = \sqrt{25 + 1 + 16} = \sqrt{42} \text{ sq. units.}$$

14. If $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$ then express $\vec{b}$ in the from of $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$. **[CBSE (AI) 2017]**

Sol. Since $\vec{b}_1 \parallel \vec{a}$

$\Rightarrow \quad \vec{b}_1 = \lambda\vec{a} = \lambda(2\hat{i} - \hat{j} - 2\hat{k}) = 2\lambda\hat{i} - \lambda\hat{j} - 2\lambda\hat{k}$

$\because \quad \vec{b}_1 + \vec{b}_2 = \vec{b}$

$$\vec{b}_2 = \vec{b} - \vec{b}_1$$

$$= (7\hat{i} + 2\hat{j} - 3\hat{k}) - (2\lambda\hat{i} - \lambda\hat{j} - 2\lambda\hat{k})$$

$$= 7\hat{i} + 2\hat{j} - 3\hat{k} - 2\lambda\hat{i} + \lambda\hat{j} + 2\lambda\hat{k}$$

$$= (7 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} - (3 - 2\lambda)\hat{k}$$

It is given that $\vec{b}_2$ is perpendicular to $\vec{a}$.

$\Rightarrow \quad \vec{b}_2 . \vec{a} = 0$

$\Rightarrow \quad (7 - 2\lambda).2 - (2 + \lambda).1 + (3 - 2\lambda).2 = 0$

$\Rightarrow \quad 14 - 4\lambda - 2 - \lambda + 6 - 4\lambda = 0$

$\Rightarrow \quad -9\lambda + 18 = 0 \quad \Rightarrow \quad \lambda = \dfrac{18}{9} = 2$

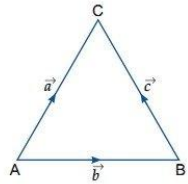
Hence, $\vec{b}_1 = 4\hat{i} - 2\hat{j} - 4\hat{k}$; $\quad \vec{b}_2 = 3\hat{i} + 4\hat{j} + \hat{k}$

Now, $7\hat{i} + 2\hat{j} - 3\hat{k} = (4\hat{i} - 2\hat{j} - 4\hat{k}) + (3\hat{i} + 4\hat{j} + \hat{k})$, *i.e.*, $\vec{b} = \vec{b}_1 + \vec{b}_2$

15. Given that vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle such that $\vec{a} = \vec{b} + \vec{c}$ .Find p, q, r, s such that area of triangle is $5\sqrt{6}$ sq. units where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}, \vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$.

[CBSE (South) 2016]

Sol. Given, $\vec{a} = \vec{b} + \vec{c}$



$\Rightarrow \quad p\hat{i} + q\hat{j} + r\hat{k} = (s\hat{i} + 3\hat{j} + 4\hat{k}) + (3\hat{i} + \hat{j} - 2\hat{k})$

$\Rightarrow \quad p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\,\hat{i} + 4\hat{j} + 2\hat{k}$

Equating the co-efficient of $\hat{i}, \hat{j}, \hat{k}$ from both sides, we get

$\Rightarrow \quad s + 3 = p;\ q = 4$ and $r = 2$ ...(*i*)

Now, area of triangle = $\frac{1}{2}|\vec{b} \times \vec{c}|$

$$\Rightarrow \quad 5\sqrt{6} = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix}\right| = \frac{1}{2}|(-6-4)\hat{i} - (-2s-12)\hat{j} + (s-9)\hat{k}|$$

$$\Rightarrow \quad 5\sqrt{6} = \frac{1}{2}\sqrt{10^2 + (2s+12)^2 + (s-9)^2} = \frac{1}{2}\sqrt{100 + 4s^2 + 144 + 48s + s^2 + 81 - 18s}$$

$$\Rightarrow \quad 5\sqrt{6} = \frac{1}{2}\sqrt{325 + 5s^2 + 30s}$$

Squaring both sides

$$\Rightarrow \quad 150 = \frac{1}{4}(325 + 5s^2 + 30s)$$

$$\Rightarrow \quad 600 - 325 = 5s^2 + 30s$$

$$\Rightarrow \quad 5s^2 + 30s - 275 = 0$$

$$\Rightarrow \quad s = \frac{-30 \pm \sqrt{900 + 4 \times 5 \times 275}}{10} = \frac{-30 \pm \sqrt{6400}}{10} = \frac{-30 \pm 80}{10}$$

$\Rightarrow \quad s = -11, 5$ ...(*ii*)

From (*i*) and (*ii*)

$s = -11, 5;\ p = -8, 8;\ q = 4$ and $r = 2$

16. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a} - \sqrt{2}\,\vec{b}$ to be a unit vector?

[CBSE South 2016]

Sol. Given, $\vec{a} - \sqrt{2}\,\vec{b}$ is an unit vector.

$\Rightarrow |\vec{a} - \sqrt{2}\,\vec{b}| = 1 \qquad \Rightarrow |\vec{a} - \sqrt{2}\,\vec{b}|^2 = 1$ (Squaring both sides)

$\Rightarrow (\vec{a} - \sqrt{2}\,\vec{b}).(\vec{a} - \sqrt{2}\,\vec{b}) = 1 \Rightarrow \vec{a}\,.\,\vec{a} - \sqrt{2}\,\vec{a}\,.\,\vec{b} - \sqrt{2}\,\vec{b}\,.\,\vec{a} + 2\,\vec{b}\,.\,\vec{b} = 1$

$\Rightarrow |a|^2 - 2\sqrt{2}\,\vec{a}.\,\vec{b} + 2|b|^2 = 1 \qquad [\because \vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}]$

$\Rightarrow 1 - 2\sqrt{2}\,\vec{a}\,.\vec{b} + 2 = 1 \qquad [\because |\vec{a}| = |\vec{b}| = 1]$

$\Rightarrow -2\sqrt{2}\,\vec{a}.\,\vec{b} = -2 \qquad \Rightarrow \vec{a}.\,\vec{b} = \frac{-2}{-2\sqrt{2}}$

$\Rightarrow \vec{a}\,.\vec{b} = \frac{1}{\sqrt{2}} \qquad \Rightarrow |\vec{a}|.|\vec{b}|\cos\theta = \frac{1}{\sqrt{2}} \qquad [\because \vec{a}.\,\vec{b} = |\vec{a}|.|\vec{b}|\cos\theta]$

$\Rightarrow 1.1.\cos\theta = \frac{1}{\sqrt{2}} \qquad \Rightarrow \cos\theta = \cos\frac{\pi}{4} \qquad \Rightarrow \theta = \frac{\pi}{4}$

17. The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle *ABC*. Find the length of the median through *A*. **[*CBSE (F) 2015*]**

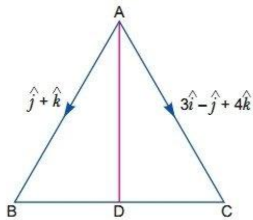
Sol. Here, $\overrightarrow{AB}=\hat{j}+\hat{k}$ and $\overrightarrow{AC}=3\hat{i}-\hat{j}+4\hat{k}$

$\therefore \quad \overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}$

$= -\overrightarrow{AB}+\overrightarrow{AC}=-\hat{j}-\hat{k}+3\hat{i}-\hat{j}+4\hat{k}=3\hat{i}-2\hat{j}+3\hat{k}$

$\because \quad \overrightarrow{BD}=\frac{1}{2}\overrightarrow{BC}$

$=\frac{1}{2}(3\hat{i}-2\hat{j}+3\hat{k})=\frac{3}{2}\hat{i}-\hat{j}+\frac{3}{2}\hat{k}$



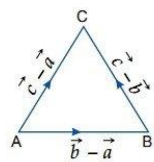
Now, $\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BD}$

$=(\hat{j}+\hat{k})+\left(\frac{3}{2}\hat{i}-\hat{j}+\frac{3}{2}\hat{k}\right)=\frac{3}{2}\hat{i}+\frac{5}{2}\hat{k}$

Length of $AD=|\overrightarrow{AD}|=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2}=\frac{\sqrt{34}}{2}$ units .

18. If $\vec{a},\vec{b}$ and $\vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})$ gives the vector area of the triangle. Hence, deduce the condition that the three point $\vec{a},\vec{b}$ and $\vec{c}$ are collinear. Also, find the unit vector normal to the plane of the triangle. **[*NCERT Exemplar*]**

Sol. Since $\vec{a},\vec{b}$ and $\vec{c}$ are the vertices of a ΔABC as shown.



$\therefore$ Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

Now, $\quad \overrightarrow{AB}=\vec{b}-\vec{a}$ and $\overrightarrow{AC}=\vec{c}-\vec{a}$

$\therefore$ Area of $\Delta ABC=\frac{1}{2}[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]$

$=\frac{1}{2}|(\vec{b}\times\vec{c})-(\vec{b}\times\vec{a})-(\vec{a}\times\vec{c})+(\vec{a}\times\vec{a})|$

$=\frac{1}{2}|(\vec{b}\times\vec{c})+(\vec{a}\times\vec{b})+(\vec{c}\times\vec{a})+\vec{0}|$

$=\frac{1}{2}|(\vec{b}\times\vec{c})+(\vec{a}\times\vec{b})+(\vec{c}\times\vec{a})| \qquad ...(i)$

For three points to be collinear, area of the ΔABC should be equal to zero.

$\Rightarrow \quad \frac{1}{2}(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})=0$

$\Rightarrow \quad \vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}=0 \qquad ...(ii)$

This is the required condition for collinearity of three points $\vec{a},\vec{b}$ and $\vec{c}$.

Let $\hat{n}$ be the unit vector normal to the plane of the ΔABC.

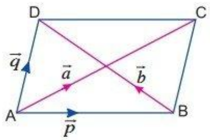
$\therefore \quad \hat{n}=\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|}$

$=\frac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

19. Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a}\times\vec{b}|}{2}$. Also, find the area of the parallelogram, whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$. [*NCERT Exemplar*]

Sol. Let *ABCD* be a parallelogram such that

$$\overrightarrow{AB}=\vec{p}, \overrightarrow{AD}=\vec{q} \Rightarrow \overrightarrow{BC}=\vec{q}$$



By triangle law of addition, we get

$$\overrightarrow{AC}=\vec{p}+\vec{q}=\vec{a} \quad \text{[say]} \quad ...(i)$$

Similarly, $\overrightarrow{BD}=-\vec{p}+\vec{q}=\vec{b}$ [say] ...(*ii*)

On adding equation (*i*) and (*ii*), we get

$$\vec{a}+\vec{b}=2\vec{q} \Rightarrow \vec{q}=\frac{1}{2}(\vec{a}+\vec{b})$$

On subtracting equation (*ii*) from equation (*i*), we get

$$\vec{a}-\vec{b}=2\vec{p} \Rightarrow \vec{p}=\frac{1}{2}(\vec{a}-\vec{b})$$

Now,
$$\vec{p}\times\vec{q}=\frac{1}{4}(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$$
$$=\frac{1}{4}(\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b})$$
$$=\frac{1}{4}[\vec{a}\times\vec{b}+\vec{a}\times\vec{b}] \quad [\because \vec{a}\times\vec{a}=0=\vec{b}\times\vec{b}]$$
$$=\frac{1}{2}(\vec{a}\times\vec{b})$$

So, area of a parallelogram $ABCD=|\vec{p}\times\vec{q}|=\frac{1}{2}|\vec{a}\times\vec{b}|$

Now, area of a parallelogram, whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

$$=\frac{1}{2}|(2\hat{i}-\hat{j}+\hat{k})\times(\hat{i}+3\hat{j}-\hat{k})|$$
$$=\frac{1}{2}\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}\right| = \frac{1}{2}|[\hat{i}(1-3)-\hat{j}(-2-1)+\hat{k}(6+1)]|$$
$$=\frac{1}{2}|-2\hat{i}+3\hat{j}+7\hat{k}| = \frac{1}{2}\sqrt{4+9+49}$$
$$=\frac{1}{2}\sqrt{62} \text{ sq. units}$$

20. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $(\vec{a}-\vec{d})$ is parallel to $(\vec{b}-\vec{c})$, it is being given that $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$. [*CBSE (F) 2016*]

Sol. Given, $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow$ $\vec{a}\times\vec{b}-\vec{a}\times\vec{c}=\vec{c}\times\vec{d}-\vec{b}\times\vec{d} \Rightarrow \vec{a}\times\vec{b}-\vec{a}\times\vec{c}+\vec{b}\times\vec{d}-\vec{c}\times\vec{d}=\vec{0}$

$\Rightarrow$ $\vec{a}\times(\vec{b}-\vec{c})+(\vec{b}-\vec{c})\times\vec{d}=\vec{0}$ [By left and right distributive law]

$\Rightarrow$ $\vec{a}\times(\vec{b}-\vec{c})-\vec{d}\times(\vec{b}-\vec{c})=\vec{0}$ [$\because$ $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$]

$\Rightarrow$ $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$ [By right distributive law]

$\Rightarrow$ $(\vec{a}-\vec{d}) \parallel (\vec{b}-\vec{c})$

21. **Using vectors, prove that the points (2, –1, 3), (3, –5, 1) and (–1, 11, 9) are collinear.**

[*CBSE* 2019 (65/5/3)] [*NCERT*]

Sol. Let A (2, – 1, 3), B (3, –5, 1) and C (–1, 11, 9) are three points.

To show that A, B, C are collinear.

$\therefore \quad \overrightarrow{AB} = (3-2)\hat{i} + (-5+1)\hat{j} + (1-3)\hat{k} = \hat{i} - 4\hat{j} - 2\hat{k}$

$\Rightarrow \quad |\overrightarrow{AB}| = \sqrt{1^2 + (-4)^2 + (-2)^2} = \sqrt{21}$

and $\quad \overrightarrow{BC} = (-1-3)\hat{i} + (11+5)\hat{j} + (9-1)\hat{k} = -4\hat{i} + 16\hat{j} + 8\hat{k}$

$\Rightarrow \quad |\overrightarrow{BC}| = \sqrt{(-4)^2 + (16)^2 + (8)^2} = 4\sqrt{21}$

and $\quad \overrightarrow{AC} = (-1-2)\hat{i} + (11+1)\hat{j} + (9-3)\hat{k} = -3\hat{i} + 12\hat{j} + 6\hat{k}$

$\Rightarrow \quad |\overrightarrow{AC}| = \sqrt{(-3)^2 + (12)^2 + (6)^2} = 3\sqrt{21}$

$\because \quad |\overrightarrow{AC}| + |\overrightarrow{AB}| = |\overrightarrow{BC}|$

$\Rightarrow \quad A, B, C$ are collinear.

Questions for Practice

Objective Type Questions

1. **Choose and write the correct option in each of the following questions.**

(*i*) If $\vec{a}\cdot\vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is [*CBSE 2020 (65/4/1)*]

(*a*) 0° (*b*) 30° (*c*) 60° (*d*) 90°

(*ii*) Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is unit vector if θ is

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{2}$ (*d*) $\frac{2\pi}{3}$

(*iii*) The magnitude of the vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is [*NCERT Exemplar*]

(*a*) 5 (*b*) 7 (*c*) 12 (*d*) 1

(*iv*) Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}\cdot\vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$ then $|\vec{b}|$ equals [*CBSE 2020 (65/4/2)*]

(*a*) 7 (*b*) 14 (*c*) $\sqrt{7}$ (*d*) 21

(*v*) If $|\vec{a}\times\vec{b}| = 4$ and $|\vec{a}\cdot\vec{b}| = 2$ then $|\vec{a}|^2|\vec{b}|^2$ is equal to

(*a*) 2 (*b*) 6 (*c*) 8 (*d*) 20

(*vi*) The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is [*CBSE 2020 (65/3/1)*]

(*a*) 0 (*b*) $\frac{1}{\sqrt{3}}$ (*c*) 1 (*d*) $\sqrt{3}$

Conceptual Questions

2. If $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, then find $\vec{a}\cdot\vec{b}$. [*CBSE (AI) 2008*]

3. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$. [CBSE Delhi 2012]

4. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $|\vec{a} \times \vec{b}| = \sqrt{3}$. [CBSE Delhi 2009]

5. Find a vector of magnitude $\sqrt{171}$, which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. [CBSE Ajmer 2015]

6. Write the distance of the point (3, –5, 12) form *X*-axis. [CBSE (F) 2017]

7. If $\vec{a} . \vec{a} = 0$ and $\vec{a} . \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$. [CBSE (F) 2011]

8. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$. [CBSE (North) 2016]

9. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then find $|\vec{a} \times \vec{b}|$. [CBSE Panchkula 2015]

10. If a unit vector $\vec{a}$ make angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ. [CBSE Delhi 2013]

11. In a triangle *OAC*, if *B* is the mid-point of side *AC* and $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$, then what is $\overrightarrow{OC}$. [CBSE Ajmer 2015]

12. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} . \vec{b}| = 400$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$. [CBSE (F) 2016]

13. If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a} + \hat{b} + \hat{c}|$. [CBSE Allahabad 2015]

14. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$. [CBSE Delhi 2008]

15. Write a vector of magnitude 9 units in the direction of vector $-2\hat{i} + \hat{j} + 2\hat{k}$. [CBSE (AI) 2010]

■ Very Short Answer Questions

16. Find a vector of magnitude 5 units and parallel to resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{i} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$. [CBSE Allahabad 2015]

17. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, find the value of $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$. [CBSE (F) 2013]

18. Find $|\vec{x}|$, if for a unit vector $\vec{a}$

$$(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15.$$ [CBSE (F) 2010]

19. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$. [CBSE (AI) 2014]

20. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units. [CBSE 2020 (65/2/1)]

21. Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\vec{a} - \vec{b}$ is also a unit vector. [CBSE 2020 (65/2/1)]

22. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$ [CBSE 2020 (65/4/1)]

23. Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$. [CBSE 2020 (65/4/1)]

■ Short Answer Questions

24. Find a unit vector perpendicular to both of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$. [CBSE (F) 2014]

25. If $\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$ then find the value of λ, so that $\vec{p}+\vec{q}$ and $\vec{p}-\vec{q}$ are perpendicular vectors. [CBSE (AI) 2013]

26. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 27$. [CBSE Ajmer 2015]

27. For three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ if $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b}$, then prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}| = |\vec{c}|$ and $|\vec{a}| = 1$. [CBSE Sample Paper 2015]

28. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram. [CBSE (Central) 2016]

29. Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$, and hence find a vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. [CBSE (East) 2016]

30. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $b = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 6$. [CBSE (F) 2017]

31. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. [CBSE 2020 (65/5/1)]

32. Using vectors, find the area of the triangle *ABC* with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$. [CBSE 2020 (65/5/1), Delhi 2017]

33. If $\vec{a}$ and $\vec{b}$ are two vectors, then prove that $|\vec{a} - \vec{b}| \le |\vec{a}| + |\vec{b}|$.

34. Given that $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - 7\hat{k}$ and $\vec{c} = 5\hat{i} + 6\hat{j} - 5\hat{k}$, verify that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}.$$

35. Find a vector whose magnitude is 3 units and which is perpendicular to the following two vectors: $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$; $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$.

Answers

1. (*i*) (*c*) (*ii*) (*d*) (*iii*) (*b*) (*iv*) (*c*) (*v*) (*d*) (*vi*) (*b*)

2. $\sqrt{3}$ 3. $-4\hat{j} - \hat{k}$ 4. $\frac{\pi}{3}$ 5. $\hat{i} - 11\hat{j} - 7\hat{k}$ 6. 13 units

7. $\vec{b}$ may be any vector 8. $\frac{1}{7}(6\hat{i} - 3\hat{j} + 2\hat{k})$ 9. $\sqrt{507}$ 10. $\frac{\pi}{3}$

11. $2\vec{b} - \vec{a}$ 12. $|\vec{b}| = 4$ 13. $\sqrt{6}$ 14. $\frac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k})$ 15. $-6\hat{i} + 3\hat{j} + 6\hat{k}$

16. $\pm\frac{5}{\sqrt{10}}(3\hat{i}+\hat{j})$

17. 0

18. 4

19. $\frac{2\pi}{3}$

20. $\vec{r}=\pm 3(\hat{i}+\hat{j}+\hat{k})$

21. $\frac{\pi}{6}$

22. $|\vec{a}|=4, |\vec{b}|=2$

23. $\frac{1}{\sqrt{185}}(7\hat{i}-6\hat{j}-10\hat{k})$

24. $-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$

25. $\lambda=\pm 1$

26. $\vec{d}=96\hat{i}-3\hat{j}-42\hat{k}$

28. $\frac{1}{2\sqrt{6}}(4\hat{i}-2\hat{j}-2\hat{k}), \frac{1}{10}(6\hat{i}+8\hat{k})$ and area of $\|^{gm}=2\sqrt{101}$ sq. units

29. $\theta=\frac{\pi}{2}; 2\hat{i}-26\hat{j}-10\hat{k}$

30. $\vec{c}=3\hat{i}+2\hat{j}+2\hat{k}$

31. $\pm\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k}), \pm\frac{1}{\sqrt{69}}(\hat{i}+2\hat{j}-8\hat{k})$

32. $\frac{1}{2}\sqrt{274}$ sq. units

35. $2\hat{i}-2\hat{j}+\hat{k}$