CBSE Board

Class IX Mathematics

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. Rationalise the denominator of $\frac{1}{3+\sqrt{2}}$.

2. Is point (2, 1) lie on a line whose equation is 2x + y = 5?

3. In Δ ABC, m $\angle$A = x, m $\angle$B = 2x, m $\angle$C = 3x. Find the value of m $\angle$C.

4. Point (–2, –5) will lie in which Quadrant?

5. If the range of the data is 28 and number of classes is 7, then find the class size of the data?

6. O is a center of a Circle and OR $\perp$ PQ, distance of a chord PQ of a circle from the center is 12 cm and the length of the chord is 10 cm, what is the length of a radius?

Section B

(Questions 7 to 12 carry 2 marks each)

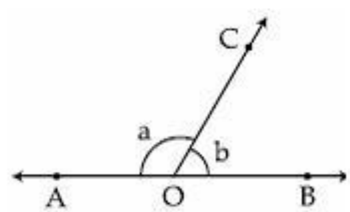
7. Express $0.\overline{975}$ in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

8. Factorise: $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

9. The perpendicular distance of a point from the x-axis is 2 units and the perpendicular distance from the y-axis is 5 units. Write the coordinates of such a point if it lies in one of the following quadrants:

   (i) I Quadrant (ii) II Quadrant (iii) III Quadrant (iv) IV Quadrant

10. In the figure, ∠AOC and ∠BOC form a linear pair. If a – b = 80°, then find the values of a and b.



11. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find
    i. Its inner curved surface area,
    ii. The cost of plastering this curved surface at the rate of Rs 40 per m^2.

12. Find the value of a and b if y = 1 and x = 2 is solution of linear equation ax + by = 3 and 3a – 2b = 1.

Section C

(Questions 13 to 22 carry 3 marks each)

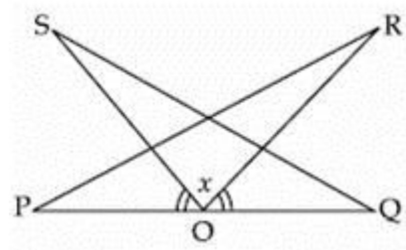
13. Simplify:

$$\frac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

14. If the polynomials $x^2 - 5x - 3a$ and $ax^2 - 5x - 7$ leave the same remainder when they are divided by $(x - 1)$, then what is the value of a?

15. Find the value of $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$.

16. In the figure, PQ is a line segment and O is the mid-point of PQ. R and S are on the same side of PQ such that $\angle PQS = \angle QPR$ and $\angle POS = \angle QOR$. Prove that:



(i) $\Delta PQR \cong \Delta QOS$

(ii) PR = QS

17. Show that the line segments joining the mid points of the opposite sides of a quadrilateral bisect each other.

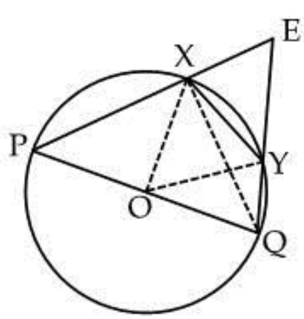
18. A company selected 2400 families at random and surveyed them to determine relationship between income level and the number of television sets at home. The information gathered is listed in the table below:

Monthly income in Rs.	Television per family			
	0	1	2	Above 2
Less than 7,000	10	160	25	0
7,000 – 10,000	0	305	27	2
10,000-13,000	1	535	29	1
13,000-16,000	2	469	59	25
16,000 or more	1	579	82	88

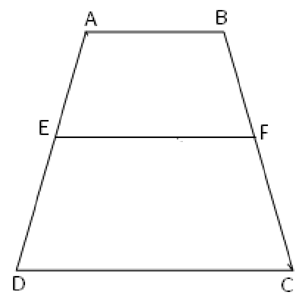
If one family is choosen at random find the probability of choosing

i. A family whose income is 16,000 or more and has more than 2 TV sets
ii. A family whose income is less than 7,000 and has 2 TV's
iii. A family whose income is between 10,000 and 13,000 and has 1 TV.

19. In the figure, PQ is the diameter of the circle and XY is chord equal to the radius of the circle. PX and QY when extended intersect at point E. Prove that $m\angle PEQ = 60°$



20. In the given figure, E is the mid-point of side AD of trapezium ABCD with AB ∥ CD, EF ∥ AB. A line through E parallel to AB meets BC in F. Show that F is the mid-point of BC.



21. Two unbiased dice are tossed 50 times. The sum of integers obtained on the dice is noted below.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	3	9	8	8	4	5	1	3	7	2	0

Find the probability that:

i. The sum of integers is more than 9.

ii. The sum of integers is exactly 7.

iii. The sum of integers is less than 6.

22. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm and its base is 7 cm, find the total surface area of the article.

(SECTION – D)

(Questions 23 to 30 carry 4 marks each)

23. Simplify: $\dfrac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}$

24. Find $x^3 + y^3$ when $x = \dfrac{1}{3-2\sqrt{2}}$ and $y = \dfrac{1}{3+2\sqrt{2}}$.

25.

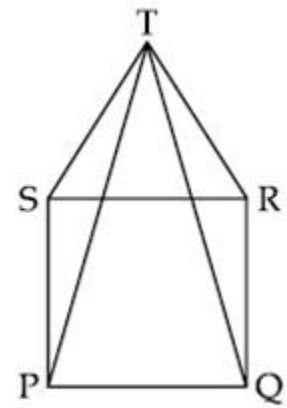
(i) Multiply $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y - 4$ using suitable identities.

(ii) Factorise: $a^2 + b^2 - 2(ab - ac + bc)$.

26. In the figure, PQRS is a square and SRT is an equilateral triangle.
Prove that:

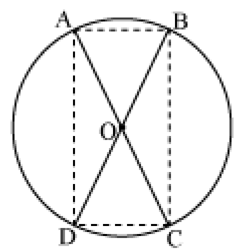
a) $\angle PST = \angle QRT$

b) $PT = QT$



27. The cost of painting the complete outside surface of a closed cylindrical oil tank at 60 paise per sq dm is Rs. 237.60. The height of the tank is 6 times the radius of the base of the tank. Find its volume corrected to two decimal places.

28. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.



29. Construct ΔABC in which $m\angle B = 60°$, $m\angle C = 45°$ and the perimeter of the triangle is 11 cm.

30. The bus fare in a city is as follows: For the first kilometre, the fare is Rs. 8 and for the subsequent distance it is Rs. 5 per kilometre. Taking the distance covered as x km and total fares as Rs. y, write a linear equation for this information and draw its graph.

CBSE Board

Class IX Mathematics

Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. Here,

$$\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$$

2. Substitute x = 2 and y = 1 in the equation 2x + y = 5, we get

   L.H.S. = 2(2) + 1 = 5 = R.H.S.

   Since L.H.S. = R.H.S.

   $\therefore$ Point (2, 1) lies on a line 2x + y = 5.

3. In $\triangle$ ABC,

   m $\angle$A + m $\angle$B + m $\angle$C = 180° (sum of the angles of a triangle is 180°)

   $\therefore$ x + 2x + 3x = 180°

   $\therefore$ 6x = 180°

   $\therefore$ x = 30°

   Since, m $\angle$C = 3x°

   $\therefore$ m $\angle$C = 3x = 3 × 30° = 90°

4. Here x = –2 and y = –5

   Since both are negative,

   $\therefore$ Point (–2, –5) will lie in 3rd Quadrant.

   (Since, both the coordinates of any point in the third quadrant are negative)

5. Here, range = 28 and number of classes = 7

   $\therefore$ Class Size = $\dfrac{\text{Range}}{\text{Number of classes}} = \dfrac{28}{7} = 4$

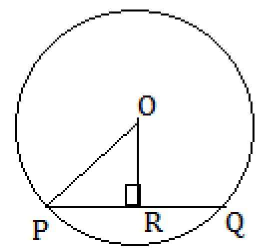
   $\therefore$ Class size is the data is 4.

6. Given, PQ = 10 cm and OR = 12 cm

Since, OR $\perp$ PQ $\therefore$ PR

$= \frac{1}{2}$ PQ

(Perpendicular from the centre of a circle to a chord bisects the chord)



Since, OR $\perp$ PQ

$\therefore$ ΔPOR is a right angled triangle.

$\therefore$ By Pythagoras theorem,

$PO^2 = OR^2 + PR^2$

$$\therefore PO = \sqrt{(12)^2 + (5)^2} = \sqrt{144+25} = \sqrt{169} = 13$$

$\therefore$ Radius = 13 cm

Section B

7. Let x = $0.\overline{975} = 0.975975975$ ....(1)

On multiplying both sides of equation (1) by 1000,

1000x = 975.975975 ....(2)

On subtracting equation (1) from equation (2),

999x = 975

$$\Rightarrow x = \frac{975}{999} = \frac{325}{333}$$

8. $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

$= 7\sqrt{2}x^2 - 14x + 4x - 4\sqrt{2}$

$= 7\sqrt{2}\,(x - \sqrt{2}) + 4(x - \sqrt{2})$

$= (7\sqrt{2} + 4)(x - \sqrt{2})$

9. (i) I quadrant: (5, 2)

(ii) II quadrant: (–5, 2)

(iii) III quadrant: (–5, –2)

(iv) IV quadrant: (5, –2)

10. $a + b = 180°$ (Linear pair)….(i)

$a – b = 80°$ (given)….(ii)

Adding (i) and (ii)

$2a = 260°$

$\Rightarrow a = 130°$

$\Rightarrow b = 180° – 130° = 50°$

11. Inner radius (r) of circular well $= \left(\frac{3.5}{2}\right) \text{ m} = 1.75 \text{ m}$

Depth (h) of circular well = 10 m

i. Inner curved surface area = $2\pi rh = \left(2 \times \frac{22}{7} \times 1.75 \times 10\right) \text{ m}^2 = (44 \times 0.25 \times 10) \text{ m}^2$

Inner curved surface area = $2\pi rh = 110 \text{ m}^2$

ii. Cost of plastering 1 m^2 = Rs. 40

Cost of plastering 110 m^2 = Rs. (110×40) = Rs.4400.

12. Substituting the values of x and y in the equation, we get

$a(2) + b(1) = 3$

$2a + b = 3$ ….(1)

Multiplying equation (1) by 2, we get

$4a + 2b = 6$ ….(2)

Also, $3a – 2b = 1$ ….(3)

Adding equations (2) and (3)

$7a = 7$

$a = 1$

Now, substituting $a = 1$ in equation (1) we get the value of $b = 1$.

Section C

13. $\dfrac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$

$= \dfrac{\left(5^2\right)^{\frac{3}{2}} \times \left(7^3\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}} \times \left(2^3\right)^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$

$= \dfrac{5^3 \times 7^{\frac{9}{5}}}{2^5 \times 2^4 \times 7^{\frac{3}{5}}}$

$= \dfrac{5^3 \times 7^{\frac{9}{5}}}{2^9 \times 7^{\frac{3}{5}}}$

$= \dfrac{5^3 \times 7^{\frac{6}{5}}}{2^9}$

14. Let $p(x) = x^2 - 5x - 3a$ and $q(x) = ax^2 - 5x - 7$.

According to remainder theorem, when the polynomial $p(x)$ is divided by a linear polynomial $(x - a)$, the remainder obtained is $p(a)$.

Remainder obtained when $p(x)$ is divided by $x - 1 = p(1)$

Remainder obtained when $q(x)$ is divided by $x - 1 = q(1)$

It is given that $p(1) = q(1)$.

$\Rightarrow (1)^2 - 5(1) - 3a = a(1)^2 - 5(1) - 7$

$\Rightarrow 1 - 5 - 3a = a - 5 - 7$

$\Rightarrow -4 - 3a = a - 12$

$\Rightarrow 8 = 4a$

$\Rightarrow a = 2$

Thus, the value of a is 2.

15. Given: $x = 2y + 6$ or $x - 2y - 6 = 0$

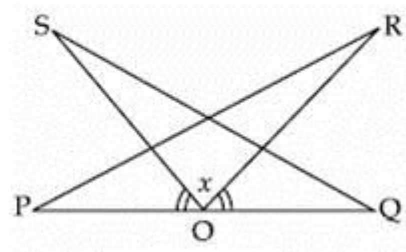
We know that if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3xyz$

Therefore, we have:

$(x)^3 + (-2y)^3 + (-6)^3 = 3x(-2y)(-6)$

Or, $x^3 - 8y^3 - 36xy - 216 = 0$

16. In ΔPOR and ΔQOS



$\angle QPR = \angle PQS$ (given)

$OP = OQ$ (O is the mid-point of PQ)

$\angle POS = \angle QOR$ (given)

$\angle POS + x^o = \angle QOR + x^o$

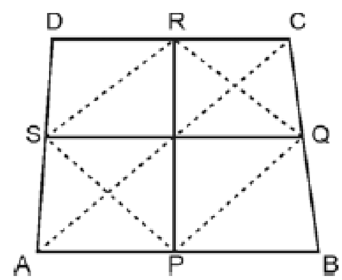
$\angle POR = \angle QOS$

By ASA congruence rule,

$\Delta PQR \cong \Delta QOS$

$\Rightarrow PR = QS$ (By CPCT)

17. Let ABCD be a quadrilateral. P, Q, R, and S are the mid points of AB, BC, CD and DA respectively.



Join PQ, QR, RS and SP. Join AC.

In ΔDAC, $SR \parallel AC$

And $SR = \frac{1}{2}AC$ (Mid-point theorem)

In ΔBAC, $PQ \parallel AC$

And $PQ = \frac{1}{2}AC$

Clearly, $PQ \parallel SR$ and $PQ = SR$

In quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other and hence it is a parallelogram.

Now, PR and SQ are the diagonals of PQRS and hence PR and SQ bisect each other.

18. i. Total number of families surveyed = 2400

No. of families with income more than 16,000 and having more than 2 TV's = 88

$$\therefore \text{Required probability} = \frac{88}{2400} = \frac{11}{300}$$

ii. Total number of families surveyed = 2400

Number of families with income less than 7,000 and having 2 TV sets = 25

$$\therefore \text{Required probability} = \frac{25}{2400} = \frac{1}{96}$$

iii. Total number of families surveyed = 2400

Number of families with income between 10,000 and 13,000 having 1 TV set = 535

$$\therefore \text{Required probability} = \frac{535}{2400} = \frac{107}{480}$$

19. PQ is the diameter of the circle, chord XY = r (radius of circle)

PX and QY extended intersect at a point E.

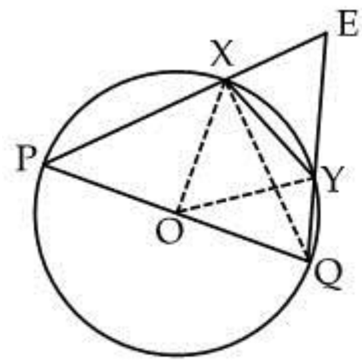
To prove $m\angle PEQ = 60°$

$XY = OX = OY$ [radii of a circle]

$\Rightarrow \Delta XOY$ is an equilateral triangle

$\therefore m\angle XOY = 60°$

$\Rightarrow \Delta XQY = 30°$ [Inscribed angle is half of the central angle]



$m\angle PXQ = 90°$ [Angle in a semi circle]

$m\angle QXE = 180° - m\angle PXQ = 90°$ [Linear pair]

In ΔXEQ,

$m\angle XEQ = 180° - (m\angle EXQ + \angle EQX)$ (Angle sum property)

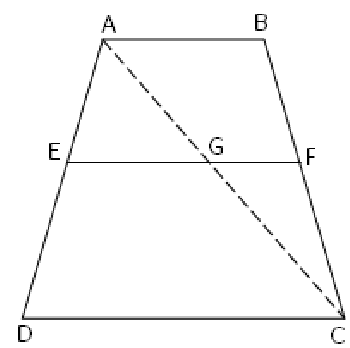
$m\angle XEQ = 180° - (90° + 30°) = 60°$

$\Rightarrow m\angle PEQ = 60°$

20. Given: ABCD is a trapezium. E is the mid-point of AD and AB || CD, EF || AB.

To prove: F is the mid-point of BC

Construction: Join AC to intersect EF at point G.



Proof: EF || DC [Given]

$\therefore$ EG || DC

Since E is the mid-point of AD.

$\therefore$ G is the mid-point of AC. [By converse of midpoint theorem]

In ΔABC, FG || AB

G is the mid-point of AC

$\therefore$ F is the mid-point of BC.

21. Total number of trials = 50

i. Number of trials when the sum of integers is more than $9 = 7 + 2 + 0 = 9$

$\therefore P(\text{the sum of integers is more than } 9) = \frac{9}{50} = 0.18$

ii. Number of trials when the sum of integers is exactly $7 = 5$

$\therefore P(\text{the sum of integers is exactly } 7) = \frac{5}{50} = \frac{1}{10} = 0.10$

iii. Number of trails when the sum of integers is less than $6 = 3 + 9 + 8 + 8 = 28$

$\therefore P(\text{the sum of integers is less than } 6) = \frac{28}{50} = \frac{14}{25} = 0.56$

22. Base radius, $r = \frac{7}{2} = 3.5$ cm

Height of the cylinder, h = 10 cm

Curved surface area of cylinder $= 2\pi rh = 2\times\pi\times3.5\times10 = 70\pi$

Curved surface area of two hemisphere $= 2\times2\pi r^2 = 4\times\pi\times3.5^2 = 49\pi$

Total surface area $= 70\pi + 49\pi = 374\ \text{cm}^2$

Section D

23. $$\frac{16\times2^{n+1}-4\times2^n}{16\times2^{n+2}-2\times2^{n+2}} = \frac{2^4\times2^{n+1}-2^2\times2^n}{2^4\times2^{n+2}-2\times2^{n+2}}$$

$$= \frac{2^{n+5}-2^{n+2}}{2^{n+6}-2^{n+3}}$$

$$= \frac{2^{n+5}-2^{n+2}}{2.2^{n+5}-2.2^{n+2}}$$

$$= \frac{\left(2^{n+5}-2^{n+2}\right)}{2\left(2^{n+5}-2^{n+2}\right)}$$

$$= \frac{1}{2}$$

24.

$$x = \frac{1}{3-2\sqrt{2}} = \frac{1}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$$

$$y = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$\Rightarrow x+y = 3+2\sqrt{2}+3-2\sqrt{2} = 6$$

$$xy = \left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right) = 9-8 = 1$$

$$x^3+y^3 = (x+y)^3-3xy(x+y)$$

$$= 6^3-3.1.6$$

$$= 216-18$$

$$= 198$$

25. (i) $(3x - 5y - 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$

$= (3x + (-5y) + (-4))\,[(3x)^2 + (-5y)^2 + (-4)^2 - (3x)(-5y) - (-5y)(-4) - (3x)(-4)]$

$= (3x)^3 + (-5y)^3 + (-4)^3 - 3(3x)(-5y)(-4)$

$[(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc]$

$= 27x^3 - 125y^3 - 64 - 180\,xy$

(ii) $a^2 + b^2 - 2(ab - ac + bc)$

$= a^2 + b^2 - 2ab + 2ac - 2bc$

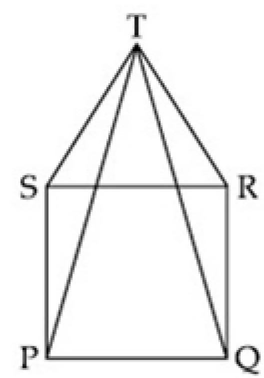
$= (a^2 + b^2 - 2ab) + 2c(a - b)$

$= (a - b)^2 + 2c(a - b)$

$= (a - b)(a - b + 2c)$

26. □PQRS is a square.

$\therefore PQ = QR = RS = SP$ ....(i)



Also $\angle RSP = \angle SRQ = \angle RQP = \angle SPQ = 90°$ ....(ii)

Also ΔTSR is equilateral.

$TS = TR = SR$......(iii)

Also $\angle STR = \angle TSR = \angle TRS = 60°$

$TR = QR$.....from (i) and (ii)

Also $\angle TSP = \angle RSP + \angle TSR$

$\angle TSP = 90° + 60° = 150°$

Similarly $\angle TRQ = 150°$

In ΔTSP and ΔTRQ,

$PS = QR$.....$(\because \text{by (i)})$

$\angle TSP = \angle TRQ$......$(\because \text{Both } 150°)$

$TS = TR$......$(\because \text{by (iii)})$

$\therefore \Delta TSP \cong \Delta TRQ$ ....(by SAS criterion)

$\therefore PT = QT$ .....(c.p.c.t)

27.

Let 'r' dm be the radius of the base and 'h' dm be the height of the cylindrical tank. Then, h = 6r (given)

Total surface area = $2\pi r(r + h) = 2\pi r(r + 6r) = 14\pi r^2$

$\Rightarrow$ Cost of painting = Rs. $14\pi r^2 \times \frac{60}{100}$ = Rs. $\frac{42}{5}\pi r^2$

It is given that the cost of painting is Rs. 237.60

$\therefore \frac{42}{5}\pi r^2 = 237.60$

$\Rightarrow \frac{42}{5} \times \frac{22}{7} \times r^2 = 237.60$

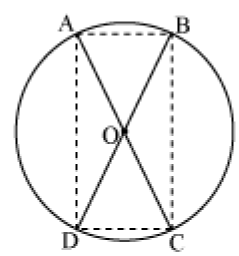
$\Rightarrow r^2 = 237.60 \times \frac{5}{42} \times \frac{7}{22} = 9 \Rightarrow r = 3$ dm

$\therefore h = 6r = 18$ dm

Volume of the cylinder = $\pi r^2 h$

$\Rightarrow$ Volume of the cylinder = $(\pi \times 3 \times 3 \times 18)$ dm^3 = $\left(\frac{22}{7} \times 9 \times 18\right)$ dm^2 = 509.14 dm^2

28. In ΔAOB and ΔCOD,



$OA = OC$	(given)
$OB = OD$	(given)
$\angle AOB = \angle COD$	(vertically opposite angles)
$\Delta AOB \cong \Delta COD$	(SAS congruence rule)
$AB = CD$	(by CPCT)

Similarly, we can prove $\Delta AOD \cong \Delta COB$

$\therefore AD = CB$ (by CPCT)

Since in □ ABCD opposite sides are equal in length. Hence, ABCD is a parallelogram.

Also, $\angle A = \angle C$ (Opposite angles of a parallelogram are equal)

But $m\angle A + m\angle C = 180°$ (ABCD is a cyclic quadrilateral)

$\Rightarrow m\angle A + m\angle A = 180°$

$\Rightarrow 2m\angle A = 180°$

$\Rightarrow m\angle A = 90°$

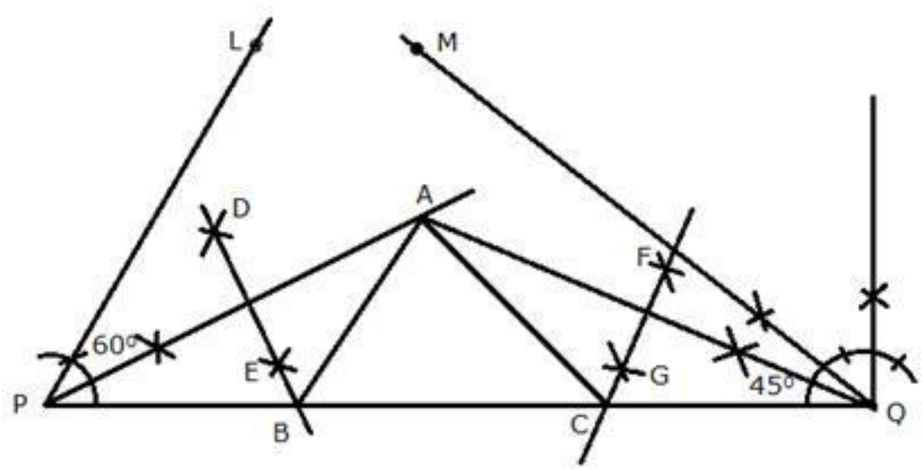
As ABCD is a parallelogram and one of its interior angles is 90°, so it is a rectangle.

$\angle A$ is the angle subtended by chord BD. And as $m\angle A = 90°$, so BD should be the diameter of the circle. Similarly AC is the diameter of the circle.

29. Steps of Construction:

1. Draw a line segment PQ = 11 cm (= AB + BC + CA).
2. At P construct an angle of 60° and at Q an angle of 45°.
3. Bisect these angles. Let the bisectors of these angles intersect at point A.
4. Draw perpendicular bisectors DE of AP to intersect PQ at B and FG of AQ to intersect PQ at C.
5. Join AB and AC

ΔABC is the required triangle.



30. Given,

Bus fare for the first kilometer = Rs. 8

Bus fare for the remaining distance = Rs. 5

Total distance covered = x

Total fare = y

Since the fare for first kilometer = Rs. 8

According to given condition,

Fare for (x – 1) kilometer = 5(x – 1)

Therefore, the total fare = 5(x – 1) + 8

$y = 5(x - 1) + 8$

$\Rightarrow y = 5x - 5 + 8$

$\Rightarrow y = 5x + 3$

Therefore, $y = 5x + 3$ is the required linear equation.

Now the equation is

$y = 5x + 3$ ----- (1)

Put the value $x = 0$ in equation (1)

$y = 5 \times 0 + 3$

$y = 0 + 3 = 3$

The solution is (0, 3).

Putting the value $x = 1$ in equation (1) we get,

$y = 5 \times 1 + 3$

$y = 5 + 3 = 8$.

The solution is (1, 8).

Putting the value $x = 2$ in equation (1)

$y = 5 \times 2 + 3$

$y = 10 + 3 = 13$.

The solution is (2, 13).

x	0	1	2
y	3	8	13

Now plot the points (0, 3), (1, 8), and (2, 13) and a draw line passing through these points.