CHAPTER 9

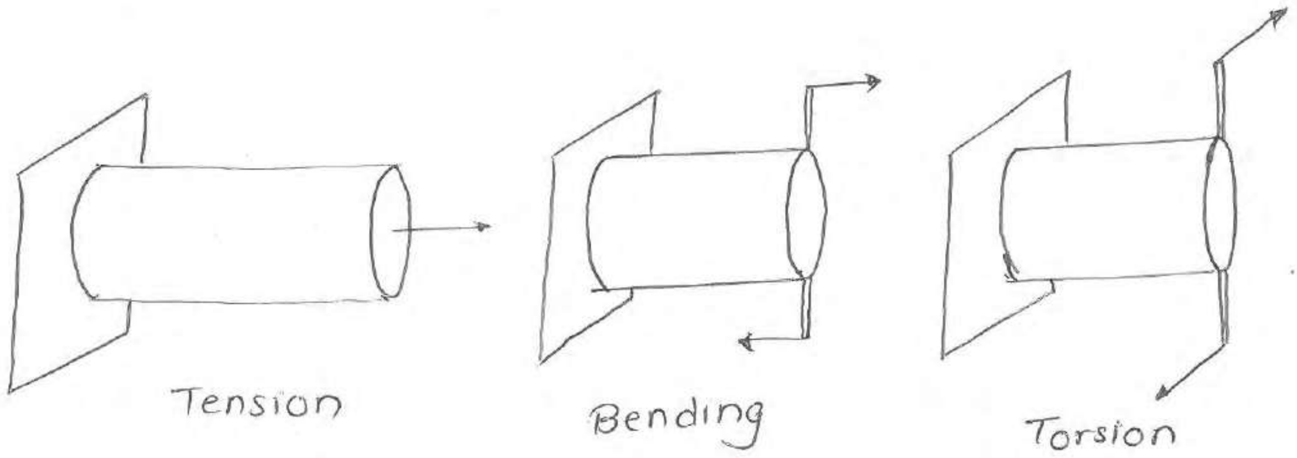
Torsion

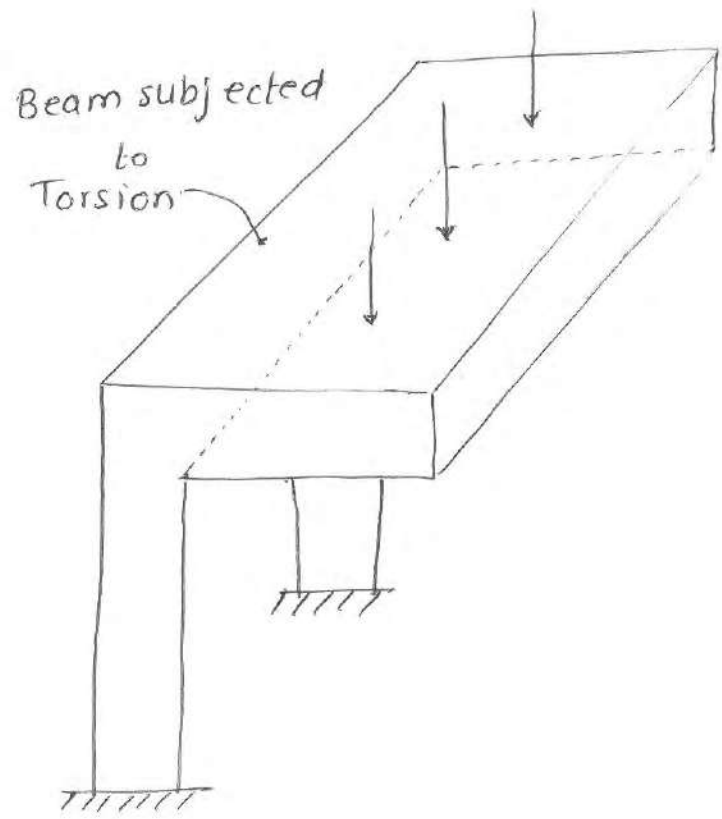
CONTENTS

9. Torsion

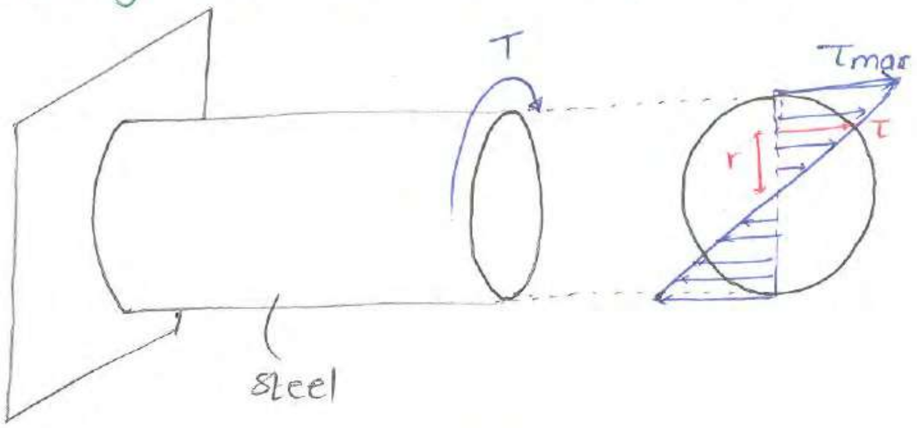
9.1 Introduction:

Torsion in a member depends on way of application of force on member





9.2 Member of Circular Section Made up of Linearly Elastic Homogeneous Material Subjected to Torsion.

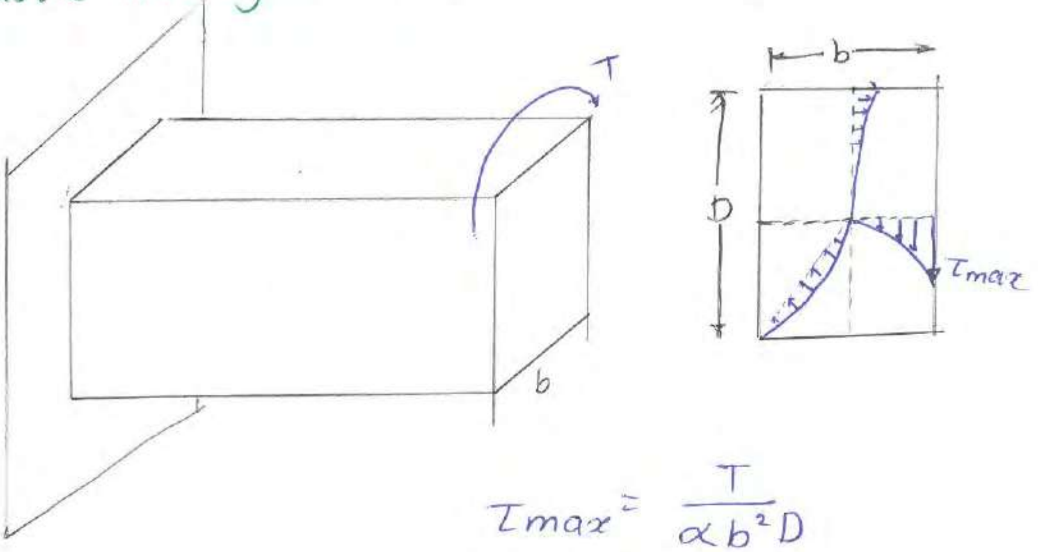


Torsion Formula,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

- Plane section remains plane after twisting.
- Shear stress variation over c/s is linear. with zero at centre and maximum at farthest point from centre
- No normal stress on section.
- Torsion formula is valid

9.3 Member of Non-circular section made up of Linearly Elastic Homogeneous Material Subjected to Torsion:



$b < D$

$$\tau_{max} = \frac{T}{\alpha b^2 D}$$

where α depends on $\frac{D}{b}$ ratio.

- Plane section no longer remains plane after twisting. Warping of section takes place.
- Shear stress variation is non-linear with maximum at mid-of longer edge and zero at centre and corners.
- Normal stress over section is also present.
- Torsion formula is no-longer valid.

9.4 RCC Member Subjected to Torsion

Due to warping of section and cracking of concrete, analysis of RCC member of non circular section subjected to torsion becomes complicated. IS456 provides a simplified approach (based on skew bending theory) to design member subjected to torsion by converting torsion into equivalent shear and equivalent moment.

9.4.1 Equivalent Shear:

$$V_{ue} = V_u + \frac{1.6 T_u}{b}$$

Now, section is designed for V_{ue} instead of V_u.

9.4.2 Equivalent Moment:

$$M_t = \frac{T_u(1 + D/b)}{1.7}$$

Case I: $M_t \le M_u$

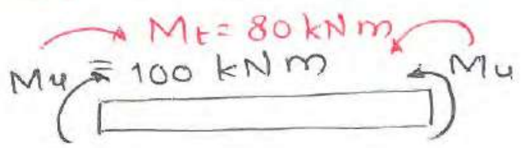
$$M_{e_1} = M_u + M_t$$

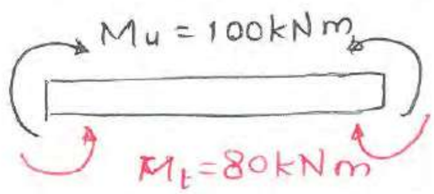
Case II $M_t > M_u$

$$M_{e_1} = M_u + M_t$$

$$M_{e_2} = M_t - M_u$$

Case I: $M_t \le M_u$



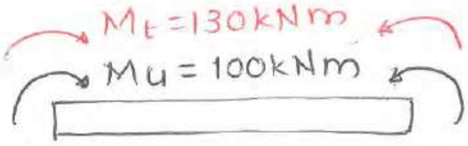


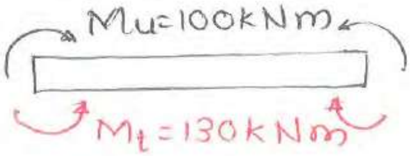
$M_u = 100$ kN·m (Sagging)
$M_t = 80$ kN·m (sagging)
$M_{e_1} = 100 + 80 = 180$ kNm (sagging)

$M_u = 100$ kN·m (sagging)
$M_t = 80$ kN (Hogging)
$M_{e_2} = 100 - 80 = 20$ kN·m (sagging).

so. design moment is M_{e_1} only.

Case II: $M_t > M_u$





$M_u = 100\,kNm$ (sagging) | $M_u = 100\,kN$ (sagging)

$M_t = 130\,kNm$ (sagging) | $M_t = 130\,kN$ (Hogging)

$M_{e_1} = M_u + M_t = 230\,kNm$. (sagging | $M_{e_2} = M_t - M_u = 30\,kNm$ (Hogging)

so design moments are M_{e_1} & M_{e_2} both.

9.5 Design Steps.

Step 1: Calculate factored SF, BM and torsional moment

Step 2: Calculate $M_{u,lim}$ of given section.

Step 3: If $M_u \le M_{u,lim}$ then calculate Ast.

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6 BM_u}{f_{ck} bd^2}}\right]$$

Step 4: Calculate equivalent SF and equivalent nominal shear stress.

$$V_{ue} = V_u + 1.6\frac{T_u}{b}$$

$$\tau_{ve} = \frac{V_{ue}}{bd}$$

Step 5: Take τ_c from Table 19 of IS456 corresponding to Ast calculated in step 3.

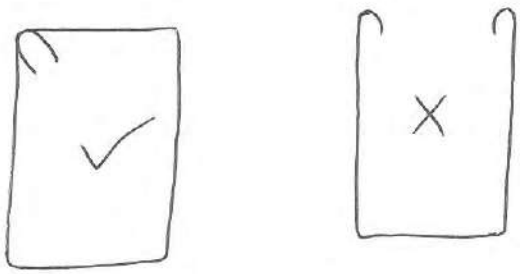
Step 6: If $\tau_{ve} < \tau_c$ then longitudinal reinforcement is provided for M_u and nominal shear stirrup is provided.

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

If $\tau_{ve} > \tau_c$ then longitudinal reinforcement is provided for M_{e1} and M_{e2} while shear design is for V_{ue}. It means shear reinforcement is for $(\tau_{ve} - \tau_c) \cdot bd$

*Note:-

- Atleast 1 longitudinal reinforcement should be provided at each corners.
- Longitudinal reinforcement should be placed at farthest possible distance from C.G. of section.
- Stirrups must be closed loop.



- D > 450 mm, 0.1% of gross web area is provided as side face reinforcement, equally distributed on both faces with spacing not more than 300 mm.

Ex. Calculate design shear force and bending moment for overall section size 350 X 750 mm which is subjected to M_u = 150 kNm Hogging, T_u = 100 kNm, V_u = 100 kN. M25, Fe 415, effective cover 50mm.

⇒

Step 1: $M_u = 150$ kNm

$T_u = 100$ kNm

$V_u = 100$ kN

Step 2: $M_{u,lim} = 0.138 f_{ck} b d^2$

$= 0.138 \times 25 \times 350 \times (750 - 50)^2$

~~$M_{u,lim}$ = 524. kNm~~

$M_{u,lim} = 591.675$ kN·m.

Step3: Since, $M_u < M_{u,lim}$, so

$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6 \times M_u}{f_{ck} b d^2}}\right]$$

$$= \frac{0.5 \times 25 \times 350 \times 700}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 150 \times 10^6}{25 \times 350 \times 700^2}}\right]$$

$$A_{st} = 619.83\ mm^2$$

Step4: $V_{ue} = V_u + 1.6 \frac{T_u}{b}$

$$= 100 \times 10^3 + 1.6 \times \frac{100 \times 10^6}{350}$$

$$V_{ue} = 557.12\ kN$$

$$\tau_{ve} = \frac{V_{ue}}{bd}$$

$$= \frac{557.12 \times 10^3}{350 \times 700}$$

$$\tau_{ve} = 2.27\ N/mm^2 < \tau_c\ (3.1\ N/mm^2)$$

Step5: $P_t = \frac{A_{st}}{bd} \times 100$

$$= \frac{619.83}{350 \times 700} \times 100$$

$$P_t = 0.25\%$$

~~Ste~~ $\tau_c = 0.36\ N/mm^2$ (from Table 19 $P_t = 0.25\%$ & M25)

Step6: Since $\tau_{ve} > \tau_c$ so

$$M_t = \frac{T_u (1 + D/b)}{1.7}$$

$$= \frac{100 \times 10^6 \left(1 + \frac{750}{350}\right)}{1.7}$$

$$M_t = 184.87\ kN \cdot m$$

Since $M_t > M_u$ so

$$M_{e1} = M_u + M_t = 150 + 184.87 = 334.87 \text{ kN·m (Hogging)}$$

$$M_{e2} = M_t - M_u = 184.87 - 150 = 34.87 \text{ kN·m (Sagging)}$$

and section is designed for $SF = V_{ue} = 557.12$ kN.

...... Chapter 9 Ends Here..