CHAPTER 13

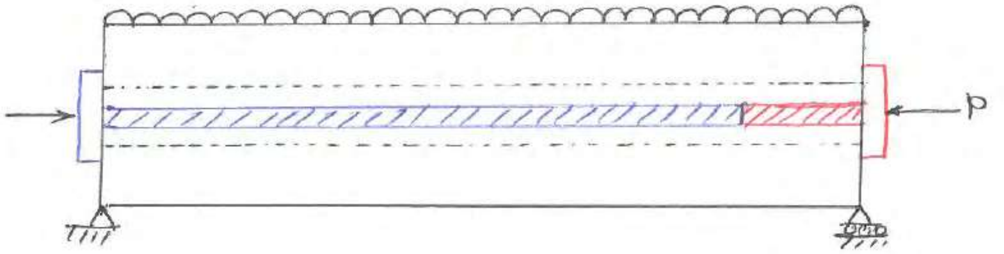
Analysis of PSC Member

CONTENTS.

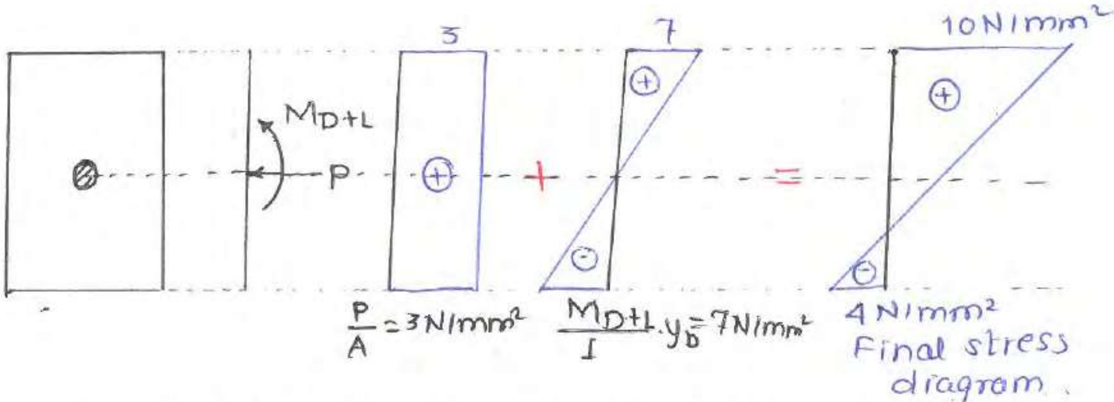
13. Analysis of PSC Member

13-1 Introduction:

PSC is basically concrete in which internal stress of suitable magnitude and distribution are introduced so that stress resulting from external loads are counteracted to a desired degree.



Permissible stress
$f_c = 13 N/mm^2$
$f_t = 4.5 N/mm^2$



- In final stress diagram, stress of top and bottom fibres are within permissible limits.
- If same BM is applied on section without prestressing then stress of bottom fibre exceeds the permissible limits ($f_t = 4.5 N/mm^2$)
- It means, member is made safe by prestressing.

13.2 Advantages and Disadvantages of Prestressing.

A) Advantages:

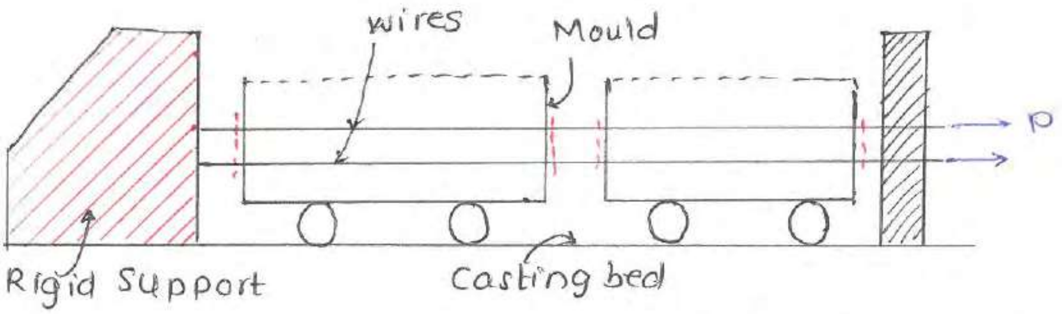
- Smaller section size of member as compared to RCC
- Concrete is more utilized.
- Less cracks as compared to RCC
- Suitable for large span beams
- Reduced DL of super structure.

B) Disadvantages:-

- Costly as compared to RCC.
- Not suitable for small work
- More technicality is involved so skilled manpower is required.
- Loss of stress of wire leads to poor performance.

13.3 Types of Prestressing.

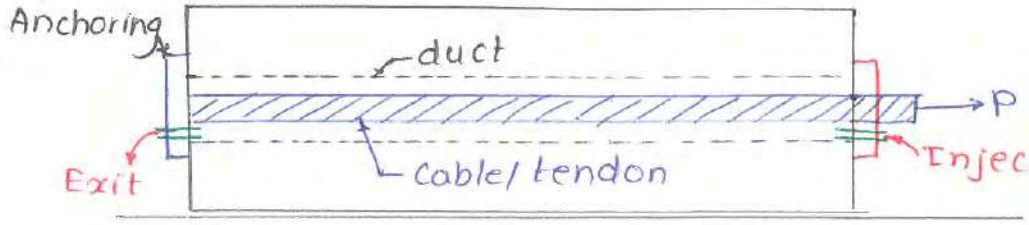
13.3.1 Pre-Tensioning:



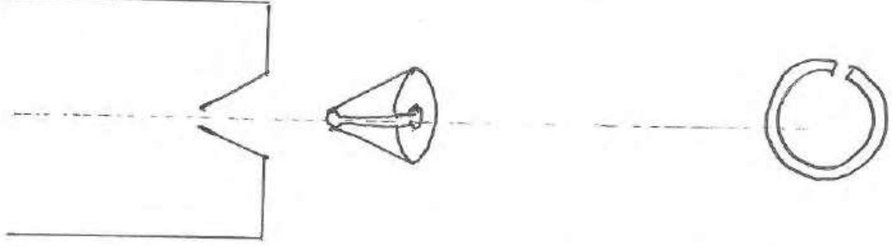
- Moulds are kept on casting bed and wires are inserted through moulds.
- Wires are fixed at position and tensioned from ends
- Concrete is casted in moulds and wires are kept in direct contact with concrete.
- Concrete is allowed to get sufficient strength.
- Now, wires are cut at ends of each mould and pre-stressing force is transfered to concrete by bond action between wire and concrete.

This method is also called as long line method or Hoyer method. It is generally used for small repeatative types of member. For e.g., railway PSC sleepers, PSC electric pole etc.

13.3.2 Post - Tensioning:



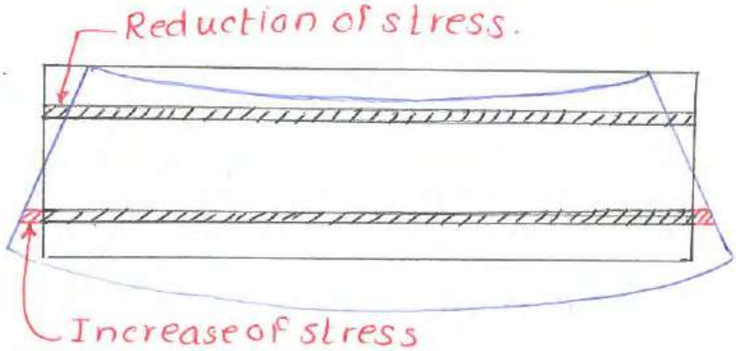
- Concrete is casted in desired shape and size with duct inside the member at desired location.
- Duct may be of PVC or Steel. It is flexible so can be provided in any shape.
- Concrete is allowed to get sufficient strength.
- Now cables are provided in duct and tensioned from ends. Cables can be provided inside the duct either before or after concreting.
- Tensioning is done either from one end or from both ends.
- After tensioning cables are anchored by any suitable arrangement
  1) Freyssinet System.
  2) Lee Mc Call System.
  3) Magnel Blaton System.
  4) Gifford Udall System.



- After anchoring, cables are cut and prestressing force is transfered on member.
- Remaining space of duct is filled with mortar of very high workability

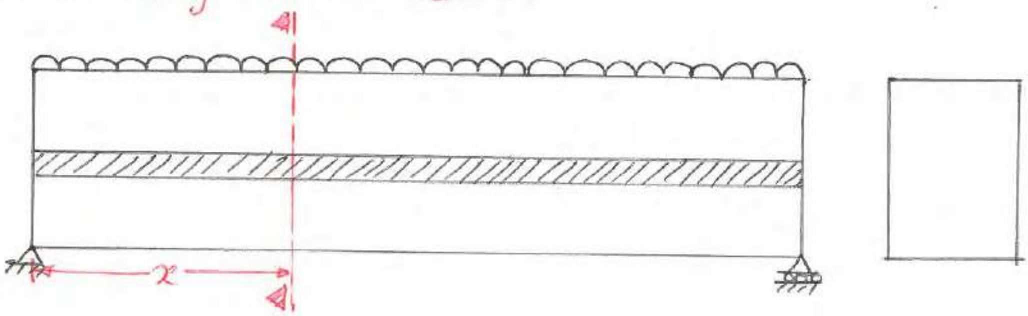
13.4 Assumptions in Analysis:

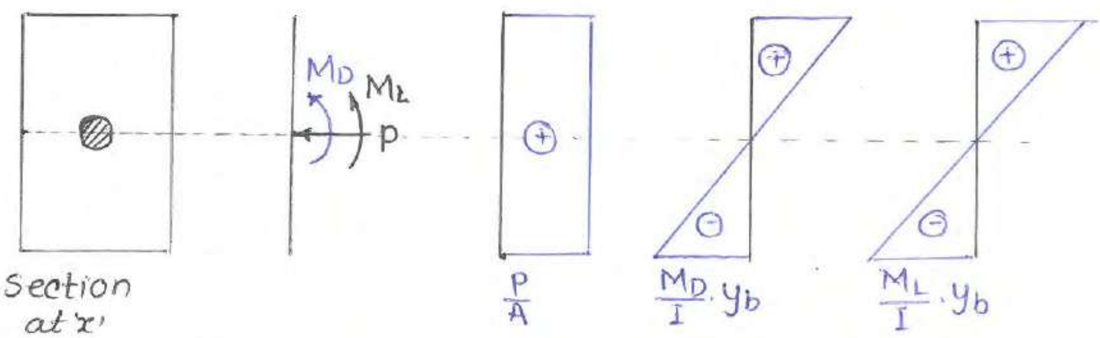
- Plane section remains plane after bending.
- Both materials are assumed to be linearly elastic.
- No variation of prestressing force along length of cable.
- No change of stress of cable due to bending of member.



13.5. Analysis of PSC Member:

13.5.1 Concentrically Placed Cable:

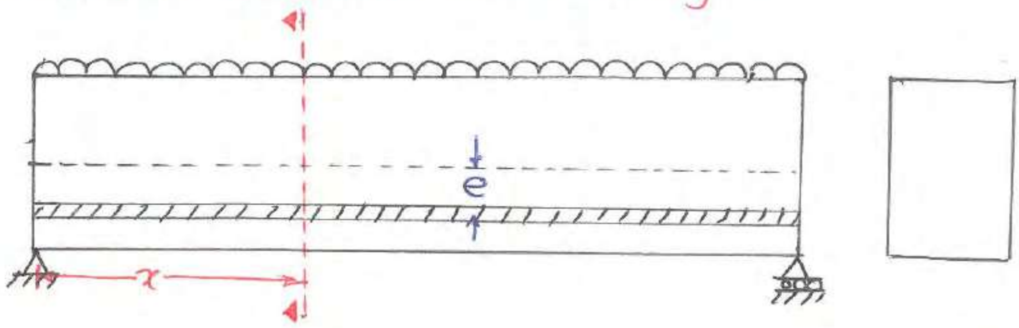


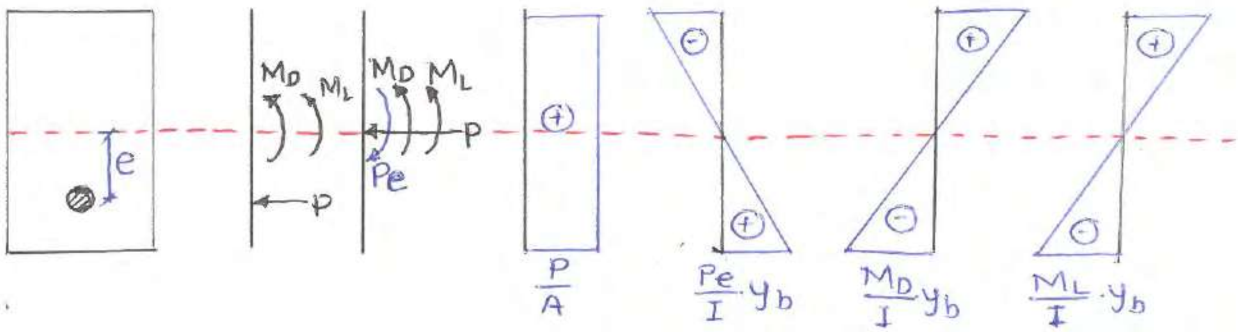


$$f_t = \frac{P}{A} + \frac{M_D}{I} \cdot y_t + \frac{M_L}{I} \cdot y_t$$

$$f_b = \frac{P}{A} - \frac{M_D}{I} \cdot y_b - \frac{M_L}{I} \cdot y_b$$

13.5.2 Cable Placed at Constant Eccentricity:

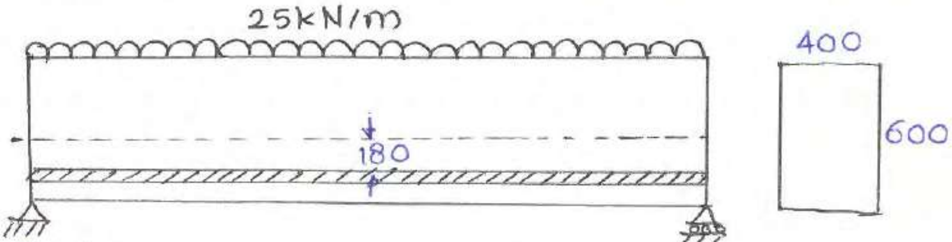




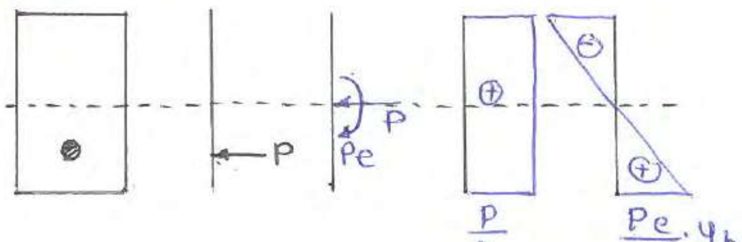
$$f_t = \frac{P}{A} - \frac{Pe}{I} y_t + \frac{M_D}{I} \cdot y_t + \frac{M_L}{I} \cdot y_t$$

$$f_b = \frac{P}{A} + \frac{Pe}{I} \cdot y_b - \frac{M_D}{I} \cdot y_b - \frac{M_L}{I} \cdot y_b$$

Ex. A simply supported beam given below is prestressed by cable area 1200 mm^2. Initial stress of cable is 1200 N/mm^2. Calculate stress of top and bottom fibres at midspan and ends of beam.



⇒ • At Ends:



$$f_t = \frac{P}{A} - \frac{Pe}{I} \cdot y_t$$

$$= \frac{1200 \times 1200}{400 \times 600} - \frac{1200 \times 1200 \times 180}{\frac{400 \times 600^3}{12}} \times \frac{600}{2}$$

$$f_t = -4.8 \ N/mm^2$$

$$f_b = \frac{P}{A} + \frac{Pe}{I} \cdot y_b$$

$$= \frac{1200 \times 1200}{400 \times 600} + \frac{1200 \times 1200 \times 180}{\frac{400 \times 600^3}{12}} \times \frac{600}{2}$$

$$f_b = 16.8 \ N/mm^2$$

- At Mid-span:

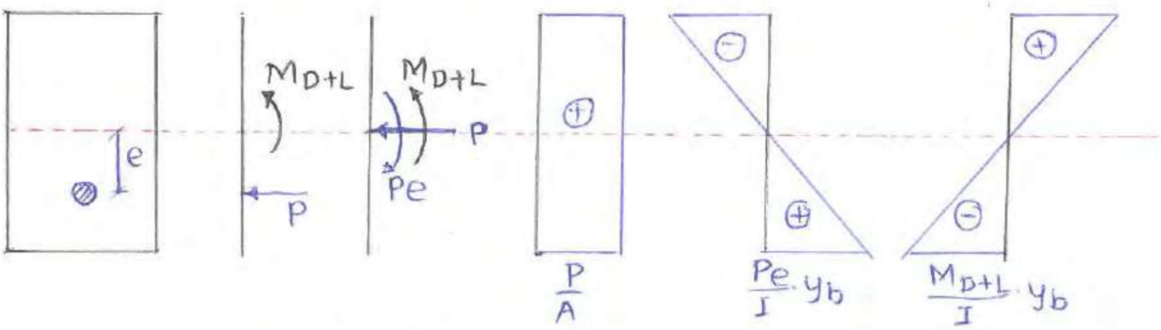
$$DL = 0.4 \times 0.6 \times 1 \times 25 = 6 \ kN/m$$

$$LL = 25 \ kN/m$$

$$M_{D+L} = \frac{WL^2}{8}$$

$$= \frac{(25+6) \times 10^2}{8}$$

$$M_{D+L} = 387.5 \ kNm$$



$$f_t = \frac{P}{A} - \frac{Pe}{I} \cdot y_t + \frac{M_{D+L}}{I} \cdot y_t$$

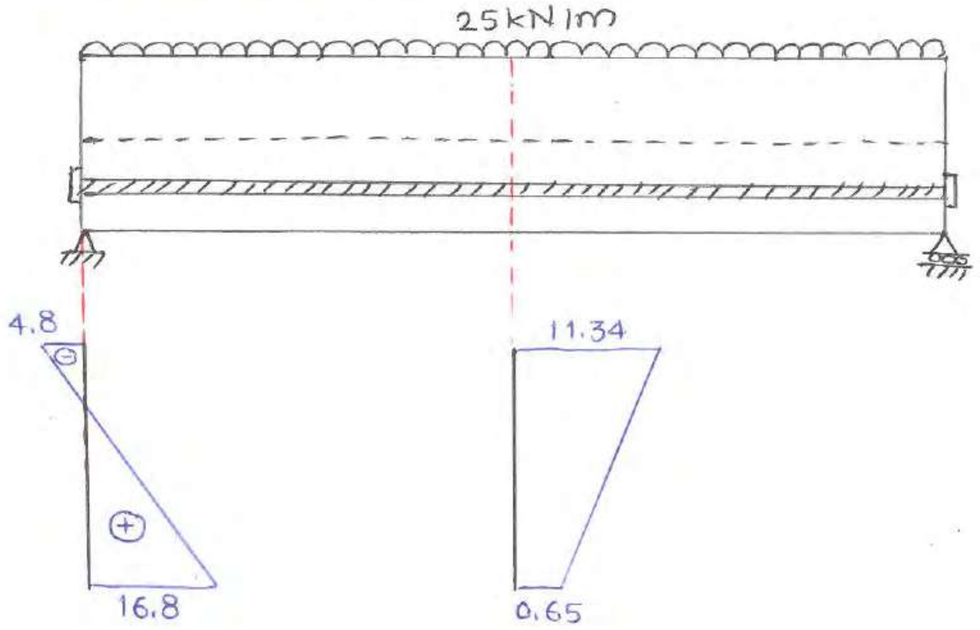
$$= \frac{1200 \times 1200}{400 \times 600} - \frac{1200 \times 1200 \times 180}{(400 \times 600^3)/12} \times \frac{600}{2} + \frac{387.5 \times 10^6 \times 300}{\frac{(400 \times 600^3)}{12}}$$

$$f_t = 11.34 \ N/mm^2$$

$$f_b = \frac{P}{A} + \frac{Pe}{I} \cdot y_b - \frac{M_{D+L}}{I} \cdot y_b$$

$$= \frac{1200 \times 1200}{400 \times 600} + \frac{1200 \times 1200 \times 180}{\frac{400 \times 600^3}{12}} \times \frac{600}{2} - \frac{387.5 \times 10^6}{\frac{400 \times 600^3}{12}} \times \frac{600}{2}$$

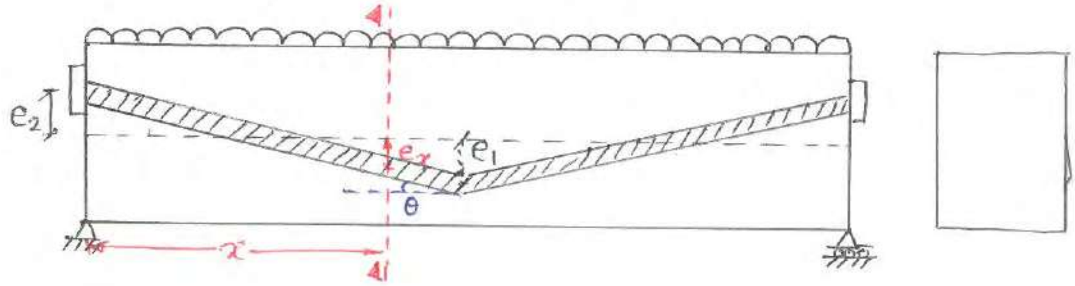
$f_b = 0.65\ N/mm^2$



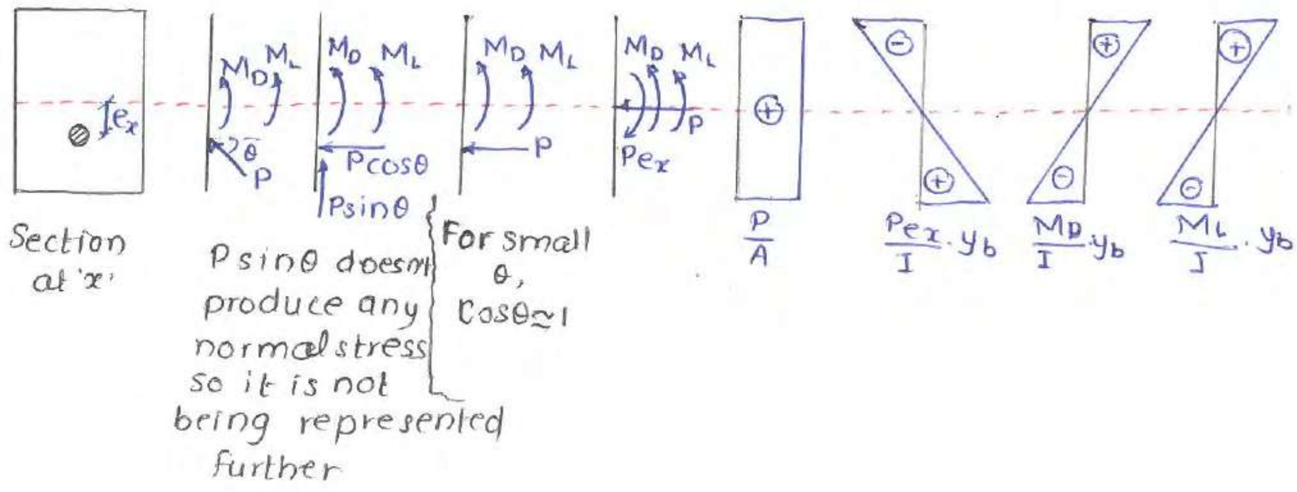
13.5.3 Cable Placed at Varying Eccentricity:

This type of proble is solved by following methods.

1. Stress Concept
2. Load Balancing Concept
3. Strength concept.



1) Stress Concept:

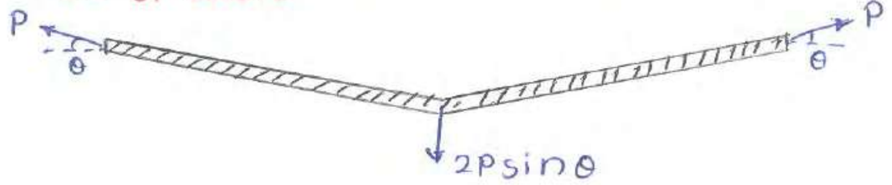


$$f_t = \frac{P}{A} - \frac{Pe_x}{I}\cdot y_t + \frac{M_D}{I}\cdot y_t + \frac{M_L}{I}\cdot y_t$$

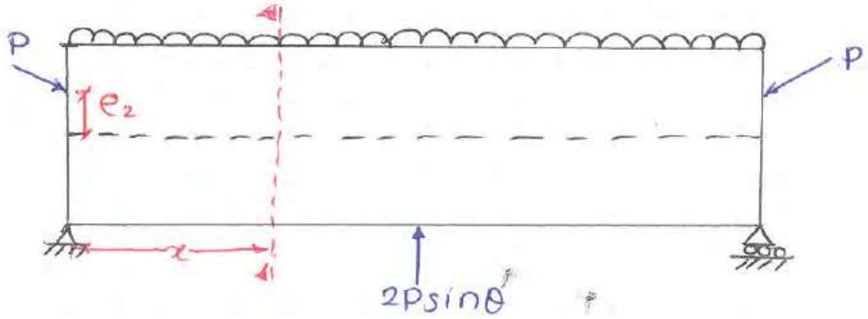
$$f_b = \frac{P}{A} + \frac{Pe_x}{I}\cdot y_b - \frac{M_D}{I}\cdot y_b - \frac{M_L}{I}\cdot y_b$$

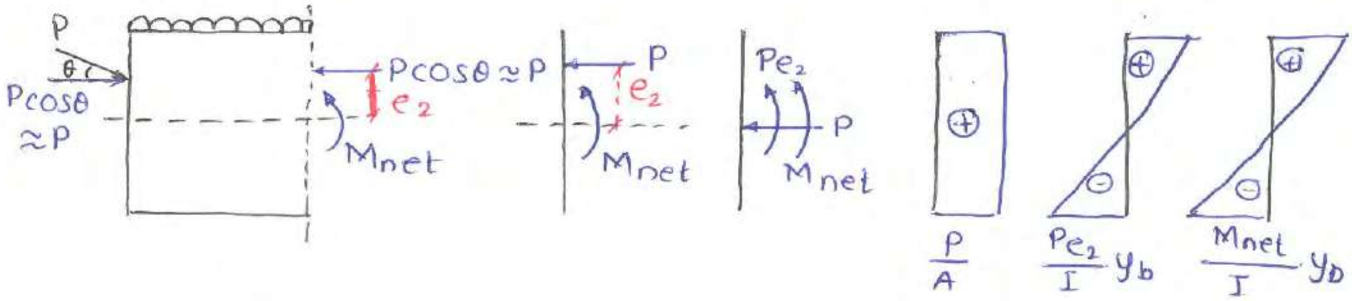
2) Load Balancing Concept:

FBD of cable



FBD of concrete beam.



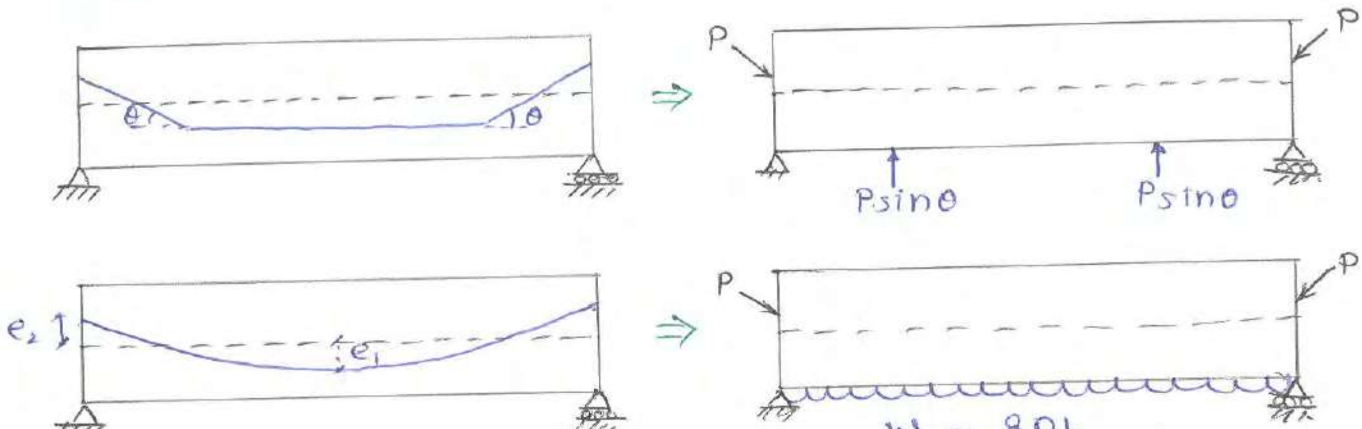


$$f_t = \frac{P}{A} + \frac{Pe_2}{I} \cdot y_t + \frac{M_{net}}{I} \cdot y_t$$

$$f_b = \frac{P}{A} - \frac{Pe_2}{I} \cdot y_b - \frac{M_{net}}{I} \cdot y_b$$

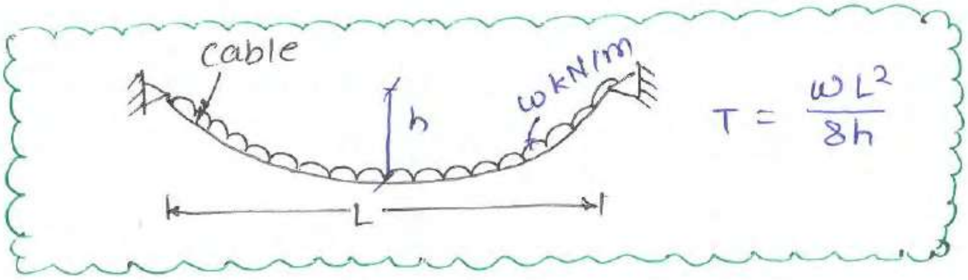
M_{net} = BM at section 'x' due to DL, LL & $2P\sin\theta$.

* Other Load Balancing Cable Profiles:



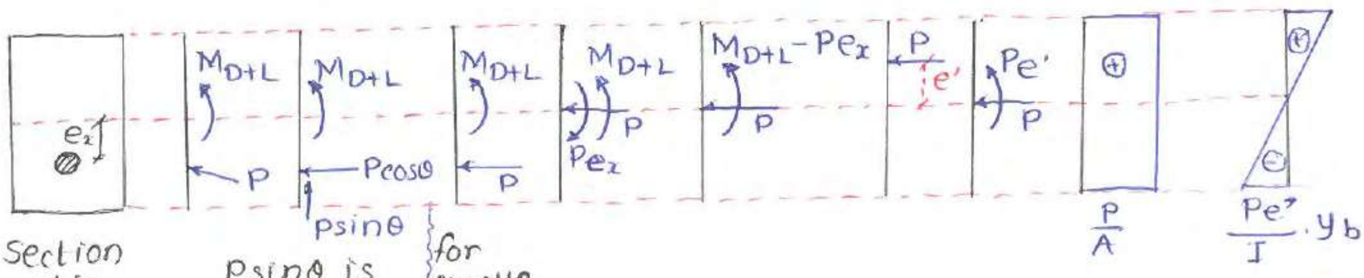
$$W_p = \frac{8Ph}{L^2}$$

where $h = e_1 + e_2$



$$T = \frac{wL^2}{8h}$$

3) Strength Concept:



Section at 'x'

Psinθ is not represented further becoz it doesn't produce any normal stress

for small θ, $\cos\theta \simeq 1$

$$f_t = \frac{P}{A} + \frac{Pe'}{I} \cdot y_t$$

$$f_b = \frac{P}{A} - \frac{Pe'}{I} \cdot y_b$$

from above diagram:

$$Pe' = M_{D+L} - Pe_x$$

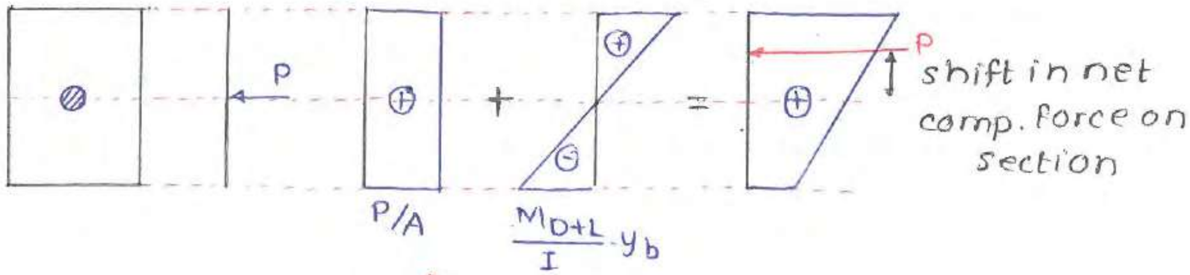
$$\Rightarrow e_x + e' = \frac{M_{D+L}}{P}$$

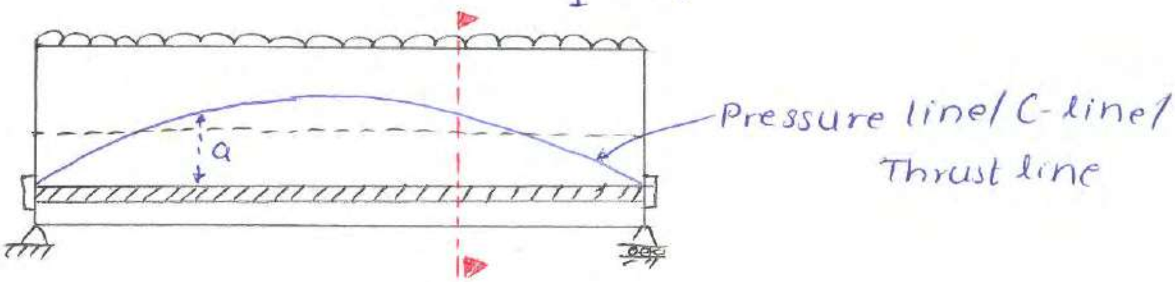
Shift in position of Net comp. force on section $= a = \frac{M_{D+L}}{P}$

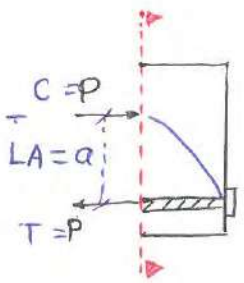
Now,

$$e' = a - e_x$$

* Meaning of shift of net compressive force on section:

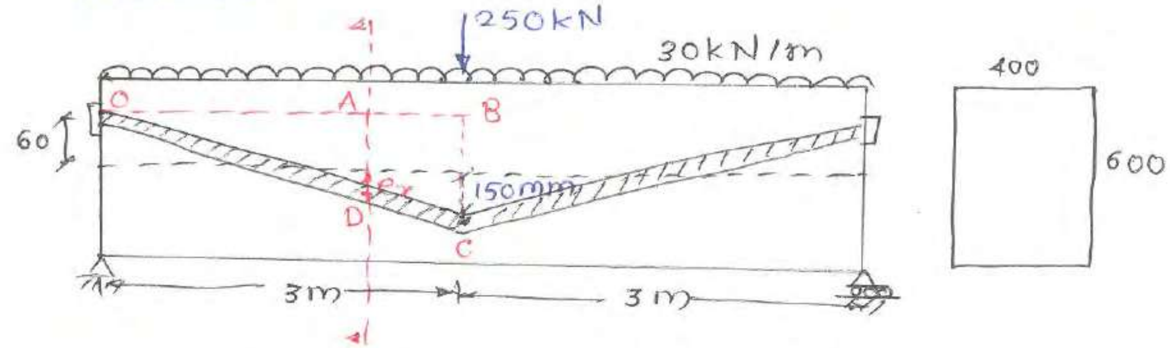






- Pressure lin is the locus of net compressive force on along section along span.
- In PSC member, C and T remains constant along the span and lever arm varies
- In RCC member, C and T vary along span while lever arm remains constant.

Ex A simply supported beam given below is prestressed by 1600kN Calculate stress in top and bottom fibres at a distance 2.5m from support. Solve by all three methods.



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1) Stress concept:

from cable profile:

$$\frac{OA}{AD} = \frac{OB}{BC}$$

$$\Rightarrow \frac{2.5}{60+e_x} = \frac{3}{60+150}$$

$e_x = 115\,mm$

DL = $0.4 \times 0.6 \times 1 \times 25 = 6\,kN/m$

UDL LL = $30\,kN/m$

Point LL = $250\,kN$

$M_{D+L}(x=2.5m)$ = Due to UDL + Due to Point load

$$= \left(\frac{WLx}{2} - \frac{wx^2}{2}\right) + \left(\frac{R}{2} \cdot x\right)$$

$$= \left(\frac{36 \times 6 \times 2.5}{2} - \frac{36 \times 2.5^2}{2}\right) + \left(\frac{250}{2} \times 2.5\right)$$

$M_{D+L} = 470\,kNm$

$$f_t = \frac{P}{A} - \frac{Pe_x}{I} \cdot y_t + \frac{M_{D+L}}{I} \cdot y_t$$

$$= \frac{1600 \times 10^3}{400 \times 600} - \frac{1600 \times 10^3 \times 115}{(400 \times 600^3)/12} \times \frac{600}{2} + \frac{470 \times 10^6}{(400 \times 600^3)/12} \times \frac{600}{2}$$

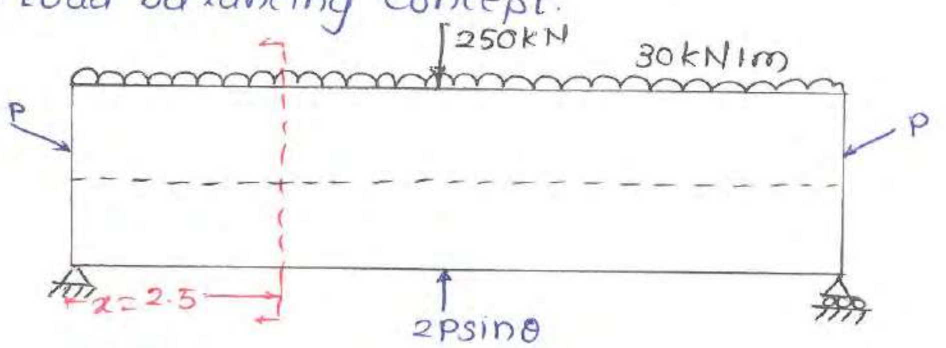
$f_t = 18.58\,N/mm^2$

$$f_b = \frac{P}{A} + \frac{Pe_x}{I} \cdot y_b - \frac{M_{D+L}}{I} \cdot y_b$$

$$= \frac{1600 \times 10^3}{400 \times 600} + \frac{1600 \times 10^3 \times 115}{\frac{400 \times 600^3}{12}} \times \frac{600}{2} - \frac{470 \times 10^6 \times}{\frac{400 \times 600^3}{12}} \frac{600}{2}$$

$f_b = -5.25\ N/mm^2$

2) Load Balancing Concept:



$2P\sin\theta$

$= 2P \cdot \theta$

$= 2P\left(\frac{e_1 + e_2}{L/2}\right)$

$= 2 \times 1600 \times \left(\frac{150 \times 60}{3000}\right)$

$= 224\ kN$

DL = 6 kN/m

UDL LL = 30 kN/m

Point Load = 250 − 224 = 26 kN

$M_{net(x=2.5m)}$ = Due to UDL + Due to Point Load

$$= \left(\frac{wLx}{2} - \frac{wx^2}{2}\right) + \left(\frac{R}{2} \cdot x\right)$$

$$= \left(\frac{36 \times 6 \times 2.5}{2} - \frac{36 \times 2.5^2}{2}\right) + \left(\frac{26}{2} \times 2.5\right)$$

$= 190\ kNm$

$$f_t = \frac{P}{A} + \frac{Pe_2}{I} \cdot y_t + \frac{M_{net}}{I} \cdot y_t$$

$$= \frac{1600 \times 10^3}{400 \times 600} + \frac{1600 \times 10^3 \times 60}{\frac{400 \times 600^3}{12}} \frac{600}{2} + \frac{190 \times 10^6}{\frac{400 \times 600^3}{12}} \times \frac{600}{2}$$

$f_t = 18.58\ N/mm^2$

$$f_b = \frac{P}{A} - \frac{Pe_2}{I}.y_b - \frac{M_{net}}{I}.y_b$$

$$= \frac{1600\times10^3}{400\times600} - \frac{1600\times10^3\times60}{\frac{400\times600^3}{12}}\times\frac{600}{2} - \frac{190\times10^6}{\frac{400\times600^3}{12}}\times\frac{600}{2}$$

$$f_b = -5.25\ N/mm^2$$

3) Strength Concept:

$$a = \frac{M_{D+L\,(x=2.5m)}}{P}$$

$$= \frac{470\times10^6}{1600\times10^3}$$

$a = 293.75mm$

$e' = a - e_x$

$= 293.75 - 115$

$e' = 178.75mm$

$$f_t = \frac{P}{A} + \frac{Pe'}{I}.y_t$$

$$= \frac{1600\times10^3}{400\times600} + \frac{1600\times10^3\times178.75}{\frac{400\times600^3}{12}}\times\frac{600}{2}$$

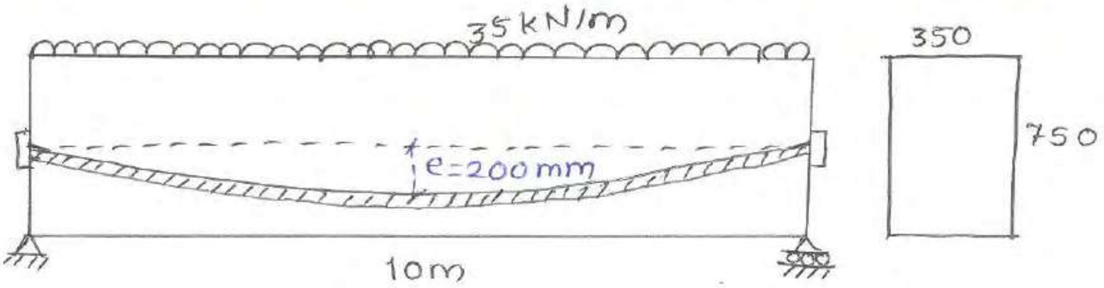
$f_t = 18.58\ N/mm^2$

$$f_b = \frac{P}{A} - \frac{Pe'}{I}.y_b$$

$$= \frac{1600\times10^3}{400\times600} - \frac{1600\times10^3\times178.75}{\frac{400\times600^3}{12}}\times\frac{600}{2}$$

$f_b = -5.25\ N/mm^2$

Ex. A simply supported beam given below is prestressed by 1750 kN force. Calculate stress in top and bottom fibres at 4m from support. Solve by all three methods.



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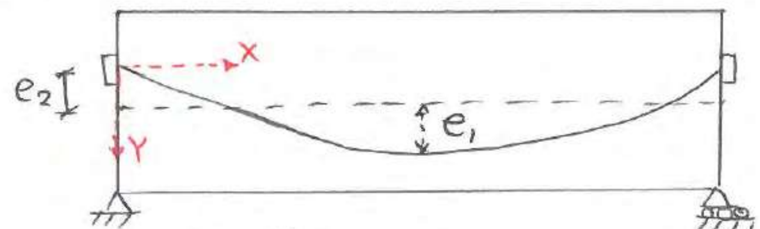
1) Stress Concept:

Equation of parabolic cable profile



$$y = \frac{4hx}{L^2}(L-x)$$

- $h = e_1 + e_2$
- Origin is at left end of cable
- Y-axis is downward.

$$e_x = \frac{4(0.2-0)}{10^2}\,4(10-4)$$

$e_x = 192$ mm

DL = 0.35 × 0.75 × 1 × 25 = 6.56 kN/m

LL = 35 kN/m

$$M_{D+L(x=4m)} = \frac{WLx}{2} - \frac{wx^2}{2}$$

$$= \frac{41.56 \times 10 \times 4}{2} - \frac{41.56 \times 4^2}{2}$$

$M_{D+L\,(x=4m)} = 498.72$ kN m

$$f_t = \frac{P}{A} - \frac{Pe_x}{I} \cdot y_t + \frac{M_{D+L}}{I} \cdot y_t$$

$$= \frac{1750 \times 10^3}{350 \times 750} - \frac{1750 \times 10^3 \times 192}{\frac{350 \times 750^3}{12}} \times \frac{750}{2} + \frac{498.72 \times 10^6}{\frac{350 \times 750^3}{12}} \times \frac{750}{2}$$

$f_t = 11.62$ N/mm²

$$f_b = \frac{P}{A} + \frac{Pe_x}{I} \cdot y_b - \frac{M_{D+L}}{I} \cdot y_b$$

$$= \frac{1750 \times 10^3}{350 \times 750} + \frac{1750 \times 10^3 \times 192}{\frac{350 \times 750^3}{12}} \times \frac{750}{2} - \frac{498.72 \times 10^6}{-}$$

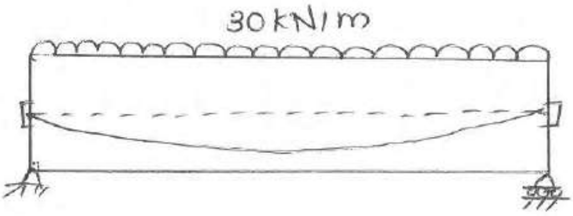
$f_b = 1.71$ N/mm²

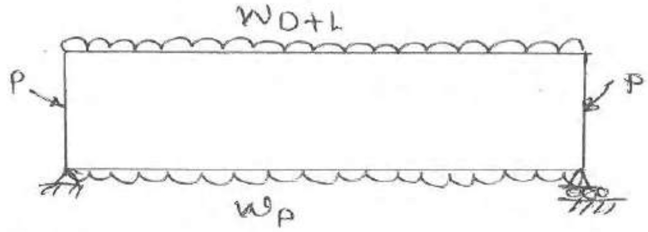
Solve yourself by other two methods.

Ex. What could be the cable profile for complete load balancing of simply supported beam subjected to 30 kN/m (including self wt.). Effective span 8m, Prestressing force 1400 kN, Section size 300×600 mm, eccentricity at ends of span is zero. Also calculate stress in extreme fibres at mid-span after complete load balancing.

⇒

Since applied load is udl so cable profile must be parabolic for complete load balancing.





For complete load balancing.

$$W_{D+L} = W_p$$

$$W_{D+L} = \frac{8Ph}{L^2}$$

$$30 = \frac{8 \times 1400 \times e}{8^2}$$

$$e = 171.42\, mm$$

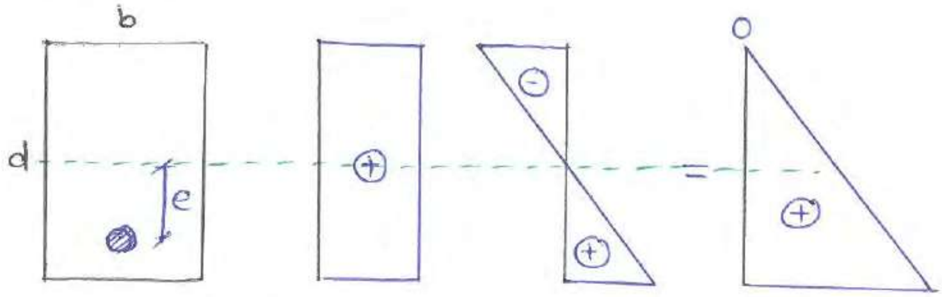
$$f_{t/b} = \frac{P}{A} \pm \frac{Pe_2}{I} \cdot y_{t/b} \pm \frac{M_{net}}{I} y_{t/b}$$

$$= \frac{1400 \times 10^3}{300 \times 600} \pm \frac{P \times 0}{I} y_{t/b} \pm \frac{0}{I} y_{t/b}$$

$$f_{t/b} = 7.77\, N/mm^2$$

13.6 Kern Distance:

It is the eccentricity of resultant prestressing force over section corresponding to which stress in one extreme fibre is just zero.



For just zero stress in one extreme fibre:

$$\frac{P}{A} - \frac{Pe}{I} y_t = 0$$

$$\Rightarrow \frac{P}{bd} - \frac{Pe}{\frac{bd^3}{12}} \times d/2 = 0$$

$$\Rightarrow \boxed{e = \frac{d}{6}}$$

*Note:-
Top kern and bottom kern are different for unsymmetrical section.

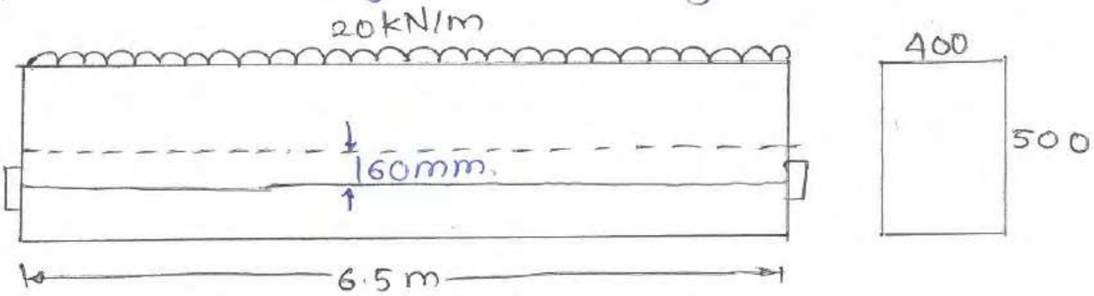
13.7 Cracking load / Cracking moment:

Load/moment corresponding to which first crack develops in extrem tension fibre is called cracking load/ cracing moment.

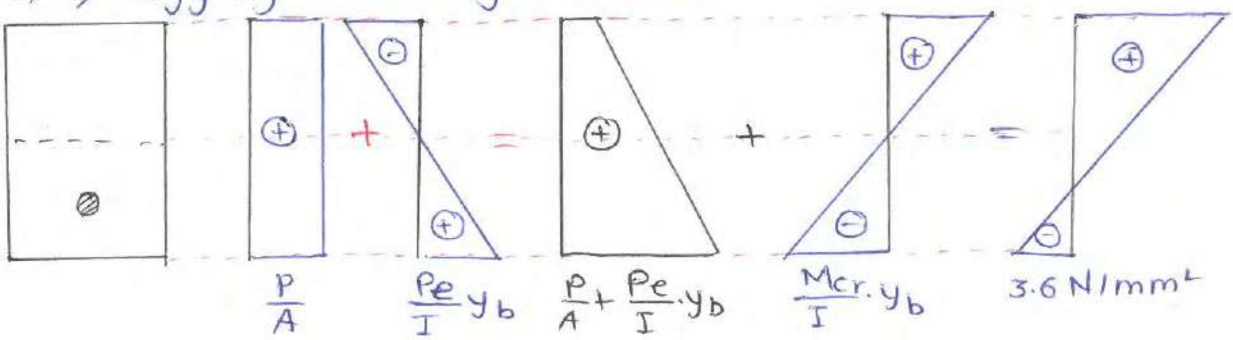
Tensile stress in extreme tension fibre $\geq$ Permissible tensile stress

$$\text{FOS against cracking} = \frac{M_{cracking}}{M_{working}} = \frac{W_{cracking}}{W_{working}}$$

Ex. A simply supported beam given below is prestressed by 1500 kN force. Modulus of rupture of concrete is 3.6 N/mm². Calculate sagging cracking moment, cracking superimposed udl, FOS against cracking.



⇒ 1) Sagging cracking Moment:



$$f_b = \frac{P}{A} + \frac{Pe}{I} \cdot y_b - \frac{M_{cr}}{I} \cdot y_b$$

$$-3.6 = \frac{1500 \times 10^3}{400 \times 500} + \frac{1500 \times 10^3 \times 160}{\frac{400 \times 500^3}{12}} \times \frac{500}{2} - \frac{M_{cr}}{\frac{400 \times 500^3}{12}} \times \frac{500}{2}$$

$M_{cr} = 425$ kNm

2) Cracking Superimposed UDL:

$DL = 0.4 \times 0.5 \times 1 \times 25$

$LL = W_{L,cr}$

For maximum cracking superimposed UDL

$BM_{max} = M_{cr}$

$$\frac{WL^2}{8} = M_{cr} \Rightarrow \frac{(5 + W_{L,cr})\, 6.5^2}{8} = 425$$

$W_{L,cr} = 75.47$ kN/m

3) F.O.S. against cracking:

$M_{cr} = 425\, kNm$

$W_{D,cr} = 5\, kN/m$

$W_{L,cr} = 75.47\, kN/m$

$W_{D,w} = 5\, kN/m$

$W_{L,w} = 20\, kN/m$

$$M_w = \frac{W_w L^2}{8} = \frac{(5+20)\times 6.5^2}{8}$$

$M_w = 132.08$

$$\text{FOS against cracking} = \frac{M_{cr}}{M_w} = \frac{W_{cr}}{W_w}$$

$$= \frac{425}{132.08} = \frac{5+75.47}{5+20}$$

$$3.21 = 3.21$$

..... Chapter 13 Ends Here..