Relations and Functions

Quick Revision

Ordered Pair

If a pair of elements written in a small brackets and grouped together in a particular order, then such a pair is called ordered pair. The ordered pair of two elements a and b is denoted by (a, b), where a is first element and b is second element.

Two ordered pairs (a, b) and (c, d) are equal, if their corresponding elements are equal i.e. $a = c$ and $b = d$.

Cartesian Products of Sets

For any two non-empty sets A and B, the set of all ordered pairs (a, b) of elements $a \in A$ and $b \in B$ is called the cartesian product of sets A and B and is denoted by $A \times B$.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If $A = \phi$ or $B = \phi$, then $A \times B = \phi$.

Note $A \times B \neq B \times A$

Number of Elements in Cartesian Product of Two Sets

(i) If there are p elements in set A and q elements in set B, then there will be pq elements in $A \times B$.

i.e. if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

(ii) If A and B are non-empty sets and either A or B is an infinite set, then $A \times B$ will also be an infinite set.

(iii) If A or B is the null set or an empty set, then $A \times B$ will also be an empty set.

i.e. $A \times B = \phi$

Relations

A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product $A \times B$, i.e. $R \subseteq A \times B$.

In $(a, b) \in A \times B$, the second element is called the image of first element. The set of all first elements in a relation R, is called the **domain** of the relation R and the set of all second elements is called the **range** of R. The set B is called the **codomain** of relation R.

Thus, if $R = \{(a, b) : a \in A, b \in B\}$, then

domain $(R) = \{a : (a, b) \in R\}$

and range $(R) = \{b : (a, b) \in R\}$

Note If $n(A) = m$, $n(B) = n$, then $n(A \times B) = mn$ and the total number of possible relations from set A to set $B = 2^{mn}$

Representation of a Relation

A relation can be represented algebraically by roster form or by set-builder form and visually, it can be represented by an arrow diagram.

(i) **Roster form** In this form, we represent the relation by the set of all ordered pairs belongs to R.

(ii) **Set-builder form** In this form, we represent the relation R from set A to set B as

$R = \{(a, b):\ a \in A, b \in B$ and the rule which relate the elements of A and $B\}$

Functions

A relation f from a non-empty set A to non-empty set B is said to be function, if every element of set A has one and only one image in set B.

If f is a function from a set A to a set B, then we write $f : A \to B$ and it is read as f is function from A to B or f map A to B and $(a, b) \in f$, then $f(a) = b$, where b is called image of a under f and a is called the pre-image of b under f.

Domain, Codomain and Range of a Function

If $f : A \to B$, then the set A is called the **domain** of function f and the set B is called the **codomain** of f. The subset of B containing the images of elements of A is called the **range** of the function.

Note Every function is a relation but converse is not true.

Real Functions

A function $f : A \to B$ is called a **real valued function**, if B is a subset of R (set of all real numbers). If A and B both are subsets of R, then f is called a real function.

Types of Functions

(i) **Identity function** The function $f : R \to R$ defined by $f(x) = x$ for each $x \in R$ is called identity function.
Domain of $f = R$
and Range of $f = R$

(ii) **Constant function** The function $f : R \to R$ defined by $f(x) = c, \forall\ x \in R$, where c is a constant $\in R$, is called a constant function.
Domain of $f = R$
and Range of $f = \{c\}$

(iii) **Polynomial function** A real function $f : R \to R$ defined by $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$, where n is a non-negative integer and $a_0, a_1, a_2, \ldots, a_n \in R$ for each $x \in R$, is called a polynomial function. If $a_n \neq 0$, then n is called the degree of the polynomial. The domain of a polynomial function is R and range depends on the polynomial representing the function.

(iv) **Rational function** A function of the form $\dfrac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of x defined in a domain and $g(x) \neq 0$, is called a rational function.

(v) **Modulus or Absolute value function**
The real function $f : R \to R$ defined by

$$f(x) = |x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

is called the modulus function.
Domain of $f = R$,
Range of $f = R^+ \cup \{0\}$ i.e. $[0, \infty)$

(vi) **Signum function** The real function $f : R \to R$ defined by

$$f(x) = \begin{cases} \dfrac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

$$= \begin{cases} -1, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 1, & \text{if } x > 0 \end{cases}$$

is called the signum function.
Domain of $f = R$; Range of $f = \{-1, 0, 1\}$

(vii) **Greatest integer or Step function** The real function $f : R \to R$ defined by $f(x) = [x]$, is called the greatest integer function, where $[x] =$ integral part of x or greatest integer less than or equal to x.
Domain of $f = R$;
Range of $f =$ The set of all integers

Objective Questions

Multiple Choice Questions

1. Which of the following is an ordered pair?

(a) (p, q), $p \in P$ and $q \in Q$

(b) $[p, q]$, $p \in P$ and $q \in Q$

(c) $\{p, q\}$, $p \in P$ and $q \in Q$

(d) All of the above

2. The values of a and b, if ordered pair is $(2a - 5, 4) = (5, b + 6)$

(a) $-2, 5$ (b) $2, 5$

(c) $5, 2$ (d) $5, -2$

3. If $A = \{a_1, a_2\}$ and $B = \{b_1, b_2, b_3, b_4\}$, then $A \times B$ is equal to

(a) $\{(a_1, b_1), (a_2, b_2)\}$

(b) $\{(a_1, b_1), (a_2, b_2), (a_3, b_3), (a_4, b_4)\}$

(c) $\{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_1, b_4)\}$

(d) $\{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_1, b_4), (a_2, b_1), (a_2, b_2), (a_2, b_3), (a_2, b_4)\}$

4. If $A = \{1, 2, 5, 6\}$ and $B = \{1, 2, 3\}$, then what is $(A \times B) \cap (B \times A)$ equal to?

(a) $\{(1, 1), (2, 1), (6, 1), (3, 2)\}$ (b) $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$

(c) $\{(1, 1), (2, 2)\}$ (d) $\{(1, 1), (1, 2), (2, 5), (2, 6)\}$

5. If $A \times B = \{(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)\}$. Then, A and B is

(a) $A = \{1, 3, 2\}$ and $B = \{a, b\}$

(b) $A = \{a, 1, 2\}$ and $B = \{b, 3\}$

(c) $A = \{a, b\}$ and $B = \{1, 2, 3\}$

(d) $A = \{a, b, 1\}$ and $B = \{a, b, 2, 3\}$

6. Let $n(A) = m$ and $n(B) = n$. Then, the total number of non-empty relations that can be defined from A to B is

(a) m^n (b) $n^m - 1$

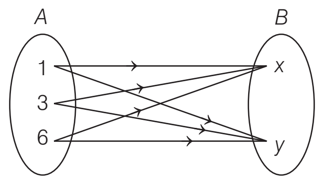
(c) $mn - 1$ (d) $2^{mn} - 1$

7. Let $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$. What is the number of elements in $A \times B$?

(a) 6 (b) 7

(c) 12 (d) 64

8. If $A = \{1, 3, 6\}$ and $B = \{x, y\}$, then representation of cartesian products by an arrow diagrams of $A \times B$ is

(a)


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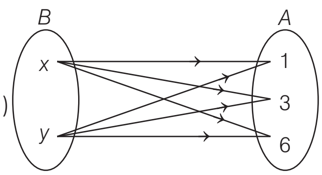

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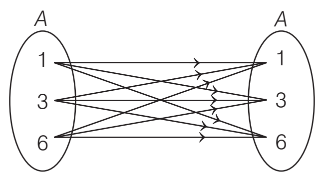

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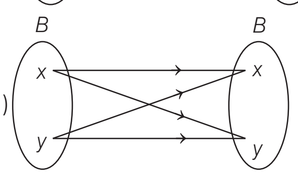


9. If $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 8\}$, then which of the following are relations from A to B?

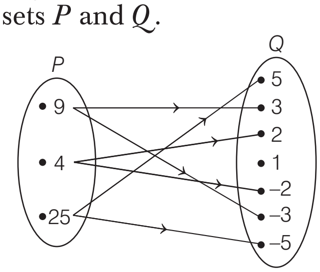
(a) $R_1 = \{(1, 5), (2, 7), (3, 8)\}$

(b) $R_2 = \{(5, 2), (3, 7), (4, 7)\}$

(c) $R_3 = \{(6, 2), (3, 7), (4, 7)\}$

(d) All are correct

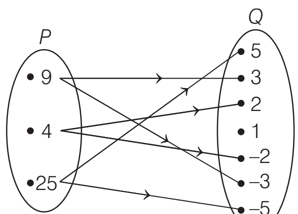
10. The figure shows a relation R between the sets P and Q.



The relation R in Roster form is

(a) {(9, 3), (4, 2), (25, 5)}
(b) {(9, −3), (4, −2), (25, −5)}
(c) {(9, −3), (9, 3), (4, −2), (4, 2), (25, −5), (25, 5)}
(d) None of the above

11. The figure shows a relation R between the sets P and Q.



The relation R in Set-builder form is

(a) $\{(x, y): x \in P, y \in Q\}$
(b) $\{(x, y): x \in Q, y \in P\}$
(c) $\{(x, y): x$ is the square of y, $x \in P, y \in Q\}$
(d) $\{(x, y): y$ is the square of x, $x \in P, y \in Q\}$

12. If a relation R is defined on the set Z of integers as follows

$$(a, b) \in R \Leftrightarrow a^2 + b^2 = 25,$$

then domain (R) is equal to

(a) {3,4,5} (b) {0, 3,4,5}
(c) $\{0, \pm 3, \pm 4, \pm 5\}$ (d) None of these

13. If $A = \{1, 2, 6\}$ and R be the relation defined on A by $R = \{(a, b): a \in A, b \in A$ and a divides $b\}$, then range of R is equal to

(a) {1, 2} (b) {2, 6}
(c) {1, 2, 6} (d) None of these

14. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16, 25\}$ and R be a relation defined from A to B, as $R = \{(x, y): x \in A, y \in B$ and $y = x^2\}$, then domain of R and codomain of R is

(a) {1, 2, 3, 4} and {1, 4, 9, 16, 25}
(b) {1, 4, 9, 16, 25} and {1, 2, 3, 4}
(c) {1, 2, 3, 4} and {1, 2, 3, 4, 9, 16, 25}
(d) None of the above

15. The inverse of the function

$f(x) = \log_a (x + \sqrt{x^2 + 1})$

(where, $a < 0, a \neq 1$) is

(a) $\frac{1}{2}(a^x - a^{-x})$ (b) not defined for all x
(c) defined for $x > 0$ (d) None of the above

16. If $f(x) = 3x + 10$ and $g(x) = x^2 - 1$, then $(fog)^{-1}$ is equal to

(a) $\left(\frac{x-7}{3}\right)^{1/2}$ (b) $\left(\frac{x+7}{3}\right)^{1/2}$
(c) $\left(\frac{x-3}{7}\right)^{1/2}$ (d) $\left(\frac{x+3}{7}\right)^{1/2}$

17. Is the given relation a function?
{(3, 3), (4, 2), (5, 1), (6, 0), (7, 7)}

(a) Yes (b) No
(c) cannot say (d) Insufficient data

18. There are three relations R_1, R_2 and R_3 such that $R_1 = \{(2, 1), (3, 1)(4, 2)\}$,
$R_2 = \{(2, 2), (2, 4), (3, 3), (4, 4)\}$ and
$R_3 = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$.

Then,

(a) R_1 and R_2 are functions
(b) R_2 and R_3 are functions
(c) R_1 and R_3 are functions
(d) Only R_1 is a function

19. Domain of $\sqrt{a^2 - x^2}$ $(a > 0)$ is

(a) $(-a, a)$ (b) $[-a, a]$
(c) $[0, a]$ (d) $(-a, 0]$

20. Range of $f(x) = \dfrac{1}{1 - 2\cos x}$ is

(a) $\left[\frac{1}{3}, 1\right]$ (b) $\left[-1, \frac{1}{3}\right]$
(c) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$ (d) $\left[-\frac{1}{3}, 1\right]$

21. $f: R - \{3\} \to R$ be defined by $f(x) = \dfrac{x^2 - 9}{x - 3}$ and $g: R \to R$ be defined by $g(x) = x + 3$. Then, $f(x)$ and $g(x)$ are

(a) Equal functions
(b) not equal (domains are same)
(c) not equal (domains are not same)
(d) None of the above

22. The domain and range of the real function f defined by $f(x) = \dfrac{4-x}{x-4}$ is given by

(a) Domain $= R$, Range $= \{-1, 1\}$
(b) Domain $= R - \{1\}$, Range $= R$
(c) Domain $= R - \{4\}$, Range $= R - \{-1\}$
(d) Domain $= R - \{-4\}$, Range $= \{-1, 1\}$

23. The domain and range of the function f given by $f(x) = 2 - |x - 5|$ is

(a) Domain $= R^+$, Range $= (-\infty, 1]$
(b) Domain $= R$, Range $= (-\infty, 2]$
(c) Domain $= R$, Range $= (-\infty, 2)$
(d) Domain $= R^+$, Range $= (-\infty, 2]$

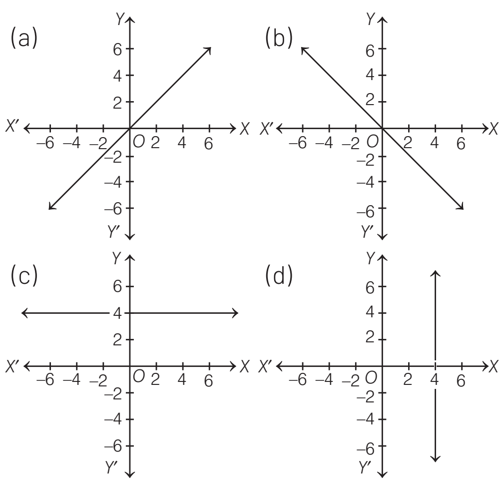
24. The domain of the function f, defined by $f(x) = \dfrac{1}{\sqrt{x - |x|}}$ is

(a) R (b) R^+
(c) R^- (d) None of these

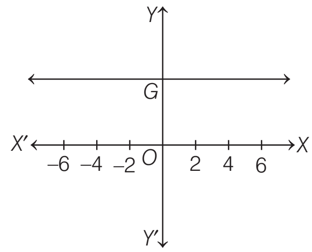
25. The range of the function $f(x) = \dfrac{x}{1+x^2}$ is

(a) $(-\infty, \infty)$ (b) $[-1, 1]$
(c) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ (d) $[\sqrt{-2}, \sqrt{2}]$

26. The graph of an identity function on R is

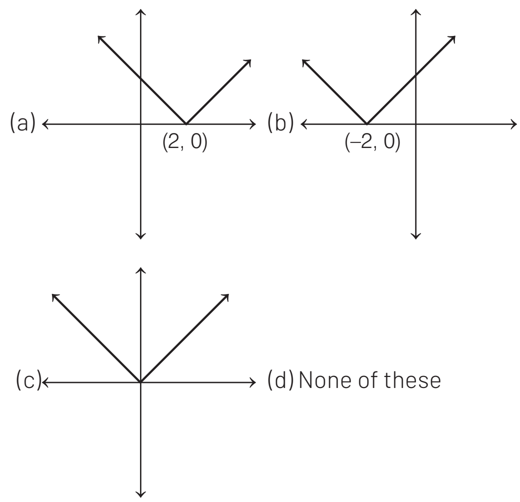


27. If G represents the name of the function in above graph, then G is a/an



(a) identity function
(b) constant function
(c) modulus function
(d) None of the above

28. The graph of the functions, $f(x) = |x - 2|$ is

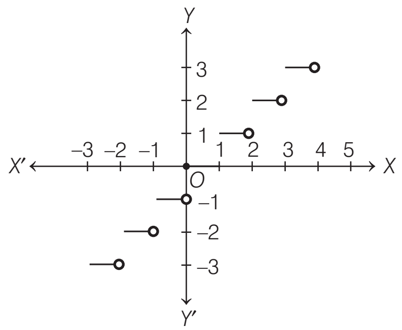


29. For each non-zero real number x, let $f(x) = \dfrac{x}{|x|}$.

The range of f is

(a) a null set
(b) a set consisting of only one element
(c) a set consisting of two elements
(d) a set consisting of infinitely many elements

30.



$f(x) = [x]$

Which of the following options identify the above graph?

(a) Modulus function
(b) Greatest integer function
(c) Signum function
(d) None of the above

31. If $[x]^2 - 5[x] + 6 = 0$, where $[\cdot]$ denote the greatest integer function, then

(a) $x \in [3, 4]$ (b) $x \in (2, 3]$
(c) $x \in [2, 3]$ (d) $x \in [2, 4)$

32. If $f(x) = \begin{cases} x^2, & x < 0 \\ x, & 0 \le x < 1, \\ \dfrac{1}{x}, & x \ge 1 \end{cases}$

then the value of $f\left(\dfrac{1}{2}\right)$ is

(a) 2 (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{2}$ (d) 4

33. If $f(x) = x^3 - \dfrac{1}{x^3}$, then $f(x) + f\left(\dfrac{1}{x}\right)$ is

(a) 0 (b) 1
(c) 2 (d) not defined

34. If $y = e^{x^3 - 2}$, then $\log y$ at $x = 5$ is

(a) 321 (b) 234
(c) 125 (d) 123

35. If $f(x) = \log_e\left(\dfrac{1+x}{1-x}\right)$, then $f\left(\dfrac{2x}{1+x^2}\right)$ is equal to

(a) $[f(x)]^2$ (b) $[f(x)]^3$
(c) $2f(x)$ (d) $3f(x)$

Assertion-Reasoning MCQs

Directions (Q. Nos. 36-50) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.
(b) A is true, R is true; R is not a correct explanation of A.
(c) A is true; R is false.
(d) A is false; R is true.

36. **Assertion** (A) If $(x+1, y-2) = (3, 1)$, then $x = 2$ and $y = 3$.

Reason (R) Two ordered pairs are equal, if their corresponding elements are equal.

37. **Assertion** (A) The cartesian product of two non-empty sets P and Q is denoted as $P \times Q$ and $P \times Q = \{(p, q) : p \in P, q \in Q\}$.

Reason (R) If $A = \{\text{red, blue}\}$ and $B = \{b, c, s\}$, then $A \times B = \{(\text{red}, b), (\text{red}, c), (\text{red}, s), (\text{blue}, b), (\text{blue}, c), (\text{blue}, s)\}$.

38. **Assertion** (A) If $(4x+3, y) = (3x+5, -2)$, then $x = 2$ and $y = -2$.

Reason (R) If $A = \{-1, 3, 4\}$, then $A \times A$ is $\{(-1, -1), (-1, 3), (-1, 4), (3, -1), (4, -1), (3, 4)\}$.

39. **Assertion** (A) If $(x, 1)$, $(y, 2)$ and $(z, 1)$ are in $A \times B$ and $n(A) = 3$, $n(B) = 2$, then $A = \{x, y, z\}$ and $B = \{1, 2\}$.

Reason (R) If $n(A) = 3$ and $n(B) = 2$, then $n(A \times B) = 6$.

40. **Assertion** (A) Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Then, number of relations from A to B is 16.

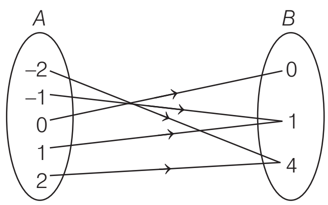
Reason (R) If $n(A) = p$ and $n(B) = q$, then number of relations is 2^{pq}.

41. Let $A = \{1, 2, 3, 4, 6\}$. If R is the relation on A defined by $\{(a, b) : a, b \in A, b$ is exactly divisible by $a\}$.

Assertion (A) The relation R in Roster form is $\{(6, 3), (6, 2), (4, 2)\}$.

Reason (R) The domain and range of R is $\{1, 2, 3, 4, 6\}$.

42. Consider the following statements



Assertion (A) The figure shows a relationship between the sets A and B. Then, the relation in Set-builder form is $\{(x, y) : y = x^2, x, y \in N$ and $-2 \le x \le 2\}$.

Reason (R) The above Relation in Roster form is $\{(-1, 1), (-2, 4), (0, 0), (1, 1), (2, 4)\}$.

43. Let R be a relation defined by
$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$
Then, consider the following

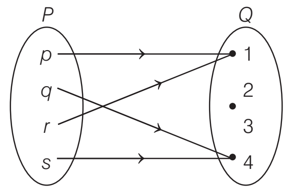
Assertion (A) The domain of R is $\{0, 1, 2, 3, 4, 5\}$.

Reason (R) The range of R is $\{0, 1, 2, 3, 4, 5\}$.

44. Assertion (A) The domain of the relation $R = \{(x + 2, x + 4) : x \in N, x < 8\}$ is $\{3, 4, 5, 6, 7, 8, 9\}$.

Reason (R) The range of the relation $R = \{(x + 2, x + 4) : x \in N, x < 8\}$ is $\{1, 2, 3, 4, 5, 6, 7\}$.

45. Assertion (A) The following arrow diagram represents a function.



Reason (R) Let $f : R - \{2\} \to R$ be defined by $f(x) = \dfrac{x^2 - 4}{x - 2}$ and $g : R \to R$ be defined by $g(x) = x + 3$. Then, $f = g$.

46. Assertion (A) The range of the function $f(x) = 2 - 3x, x \in R, x > 0$ is R.

Reason (R) The range of the function $f(x) = x^2 + 2$ is $[2, \infty)$.

47. Assertion (A) Let $A = \{1, 2, 3, 5\}$, $B = \{4, 6, 9\}$ and $R = \{(x, y) : |x - y|$ is odd, $x \in A, y \in B\}$. Then, domain of R is $\{1, 2, 3, 5\}$.

Reason (R) $|x|$ is always positive $\forall x \in R$.

48. Assertion (A) The domain of the real function f defined by $f(x) = \sqrt{x - 1}$ is $R - \{1\}$.

Reason (R) The range of the function defined by $f(x) = \sqrt{x - 1}$ is $[0, \infty)$.

49. Assertion (A) If $f(x) = x + \dfrac{1}{x}$, then $[f(x)]^3 = f(x^3) + 3f\left(\dfrac{1}{x}\right)$.

Reason (R) If $f(x) = (x - a)^2 (x - b)^2$, then $f(a + b)$ is 0.

50. Assertion (A) If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the values of x such that $g\{f(x)\} = 8$ are -1 and 2.

Reason (R) If $f : R \to R$ be given by $f(x) = \dfrac{4^x}{4^x + 2}$ for all $x \in R$, then $f(x) + f(1 - x) = 1$.

Case Based MCQs

51. Method to Find the Sets When Cartesian Product is Given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).

Number of Elements in Cartesian Product of Two Sets

If there are p elements in set A and q elements in set B, then there will be pq elements in $A \times B$ i.e. if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

Based on the above two topic, answer the following questions.

(i) If $A \times B = \{(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)\}$. Then, A and B are

(a) $\{1, 3, 2\}, \{a, b\}$ (b) $\{a, b\}, \{1, 3\}$
(c) $\{a, b\}, \{1, 3, 2\}$ (d) None of these

(ii) If the set A has 3 elements and set B has 4 elements, then the number of elements in $A \times B$ is

(a) 3 (b) 4
(c) 7 (d) 12

(iii) A and B are two sets given in such a way that $A \times B$ contains 6 elements. If three elements of $A \times B$ are $(1, 3)$, $(2, 5)$ and $(3, 3)$, then A, B are

(a) $\{1, 2, 3\}, \{3, 5\}$ (b) $\{3, 5,\}, \{1, 2, 3\}$
(c) $\{1, 2\}, \{3, 5\}$ (d) $\{1, 2, 3\}, \{5\}$

(iv) The remaining elements of $A \times B$ in (iii) is

(a) $(5, 1), (3, 2), (3, 5)$
(b) $(1, 5), (2, 3), (3, 5)$
(c) $(1, 5), (3, 2), (5, 3)$
(d) None of the above

(v) The cartesian product $P \times P$ has 16 elements among which are found $(a, 1)$ and $(b, 2)$. Then, the set P is

(a) $\{a, b\}$ (b) $\{1, 2\}$
(c) $\{a, b, 1, 2\}$ (d) $\{a, b, 1, 2, 4\}$

52. Ordered Pairs The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).

Two ordered pairs are equal if their corresponding elements are equal.

i.e. $(a, b) = (c, d) \Rightarrow a = c$ and $b = d$

Cartesian Product of Two Sets For two non-empty sets A and B, the cartesian product $A \times B$ is the set of all ordered pairs of elements from sets A and B.

In symbolic form, it can be written as

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Based on the above topics, answer the following questions.

(i) If $(a - 3, b + 7) = (3, 7)$, then the value of a and b are

(a) 6, 0 (b) 3, 7
(c) 7, 0 (d) 3, -7

(ii) If $(x + 6, y - 2) = (0, 6)$, then the value of x and y are

(a) 6, 8 (b) $-6, -8$
(c) $-6, 8$ (d) $6, -8$

(iii) If $(x + 2, 4) = (5, 2x + y)$, then the value of x and y are

(a) $-3, 2$ (b) 3, 2
(c) $-3, -2$ (d) $3, -2$

(iv) Let A and B be two sets such that $A \times B$ consists of 6 elements. If three elements of $A \times B$ are $(1, 4)$, $(2, 6)$ and $(3, 6)$, then

(a) $(A \times B) = (B \times A)$
(b) $(A \times B) \neq (B \times A)$
(c) $A \times B = \{(1, 4), (1, 6), (2, 4)\}$
(d) None of the above

(v) If $n(A \times B) = 45$, then $n(A)$ cannot be

(a) 15 (b) 17
(c) 5 (d) 9

53. Representation of a Relation A relation can be represented algebraically by roster form or by set-builder form and visually it can be represented by an arrow diagram which are given below

(i) **Roster form** In this form, we represent the relation by the set of all ordered pairs belongs to R.

(ii) **Set-builder form** In this form, we represent the relation R from set A to set B as $R = \{(a, b) : a \in A, b \in B$ and the rule which relate the elements of A and $B\}$.

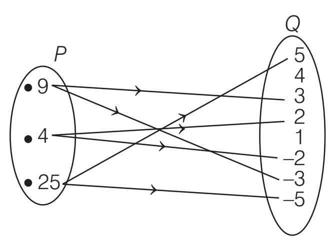
(iii) **Arrow diagram** To represent a relation by an arrow diagram, we draw arrows from first element to second element of all ordered pairs belonging to relation R.

Based on the above topics, answer the following questions.

(i) Expression of $R = \{(a, b) : 2a + b = 5; a, b \in W\}$ as the set of ordered pairs (in roster form) is

(a) $R = \{(5, 0), (3, 1), (1, 2)\}$
(b) $R = \{(0, 5), (1, 3), (1, 2)\}$
(c) $R = \{(0, 5), (1, 3), (2, 1)\}$
(d) None of the above

(ii) The relation between sets P and Q given by an arrow diagram in roster form will be



(a) $R = \{(9,3),(9,-3),(4,2),(4,-2),(25,5),(25,-5)\}$
(b) $R = \{(9, 3), (4, 2), (25, 5)\}$
(c) $R = \{(9,-3),(4,-2),(25,-5)\}$
(d) None of the above

(iii) The relation given in (ii) can be written in set-builder form as

(a) $R = \{(x, y) : x \in P, y \in Q$ and x is the square of $y\}$
(b) $R = \{(x, y) : x \in P, y \in Q$ and y is the square of $x\}$
(c) $R = \{(x, y) : x \in P, y \in Q$ and $x = \pm y\}$
(d) None of the above

(iv) If $A = \{a, b\}$ and $B = \{2, 3\}$, then the number of relations from A to B is

(a) 4 (b) 8 (c) 6 (d) 16

(v) If $n(A) = 3$ and $B = \{2, 3, 4, 6, 7, 8\}$, then the number of relations from A to B is

(a) 2^3 (b) 2^6 (c) 2^{18} (d) 2^9

54. Function as a Relation A relation f from a non-empty set A to a non-empty set B is said to be a function, if every element of set A has one and only one image in set B.

In other words, we can say that a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element or component.

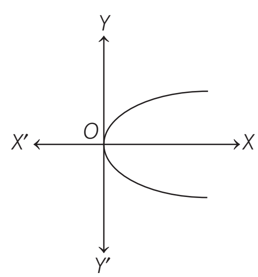
If f is a function from a set A to a set B, then we write

$$f : A \to B \text{ or } A \xrightarrow{f} B$$

and it is read as f is a function from A to B or f maps A to B.

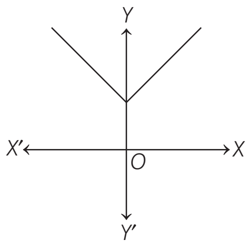
Based on the above topic, answer the following questions.

(i) The given curve is a



(a) Function
(b) Relation
(c) Can't say anything
(d) Data not sufficient

(ii) The given curve is a



(a) Function
(b) Relation
(c) Can't say anything
(d) Data not sufficient

(iii) If $f(x) = x^2 + 2x + 3$, then among $f(1)$, $f(2)$ and $f(3)$, which one gives the maximum value.

(a) $f(1)$ (b) $f(2)$
(c) $f(3)$ (d) $f(1) = f(2) = f(3)$

(iv) If $f(1 + x) = x^2 + 1$, then $f(2 - h)$ is

(a) $h^2 - 2h + 2$ (b) $h^2 - 2h + 1$
(c) $h^2 - 2h - 2$ (d) $h^2 + 2h + 2$

(v) If $f(x) = \dfrac{1}{2 - \sin 3x}$, then range (f) is equal to

(a) $[-1, 1]$ (b) $\left[-\frac{1}{3}, \frac{1}{3}\right]$
(c) $\left[\frac{1}{3}, 1\right]$ (d) $\left[-1, \frac{-1}{3}\right]$

ANSWERS

Multiple Choice Questions

1. (a)	*2. (d)*	*3. (d)*	*4. (b)*	*5. (c)*	*6. (d)*	*7. (c)*	*8. (a)*	*9. (a)*	*10. (c)*
11. (c)	*12. (c)*	*13. (c)*	*14. (a)*	*15. (a)*	*16. (a)*	*17. (a)*	*18. (c)*	*19. (b)*	*20. (b)*
21. (c)	*22. (c)*	*23. (b)*	*24. (d)*	*25. (c)*	*26. (a)*	*27. (b)*	*28. (a)*	*29. (c)*	*30. (b)*
31. (c)	*32. (c)*	*33. (a)*	*34. (d)*	*35. (c)*					

Assertion-Reasoning MCQs

36. (a)	*37. (a)*	*38. (c)*	*39. (b)*	*40. (a)*	*41. (d)*	*42. (d)*	*43. (c)*	*44. (c)*	*45. (c)*
46. (d)	*47. (b)*	*48. (d)*	*49. (c)*	*50. (d)*					

Case Based MCQs

51. (i) - (c); (ii) - (d); (iii) - (a); (iv) - (b); (v) - (c)
52. (i) - (a); (ii) - (c); (iii) - (d); (iv) - (b); (v) - (b)
53. (i) - (c); (ii) - (a); (iii) - (a); (iv) - (d); (v) - (c)
54. (i) - (b); (ii) - (a); (iii) - (c); (iv) - (a); (v) - (c)

SOLUTIONS

1. We know that, an ordered pair of elements taken from any two sets P and Q is a pair of elements written in small brackets and grouped together in a particular order, i.e. (p, q), $p \in P$ and $q \in Q$.

2. We know that, two ordered pairs are equal, if their corresponding elements are equal.

$$(2a - 5, 4) = (5, b + 6)$$

$\Rightarrow \quad 2a - 5 = 5$ and $4 = b + 6$ [equating corresponding elements]

$\Rightarrow \quad 2a = 5 + 5$ and $4 - 6 = b$

$\Rightarrow \quad 2a = 10$ and $-2 = b$

$\Rightarrow \quad a = 5$ and $b = -2$

3. If $A = \{a_1, a_2\}$, $B = \{b_1, b_2, b_3, b_4\}$, then.

$$A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_1, b_4), (a_2, b_1), (a_2, b_2), (a_2, b_3), (a_2, b_4)\}$$

4. Given, $A = \{1, 2, 5, 6\}$

and $B = \{1, 2, 3\}$

$$A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$$

$$B \times A = \{(1, 1), (1, 2), (1, 5), (1, 6), (2, 1), (2, 2), (2, 5), (2, 6), (3, 1), (3, 2), (3, 5), (3, 6)\}$$

$\therefore (A \times B) \cap (B \times A) = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

5. Here, first element of each ordered pair of $A \times B$ gives the elements of set A and corresponding second element gives the elements of set B.

$\therefore \quad A = \{a, b\}$ and $B = \{1, 3, 2\}$

6. We have, $n(A) = m$ and $n(B) = n$

$$n(A \times B) = n(A) \cdot n(B) = mn$$

Total number of relation from A to B

$$= 2^{mn} - 1 = 2^{n(A \times B)} - 1$$

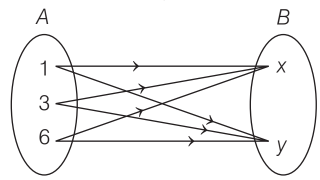
7. Here, $n(A) = 4$ and $n(B) = 3$

$\therefore n(A \times B) = n(A) \times n(B) = 4 \times 3 = 12$

8. We have, $A = \{1, 3, 6\}$, $B = \{x, y\}$

$$A \times B = \{1, 3, 6\} \times \{x, y\} = \{(1, x), (1, y), (3, x), (3, y), (6, x), (6, y)\}$$

Required arrow diagram is



9. We have, $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 8\}$

$$A \times B = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

(a) Since, $R_1 \subseteq A \times B$, therefore R_1 is a relation from A to B.

(b) Since, $(5, 2) \in R_2$ but $(5, 2) \notin A \times B$, therefore $R_2 \not\subseteq A \times B$. Thus, R_2 is not a relation from A to B.

(c) Since, $(6, 2) \in R_3$ but $(6, 2) \notin A \times B$, therefore $R_3 \not\subseteq A \times B$. Thus, R_3 is not a relation from A to B.

10. In Roster form, $R = \{(9, 3), (9, -3), (4, 2), (4, -2), (25, 5), (25, -5)\}$.

11. It is obvious that the relation R is 'x is the square of y'.
In Set-builder form, $R = \{(x, y): x$ is the square of y, $x \in P$, $y \in Q\}$

12. We have, $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$

$\Rightarrow \quad b = \pm\sqrt{25 - a^2}$

Clearly, $\quad a = 0 \Rightarrow b = \pm 5$

$a = \pm 3 \Rightarrow b = \pm 4$

$a = \pm 4 \Rightarrow b = \pm 3$

and $\quad a = \pm 5 \Rightarrow b = 0$

Hence, domain $(R) = \{0, \pm 3, \pm 4, \pm 5\}$.

13. Let $R = \{(a, b) : a \in A, b \in A$, and a divides $b\}$

$\therefore \quad R = \{(1, 1), (1, 2), (1, 6), (2, 2), (2, 6), (6, 6)\}$

$\therefore$ Range $= \{1, 2, 6\} = A$

14. Relation $R = \{(1, 1), (2, 4), (3, 9), (4, 16)\}$

Domain $(R) = \{1, 2, 3, 4\}$

Codomain $(R) = \{1, 4, 9, 16, 25\}$

15. Let $f(x) = y$, then

$$a^y = x + \sqrt{x^2+1}$$

$$\Rightarrow \quad a^{-y} = \frac{1}{x+\sqrt{x^2+1}} = \frac{x-\sqrt{x^2+1}}{-1}$$

$$\therefore a^y - a^{-y} = 2x \Rightarrow x = \frac{1}{2}(a^y - a^{-y})$$

$$\therefore \quad f^{-1}(x) = \frac{1}{2}(a^x - a^{-x})$$

16. $f(x) = 3x + 10$ and $g(x) = x^2 - 1$

$$\therefore fog = f[g(x)] = 3[g(x)] + 10$$

$$= 3(x^2-1) + 10 = 3x^2 + 7$$

Let $3x^2 + 7 = y \Rightarrow x^2 = \frac{y-7}{3}$

$$\Rightarrow \quad x = \left(\frac{y-7}{3}\right)^{1/2}$$

So, $(fog)^{-1} = \left(\frac{x-7}{3}\right)^{1/2}$

17. $\{(3, 3), (4, 2), (5, 1), (6, 0), (7, 7)\}$

It is a function, because first element of each ordered pair is different.

18. Since 2, 3, 4 are the elements of domain of R_1 having their unique images, this relation R_1 is a function.

Since, the same first element 2 corresponds to two different images 2 and 4, this relation R_2 is not a function.

Since, every element has one and only one image, this relation R_3 is a function.

19. Let

$$f(x) = \sqrt{a^2 - x^2}$$

$f(x)$ is defined, if $a^2 - x^2 \geq 0$

$$\Rightarrow \quad x^2 - a^2 \leq 0$$

$$\Rightarrow \quad (x-a)(x+a) \leq 0$$

$$\Rightarrow \quad -a \leq x \leq a \qquad [\because a > 0]$$

$$\therefore \quad \text{Domain of } f = [-a, a]$$

20. We know that,

$$-1 \leq -\cos x \leq 1$$

$$\Rightarrow \quad -2 \leq -2\cos x \leq 2$$

$$\Rightarrow \quad 1-2 \leq 1-2\cos x \leq 1+2$$

$$\Rightarrow \quad -1 \leq 1-2\cos x \leq 3$$

$$\Rightarrow \quad -1 \leq \frac{1}{1-2\cos x} \leq \frac{1}{3}$$

$$\Rightarrow \quad -1 \leq f(x) \leq \frac{1}{3}$$

$$\therefore \quad \text{Range of } f = \left[-1, \frac{1}{3}\right]$$

21. Here, $f(x) = \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = (x+3)$

and $g(x) = x + 3 \Rightarrow f(x) = g(x) = x + 3$

But $f(x) \neq g(x)$ as domain of $f(x)$ is $R - \{3\}$ and domain of $g(x)$ is R.

$\Rightarrow$ Domain of $f(x) \neq$ Domain of $g(x)$

$$\Rightarrow \quad f \neq g$$

22. We have, $f(x) = \frac{4-x}{x-4}$

$f(x)$ is defined, if $x - 4 \neq 0$ i.e. $x \neq 4$

$\therefore$ Domain of $f = R - \{4\}$

Let $f(x) = y$

$$\therefore \quad y = \frac{4-x}{x-4} \Rightarrow xy - 4y = 4 - x$$

$$\Rightarrow xy + x = 4 + 4y \Rightarrow x(y+1) = 4(1+y)$$

$$\therefore \quad x = \frac{4(1+y)}{y+1}$$

x assumes real values, if $y + 1 \neq 0$ i.e. $y \neq -1$.

$\therefore$ Range of $f = R - \{-1\}$

23. We have, $f(x) = 2 - |x-5|$

$f(x)$ is defined for all $x \in R$

$\therefore$ Domain of $f = R$

We know that,

$$|x-5| \geq 0 \Rightarrow -|x-5| \leq 0 \Rightarrow 2 - |x-5| \leq 2$$

$\therefore \quad f(x) \leq 2$

$\therefore \quad$ Range of $f = (-\infty, 2]$

24. We have, $f(x) = \frac{1}{\sqrt{x-|x|}}$

where, $x - |x| = x - x = 0$, if $x \geq 0$

$x - (-x) = 2x$, if $x < 0$

Thus, $\frac{1}{\sqrt{x-|x|}}$ is not defined for $x \in R$.

Hence, f is not defined for any $x \in R$.

25. We have, $f(x) = \frac{x}{1+x^2}$

Let, $f(x) = y$

$$\therefore \quad y = \frac{x}{1+x^2}$$

$$\Rightarrow \quad x^2y + y = x \Rightarrow yx^2 - x + y = 0$$

x assumes real values, if $\Delta \geq 0$

$$(-1)^2 - 4(y)(y) \geq 0$$

$$\Rightarrow \quad 1 - 4y^2 \geq 0$$

$$\Rightarrow \quad 4y^2 - 1 \leq 0$$

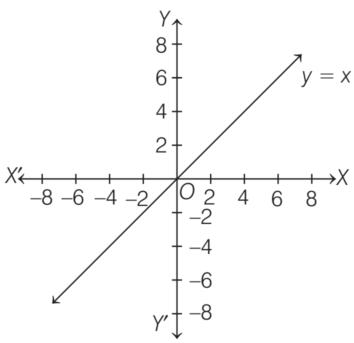
$$\Rightarrow \quad (2y+1)(2y-1) \leq 0$$

$$\Rightarrow \quad y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$\therefore$ Range of $f = \left[-\frac{1}{2}, \frac{1}{2}\right]$

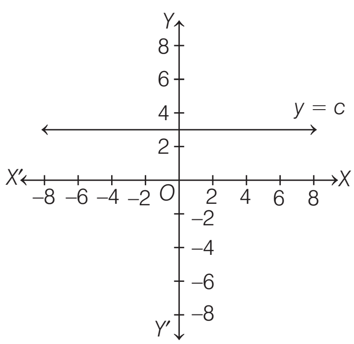
26. Let R be the set of real numbers. Define the real valued function $f : R \to R$ by $y = f(x) = x$ for each $x \in R$.

Such a function is called the identity function. Here, the domain and range of f are R. The graph is a straight line as shown in figure given below. It passes through the origin.



27. **Constant function** Define the function $f : R \to R$ by $y = f(x) = c$, $x \in R$, where c is a constant.

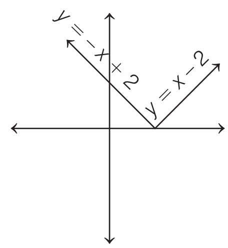
Here, domain of f is R and its range is $\{c\}$.



e.g. The graph of $f(x) = 3$, is a line passing through $(0, 3)$ and parallel to X-axis.

28. $f(x) = |x - 2|$

$$f(x) = \begin{cases} -x + 2 & x \leq 2 \\ x - 2 & x > 2 \end{cases}$$



29. We have, $f(x) = \dfrac{x}{|x|}$, for $x \neq 0$

i.e. $f(x) = \begin{cases} \dfrac{x}{x}, & \text{if } x > 0 \\ \dfrac{x}{-x}, & \text{if } x < 0 \end{cases} = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases}$

Thus, range of $f = \{1, -1\}$.

30. **Greatest integer function** The function $f : R \to R$ defined by $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x. Such a function is called the greatest integer function.

From the definition of $[x]$, we can see that

$$[x] = -1 \text{ for } -1 \leq x < 0$$

$$[x] = 0 \text{ for } 0 \leq x < 1$$

$$[x] = 1 \text{ for } 1 \leq x < 2$$

$$[x] = 2 \text{ for } 2 \leq x < 3 \text{ and so on.}$$

The graph of the function is given in the question.

31. We have, $[x]^2 - 5[x] + 6 = 0$

$$\Rightarrow \quad [x]^2 - 3[x] - 2[x] + 6 = 0$$

$$\Rightarrow \quad [x]([x] - 3) - 2([x] - 3) = 0$$

$$\Rightarrow \quad ([x] - 3)([x] - 2) = 0$$

$$\Rightarrow \quad [x] = 2, 3$$

$$\therefore \quad x \in [2, 3]$$

32. Given, $f(x) = \begin{cases} x^2, & x < 0 \\ x, & 0 \leq x < 1 \\ \dfrac{1}{x}, & x \geq 1 \end{cases}$

At $x = \dfrac{1}{2} \Rightarrow f(x) = x$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

33. Given, $f(x) = x^3 - \dfrac{1}{x^3}$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\left(\frac{1}{x}\right)^3} = \frac{1}{x^3} - x^3$$

$$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3$$

$$= 0$$

34. Given, $y = e^{x^3 - 2}$

$$\log y = \log e^{x^3 - 2}$$

$$\log y = (x^3 - 2)\log e$$

$$\log y = x^3 - 2$$

$$\log y|_{x=5} = 5^3 - 2 = 125 - 2 = 123$$

35. We have, $f(x) = \log_e\left(\dfrac{1+x}{1-x}\right)$

$$f\left(\frac{2x}{1+x^2}\right) = \log_e\left(\frac{1 + \frac{2x}{1+x^2}}{1 - \frac{2x}{1+x^2}}\right)$$

$$= \log_e\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log_e\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log_e\left(\frac{1+x}{1-x}\right)$$

$[\because \log a^b = b\log a]$

$$= 2f(x)$$

36. **Assertion** Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

Given, $(x+1, y-2) = (3, 1)$.

Then, by the definition

$$x + 1 = 3 \text{ and } y - 2 = 1$$

$$\Rightarrow \quad x = 2 \text{ and } y = 3$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

37. **Assertion** P and Q are two non-empty sets. The cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q, i.e. $P \times Q = \{(p, q) : p \in P \text{ and } q \in Q\}$

Reason Now, $A = \{\text{red, blue}\}$, $B = \{b, c, s\}$

$A \times B$ = Set of all ordered pairs

$= \{(\text{red}, b), (\text{red}, c), (\text{red}, s), (\text{blue}, b), (\text{blue}, c), (\text{blue}, s)\}$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

38. **Assertion** Given, $(4x + 3, y) = (3x + 5, -2)$

Two ordered pairs are equal when their corresponding elements are equal.

$$4x + 3 = 3x + 5 \text{ and } y = -2$$

$$4x - 3x = 5 - 3$$

$$x = 2$$

Reason Now, $A = \{-1, 3, 4\}$

$\therefore A \times A = \{(-1, -1), (-1, 3), (-1, 4), (3, -1), (3, 3), (3, 4), (4, -1), (4, 3), (4, 4)\}$

$\therefore$ Assertion is true and Reason is false.

39. **Assertion** A = Set of first elements $= \{x, y, z\}$

B = Set of second elements $= \{1, 2\}$

$\therefore A$ is correct.

Reason $n(A) = 3$, $n(B) = 2$

$$n(A \times B) = n(A) \times n(B) = 3 \times 2 = 6$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

40. **Assertion** The total number of relation that can be defined from a set A to a set B is the number of possible subset of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total number of relation is 2^{pq}.

Given, $A = \{1, 2\}$ and $B = \{3, 4\}$

$\therefore \quad A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

Since, $n(A \times B) = 4$, the number of subsets of $A \times B$ is 2^4. Therefore, the number of relation from A to B will be $2^4 = 16$.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

41. **Assertion** In Roster form $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 4), (2, 6), (2, 2), (4, 4), (6, 6), (3, 3), (3, 6)\}$

Reason Domain of R = Set of first element of ordered pairs in $R = \{1, 2, 3, 4, 6\}$

Range of $R = \{1, 2, 3, 4, 6\}$

Hence, Assertion is false, Reason is true.

42. Assertion In Set-builder form,

$$R = \{(x, y) : y = x^2, x, y \in Z \text{ and } -2 \le x \le 2\}$$

$[\because 0 \notin N]$

Reason The relation shown in figure is represented in Roster form as

$$R = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$$

We observe that, second element of each ordered pair is the square of first element.

Hence, Assertion is false, Reason is true.

43. Assertion The given relation in Roster form is

$$R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$$

Domain of $R = \{0, 1, 2, 3, 4, 5\}$

Reason Range of $R = \{5, 6, 7, 8, 9, 10\}$

Hence, Assertion is true, Reason is false.

44. Assertion The given relation in Roster form is, $R = \{(3, 5), (4, 6), (5, 7), (6, 8), (7, 9), (8, 10), (9, 11)\}$

$\therefore$ Domain of $R = \{3, 4, 5, 6, 7, 8, 9\}$

So, A is true.

Reason Range of $R = \{5, 6, 7, 8, 9, 10, 11\}$

So, R is false.

Hence, Assertion is true, Reason is false.

45. Assertion In arrow diagram, every element of P has its unique image in Q. So, it represent a function.

Reason Domain of $f = R - \{2\}$.

Domain of $g = R$

$\because \quad D_f \ne D_g$

We know that, two functions are equal when their domain and range are equal and same element in their domain produce same image.

$\therefore \quad f \ne g$

Hence, Assertion is true and Reason is false.

46. Assertion We have,

$$f(x) = 2 - 3x, x \in R, x > 0$$

Let $f(x) = y$, then $y = 2 - 3x$

$\Rightarrow \quad 3x = 2 - y$

$\Rightarrow \quad x = \dfrac{2 - y}{3}$

$\because \quad x > 0$

$\Rightarrow \quad \dfrac{2 - y}{3} > 0 \Rightarrow 2 - y > 0 \Rightarrow 2 > y$

$\therefore \quad y < 2$

Hence, range of $f = (-\infty, 2)$

Reason Now, $f(x) = x^2 + 2$

Let $y = f(x)$, then

$$y = x^2 + 2 \Rightarrow x = \sqrt{y - 2}$$

x assumes real values, if $y - 2 \ge 0$

$\Rightarrow \quad y \ge 2 \Rightarrow y \in [2, \infty)$

$\therefore$ Range of $f = [2, \infty)$

Hence, Assertion is false, Reason is true.

47. Assertion Given,

$$R = \{(x, y) : |x - y| \text{ is odd}, x \in A, y \in B\}$$

The relation R in Roster form is

$$R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$$

$\therefore$ Domain of $R = \{1, 2, 3, 5\}$

So, A is true.

Reason It is also true $|x|$ is always positive.

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

48. Assertion We have, $f(x) = \sqrt{x - 1}$

$f(x)$ is defined, if $x - 1 \ge 0$ i.e. $x \ge 1$

$\therefore$ Domain of $f = [1, \infty)$

Hence, A is incorrect.

Reason Let $f(x) = y$

Then, $\quad y = \sqrt{x - 1}$

$\because \quad x \ge 1$

$\therefore$ Range of $f = [0, \infty)$.

Hence, Assertion is false, Reason is true.

49. Assertion Given,

$$f(x) = x + \frac{1}{x}$$

$$f(x^3) = x^3 + \frac{1}{x^3}$$

$$[f(x)]^3 = \left(x + \frac{1}{x}\right)^3$$

$$= x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$= f(x^3) + 3f(x)$$

$$= f(x^3) + 3f\left(\frac{1}{x}\right)$$

$$\left[\because f\left(\frac{1}{x}\right) = \frac{1}{x} + x = f(x)\right]$$

Reason Now, we have,

$$f(x) = (x-a)^2 (x-b)^2$$
$$f(a+b) = (a+b-a)^2 (a+b-b)^2 = b^2a^2$$

Hence, Assertion is true, Reason is false.

50. **Assertion** We have,

$$f(x) = 2x+3,\ g(x) = x^2+7$$
$$g[f(x)] = 8$$
$$\Rightarrow \quad g(2x+3) = 8$$
$$\Rightarrow \quad (2x+3)^2 + 7 = 8$$
$$\Rightarrow \quad (2x+3)^2 = 1$$
$$\Rightarrow \quad 2x+3 = \pm 1,$$
$$2x+3 = -1$$
$$\text{or} \quad 2x+3 = 1, \text{ then}$$
$$\Rightarrow \quad x = -1,\ x = -2$$

Reason Now, $f(x) = \dfrac{4^x}{4^x+2}$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2}$$

$$\therefore \quad f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2}$$
$$= \frac{4^x}{4^x+2} + \frac{4/4^x}{\frac{4}{4^x}+2}$$
$$= \frac{4^x}{4^x+2} + \frac{4}{4+2\cdot 4^x}$$
$$= \frac{4^x}{4^x+2} + \frac{2}{4^x+2}$$
$$= \frac{4^x+2}{4^x+2} = 1$$

Hence, Assertion is false, Reason is true.

51. (i) Here, first element of each ordered pair of $A \times B$ gives the elements of set A and corresponding second element gives the elements of set B.

$\therefore \quad A = \{a, b\}$ and $B = \{1, 3, 2\}$

Note We write each element only one time in set, if it occurs more than one time.

(ii) Given, $n(A) = 3$ and $n(B) = 4$.

$\therefore$ The number of elements in $A \times B$ is

$$n(A \times B) = n(A) \times n(B) = 3 \times 4 = 12$$

(iii) It is given that (1, 3), (2, 5) and (3, 3) are in $A \times B$. It follows that 1, 2, 3 are elements of A and 3, 5 are elements of B.

$\therefore \quad A = \{1, 2, 3\}$ and $B = \{3, 5\}$

(iv) $\because \quad A = \{1, 2, 3\}$ and $B = \{3, 5\}$

$$\therefore \quad A \times B = \{1, 2, 3\} \times \{3, 5\}$$
$$= \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$$

Hence, the remaining elements of $(A \times B)$ are (1, 5), (2, 3), (3, 5).

(v) Given, $\quad n(P \times P) = 16$

$$\Rightarrow \quad n(P) \cdot n(P) = 16$$
$$\Rightarrow \quad n(P) = 4 \quad \text{... (i)}$$

Now, as $\quad (a, 1) \in P \times P$

$\therefore \quad a \in P$ and $1 \in P$

Again, $\quad (b, 2) \in P \times P$

$\therefore \quad b \in P$ and $2 \in P$

$\Rightarrow \quad a, b, 1, 2 \in P$

From Eq. (i), it is clear that P has exactly four elements.

52. (i) We know that, two ordered pairs are equal, if their corresponding elements are equal.

$$(a-3, b+7) = (3, 7) \Rightarrow a-3 = 3$$

and $\quad b+7 = 7$

[equating corresponding elements]

$\Rightarrow \quad a = 3+3$ and $b = 7-7$

$\Rightarrow \quad a = 6$ and $b = 0$

(ii) $(x+6, y-2) = (0, 6)$

$$x+6 = 0 \Rightarrow x = -6$$
$$y-2 = 6 \Rightarrow y = 6+2 = 8$$

(iii) $(x+2, 4) = (5, 2x+y)$

$$x+2 = 5 \Rightarrow x = 5-2 = 3$$
$$4 = 2x+y \Rightarrow 4 = 2 \times 3 + y$$
$$\Rightarrow \quad y = 4-6 = -2$$

(iv) Since, (1, 4), (2, 6) and (3, 6) are elements of $A \times B$, it follows that 1, 2, 3 are elements of A and 4, 6 are elements of B. It is given that $A \times B$ has 6 elements.

So, $A = \{1, 2, 3\}$ and $B = \{4, 6\}$

Hence, $A \times B = \{1, 2, 3\} \times \{4, 6\}$

$$= \{(1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6)\}$$

and $B \times A = \{4, 6\} \times \{1, 2, 3\}$

$= \{(4, 1), (4, 2), (4, 3), (6, 1), (6, 2), (6, 3)\}$

(v) We have, $n(A \times B) = 45$

$\Rightarrow n(A) \times n(B) = 45$

$\Rightarrow n(A)$ and $n(B)$ are factors of 45 such that their product is 45.

Here, $n(A)$ cannot be 17.

53. (i) Given, $R = \{(a, b) : 2a + b = 5; a, b \in W\}$

Here, W represent set of whole numbers.

When $a = 0, b = 5$

When $a = 1, b = 3$

When $a = 2, b = 1$

For $a \geq 3$, the value of b given by the above relation are not whole numbers.

$\therefore \quad R = \{(0, 5), (1, 3), (2, 1)\}$

(ii) From arrow diagram, we have

$P = \{9, 4, 25\}$

and $Q = \{5, 4, 3, 2, 1, -2, -3, -5\}$

Here, the relation R is 'x is the square of y', where $x \in P$ and $y \in Q$.

In roster form, it can be written as

$R = \{(9, 3) (9, -3) (4, 2) (4, -2) (25, 5) (25, -5)\}$

(iii) In set-builder form, R can be written as

$R = \{(x, y) : x \in P, y \in Q$ and x is the square of $y\}$

(iv) We have, $A = \{a, b\}$ and $B = \{2, 3\}$,

$\therefore \quad n(A \times B) = n(A) \times n(B)$

$= 2 \times 2 = 4$

Now, number of subsets of $A \times B$

$= 2^{n(A \times B)}$

$= 2^4$

$= 16$

Thus, the number of relations from A to B is 16.

(v) Given, $n(A) = 3$

and $B = \{2, 3, 4, 6, 7, 8\}$

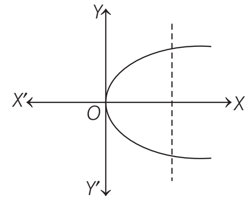
$\Rightarrow n(B) = 6$

$\therefore$ Number of relations from A to B

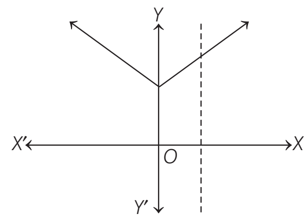
$= 2^{n(A) \times n(B)}$

$= 2^{3 \times 6} = 2^{18}$

54. (i) If we draw a vertical line, then it will intersect the curve at two points. It shows that given curve is a relation.



(ii) If we draw a vertical line, then it will intersect the curve at only one point. It shows that given curve is a function.



(iii) We have, $f(x) = x^2 + 2x + 3$ ...(i)

$f(1) = (1)^2 + 2(1) + 3$ [putting $x = 1$ in Eq. (i)]

$= 1 + 2 + 3 = 6$

$f(2) = (2)^2 + 2(2) + 3$ [putting $x = 2$ in Eq. (i)]

$= 4 + 4 + 3 = 11$

$f(3) = (3)^2 + 2(3) + 3$ [putting $x = 3$ in Eq. (i)]

$= 9 + 6 + 3 = 18$

(iv) We have, $f(1 + x) = x^2 + 1$ ...(i)

On substituting $x = (1 - h)$ in Eq. (i), we get

$f(1 + 1 - h) = (1 - h)^2 + 1$

$f(2 - h) = 1 + h^2 - 2h + 1 = h^2 - 2h + 2$

(v) We know that, $-1 \leq \sin 3x \leq 1$

$\Rightarrow -1 \leq -\sin 3x \leq 1$

$\Rightarrow 1 \leq 2 - \sin 3x \leq 3$

$\Rightarrow \frac{1}{3} \leq \frac{1}{2 - \sin 3x} \leq 1$

$\Rightarrow \frac{1}{3} \leq f(x) \leq 1$

$\therefore$ Range $(f) = \left[\frac{1}{3}, 1\right]$