

Chapter – 10

Circles

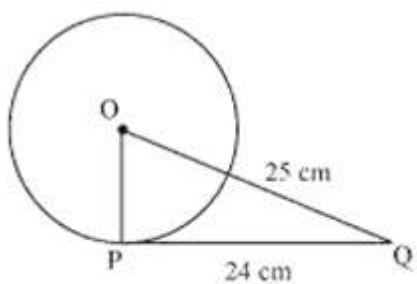
Exercise 10.2

Q. 1 From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

- A. 7 cm B. 12 cm
C. 15 cm D. 24.5 cm

Answer:

(A)



Let O be the centre of the circle.

Given that,

$$OQ = 25\text{cm and } PQ = 24\text{ cm}$$

As the radius is perpendicular to the tangent at the point of contact,

Therefore, $OP \perp PQ$

Applying Pythagoras theorem in $\triangle OPQ$, we obtain

$$OP^2 + PQ^2 = OQ^2$$

$$OP^2 + 24^2 = 25^2$$

$$OP^2 = 625 - 576$$

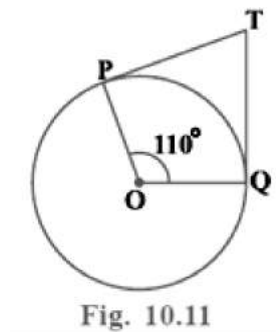
$$OP^2 = 49$$

$$OP = \sqrt{49} = 7$$

Therefore, the radius of the circle is 7 cm.

Q. 2 In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to

- A. 60° B. 70°
C. 80° D. 90°



Answer:

Given: TP and TQ are tangents.

Therefore, radius drawn to these tangents from centre of the circle will be perpendicular to the tangents.

Thus, $OP \perp TP$ and $OQ \perp TQ$

And therefore,

$$\angle OPT = 90^\circ$$

$$\angle OQT = 90^\circ$$

In quadrilateral POQT,

$$\text{Sum of all interior angles} = 360^\circ$$

$$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$$

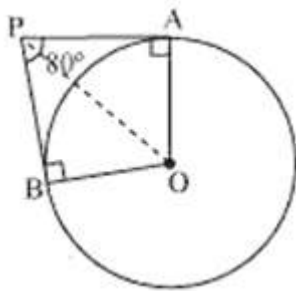
$$90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$$

$$\angle PTQ = 70^\circ$$

Q. 3 If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to

- A. 50° B. 60°
C. 70° D. 80°

Answer: Given: PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp PA$ and $OB \perp PB$

$$\angle OBP = 90^\circ$$

$$\angle OAP = 90^\circ$$

In quadrilateral AOBP,

$$\text{Sum of all interior angles} = 360^\circ$$

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ \quad 90^\circ + 80^\circ + 90^\circ + \angle BOA = 360^\circ$$

$$\angle BOA = 100^\circ$$

In $\triangle OPB$ and $\triangle OPA$,

$$AP = BP \text{ (Tangents from a point)}$$

$$OA = OB \text{ (Radii of the circle)}$$

$$OP = OP \text{ (Common side)}$$

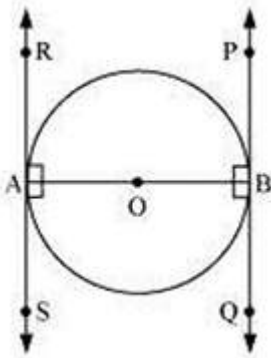
Therefore, $\triangle OPB \cong \triangle OPA$ (SSS congruence criterion)

And thus, $\angle POB = \angle POA$

$$\angle POA = \frac{1}{2} \angle AOB = \frac{100}{2} = 50^\circ$$

Q. 4 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer:



Let AB is diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively. Radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp RS$ and $OB \perp PQ$

$$\angle OAR = 90^\circ$$

$$\angle OAS = 90^\circ$$

$$\angle OBP = 90^\circ$$

$$\angle OBQ = 90^\circ$$

It can be observed that

$$\angle OAR = \angle OBQ \text{ (Alternate interior angles)}$$

$$\angle OAS = \angle OBP \text{ (Alternate interior angles)}$$

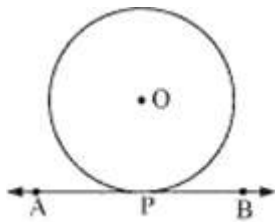
Since alternate interior angles are equal, lines PQ and RS will be parallel.

Q. 5 Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer: To Prove: A line drawn perpendicular from P will pass from O.

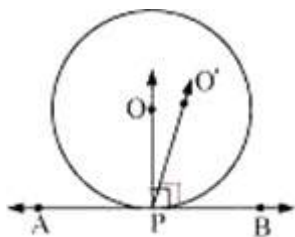
Given: APB is tangent to circle with centre O.

Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



Proof:

Assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.



As perpendicular to AB at P passes through O', therefore,

$$\angle O'PB = 90^\circ \dots (1)$$

O is the centre of the circle and P is the point of contact.

We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$$\therefore \angle OPB = 90^\circ \dots (2)$$

Comparing equations (1) and (2), we obtain

$$\angle O'PB = \angle OPB \dots (3)$$

From the figure, it can be observed that,

$$\angle O'PB < \angle OPB \dots (4)$$

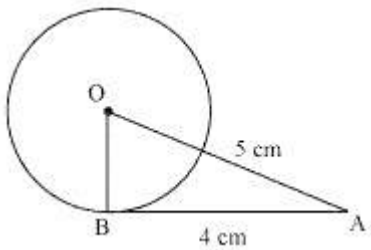
Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O'P coincides with OP.

Therefore, the perpendicular to AB through P passes through centre O.

Hence, Proved.

Q. 6 The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4cm. Find the radius of the circle.

Answer:



Let us consider a circle centred at point O.

AB is a tangent drawn on this circle from point A.

Given that,

$$OA = 5\text{cm and } AB = 4\text{ cm}$$

In $\triangle ABO$,

$OB \perp AB$ (Radius \perp tangent at the point of contact)

Applying Pythagoras theorem in $\triangle ABO$, we obtain

$$AB^2 + BO^2 = OA^2$$

$$4^2 + BO^2 = 5^2$$

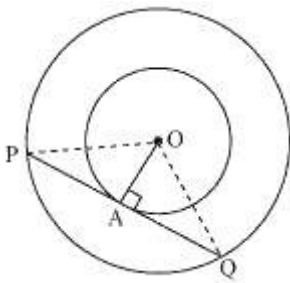
$$16 + BO^2 = 25 \quad BO^2 = 9$$

$$BO = 3$$

Hence, the radius of the circle is 3 cm.

Q. 7 Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer:



Let the two concentric circles be centred at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

$OA \perp PQ$ (As OA is the radius of the circle)

Applying Pythagoras theorem in $\triangle OAP$, we obtain

$$OA^2 + AP^2 = OP^2$$

$$3^2 + AP^2 = 5^2$$

$$9 + AP^2 = 25$$

$$AP^2 = 16$$

$$AP = 4$$

In $\triangle OPQ$,

Since $OA \perp PQ$,

$AP = AQ$ (Perpendicular from the centre of the circle bisects the chord)

$$PQ = 2AP = 2 \times 4 = 8$$

Therefore, the length of the chord of the larger circle is 8 cm.

Q. 8 A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that

$$AB + CD = AD + BC$$

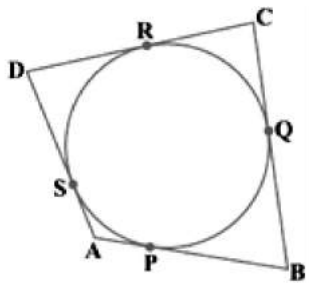


Fig. 10.12

Answer:

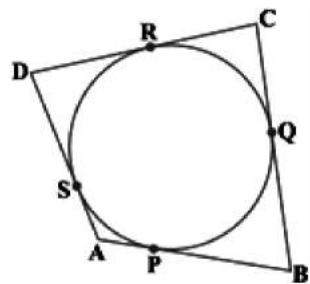


Fig. 10.12

To Prove: $AB + CD = AD + BC$

Proof:

In the given figure, it can be observed that AB touches the circle at point P; BC touches the circle at point Q; CD touches the circle at point R; DA touches the circle at point S.

Then, we can say from a theorem that "*Length of tangents drawn from an external point to the circle are same*",

Therefore,

$$DR = DS \text{ (Tangents on the circle from point D) } \dots (1)$$

$$CR = CQ \text{ (Tangents on the circle from point C) } \dots (2)$$

$BP = BQ$ (Tangents on the circle from point B) (3)

$AP = AS$ (Tangents on the circle from point A) (4)

Adding all these equations, we obtain

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC$$

Hence, Proved.

Q. 9 In Fig. 10.13, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B . Prove that $\angle AOB = 90^\circ$

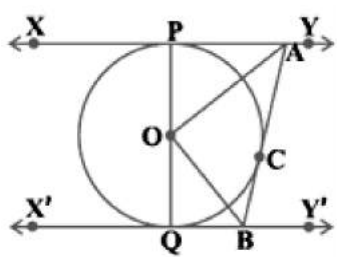


Fig. 10.13

Answer: Let us join point O to C .

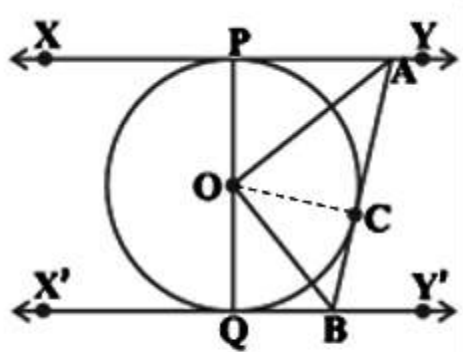


Fig. 10.13

In $\triangle OPA$ and $\triangle OCA$,

$OP = OC$ (Radius of the same circle)

$AP = AC$ (Tangents from point A)

$AO = AO$ (Common side)

$\triangle OPA \cong \triangle OCA$ (SSS congruence criterion)

$\angle POA = \angle COA \dots (i)$

Similarly, $\triangle OQB \cong \triangle OCB$

$\angle QOB = \angle COB \dots (ii)$

Since POQ is a diameter of the circle, it is a straight line.

Therefore, $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

From equations (i) and (ii), it can be observed that $2\angle COA + 2\angle COB = 180^\circ$

$\angle COA + \angle COB = 180^\circ / 2$

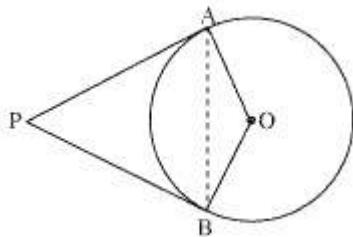
$\angle COA + \angle COB = 90^\circ$

$\angle AOB = 90^\circ$

Hence Proved.

Q. 10 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Answer:



To Prove: $\angle APB + \angle BOA = 180^\circ$

Proof: Let us consider a circle centred at point O.

Let P be an external point from which two tangents PA and PB are drawn to the circle which touches the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends

$\angle AOB$ at centre O of the circle.

It can be observed that

$OA \perp PA$ (radius of circle is always perpendicular to tangent)

Therefore, $\angle OAP = 90^\circ$

Similarly, $OB \perp PB$

$\angle OBP = 90^\circ$

In quadrilateral OAPB,

Sum of all interior angles = 360°

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$

$90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ$

$\angle APB + \angle BOA = 180^\circ$

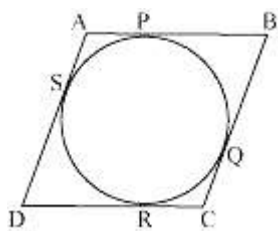
Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Q. 11 Prove that the parallelogram circumscribing a circle is a rhombus.

Answer: Since ABCD is a parallelogram,

$AB = CD$... (1)

$BC = AD$... (2)



It can be observed that

$DR = DS$ (Tangents on the circle from point D)

$CR = CQ$ (Tangents on the circle from point C)

$BP = BQ$ (Tangents on the circle from point B)

$AP = AS$ (Tangents on the circle from point A)

Adding all these equations, we obtain

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC$$

On putting the values of equations (1) and (2) in this equation, we obtain $2AB = 2BC$

$$AB = BC \dots(3)$$

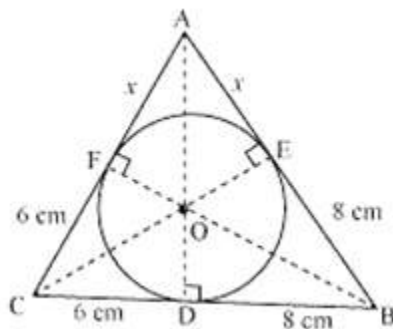
Comparing equations (1), (2), and (3), we obtain

$$AB = BC = CD = DA$$

Hence, ABCD is a rhombus.

Q. 12 A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

Answer:



To Find: AB and AC

Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be x .

In $\triangle ABC$,

From the theorem which states that the lengths of two tangents drawn from an external point to a circle are equal.

So,

$$CF = CD = 6\text{cm (Tangents on the circle from point C)}$$

$$BE = BD = 8\text{cm (Tangents on the circle from point B)}$$

$$AE = AF = x \text{ (Tangents on the circle from point A)}$$

$$AB = AE + EB = x + 8$$

$$BC = BD + DC = 8 + 6 = 14$$

$$CA = CF + FA = 6 + x$$

By heron's formula

$$\text{area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

where, a , b and c are sides of triangle and s is semi perimeter.

$$2S = AB + BC + CA$$

$$= x + 8 + 14 + 6 + x$$

$$= 28 + 2x$$

$$S = \frac{28+2x}{2} = 14 + x$$

$$\text{area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\{14+x\}\{x\}(8)(6)}$$

$$= 4\sqrt{3(14x+x^2)}$$

$$\text{Also, area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

And from the given figure it is clear that Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$\text{Area of } \Delta OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$$

$$\text{Area of } \Delta OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6 + x) = 12 + 2x$$

$$\text{Area of } \Delta OCA = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8 + x) = 16 + 2x$$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$\Rightarrow 4\sqrt{3(14x + x^2)} = 28 + 12 + 2x + 16 + 2x$$

$$\Rightarrow 4\sqrt{3(14x + x^2)} = 56 + 4x$$

$$= \sqrt{3(14x + x^2)} = 14 + x$$

$$\Rightarrow 3x(14 + x) = (14 + x)^2$$

$$\Rightarrow 3x = 14 + x$$

$$\Rightarrow 2x = 14$$

$$\Rightarrow x = 7$$

Therefore, $x = 7$

Hence,

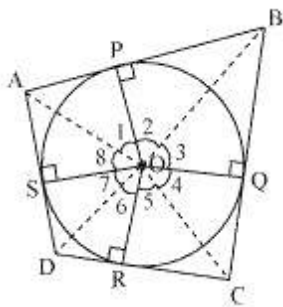
$$AB = x + 8 = 7 + 8 = 15 \text{ cm}$$

$$CA = 6 + x = 6 + 7 = 13 \text{ cm}$$

Hence, $AB = 15 \text{ cm}$ and $AC = 13 \text{ cm}$

Q. 13 Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer:



To Prove: $\angle AOB + \angle COD = 180^\circ$, $\angle BOC + \angle DOA = 180^\circ$

Given: ABCD is circumscribing the circle.

Proof: Let ABCD be a quadrilateral circumscribing a circle centred at O such that it touches the circle at point P, Q, R, S.

Join the vertices of the quadrilateral ABCD to the centre of the circle.

Consider $\triangle OAP$ and $\triangle OAS$,

$AP = AS$ (Tangents from the same point)

$OP = OS$ (Radii of the same circle)

$OA = OA$ (Common side)

$\triangle OAP \cong \triangle OAS$ (SSS congruence criterion)

And thus, $\angle POA = \angle AOS$

$\angle 1 = \angle 8$ Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^\circ$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.