CBSE Board
Class X Mathematics

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

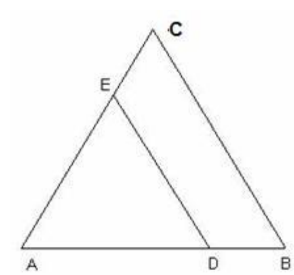
Section A
(Questions 1 to 6 carry 1 mark each)

1. What is the probability of getting a prime number when a die is thrown once?

2. Find the roots of the equation $x^2 - 3\sqrt{3}x + 6 = 0$.

3. The ratio of the length of a pole and its shadow is $\sqrt{3} : 1$. Find the angle of elevation of the Sun.

4. In the adjoining figure, DE is parallel to BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.



5. Is number '7 × 11 × 13 + 13 + 13 × 2' a composite number?

6. In ΔABC, ∠A = 80° and ∠B = 60°. If ΔABC ~ ΔRQP, find the value of ∠P.

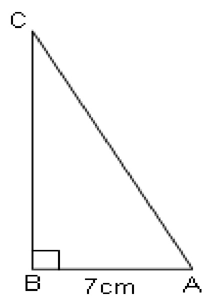
Section B
(Questions 7 to 12 carry 2 marks each)

7. In two concentric circles, the radius of the inner circle is 5 m. A chord of length 24 m of the outer circle becomes a tangent to the inner circle. Find the radius of the larger circle.

8. Find the roots of the quadratic equation $2x^2 - \sqrt{5}x - 2 = 0$ using the quadratic formula.

9. Can the number 4^n, n being a natural number end with the digit 0? Given reasons.

10. In $\triangle ABC$, $m\angle B = 90^\circ$, AB = 7 cm and AC – BC = 1 cm. Determine the values of sin C and cos C.



11. Two tangents PQ and PR are drawn from an external point P to a circle with center O. Prove that PROQ is a cyclic quadrilateral.

12. Prove that: $\frac{\sec A + \tan A}{\sec A - \tan A} = \left(\frac{1 + \sin A}{\cos A}\right)^2$

Section C
(Questions 13 to 22 carry 3 marks each)

13. If the point (x, y) is equidistant from the points (a + b, b − a) and (a − b, a + b), then prove that bx = ay.

14. If the roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal then prove that $2a = b + c$.

15. Solve for x and y:

$\frac{x}{a} + \frac{y}{b} = 2$; $\quad ax - by = a^2 - b^2$

16. If $0° < A < 90°$; find A, if $\frac{\cos A}{1-\sin A}+\frac{\cos A}{1+\sin A}=4$

17. Prove that $\frac{3}{2\sqrt{5}}$ is an irrational number.

18. The sum of the numerator and denominator of a fraction is 8. If 3 is added to both the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction.

19. Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7).

20. Find the mode for the following data which gives the literacy rate (in %) in 40 cities of India.

Literacy rate (%)	45-55	55-65	65-75	75-85	85-95
No. of cities	4	11	12	9	4

21. In ΔABC, if AD is the median, then show that $AB^2 + AC^2 = 2[AD^2 + BD^2]$.

22. 17 cards numbered 1, 2, 3, 4, ……………, 16, and 17, are put in a box and mixed thoroughly. A girl draws a card from the box. Find the probability that the number on the card is

i. Prime
ii. Divisible by 3
iii. Divisible by both 2 and 3

Section D
(Questions 23 to 30 carry 4 marks each)

23. In November 2009, the number of visitors to a zoo increased daily by 20. If a total of 12300 people visited the zoo in that month, find the number of visitors on 1st Nov. 2009.

24. In triangle ABC, D is the mid-point of BC and AE $\perp$ BC. If AC > AB, then show that:

$AB^2 = AD^2 - BC \times DE + \frac{BC^2}{4}$.

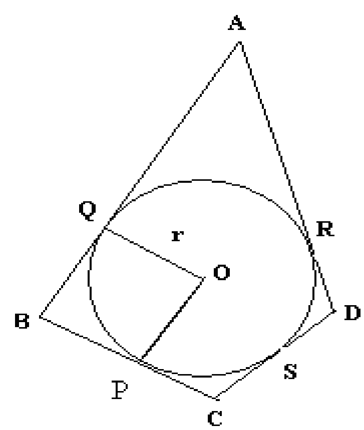
25. Draw a circle of radius 2.9 cm and draw its diameter. Through one of the end points of the diameter, construct tangent to the circle.

26. Form a pair of linear equations for the following problem, and find the solution graphically.

"10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz."

27. A bucket is in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the capacity and surface area of the bucket. Also, find the cost of the milk required to completely fill the bucket, at the rate of Rs. 25 per litre (use $\pi = 3.14$).

28. A circle is inscribed in a quadrilateral ABCD in which $m\angle B = 90°$. If AD = 23 cm, AB = 29 cm and DS = 5 cm. Find the radius of the circle.



29. The following table shows the number of runs scored by a certain batsman in different overs:

Over	50-55	55-60	60-65	65-70	70-75	75-80
No. of runs	2	8	12	24	38	16

Change the distribution to a 'more than' type distribution and draw its OGIVE on the graph.

30. A bucket is raised from a well by means of a rope which is wound around a wheel of radius 38.5 cm. Given that the bucket ascends in 1 min 28 seconds with a uniform speed of 1.1 m/ sec, calculate the number of complete revolutions the wheel makes in raising the bucket.

CBSE Board
Class X Mathematics
Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. When a die is thrown once the outcomes are 1, 2, 3, 4, 5 and 6.
Thus, total number of outcomes = 6
Out of these outcomes, 2, 3 and 5 are prime numbers.
Thus, number of favourable outcomes = 3
Therefore, P(prime number) = $\frac{3}{6}=\frac{1}{2}$

2. $x^2-3\sqrt{3}x+6=0$

$=x^2-\sqrt{3}x-2\sqrt{3}x+6=0$

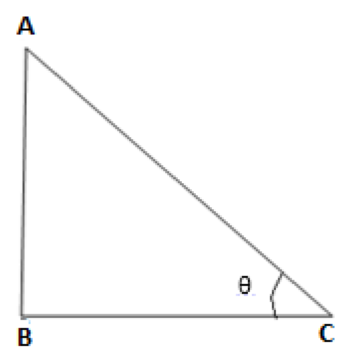
$\Rightarrow x\left(x-\sqrt{3}\right)-2\sqrt{3}\left(x-\sqrt{3}\right)=0$

$\Rightarrow \left(x-\sqrt{3}\right)\left(x-2\sqrt{3}\right)=0$

$\Rightarrow x-\sqrt{3}=0 \text{ or } x-2\sqrt{3}=0$

$\Rightarrow x=\sqrt{3} \text{ or } x=2\sqrt{3}$

3. Consider the following figure.
Let AB be the pole and BC be its shadow.



Given that $\frac{AB}{BC}=\frac{\sqrt{3}}{1}$

Let $\theta=\angle ACB$ be the angle of elevation.

In ΔABC,

$\tan\theta=\frac{AB}{BC}$

$\Rightarrow \tan\theta=\frac{\sqrt{3}}{1}$

$\Rightarrow \tan\theta=\tan 60^\circ$

$\Rightarrow \theta=60^\circ$

4. In ΔABC, DE ∥ BC.

Then, by Basic Proportionality Theorem,

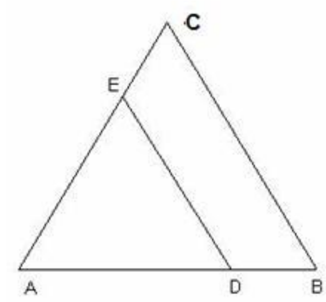
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

$$\Rightarrow x(x-1) = (x-2)(x+2)$$

$$\Rightarrow x^2 - x = x^2 - 4$$

$$\Rightarrow x = 4$$



5. 7 × 11 × 13 + 13 + 13 × 2

= 13(7 × 11 + 1 + 2)

= 13(80)

Thus, the given number is a composite number.

6. Since ΔABC ~ ΔRQP,

$\angle A = \angle R = 80^\circ$

$\angle B = \angle Q = 60^\circ$

Therefore, using the angle sum property in ΔRQP, we have

$\angle P = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$

Section B

7. Let O be the centre of circle and AB be the chord of larger circle and OT be the radius of smaller circle.

So $OT \perp AB$ since tangent is $\perp$ to radius at its point of contact.

AT = TB = 12 m

(Since perpendicular from centre to the chord bisects it)

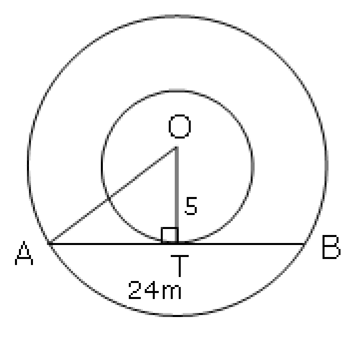
So, in ΔOAT,

$OA^2 = OT^2 + AT^2$

$OA^2 = 5^2 + 12^2 = 169$

$\Rightarrow$ OA = 13 cm

Thus, the radius of the larger circle is 13 cm.



8. The given quadratic equation is $2x^2 - \sqrt{5}x - 2 = 0$.

Comparing with general form of quadratic equation $ax^2 + bx + c = 0$, we have

$a = 2, b = -\sqrt{5}$ and $c = -2$

Now, $b^2 - 4ac = \left(-\sqrt{5}\right)^2 - 4 \times 2 \times (-2) = 5 + 16 = 21$

Thus, the roots of the given equation are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{5} \pm \sqrt{21}}{4}$$

Hence, roots are $\frac{\sqrt{5}+\sqrt{21}}{4}$ and $\frac{\sqrt{5}-\sqrt{21}}{4}$.

9. $4^n = (2^2)^n = 2^{2n}$

The only prime in the factorisation of 4^n is 2.

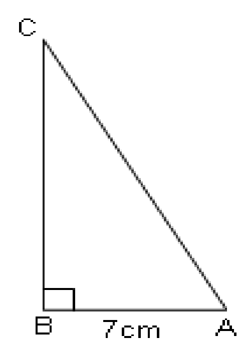
There is no other primes in the factorisation of $4^n = 2^{2n}$

[By uniqueness of the Fundamental Theorem of Arithmetic]

$\Rightarrow$ 5 does not occur in the prime factorisation of 4^n for any n.

$\Rightarrow$ 4^n does not end with the dizit 0 for any natural number n.

10. In ABC, we have



$AC^2 = BC^2 + AB^2$

$(1 + BC)^2 = BC^2 + AB^2$

$\Rightarrow 1 + BC^2 + 2BC = BC^2 + AB^2$

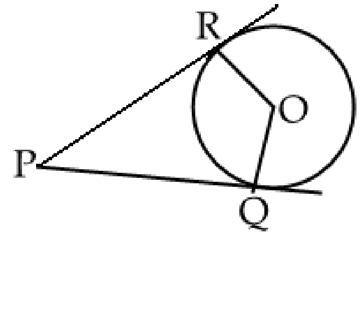
$\Rightarrow 1 + 2BC = 7^2$

$\Rightarrow 2BC = 48 \Rightarrow BC = 24$ cm

$\Rightarrow AC = 1 + BC = 1 + 24 = 25$ cm

Hence, $\sin C = \frac{AB}{AC} = \frac{7}{25}$ and $\cos C = \frac{BC}{AC} = \frac{24}{25}$

11. It can be observed that:



OR (radius) $\perp$ PR (tangent)

Therefore, $\angle ORP = 90°$

Similarly, OQ (radius) $\perp$ PQ (tangent)

$\angle OQP = 90°$

In quadrilateral ORPQ,

Sum of all interior angles = 360°

$\angle ORP + \angle RPQ + \angle PQO + \angle QOR = 360°$

$\Rightarrow 90° + \angle RPQ + 90° + \angle QOR = 360°$

$\Rightarrow \angle RPQ + \angle QOR = 180°$

Hence, PROQ is a cyclic quadrilateral.

12.

$$\text{L.H.S.} = \frac{\sec A + \tan A}{\sec A - \tan A}$$

$$= \frac{\sec A + \tan A}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A}$$

$$= \frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}$$

$$= (\sec A + \tan A)^2 \quad \left(\because \sec^2\theta = 1 + \tan^2\theta \therefore \sec^2\theta - \tan^2\theta = 1\right)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)^2$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)^2$$

$$= \text{R.H.S.}$$

Section C

13. Let P(x, y), Q(a + b, b – a) and R(a – b, a + b) be the given points.

It is given that PQ = PR $\Rightarrow PQ^2 = PR^2$

$\{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow x^2 - 2x(a + b) + (a + b)^2 + y^2 - 2y(b - a) + (b - a)^2$

$= x^2 + (a - b)^2 - 2x(a - b) + y^2 - 2y(a + b) + (a + b)^2$

$\Rightarrow -2x(a + b) - 2y(b - a) = -2x(a - b) - 2y(a + b)$

$\Rightarrow -ax - bx - by + ay = -ax + bx - ay - by$

$\Rightarrow 2bx = 2ay$

$\Rightarrow bx = ay$

14. $(a - b)x^2 + (b - c)x + (c - a) = 0$

The given equation will have equal roots, if

$(b - c)^2 - 4(a - b)(c - a) = 0$

$\Rightarrow b^2 + c^2 - 2bc - 4(ac - bc - a^2 + ab) = 0$

$\Rightarrow b^2 + c^2 + 4a^2 + 2bc - 4ab - 4ac = 0$

$\Rightarrow (b + c - 2a)^2 = 0$

$\Rightarrow b + c - 2a = 0$

$\Rightarrow 2a = b + c$

15. $\frac{x}{a}+\frac{y}{b} = 2$

$\Rightarrow$ bx + ay = 2ab ....(1)

ax – by = $a^2 - b^2$ ....(2)

Multiplying (1) with a and (2) with b, we get

$$\begin{aligned} \cancel{abx} + a^2y &= 2a^2b \\ \cancel{abx} - b^2y &= a^2b - b^3 \\ - \quad + \quad &\quad - \quad + \\ \hline y(a^2+b^2) &= a^2b + b^3 \\ \Rightarrow y(a^2+b^2) &= b(a^2+b^2) \\ \Rightarrow y &= b \end{aligned}$$

From (1), bx + ab = 2ab

$\Rightarrow$ bx = ab

$\Rightarrow$ x = a

Hence, x = a and y = b.

16.

$$\frac{\cos A}{1-\sin A}+\frac{\cos A}{1+\sin A}=4$$

$$\Rightarrow \frac{\cos A+\cos A\sin A+\cos A-\sin A\cos A}{(1-\sin A)(1+\sin A)}=4$$

$$\Rightarrow \frac{2\cos A}{1-\sin^2 A}=4$$

$$\Rightarrow \frac{2\cos A}{\cos^2 A}=4$$

$$\Rightarrow \frac{1}{\cos A}=2$$

$$\Rightarrow \cos A=\frac{1}{2}$$

We know cos 60° = $\frac{1}{2}$

Hence, A = 60°

17. Let $\frac{3}{2\sqrt{5}}$ be a rational number.

$\Rightarrow \frac{3}{2\sqrt{5}}=\frac{a}{b}$, where a and b are co-prime integers and b ≠ 0.

$\Rightarrow \sqrt{5}=\frac{3b}{2a}$

Now, a, b, 2 and 3 are integers.

Therefore, $\frac{3b}{2a}$ is a rational number.

$\Rightarrow \sqrt{5}$ is a rational number.

This is a contradiction as we know that $\sqrt{5}$ is irrational.

Therefore, our assumption is wrong.

Hence, $\frac{3}{2\sqrt{5}}$ is an irrational number.

18. Let the fraction be $\frac{x}{y}$.

According to the question,

$x + y = 8$ ....(1)

$\frac{x+3}{y+3} = \frac{3}{4}$

$\Rightarrow 4x + 12 = 3y + 9$

$\Rightarrow 4x - 3y = -3$ ....(2)

Multiplying (1) by 3, we get

$3x + 3y = 24$ ....(3)

Adding (2) and (3), we get

$7x = 21$

$\Rightarrow x = 3$

$\Rightarrow y = 8 - x = 8 - 3 = 5$

Thus, the fraction is $\frac{3}{5}$.

19. Suppose the line $3x + y - 9 = 0$ divides the line segment joining the points A(1, 3) and B(2, 7) in the ratio k : 1 at point C.

Then, the co-ordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

But, C lies on $3x + y - 9 = 0$.

Therefore,

$$\left[3\left(\frac{2k+1}{k+1}\right)\right] + \left[\frac{7k+3}{k+1}\right] - 9 = 0$$

$\Rightarrow 6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow 4k - 3 = 0$

$\Rightarrow k = \frac{3}{4}$

So, the required ratio is 3 : 4.

20. From the data given as above we may observe that maximum class frequency is 12 belonging to class interval 65 – 75.

So, modal class = 65 – 75

Lower class limit (l) of modal class = 65

Frequency (f_1) of modal class = 12

Frequency (f_0) of class preceding the modal class = 11

Frequency (f_2) of class succeeding the modal class = 9

Class size (h) = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

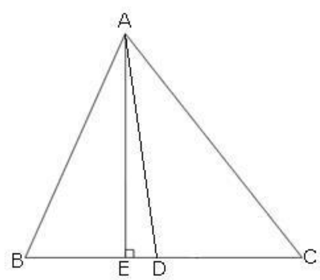
$$= 65 + \left(\frac{12 - 11}{2(12) - 11 - 9}\right) \times 10$$

$$= 65 + \frac{1}{4} \times 10$$

$$= 65 + 2.5$$

$$= 67.5$$

21. Given: AD is a median in $\triangle$ ABC.
To prove: $AB^2 + AC^2 = 2[AD^2 + BD^2]$
Construction: Draw AE $\perp$ BC



Proof:

In right angled triangles AEB and AEC using Pythagoras theorem we get:

$AB^2 + AC^2 = BE^2 + AE^2 + EC^2 + AE^2 = 2AE^2 + (BD - ED)^2 + (ED + DC)^2$

$= 2AE^2 + BD^2 + ED^2 - 2BD.ED + ED^2 + DC^2 + 2ED.DC$

$= 2AE^2 + 2ED^2 + BD^2 + DC^2$ [since BD = DC]

$= 2AE^2 + 2ED^2 + 2BD^2$ [since BD = DC]

$= 2(AE^2 + ED^2 + BD^2)$

$= 2(AD^2 + BD^2)$ [Using Pythagoras theorem in ΔAED]

22. Total number of outcomes = 17

i. Prime numbers from 1 to 17 are 2, 3, 5, 7, 11, 13, and 17.
Number of favourable outcomes = 7
P(prime number) = $\frac{7}{17}$

ii. Numbers from 1 to 17 which are divisible by 3 are 3, 6, 9, 12, 15.
Number of favourable outcomes = 5
P(number divisible by 3) = $\frac{5}{17}$

ii. Numbers from 1 to 17 which are divisible by 2 and 3 both are 6 and 12 only.
Number of favourable outcomes = 2
P(number divisible by 2 and 3 both) = $\frac{2}{17}$

Section D

23. Let number of visitors to the zoo on 1st November be x. Then the daily visitors in November in the zoo are: x, x + 20 ...

This will form an A.P. with the first term x and common difference 20.

Total no. of visitors in Nov. = 12300 (Given)

$\therefore S_{30} = 12300$

$$S_{30} = \frac{n}{2}[2a + (n-1)d] = \frac{30}{2}\left[2x + (29)(20)\right]$$

$\Rightarrow$ 12300 = 15(2x + 580)

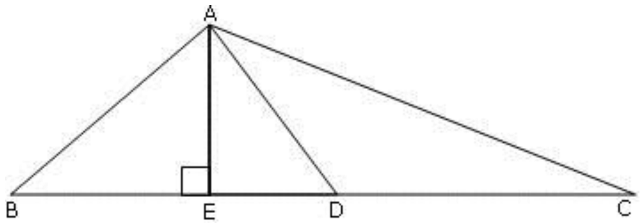
$\Rightarrow$ 820 = 2x + 580

$\Rightarrow$ 2x = 240

$\Rightarrow$ x = 120

$\therefore$ Number of visitors on 1st Nov. 2009 = 120

24. AD is the median of triangle ABC since D is the mid-point of BC.



$$\Rightarrow BD = DC = \frac{BC}{2} \text{....(i)}$$

In right ΔAEB,

$$AB^2 = AE^2 + BE^2 \text{ (Pythagoras theorem)}$$

$$\Rightarrow AB^2 = (AD^2 - DE^2) + (BD - DE)^2$$

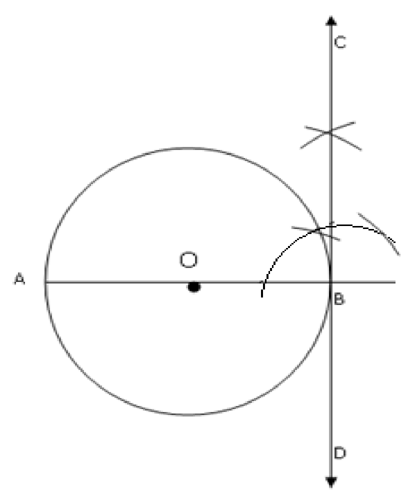
$$\Rightarrow AB^2 = AD^2 - DE^2 + \left(\frac{BC}{2} - DE\right)^2 \text{ ....(From (i))}$$

$$\Rightarrow AB^2 = AD^2 - DE^2 + \frac{BC^2}{4} + DE^2 - 2\left(\frac{BC \times DE}{2}\right)$$

$$\Rightarrow AB^2 = AD^2 - BC \times DE + \frac{BC^2}{4} \quad \text{(Hence proved)}$$

25. The steps of construction are as follows:

1. Take a point O on the given plane as the centre of the circle. Through O, draw a circle of radius 2.9 cm.
   (Since, diameter of the circle is 5.8 cm).
2. Draw a diameter AOB.
3. Draw CD $\perp$ AB at B.
4. CD is the required tangent to the circle.



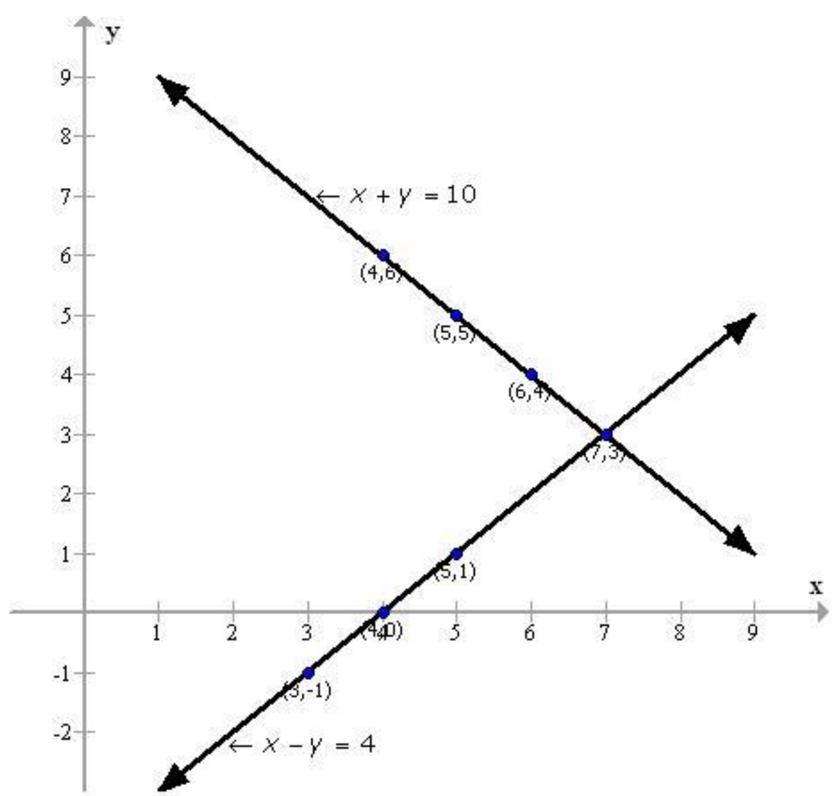
26. Let the number of girls and boys in the class be x and y, respectively.
According to the given conditions, we have:
x + y = 10
⇒ x = 10 – y
Three solutions of this equation can be written in a table as follows:

x	5	4	6
y	5	6	4

x – y = 4
⇒x = 4 + y
Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	−1

The graphs of the two equations can be drawn as follows:



From the graph, it can be observed that the two lines intersect each other at the point (7, 3).
So, x = 7 and y = 3 is the required solution of the given pair of equations.

27. Let R and r be the radii of the top and base of the bucket respectively.

Let h be its height.

Then, $R = 20$ cm, $r = 10$ cm, $h = 30$ cm

Capacity of the bucket = Volume of frustum of the cone

$$= \frac{1}{3}\pi h\left[R^2 + r^2 + Rr\right]$$

$$= \frac{1}{3} \times 3.14 \times 30\left[(20)^2 + (10)^2 + 20 \times 10\right] \text{cm}^3$$

$$= 3.14 \times 10[400 + 100 + 200] \text{ cm}^3$$

$$= 3.14 \times 10 \times 700 \text{ cm}^3$$

$$= 21980 \text{ cm}^3$$

$$= 21.98 \text{ litres}$$

Surface area of the bucket = CSA of the bucket + Surface area of the bottom

$$= \pi l(R + r) + \pi r^2$$

Now, $l = \sqrt{h^2 + (R - r)^2} = \sqrt{(30)^2 + (20 - 10)^2} = \sqrt{900 + 100} = \sqrt{1000} = 31.62$

$\therefore$ Surface area of the bucket $= 3.14 \times 31.62(20+10) + 3.14 \times (10)^2 \text{ cm}^2$

$$= 3.14 \times 31.62 \times 30 + 3.14 \times 100 \text{ cm}^3$$

$$= 2978.60 + 314 \text{ cm}^3$$

$$= 3292.60 \text{ cm}^2$$

Now, cost of 1 litre milk = Rs. 25

$\therefore$ Cost of 21.98 litres of milk = Rs. (25×21.98) = Rs. 549.50

28. Since tangents drawn from an external point to a circle are equal.

DR = DS = 5 cm

Now, AR = AD − DR = 23 − 5 = 18 cm

But, AR = AQ

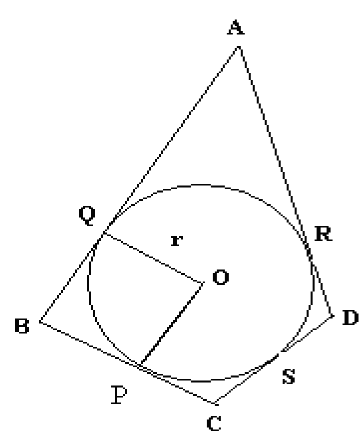
$\therefore$ AQ = 18 cm

Also, BQ = AB − AQ = 29 − 18 = 11 cm

But, BP = BQ

$\therefore$ BP = 11 cm

Also, m $\angle$ Q = m $\angle$ P = 90°.



In quadrilateral OQBP,

m$\angle$QOP + m$\angle$P + m$\angle$Q + m$\angle$B = 360°

m$\angle$QOP = 360° − ($\angle$P + $\angle$Q + $\angle$B) = 360° − (90° + 90° + 90°) = 90°

Hence, OQBP is a square.

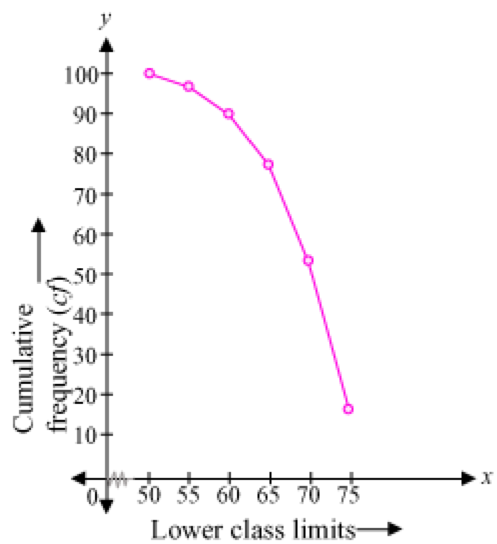
$\therefore$ BQ = OQ = OP = BP = 11 cm

Hence, the radius of the circle is 11 cm.

29. We can obtain cumulative frequency distribution of more than type as following –

Over (lower class limits)	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	100 – 2 = 98
More than or equal to 60	98 – 8 = 90
More than or equal to 65	90 – 12 = 78
More than or equal to 70	78 – 24 = 54
More than or equal to 75	54 – 38 = 16

Now taking lower class limits on *x*-axis and their respective cumulative frequencies on *y*-axis, we can obtain its ogive as follows.



30. Time taken by bucket to ascend = 1 min 28 secs = 88 secs

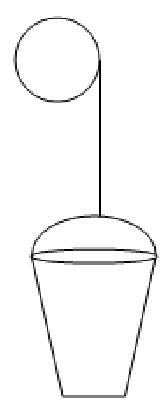
Speed = 1.1 m/ sec

Length of the rope = distance covered by bucket to ascend

= (1.1 × 88) m = (1.1 × 88 × 100) cm = 9680 cm

Radius of the wheel = 38.5 cm = $\frac{77}{2}$ cm

Circumference of the wheel = $2\pi r$ = $\left(2\times\frac{22}{7}\times\frac{77}{2}\right)$ cm = 242 cm



$\therefore$ Number of revolutions =

$$\frac{\text{Length of the rope}}{\text{Circumference of the wheel}} = \left(\frac{9680}{242}\right) = 40$$

Hence, the wheel makes 40 revolutions to raise the bucket.