47. The Special Theory of Relativity

Short Answer

Answer.1

The second postulates describe that the speed of light (for medium vacuum) has same value in all inertial frame. Speed of light in any other medium will be less than that of vacuum using Snell's law. Snell's law is given by,

$$v = \frac{c}{\mu}$$

Where,

v=Speed of light in glass

c=Speed of light in vacuum

μ=Refractive index

Thus, it doesn't violets second postulate of the refractive index because the speed of light in glass is not more than that of vacuum.

Answer.2

As the platform is stationary it is considered as the proper frame for this event.

The train travels with a speed which is very less as compared with the speed of light. So, it does not violate the second postulate of relativity.

Thus, both train and platform are proper frame.

Answer.3

Let the speed of rod is v, the length is given by

$$l = l_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

Now assume that the observer and the rod are moving with the speed v in same direction. The length of the rod is given by,

$$l = l_0 \sqrt{1 - \left(\frac{(v - [-v])^2}{c^2}\right)}$$

$$l = l_0 \sqrt{1 - \left(\frac{4v^2}{c^2}\right)}$$

Thus, it is true that an object may be regarded to be at rest or in motion depending on the frame of reference chosen to view the object.

Answer.4

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

The gravitational force of Earth is given by,

$$F = \frac{GMm_0}{r^2}$$

$$F = \frac{GMm_o}{r^2\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

Thus, both mass and the gravitational attraction depends on the particle’s speed.

Answer.5

There are two ways to measure the distance between the Earth and the Moon which are using meter scale and using light pulse.

In the first method the distance is measured using meter scale, the length gets contracted in moving frame. Thus, the length measured is smaller.

In second method, the time difference between emission of light pulse and reception is noted. So, the distance measured in this case is larger.

Thus, the distance measured is either smaller or larger than the actual distance.

Objective I

Answer.1

The magnitude of linear momentum is given by,

P=mv

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

Thus, magnitude of linear momentum is,

$$p = \frac{m_o v}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

This value of the linear momentum does no match with any option.

Thus, correct option is D

Answer.2

As the mass is at rest, its value is not changed when observed with the frame moving with the velocity v.

Thus, correct option is C.

Answer.3

The new length is given by,

$$l = l_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

As v << C

$\therefore \sqrt{1 - \left(\frac{v^2}{c^2}\right)} < 1$

Thus, $l < l_0$

Consider the rod at rest and observer moves with the speed of v then the new length is given by,

$$l = l_0 \sqrt{1 - \left(\frac{(-v)^2}{c^2}\right)}$$

$$l = l_0 \sqrt{1 - \left(\frac{(v)^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

As v << C

$\therefore \sqrt{1 - \left(\frac{v^2}{c^2}\right)} < 1$

Thus, $l < l_0$

Thus, the length will be reduced in both the cases.

So, correct option is C.

Answer.4

The new length is given by,

$$l = l_0 \sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

Consider the rod at rest and observer moves with the speed of v parallel to the measured length then the new length is given by,

$$l = l_0 \sqrt{1 - \left(\frac{(-v)^2}{c^2}\right)}$$

$$l = l_0 \sqrt{1 - \left(\frac{(v)^2}{c^2}\right)}$$

Where, l_0 = Length of rod at rest

` c = Speed of light >> v

Consider the rod at rest and observer moves with the speed of v opposite to the measured length then the new length is given by,

$$l = l_0 \sqrt{1 - \left(\frac{(v - [-v])^2}{c^2}\right)}$$

$$l = l_0 \sqrt{1 - \left(\frac{4v^2}{c^2}\right)}$$

As v << C

$\therefore \sqrt{1 - \left(\frac{v^2}{c^2}\right)} < 1$

Thus, $l < l_0$

Thus, the length will be minimum for the rod moving in the opposite direction.

So, correct option is B.

Answer.5

The magnitude of linear momentum is given by, P=mv

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

Thus, magnitude of linear momentum is,

$$P = \frac{m_o v}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$$

$$P = m_0 v\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Using binomial expansion, we get,

$$P = m_0 v + \frac{m_0 v^3}{2c^2}$$

When the speed of the particle is doubled then,

$$P' = \frac{m_o 2v}{\sqrt{1-\left(\frac{4v^2}{c^2}\right)}}$$

$$P = 2m_0 v + \frac{4m_0 v^3}{c^2}$$

Thus, the new value of linear momentum is more than double.

So, correct option is B.

Answer.6

When the force acting on the particle is constant, the speed of the particle increases due to the force.

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

So, the mass of the particle increases, this results in decreasing the acceleration gradually.

Thus, correct option is B.

Answer.7

The charged particle is projected at a very high speed perpendicular to the uniform magnetic field. The mass will increase using the relativistic relation.

The relativistic mass of the particle is,

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

So, its radius will increase.

Thus, correct option is B.

Objective II

Answer.1

Statement (a) is correct because the equation of special relativity is only applicable when the speed of object is comparable to the speed of light. Newton's equations of motion are used for small speeds.

When the body is travelling with the relativistic speed, then

mass becomes $m' = \gamma m$, length becomes $L' = \frac{L}{\gamma}$, change in time becomes $\Delta T' = \gamma \Delta T$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{v^2}{2c^2} + \ldots\ldots.. = \text{sum} << 1 \text{ when } v << c.$$

Still the sum of the series will be greater than 0

Hence, non-relativistic equations in which γ factor is taken to be exactly 1 never give exact results.

Option (B) and (D) are correct

Answer.2

If the speed of rod is relativistic speed then, its mas will be

$$m' = \gamma m = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{v^2}{2c^2} + \ldots\ldots.. > 1 \text{ as } v < c$$

And its length will be,

$$L' = \frac{L}{\gamma} = L\sqrt{1-\frac{v^2}{c^2}}$$

When speed v is doubled,

$$\gamma' = \frac{1}{\sqrt{1 - \frac{4v^2}{c^2}}}$$

$$\Rightarrow \gamma' = \left(1 - \frac{4v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{2v^2}{c^2} + \ldots\ldots\ldots > 2\gamma$$

$$\textit{and}\ m' = \gamma' m > 2\gamma m, L' = \frac{l}{\gamma'} < \frac{l}{2\gamma}$$

Hence, mass will be increased and it will be more than double. Length will decrease but not exactly half of the original length.

Option (C) and (D) are correct.

Answer.3

The two events occur at the same time at points A and B in rest frame. If they occur simultaneously in a frame moving with respect to the lab, the frame must be moving in the direction perpendicular to AB.

Option (b) is correct.

Answer.4

the rest length of the rod= L

Relativistic speed = v

Length contraction= L'

$$L' = \frac{L}{\gamma} = L\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If the rod is moving with relativistic speed, then γ will be approximately equal to 1. This means that the length of the rod L' will also be almost equal to L. however, the length of rod in rest frame L, can be more than L' depending upon the frame of observer.

Option (B) and (c) are correct

Answer.5

if the rod is moving with relativistic speed v, then mass will be given by,

$$m' = \gamma m = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{here, } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

thus, the mass will increase by the factor of γ

option (A) is correct

Exercises

Answer.1

The house of yogi is 1000km away from his guru in Himalayan cave. The velocity of travelling of the thoughts should be maximum so that it can reach to his guru in minimum time. The maximum velocity that can be attained is speed of light i.e. 3 x ^{8} m/sec.

Distance= 1000 km = 10^6 m

Velocity= 3 x 10^8 m/sec

$$\text{Time} = \frac{\text{Distance}}{\text{velocity}} = \frac{10^6}{3 \times 10^8} = \frac{1}{300} \text{ sec}$$

Answer.2

(a) The traveler is traveling in the same frame in which suitcase is travelling. So, the dimensions of the suitcase for the traveler will remain same.

(b) The observer who is on the ground will not see the same dimensions of the suitcase. It is because the frame of reference of observer and the suitcase is different. The suitcase is in the frame of reference which is moving with the speed of 0.6c. Since, the suitcase is placed with its length along the train's velocity, the changes will be observed in the length only.

Length of the suitcase= 50cm

Breadth = 25cm

Height = 10cm

Velocity= 0.6c

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 50\sqrt{1-\frac{(0.6\,c)^2}{c^2}} = 50\sqrt{1-0.36} = 50 \times 0.8 = 40\text{cm}$$

Therefore, the length observed by the observer on the ground will be less than the original length of the suitcase. The dimensions will be 40cm x 25cm x 10cm.

Answer.3

Whenever the object moves with the speed comparable to the speed of light, the length of the object contracts. So, let us consider the following cases when the speed of rod is comparable to the speed of light.

(a) length of the rod= 1m

Speed of the rod= 3×10^5 m s^{-1}

Using the relativistic relation of length contraction

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{9\text{ x }10^{10}}{9\text{ x }10^{16}}} = \sqrt{1-10^{-6}} = 0.99999995\text{m}$$

The length observed will be 0.99999995m

(b) length = 1m

Speed= 3×10^6 m s^{-1}

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{9\text{ x }10^{12}}{9\text{ x }10^{16}}} = \sqrt{1-10^{-4}} = 0.99995\text{m}$$

(c) length= 1m

Speed = 3×10^7 m s^{-1}

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{9\text{ x }10^{14}}{9\text{ x }10^{16}}} = \sqrt{1-10^{-2}} = 0.9949\text{m}$$

Answer.4

The velocity of the train, v = 0.6c

Time taken by the train to pass the man standing on platform, t= 1sec

(a) the length of the train as seen by the person, L' = vt = 0.6 x 3 × 10^8 = 1.8 x 10^8m

(b) the rest length of the train L

$$L' = L\sqrt{1-\frac{v^2}{c^2}}$$

$$1.8\text{ x }10^8 = L\sqrt{1-\frac{(0.6\text{ c})^2}{c^2}}$$

$$L = \frac{1.8\text{ x }10^8}{0.8} = 2.25\text{ x }10^8\ m/sec$$

Answer.5

The field becomes square in the plane frame only when its length and breadth become equal. Originally, the field is in rectangular shape, with length = 100m and breadth = 50 m. In order to make the rectangular field into square field, either the breadth can be increased or length can be reduced. But we know that with the speed comparable to the speed of light, the object's length contracts. Hence, the only possible way to make the rectangular field into square field is that the plane must travel along the length of the field with the speed comparable to the speed of light.

$$L' = L\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow 50 = 100\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow \frac{1}{2} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow v = 0.866c$$

Answer.6

The distance between Patna and Delhi = 1000km

Speed of the train= 360 km h^{-1} =100 m/s

(a) Distance between the two cities in the train frame d

$$D' = D\sqrt{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow D' = 10^6\sqrt{1 - \frac{10^4}{9 \text{ x } 10^{16}}}$$

$\Rightarrow D' = 10^9$

(b) time elapses in the train frame

$$\Delta t = \frac{\Delta L}{v}$$

To solve for ΔL

$$L' = L\sqrt{1 - \frac{v^2}{c^2}}$$

$$L' = 10^6 \sqrt{1 - \frac{100^2}{c^2}}$$

$\Rightarrow L' = 56 \times 10^{-9}$ m

Change in length ΔL = 56 nm

therefore, the distance between Patna and Delhi in the train frame is 56 nm less than 1000 km.

now, actual time taken by train= time = distance/ speed

$$t = \frac{1000 \times 10^3}{100} = 10^4 \text{ sec}$$

change in time, $\Delta T = \frac{\Delta L}{v} = \frac{56 \times 10^{-9}}{100} = 0.56\text{ns}$

So, the time lapse in the train between Patna and Delhi will be 0.56ns less than 10^4 sec

Answer.7

Speed of the car= 180 km h^{-1}= 50 m/sec

Time taken by car to reach from station A to station B = 10 hours

The distance travelled by person in a car L'= speed X time= 180 x 10= 1800km

$$L' = L\sqrt{1-\frac{v^2}{c^2}}$$

$$1800 = L\sqrt{1-\frac{(180)^2}{9 \text{ x } 10^{16}}}$$

$$\Rightarrow L = 1800 + 25 \text{ x } 10^{-9}$$

So, the rest distance between the two station is 25nm more than 1800km.

(b) time taken by car to cover the distance in the road= distance/ speed

$$\Rightarrow \frac{1.8 \text{ X } 10^6 + 25 \text{ x } 10^{-9}}{50}$$

$$\Rightarrow 0.36 \text{ x } 10^5 + 5\text{x}10^{-8} = 10 \text{ hours } 0.5 \text{ ns}$$

Answer.8

The speed of spaceship =5c/13

(a) consecutive birthday celebration means the time period t= 1 year.

But the time interval observed by the person is spacecraft will be

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow t' = \frac{1}{\sqrt{1-\frac{(5c/13)^2}{c^2}}}$$

$$\Rightarrow t' = \frac{1}{\sqrt{1-\frac{25}{169}}}$$

$$\Rightarrow t' = \frac{13}{12}\text{years}$$

Thus, the time interval observed by the person will be more than one year i.e. 1.08 years (approx.)

(b) The person on Earth will also calculate the same time interval because for him also, the person in spacecraft is travelling with same speed.

Answer.9

Both the stations are in ground frame; hence the station clocks will record the proper time interval. But the clocks in train will record improper time because they are at different places travelling with different speed. The proper time interval ΔT is less than improper i.e. $\Delta T' = v\Delta T$. Hence, in case (a) In the train going from Howrah to Delhi, the baby born in Delhi will be elder. (b) In the train going from Delhi to Howrah, Howrah baby will be elder.

Answer.10

Since, the frame is moving, the clocks will not record the synchronized time. The clock at the rear end leads the clock which is at the other end by $L\,v/c^2$ where, L is the rest separation between the clocks and v is the speed of the moving frame i.e. train. Thus, the baby born adjacent to the guard cell will be elder.

Answer.11

velocity of Swarglok with respect to Earth = 0.9999c

According to the Earth' frame the velocity will be

$$v' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \ v' = \frac{1}{\sqrt{1 - \frac{(0.9999c)^2}{c^2}}} = \frac{1}{0.01414} = 70.712$$

Let one day on Earth = ΔT

One day on heaven= $\Delta T'$

$\Delta \ T' = v\Delta T$

$\Rightarrow \ \Delta T' = 70.71$ days in heaven

Answer.12

As the Lorentz transformation suggests there will be time dilation for a moving object with respect to rest frame which is given by γT_o

Where $\gamma = \frac{1}{\sqrt{1 - V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8 \ ^{m}/_{s}$

And T_o is time in rest frame.

ATQ, $V = 60$ % of C i.e. $\frac{V}{C} = \frac{3}{5}$

$$\therefore \ \gamma = \frac{1}{\sqrt{1 - (\frac{3}{5})^2}} = \frac{5}{4}$$

$\therefore$ The person will live $\frac{5}{4} \times 100 \ years = 125 \ years$ in the spaceship with respect to earth frame.

Answer.13

As the Lorentz transformation suggests there will be time dilation for a moving object with respect to rest frame which is given by γT_o

Where $\gamma = \frac{1}{\sqrt{1-V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ ^m/_s$

And T_o is time in rest frame.

ATQ, $\frac{V}{C} = \frac{4}{5}$

$$\therefore\ \gamma = \frac{1}{\sqrt{1-(\frac{4}{5})^2}} = \frac{5}{3}$$

$\therefore$ Time taken by the bulb to switch off and switch on with respect to spaceship will be $\frac{5}{3} \times 1s$

$\therefore$ Frequency will be $\frac{1}{T} = \frac{3}{5}\ Hz$

Answer.14

Time taken by the car to reach the tower B from tower A $= \frac{1000}{100} = 10\ Hour$.

As the frame of wrist watch is moving $\therefore$ the wrist watch will lag by $\gamma T_o - T_o$.

Where $\gamma = \frac{1}{\sqrt{1-V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ ^m/_s$

And T_o is time in rest frame .

V=$100\ ^{Km}/_{hr} = 100 \times \frac{5}{18}\ ^m/_s = 27.78\ m/s$

$\frac{V}{C} = \frac{27.78}{3\times10^8}$ as $\frac{V}{C} << 1$ using binomial expansion we know we can write

$(1+x)^n = 1 + nx$ when $x << 1$

Similarly, $\gamma = \frac{1}{\sqrt{1-(\frac{27.78}{3\times10^8})^2}} = (1-(\frac{27.78}{3\times10^8})^2)^{-\frac{1}{2}}$=$1 + \frac{1}{2} \times 86.0254 \times 10^{-16}$

$\therefore T_o(\gamma - 1) = 10\, hour \times 43 \times 10^{-16} = 1.548 \times 10^{-12} s$

$Since\ 1\ hour = 3600\ s$

And as we can see the calculated value is too small to make drastic affect that's why we don't face any trouble in real life situation.

Answer.15

As we know the moving object appears to be contracted with the earth aur rest frame that's because of Lorentz contraction known as length contraction which is given by $l = \frac{l_O}{\gamma}$

Where $\gamma = \frac{1}{\sqrt{1 - V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ ^m/_s$

And l_O is length at rest .

As volume always include power of 3 weather it is cube sphere we will take a simple case of cube where all of its length is given by l_O at rest

$\therefore$ Volume at rest is $= {l_O}^3$

ATQ Volume seems to be half at certain speed

$$\therefore l^3 = \frac{{l_O}^3}{2}$$

$$\Rightarrow l = \frac{l_O}{\sqrt[3]{2}} = 0.794 l_O$$

$$\therefore \frac{l_O}{\gamma} = 0.794 l_o$$

$$\Rightarrow \gamma = 1.26$$

And $1.26 = \frac{1}{\sqrt{1 - V^2/_{C^2}}}$

$$\Rightarrow \sqrt{1 - V^2/_{C^2}} = 0.63$$

$$\Rightarrow 0.604 \times C^2 = V^2$$

$$\therefore V = 0.78C$$

Answer.16

a)We know time taken = $\frac{Distance\ traversed}{Speed\ of\ the\ particle}$

$$\therefore life\ of\ particle\ will\ be = \frac{1 \times 10^{-2}\ m}{0.995 \times 3 \times 10^{8}\ {}^{m}/_{s}} = 3.35 \times 10^{-11} s$$

b)In the frame of particle there will be length contraction or we can say time dilation we can solve the question with anyone of the approach.

As the Lorentz transformation suggests there will be time dilation for a moving object with respect to rest frame which is given by γT_O

Where $\gamma = \frac{1}{\sqrt{1 - {}^{V^2}/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^{8}\ {}^{m}/_{s}$

And T_O is time in rest frame.

$\therefore T_O$ was found in previous case as $3.35 \times 10^{-11} s$

$$\therefore Life\ of\ particle\ in\ particle\ frame\ will\ be = T_O \times \frac{1}{\sqrt{1 - {}^{V^2}/_{C^2}}}$$

$$\Rightarrow 3.35 \times 10^{-11} s \times \frac{1}{\sqrt{1 - {}^{(0.995C)^2}/_{C^2}}} = 3.35 \times 10^{-11} s \times 10$$

$$= 3.35 \times 10^{-10} s$$

Answer.17

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

m

$\therefore$ energy possessed by the compressed spring as potential energy is due to loss of the mass

$$\therefore \frac{1}{2}kx^2 = mC^2$$

Given:

$$k = 500\ {}^{N}/_{m}$$

$$x = 1\ cm = 10^{-2}\ m$$

$$\Rightarrow \frac{1}{2} \times 500 \times (10^{-2}) = m \times 9 \times 10^{16}$$

$$\therefore m = 2.78 \times 10^{-19} Kg$$

So, the change in mass of spring will be $2.78 \times 10^{-19} Kg$

Answer.18

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

$\therefore$ Heat energy given will be converted in the mass

$$\Delta Q = mc\Delta T = \Delta mC^2$$

Given:

c =Specific heat capacity of water = 4200 J kg^{-1} K^{-1}.

$$m = 1kg$$

$$\Delta T = 373K - 273K = 100K$$

$$\therefore 1 \times 4200 \times 100 = \Delta m \times 9 \times 10^{16}$$

$$\Rightarrow \Delta m = \frac{4.2 \times 10^5}{9 \times 10^{16}} = 4.7 \times 10^{-12} Kg$$

Answer.19

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴ Heat energy extracted will result in loss of the mass

$$\Delta Q = C_V \Delta T = \Delta m C^2$$

Given:

$$C_V = \frac{3}{2}R \; where \; R = 8.314 \; {}^{J}/_{K.mol}$$

$$\Delta T = 10K$$

$$\therefore \frac{3}{2} \times 8.314 \times 10 = \Delta m \times 9 \times 10^{16}$$

$$\Rightarrow \Delta m = 13.86 \times 10^{-16} Kg$$

Answer.20

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴Gain in kinetic energy by the boy is possessed by the mass gain for the boy.

$$\therefore \frac{1}{2} \times m \times V^2 = \Delta m C^2$$

Given:

V=$12 \; {}^{km}/_{h} = 12 \times \frac{5}{18} {}^{m}/_{s}$

$$\therefore \frac{1}{2} \times m \times (12 \times \frac{5}{18})^2 = \Delta m \times 9 \times 10^{16}$$

$$\Rightarrow \frac{\Delta m}{m} = \frac{1}{2} \times \frac{1}{9 \times 10^{16}} \times (12 \times \frac{5}{18})^2$$

$$\Rightarrow \frac{\Delta m}{m} = 6.17 \times 10^{-17}$$

Answer.21

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E=mC^2

∴ Loss in the mass due to continuous illumination of bulb

ATQ, the bulb illuminates for 1 year i.e $365 \times 24 \times 3600\ s$ with power of 1 W or 1 $^{J}/_{s}$.

Total energy dissipated by the bulb in the form of light energy will

be $100 \times 365 \times 24 \times 3600\ J = 3.15 \times 10^9 J$

∴ $3.15 \times 10^9 J = \Delta m \times 9 \times 10^{16}$

$\Longrightarrow \Delta m = 3.5 \times 10^{-8} kg$

Answer.22

a)

The amount of energy radiated by the sun in $1m^2 = 1400\ W$

As the energy radiated is dependent on the surface area it covers which is given by $4 \times \pi \times r^2$

∴ Energy loosed by the sun till it reaches earth in 1 second is given by $4 \times \pi \times (1.5\ \times 10^{11}\ \text{m})^2 \times 1400 = 3.96 \times 10^{26}\ J$

So, the mass loosed by the sun in 1 second is $\frac{3.96 \times 10^{26}}{9 \times 10^{16}} = 4.4 \times 10^9 Kg$

b)

As the rate of decay of the mass is constant it will last up to $\frac{Total\ mass\ of\ sun}{Rate\ at\ which\ mass\ is\ decaying} = \frac{2 \times 10^{30}\ Kg}{4.4 \times \frac{10^9 Kg}{s}} = 4.55 \times 10^{10}\ s$

In years $\frac{4.55 \times 10^{10}\ s}{365 \times 24 \times 3600\ s} = 1.44 \times 10^3\ Years$

Answer.23

As we know electron and proton has same mass i.e. $9.1 \times 10^{-31} Kg$

As they annihilate and produces gamma particle with some energy

$\Delta m = 2 \times 9.1 \times 10^{-31} Kg$

There is a physical significance of special theory of relativity that Energy and mass are interchangeable and that is given by Sir Einstein famous equation E= $\Delta m C^2$

$$\therefore Energy\ of\ gamma\ particle = 2 \times 9.1 \times 10^{-31} Kg \times 9 \times 10^{16} \frac{m}{s}$$
$$= 1.66 \times 10^{-13} J$$

Answer.24

As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_o \gamma$

Where $\gamma = \frac{1}{\sqrt{1 - V^2/C^2}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ m/s$

And m_o is mass at rest

$$\therefore \ \gamma = \frac{1}{\sqrt{1 - (0.8C)^2/C^2}} = \frac{1}{0.6}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.6} = 1.52 \times 10^{-30}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$$(m - m_o)C^2 = (15.2 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 5.45 \times 10^{-15} J$$

Momentum of electron will be is equal to = Velocity $\times$ apparent mass

$$\Rightarrow 1.52 \times 10^{-30}\ Kg \times 0.8 \times 3 \times 10^8\ m/s = 3.65 \times 10^{-22}\ kg.m/s$$

Answer.25

a) As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_o\, \gamma$

Where $\gamma = \frac{1}{\sqrt{1-V^2/C^2}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ m/s$

And m_o is mass at rest

$$\therefore\ \gamma = \frac{1}{\sqrt{1-(0.6C)^2/C^2}} = \frac{1}{0.8}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.8} = 11.375 \times 10^{-31}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$$(m - m_o)C^2 = (11.375 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 20.475 \times 10^{-15} J$$

As the gain in kinetic energy is given by charge $\times$ potential difference applied i.e. q $\times V$

As $q = 1.6 \times 10^{-19} c$

$$\therefore\ 1.6 \times 10^{-19} c \times V = 20.475 \times 10^{-15} J$$

$$\Rightarrow V = 12.8 \times 10^4\ V$$

b)Similarly for 0.9c

$$\gamma = \frac{1}{\sqrt{1-(0.9C)^2/C^2}} = \frac{1}{0.44}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.44} = 20.68 \times 10^{-31}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$$(m - m_o)C^2 = (20.68 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 1 \times 10^{-13} J$$

As the gain in kinetic energy is given by charge $\times$ potential difference applied i.e. q $\times V$

As $q = 1.6 \times 10^{-19} c$

$$\therefore\ 1.6 \times 10^{-19} c \times V = 1 \times 10^{-13} J$$

$\Rightarrow V = 6.25 \times 10^5\ V$

c) Similarly for 0.99c

$$\gamma = \frac{1}{\sqrt{1 - {(0.99C)^2}/{C^2}}} = \frac{1}{0.1}$$

$$\therefore Apparent\ mass\ = \frac{9.1 \times 10^{-31} Kg}{0.1} = 9.1 \times 10^{-30}\ Kg$$

Kinetic energy gained by the electron will be due to change in mass of electron.

$$(m - m_o)C^2 = (91 - 9.1) \times 10^{-31} \times 9 \times 10^{16} = 7.37 \times 10^{-13} J$$

As the gain in kinetic energy is given by charge $\times$ potential difference applied i.e. q $\times V$

As $q = 1.6 \times 10^{-19} c$

$\therefore\ 1.6 \times 10^{-19} c \times V = 7.37 \times 10^{-13} J$

$\Rightarrow V = 4.6 \times 10^6\ V$

Answer.26

We can't deal the question with classical mechanics approach as $\frac{1}{2} \times m_e \times v^2$ as when you calculate v with this approach it will come out to be more than speed of light in vacuum i.e. $3 \times 10^8\ {m}/{s}$ that violates the special theory of relativity.

So we have to solve these types of question taking relativistic approach.

As we know the gain in Kinetic energy is due to change in mass

As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_O\ \gamma$

Where $\gamma = \frac{1}{\sqrt{1 - {V^2}/{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8\ {m}/{s}$

And m_O is mass at rest i.e.= $9.1 \times 10^{-31} Kg$

K.E= $m_O(\gamma - 1)C^2$

$\therefore m_o \times C^2 = 9.1 \times 10^{-31} \times 9 \times 10^{16} = 8.19 \times 10^{-14} kg\ {m^2}/{s^2}$

a) So 10for 1 eV i.e. $1.6 \times 10^{-19} J$

$8.19 \times 10^{-14} kg\ {m^2}/{s^2} \times (\gamma - 1) = 1.6 \times 10^{-19} J$

$\Longrightarrow \gamma - 1 = 1.95 \times 10^{-6}$

$\Longrightarrow \gamma = 1 + 1.95 \times 10^{-6}$

As $\gamma = \frac{1}{\sqrt{1 - {V^2}/{C^2}}}$

Squaring both side and using binomial as we know using binomial expansion we know we can write

$(1 + x)^n = 1 + nx$ when $x << 1$

$$\therefore 1 + 3.9 \times 10^{-6} = \frac{1}{1 - {V^2}/{C^2}}$$

$$\Longrightarrow 1 - {V^2}/{C^2} = \frac{1}{1 + 3.9 \times 10^{-6}}$$

$\Longrightarrow V^2 = C^2 \times 3.9 \times 10^{-6}$

V=$6 \times 10^5\ m/s$

b) Similarly for 10KeV i.e. $1.6 \times 10^{-15} J$

$8.19 \times 10^{-14} kg\ {m^2}/{s^2} \times (\gamma - 1) = 1.6 \times 10^{-15} J$

$\Longrightarrow \gamma - 1 = 1.95 \times 10^{-2}$

$\Longrightarrow \gamma = 1 + 1.95 \times 10^{-2}$

As $\gamma = \frac{1}{\sqrt{1 - {V^2}/{C^2}}}$

Squaring both side and using binomial as we know using binomial expansion we know we can write

$(1 + x)^n = 1 + nx$ when $x << 1$

$$\therefore 1 + = \frac{1}{1 - {V^2}/{C^2}}$$

$$\Longrightarrow 1 - {V^2}/{C^2} = \frac{1}{1 + 3.8 \times 10^{-2}}$$

$\Rightarrow V^2 = C^2 \times 3.9 \times 10^{-2}$

V=$6 \times 10^7\ m/s$

c) Similarly for 10KeV i.e. $1.6 \times 10^{-12} J$

$8.19 \times 10^{-14} kg\ {m^2}/{s^2} \times (\gamma - 1) = 1.6 \times 10^{-12} J$

$\Rightarrow \gamma - 1 = 19.5$

$\Rightarrow \gamma = 20.5$

As $\gamma = \frac{1}{\sqrt{1 - {V^2}/{C^2}}}$

$\therefore 420.5 = \frac{1}{1 - {V^2}/{C^2}}$

$\Rightarrow 1 - {V^2}/{C^2} = 0.0024$

$\Rightarrow V^2 = C^2 \times 0.9976$

$\therefore V = 0.998C$

Answer.27

As we know that Kinetic energy gained by the electron is due to change in mass of electron.

ATQ mass becomes double $\therefore \Delta m = 2m_o - m_o$

And m_o is mass at rest i.e.= $9.1 \times 10^{-31} Kg$

As E=K.E=ΔmC^2=$m_o \times 9 \times 10^{16} = 9.1 \times 10^{-31} \times 9 \times 10^{16} = 8.2 \times 10^{-14} J$

Answer.28

As we know that Kinetic energy gained by the electron is due to change in mass of electron.

As E=K.E=ΔmC^2

As we know relativistic Energy is always more than non-relativistic kinetic energy because Rest energy is always less then apparent mass energy

$\therefore$The relativistic value of Kinetic energy will be $\frac{101}{200} \times m_o \times V^2$

As we know the moving object appears to be heavier in the moving frame that's because of Lorentz transformation known as apparent mass which is given by $m = m_o \gamma$

Where $\gamma = \frac{1}{\sqrt{1-V^2/_{C^2}}}$, V is velocity of moving object and C is speed of light in vacuum i.e. $3 \times 10^8 \; {}^{m}/_{s}$

K.E= $m_o(\gamma - 1)C^2 = \frac{101}{200} \times m_o \times V^2$

$$\Longrightarrow \frac{1}{\sqrt{1 - V^2/_{C^2}}} - 1 = \frac{101}{200} \times m_o \times \frac{V^2}{C^2}$$

Let $\frac{V^2}{C^2} = K$

$$\therefore \frac{1}{\sqrt{1 - K}} - 1 = \frac{101}{200} \times K$$

$$\Longrightarrow \frac{1}{\sqrt{1 - K}} = \frac{301}{200} \times K$$

Squaring both side we get

$$\Longrightarrow \frac{1}{1 - K} = 2.265 \times K^2$$

$$\Longrightarrow 1 = 2.265K^2 - 2.265K^3$$

As K=$\frac{V^2}{C^2}$, $K^3 = \frac{V^6}{C^6}$ which is $<< 1$ $\therefore$we can neglect K^3 *term.*

$$\Longrightarrow K^2 = \frac{1}{2.265} \Longrightarrow K = 0.441$$

$$\Longrightarrow \frac{V^2}{C^2} = 0.441$$

$$\therefore V = 0.66\, C$$