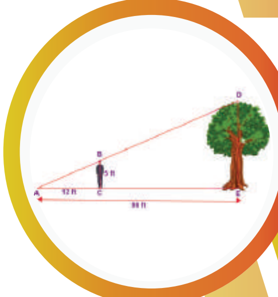
5 GEOMETRY



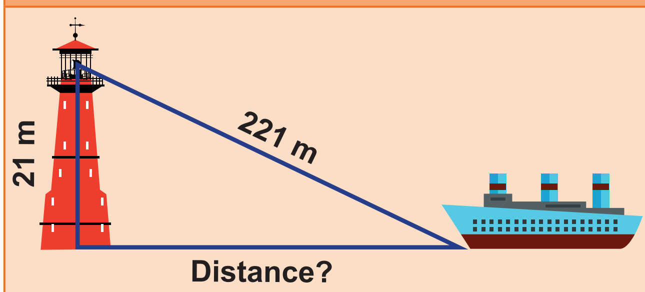
Learning Objectives

- To recall the similar and congruence properties and also the basic properties of triangles.
- To understand the theorems based on these properties and apply them appropriately to problems.
- To understand the Pythagorean theorem and solve problems using it.
- To know and understand the concurrency of medians, altitudes, angle bisectors and perpendicular bisectors in a triangle.
- To construct quadrilaterals of various types.

K5E9W5

5.1 Introduction

Geometry, as we all know studies shapes by looking at the properties and relations of points, circles, triangles of two dimensions and solids. In the earlier classes, we have seen a few properties of triangles. In this class, we are going to recall them and also learn the congruence and the similarity properties in triangles. Also, we shall learn about the Pythagorean theorem and the concurrency of medians, altitudes, angle bisectors and perpendicular bisectors in a triangle. Also, we will also see how to construct quadrilaterals of various types.

MATHEMATICS ALIVE – GEOMETRY IN REAL LIFE	
	
The Pythagoras theorem is useful in finding the distance and the heights of objects.	For better strength and stability, congruent triangles in the construction of buildings.

Answer the following questions by recalling the properties of triangles:

1. The sum of the three angles of a triangle is ______________ .
2. The exterior angle of a triangle is equal to the sum of the _____________ angles to it.
3. In a triangle, the sum of any two sides is _____________ than the third side.
4. Angles opposite to equal sides are _______ and vice – versa.
5. What is $\angle A$ in the triangle ABC?

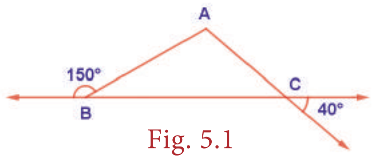
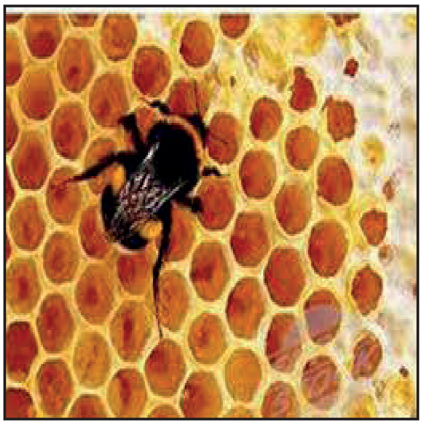
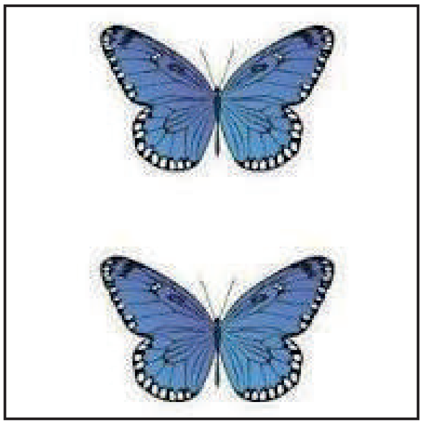
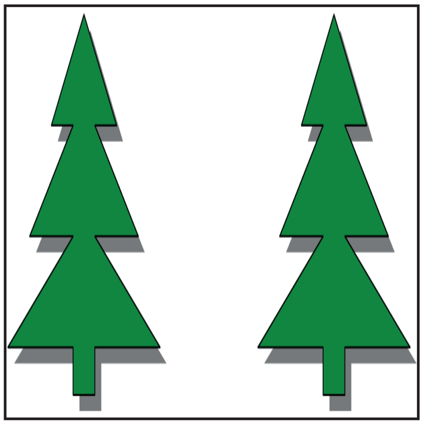


Fig. 5.1

5.2 Congruent and Similar Shapes

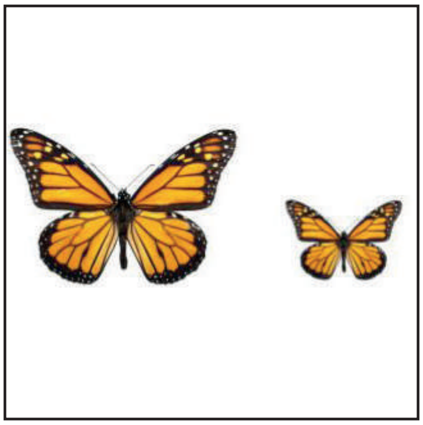
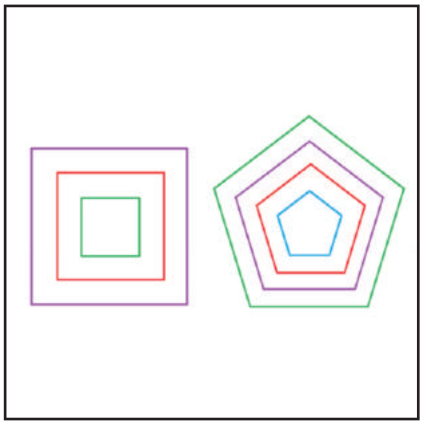
Congruent figures are exactly the same in shape and size. In other words, shapes are congruent if one fits exactly over the other.

Examples

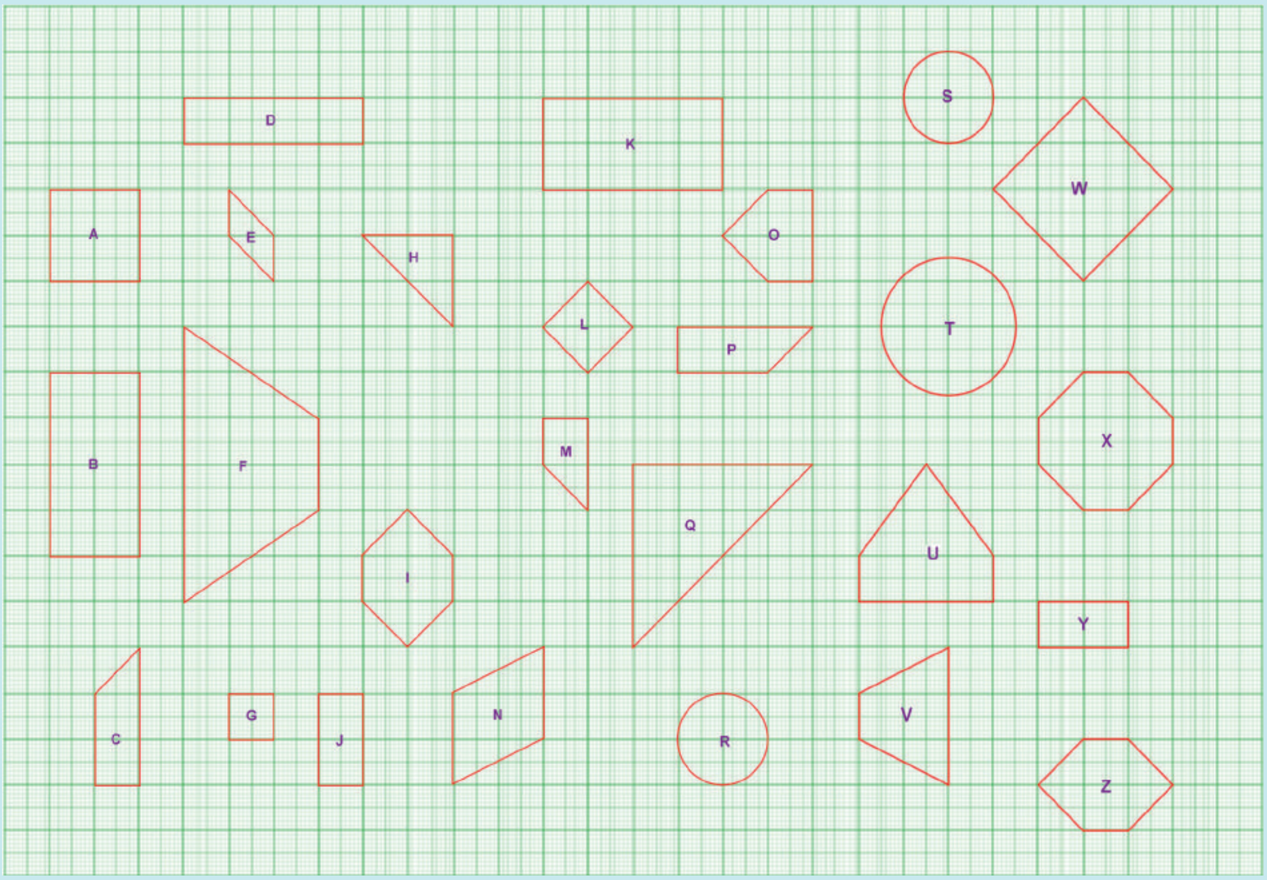
Similar figures mathematically have the same shape but different sizes. Two geometrical figures are said to be similar $(\sim)$ if the measures of one to the corresponding measures of the other are in a constant ratio. In other words, every part of a photographic enlargement is similar to the corresponding part of the original.

Examples

Try these

Identify the pairs of figures which are similar and congruent and write the letter pairs.



5.2.1 Congruent Triangles

Consider two given triangles PQR and ABC. They are said to be congruent (≡) if their corresponding parts are congruent. That is PQ=AB, QR=BC and PR=AC and also $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$. This is denoted as $\Delta PQR \equiv \Delta ABC$.

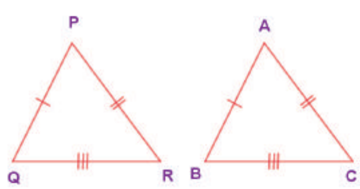


Fig. 5.2

There are 4 ways by which one can prove that two triangles are congruent.

(i) SSS (Side – Side – Side) Congruence

If the three sides of a triangle are congruent to the three sides of another triangle, then the triangles are congruent. That is $AB = PQ$, $BC = QR$ and $AC = PR$

$\Rightarrow \Delta ABC \equiv \Delta PQR.$

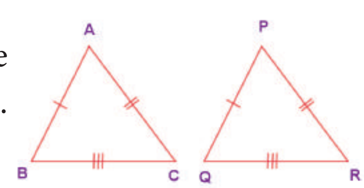


Fig. 5.3

(ii) SAS (Side – Angle – Side) Congruence

If two sides and the included angle (the angle between them) of a triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. Here, $AC = PQ$, $\angle A = \angle P$ and $AB = PR$ and hence $\Delta ACB \equiv \Delta PQR$.

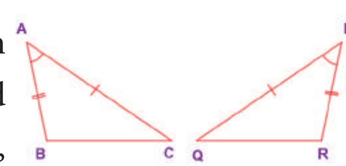


Fig. 5.4

(iii) ASA (Angle-Side-Angle) Congruence

If two angles and the included side of a triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. Here, $\angle A = \angle R$, CA=PR and $\angle C = \angle P$ and hence $\Delta ABC \equiv \Delta RQP$

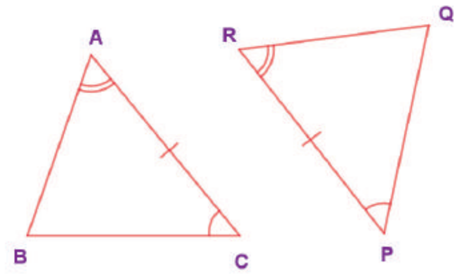


Fig. 5.5

(iv) RHS (Right Angle – Hypotenuse – Side) Congruence

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. Here, $\angle B = \angle Q = 90°$, $BC = QR$ and $AC = PR$
(right angle) (leg) (hypotenuse)

and hence $\Delta ABC \equiv \Delta PQR$.

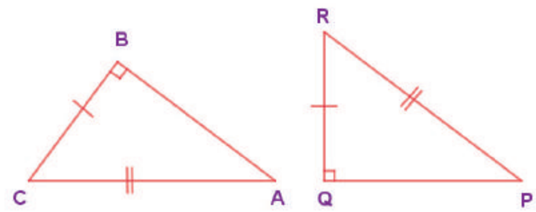


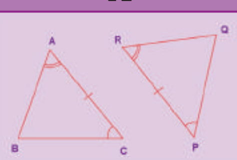
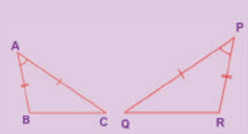
Fig. 5.6

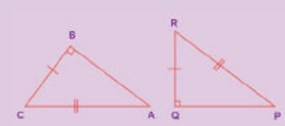
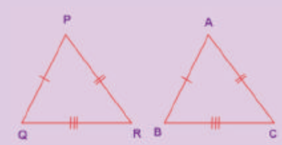
Note

- Any segment or angle is congruent to itself! This is called Reflexive property
- If two triangles are congruent, then their corresponding parts are congruent. This is called **CPCTC (Corresponding parts of Congruent Triangles are Congruent).**
- *If angles then sides* means if two angles are equal in a triangle, then the sides opposite to them are equal.
- *If sides then angles* means if two sides are equal in a triangle, then the angles opposite to them are equal.

Try these

Match the following by their congruence property

S.No.	A	B
1		RHS
2		SSS

S.No.	A	B
3		SAS
4		ASA

Example 5.1

Find the unknowns in the following figures

(i)

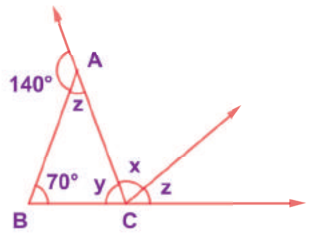


Fig. 5.7(i)

(ii)

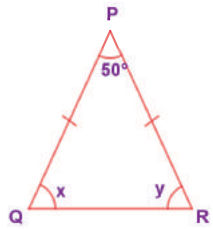


Fig. 5.7(ii)

(iii)

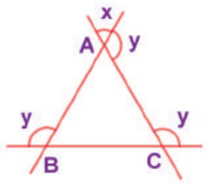


Fig. 5.7(iii)

Solution:

(i) Now, from Fig. 5.7(i), $140^\circ + \angle z = 180^\circ$ (linear pair)

$\Rightarrow \angle z = 180^\circ - 140^\circ = 40^\circ$

Also $\angle x + \angle z = 70^\circ + \angle z$ (exterior angle property)

$\Rightarrow \angle x = 70^\circ$

Also $\angle z + \angle y + 70^\circ = 180^\circ$ (angle sum property in Δ ABC)

$\Rightarrow 40^\circ + \angle y + 70^\circ = 180^\circ$

$\Rightarrow \angle y = 180^\circ - 110^\circ = 70^\circ$

(ii) Now, from Fig. 5.7(ii), $PQ = PR$

$\Rightarrow \angle Q = \angle R$ (angles opposite to equal sides are equal)

$\Rightarrow \angle x = \angle y$

$\Rightarrow \angle x + \angle y + 50^\circ = 180^\circ$ (angle sum property in ΔPQR)

$\Rightarrow 2\angle x = 130^\circ$

$\Rightarrow \angle x = 65^\circ$

$\Rightarrow \angle y = 65^\circ$

(iii) Now, from Fig. 5.7(iii), in ΔABC $\angle A = x$ (vertically opposite angles)

Similarly $\angle B = \angle C = x$ (Why?)

$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$ (angle sum property in ΔABC)

$\Rightarrow 3x = 180^\circ$

$\Rightarrow x = 60^\circ$

$\Rightarrow y = 180^\circ - 60^\circ = 120^\circ$

Example 5.2 (Illustrating SSS and SAS Congruence)

If $\angle E = \angle S$ and G is the midpoint of ES,

prove that $\Delta GET \equiv \Delta GST$.

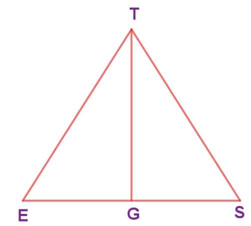
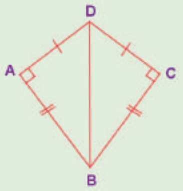


Fig. 5.8

Proof:

	Statements	Reasons
1	$\angle E \equiv \angle S$	given
2	$ET \equiv ST$	if angles, then sides
3	G is the midpoint of ES	given
4	$EG \equiv SG$	follows from 3
5	$TG \equiv TG$	reflexive property
6	$\Delta GET \equiv \Delta GST$	by SSS (2,4,5) & also by SAS (2,1,4)

Think



In the figure, $DA = DC$ and $BA = BC$.
Are the triangles DBA and DBC congruent?
Why?

Example 5.3 (Illustrating ASA Congruence)

If $\angle YTB \equiv \angle YBT$ and $\angle BOY \equiv \angle TRY$,

prove that $\Delta BOY \equiv \Delta TRY$

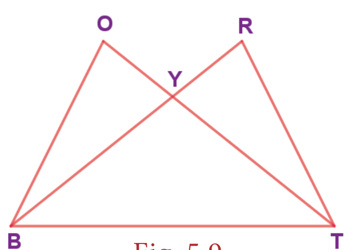


Fig. 5.9

Proof:

	Statements	Reasons
1	$\angle BYO \equiv \angle TYR$	vertical angles are congruent
2	$\angle YTB \equiv \angle YBT$	given
3	$BY \equiv TY$	if angles, then sides
4	$\angle BOY \equiv \angle TRY$	given
5	$\angle OBY \equiv \angle RTY$	follows from 1 and 4
6	$\Delta BOY \equiv \Delta$TRY	by ASA (1,3,5)

Example 5.4 (Illustrating RHS Congruence)

If TAP is an isosceles triangle with $TA = TP$ and $\angle TSA = 90°$.

(i) Is $\Delta TAS \equiv \Delta TPS$? Why?

(ii) Is $\angle P = \angle A$? Why?

(iii) Is $AS = PS$? Why?

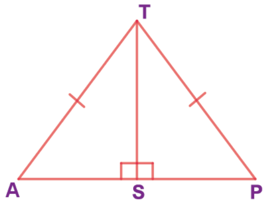


Fig. 5.10

Proof:

(i) TA = TP (hypotenuse) and $\angle TSA = 90$

TS is common (leg)

Hence, by RHS congruence, $\triangle TAS \equiv \triangle TPS$

(ii) Given TA = TP

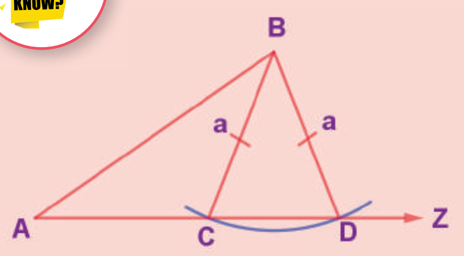
$\therefore \quad \angle P = \angle A$ (if angles then sides)

(iii) From (i) $\triangle TAS \equiv \triangle TPS$,

By CPCTC

$AS = PS$

DO YOU KNOW?



SSA and ASS properties are not sufficient to prove that two triangles are congruent. This is explained in the given figure. By construction, in triangles ABD and ABC, BC = BD = a. Also, AB and ∠BAZ are common. But AC ≠ AD. So, ΔABD is not congruent to ΔABC and so SSA fails.

5.2.2 Similar Triangles

Consider two given triangles PQR and ABC. They are said to be similar ($\sim$) if their corresponding angles are equal and corresponding sides are proportional. That is $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$ and also $\frac{PQ}{AB} = \frac{PR}{AC} = \frac{QR}{BC}$. This is denoted as $\Delta PQR \sim \Delta ABC$.

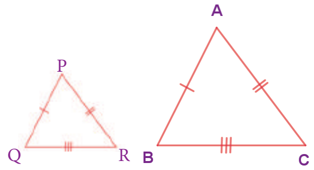


Fig. 5.11

There are 4 ways by which one can prove that two triangles are similar.

(i) AAA (Angle – Angle – Angle) or AA (Angle – Angle) Similarity

Two triangles are similar if two angles of one triangle are equal respectively to two angles of the other triangle. In the given figure, $\angle A = \angle P$, $\angle B = \angle Q$. Therefore, ΔABC ~ ΔPQR.

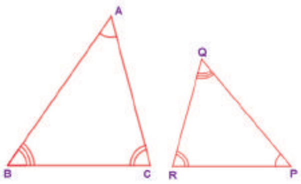


Fig. 5.12

(ii) SAS (Side – Angle – Side) Similarity

Two triangles are similar if two sides of one triangle are proportional to two sides of the other triangle and the included angles are equal. In the given figure, $\frac{AC}{PQ} = \frac{AB}{PR}$ and $\angle A = \angle P$ and hence ΔACB ~ ΔPQR.

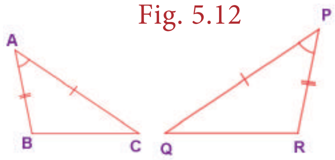


Fig. 5.13

(iii) SSS (Side-Side-Side) Similarity

Two triangles are similar if their corresponding sides are in the same ratio. That is, if $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$, then $\Delta ABC \sim \Delta PQR$.

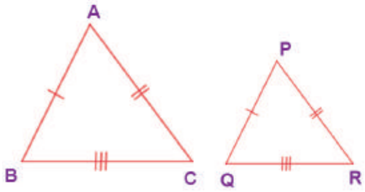


Fig. 5.14

(iv) RHS (Right Angle – Hypotenuse – Side) Similarity

Two right triangles are similar if the hypotenuse and a leg of one triangle are respectively proportional to the hypotenuse and a leg of the other triangle. That is, if $\angle B=\angle Q=90^\circ$ and $\frac{AC}{PR}=\frac{BC}{QR}$ then, $\Delta ABC \sim \Delta PQR$.

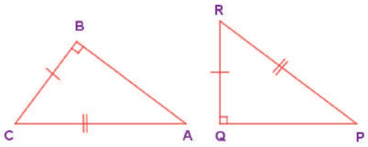
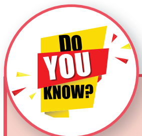
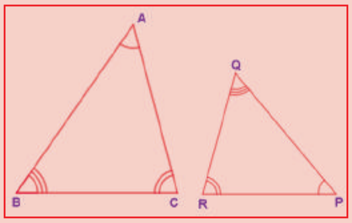


Fig. 5.15



If $\Delta ABC \sim \Delta PQR$, then the corresponding sides to AB, BC and AC of ΔABC are PQ, QR and PR respectively and the corresponding angles to A, B and C are P, Q and R respectively. Naming a triangle has a significance. For example, if $\Delta ABC \sim \Delta PQR$ then, ΔBAC is not similar to ΔPQR.



Example 5.5

In the Fig. 5.16, if $\Delta PQR \sim \Delta XYZ$, find a and b.

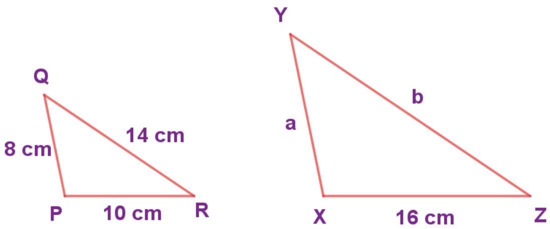


Fig. 5.16

Solution:

Given that $\Delta PQR \sim \Delta XYZ$

$\therefore$ Their corresponding sides are proportional.

$$\Rightarrow \frac{PQ}{XY}=\frac{QR}{YZ}=\frac{PR}{XZ}$$

$$\Rightarrow \frac{8}{a}=\frac{14}{b}=\frac{10}{16}$$

$$\Rightarrow \frac{8}{a}=\frac{10}{16}$$

$$\Rightarrow a=\frac{8\times 16}{10}=\frac{128}{10}$$

$$a = 12.8\ cm$$

Also, $$\frac{14}{b}=\frac{10}{16}$$

$$\Rightarrow b=\frac{14\times 16}{10}=\frac{224}{10}$$

$$\therefore\ b=22.4\ cm$$

Note

- All circles and squares are similar to each other.
- Not all rectangles need to be similar always.
- If two angles are both congruent and supplementary then, they are right angles.
- All congruent triangles are similar.
- The symbol $\sim$ is used to denote similarity.

Example 5.6 (Illustrating AA Similarity)

In the Fig. 5.17 , $\angle ABC \equiv \angle EDC$ and the perimeter of ΔCDE is 27 *units*, prove that $AB \equiv EC$.

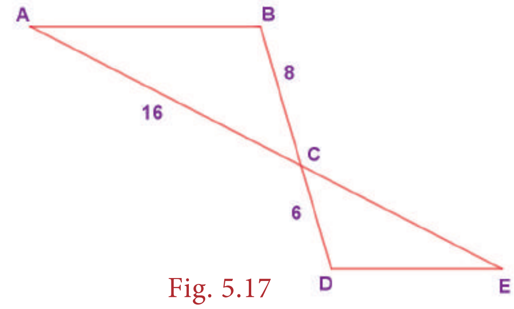


Fig. 5.17

Proof:

	Statements	Reasons
1	$\angle ABC \equiv \angle EDC$	given
2	$\angle BCA \equiv \angle DCE$	vertically opposite angles are equal
3	$\Delta ABC \sim \Delta EDC$	by AA property (1, 2)
4	$\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$	corresponding sides are proportional by 3

$$\Rightarrow \frac{8}{6} = \frac{16}{EC} \Rightarrow EC = 12 \ units$$

Given, the perimeter of $\Delta CDE = 27$ *units*,

$\therefore$ $ED + DC + EC = 27 \Rightarrow ED + 6 + 12 = 27 \Rightarrow ED = 27 - 18 = 9$ *units*

$\therefore$ $\frac{AB}{9} = \frac{8}{6} \Rightarrow AB = 12$ units and hence $AB = EC$.

Example 5.7 (Illustrating AA Similarity)

In the given Fig. 5.18,

if $\angle 1 \equiv \angle 3$ and $\angle 2 \equiv \angle 4$ then,

prove that $\Delta BIG \sim \Delta FAT$. Also find FA.

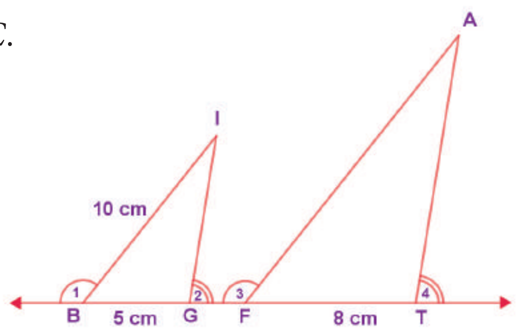


Fig. 5.18

Proof:

	Statements	Reasons
1	$\angle 1 \equiv \angle 3$	given
2	$\angle IBG \equiv \angle AFT$	supplements of congruent angles are congruent.
3	$\angle 2 \equiv \angle 4$	given
4	$\angle IGB \equiv \angle ATF$	supplement of congruent angles are congruent
5	$\Delta BIG \sim \Delta FAT$	by AA property (2, 4)

Also, their corresponding sides are proportional

$$\Rightarrow \frac{BI}{FA} = \frac{BG}{FT} \Rightarrow \frac{10}{FA} = \frac{5}{8} \Rightarrow FA = \frac{10 \times 8}{5} = \frac{80}{5} = 16\,cm$$

Example 5.8 (Illustrating SAS Similarity)

If A is the midpoint of RU and T is the midpoint of RN, prove that $\Delta RAT \sim \Delta RUN$.

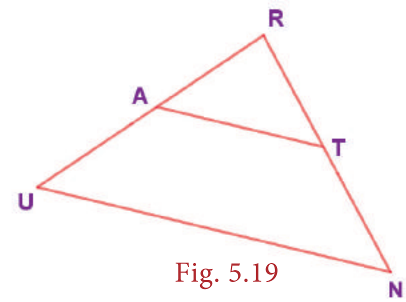


Fig. 5.19

Proof:

	Statements	Reasons
1	$\angle ART = \angle URN$	$\angle R$ is common in ΔRAT and ΔRUN
2	$RA = AU = \frac{1}{2}RU$	A is the midpoint of RU
3	$RT = TN = \frac{1}{2}RN$	T is the midpoint of RN
4	$\frac{RA}{RU} = \frac{RT}{RN} = \frac{1}{2}$	the sides are proportional from 2 and 3
5	$\Delta RAT \sim \Delta RUN$	by SAS (1 and 4)

Example 5.9 (Illustrating SSS similarity)

Prove that $\Delta PQR \sim \Delta PRS$ in the given Fig. 5.20.

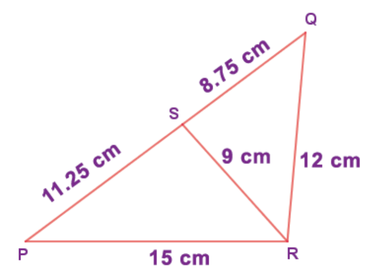
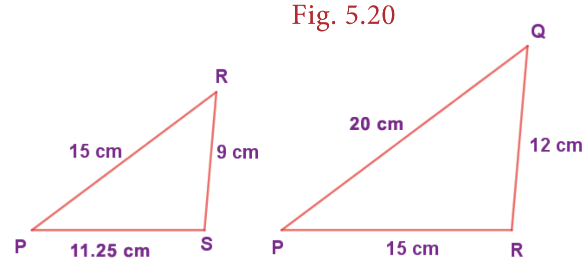


Fig. 5.20



Solution:

Now, $\frac{PQ}{PR} = \frac{20}{15} = \frac{4}{3}$

$\frac{PR}{PS} = \frac{15}{11.25} = \frac{4}{3}$

Also, $\frac{QR}{RS} = \frac{12}{9} = \frac{4}{3}$

We find $\frac{PQ}{PR} = \frac{PR}{PS} = \frac{QR}{RS}$

That is, their corresponding sides are proportional.

$\therefore$ By SSS Similarity, $\Delta PQR \sim \Delta PRS$

Example 5.10 (Illustrating RHS similarity)

The height of a man and his shadow form a triangle similar to that formed by a nearby tree and its shadow. What is the height of the tree?

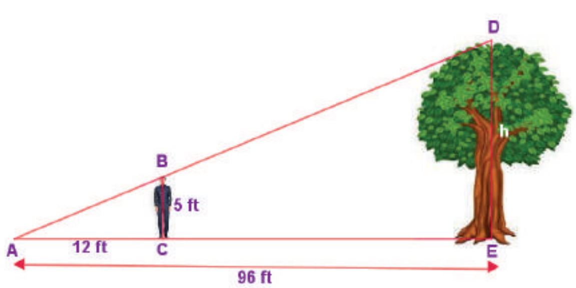


Fig. 5.21

Solution:

Here, $\Delta ABC \sim \Delta ADE$ (given)

$\therefore$ Their corresponding sides are proportional (by RHS similarity).

$$\therefore \quad \frac{AC}{AE} = \frac{BC}{DE}$$

$$\Rightarrow \frac{12}{96} = \frac{5}{h}$$

$$\Rightarrow h = \frac{5 \times 96}{12} = 40 \text{ } feet$$

$\therefore$ The height of the tree is 40 *feet*.

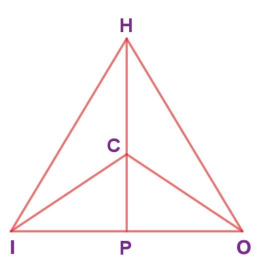
Activity

The teacher cuts many triangles that are similar or congruent from a card board (or) chart sheet. The students are asked to find which pair of triangles are similar or congruent based on the measures indicated in the triangles.

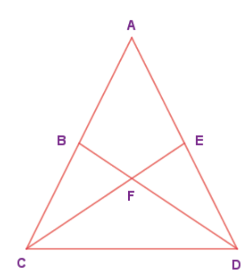
Exercise 5.1

1. Fill in the blanks with the correct term from the given list.
   (in proportion, similar, corresponding, congruent, shape, area, equal)
   (i) Corresponding sides of similar triangles are ________.
   (ii) Similar triangles have the same __________ but not necessarily the same size.
   (iii) In any triangle _______ sides are opposite to equal angles.
   (iv) The symbol $\equiv$ is used to represent ________ triangles.
   (v) The symbol $\sim$ is used to represent _________ triangles.

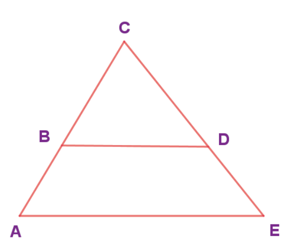
2. In the given figure, $\angle CIP \equiv \angle COP$ and $\angle HIP \equiv \angle HOP$. Prove that $IP \equiv OP$.



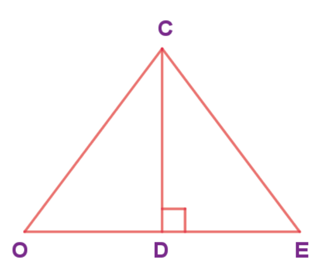
3. In the given figure, $AC \equiv AD$ and $\angle CBD \equiv \angle DEC$. Prove that $\Delta BCF \equiv \Delta EDF$.



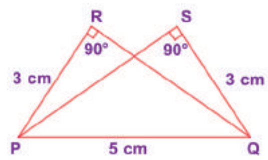
4. In the given figure, ΔBCD is isosceles with base BD and $\angle BAE \equiv \angle DEA$. Prove that $AB \equiv ED$.



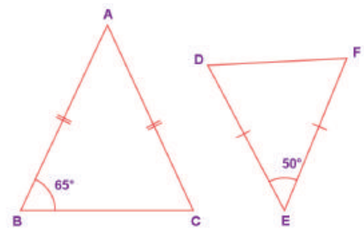
5. In the given figure, D is the midpoint of OE and $\angle CDE = 90°$. Prove that $\Delta ODC \equiv \Delta EDC$



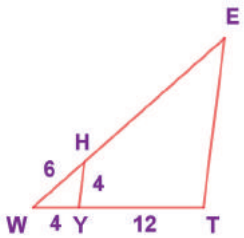
6. Is $\triangle PRQ \equiv \triangle QSP$? Why?



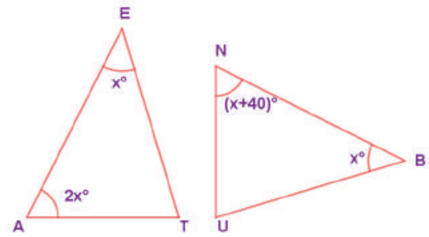
7. From the given figure, prove that $\Delta ABC \sim \Delta EDF$



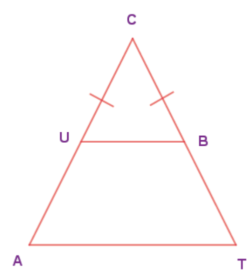
8. In the given figure $YH \parallel TE$. Prove that $\Delta WHY \sim \Delta WET$ and also find HE and TE.



9. In the given figure, if $\Delta EAT \sim \Delta BUN$, find the measure of all angles.



10. In the given figure, $UB \parallel AT$ and $CU \equiv CB$ Prove that $\Delta CUB \sim \Delta CAT$ and hence ΔCAT is isosceles.



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Objective Type Questions

11. Two similar triangles will always have _________angles

(A) acute (B) obtuse (C) right (D) matching

12. If in triangles PQR and XYZ, $\frac{PQ}{XY} = \frac{QR}{YZ}$ then they will be similar if

(A) $\angle Q = \angle Y$ (B) $\angle P = \angle Y$ (C) $\angle Q = \angle X$ (D) $\angle P = \angle Z$

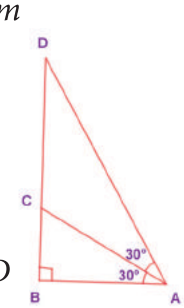
13. A flag pole 15 m high casts a shadow of 3 m at 10 a.m. The shadow cast by a building at the same time is 18.6 m. The height of the building is

(A) 90 m (B) 91 m (C) 92 m (D) 93 m

14. If $\Delta ABC \sim \Delta PQR$ in which $\angle A = 53^\circ$ and $\angle Q = 77^\circ$, then $\angle R$ is

(A) 50° (B) 60° (C) 70° (D) 80°

15. In the figure, which of the following statements is true?



(A) $AB = BD$ (B) $BD < CD$ (C) $AC = CD$ (D) $BC = CD$

5.3 The Pythagorean Theorem

The Pythagorean theorem or simply Pythagoras theorem, named after the ancient Greek Mathematician Pythagoras BC 570-495 (BCE) is one of the most famous and celebrated theorems in Mathematics. People have proved this theorem in numerous ways possibly the most for any mathematical theorem. The proofs are very diverse which include both geometric and algebraic methods dating back to thousands of years.

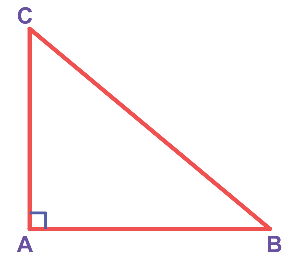


Fig. 5.22

Statement of the theorem

In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

In $\varnothing ABC$, $BC^2 = AB^2 + AC^2$

Visual Illustraton:

The given figure contains a triangle of sides of measures 3 units, 4 units and 5 units. From this well known $3-4-5$ triangle, one can easily visualise and understand the meaning of the Pythagorean theorem.

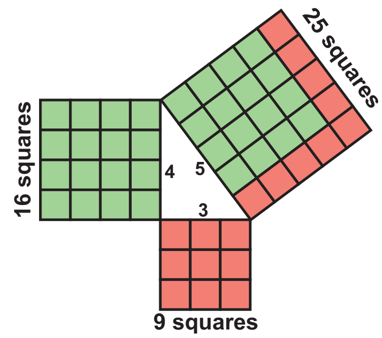


Fig. 5.23

In the figure, the sides of measure 3 units and 4 units are called the legs or sides of the right angled triangle. The side of measure 5 units is called the hypotenuse. Recall that the hypotenuse is the greatest side in a right angled triangle

Now, it is easily seen that a square formed with side 5 units (hypotenuse) has $5 \times 5 = 25$ unit squares (small squares) and the squares formed with side 3 units and 4 units have $3\times3=9$ unit squares and $4 \times 4 = 16$ unit squares respectively. As per the statement of the theorem, the number of unit squares on the hypotenuse is exactly the sum of the unit squares on the other two legs (sides) of the right angled triangle. Isn't this amazing?

Yes, we find that

$5\times5 = 3\times3+4\times4$

i.e. $25 = 9+16$ (True)

M3J7F1

5.4 Converse of Pythagoras Theorem

If in a triangle, the square on the greatest side is equal to the sum of squares on the other two sides, then the triangle is right angled triangle.

Example:

In the triangle ABC,

$AB^2 + AC^2 = 11^2 + 60^2 = 3721 = 61^2 = BC^2$

Hence, ΔABC is a right angled triangle.

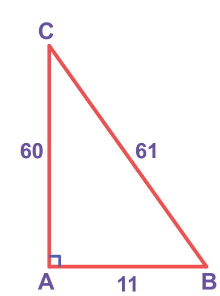


Fig. 5.24

DO YOU KNOW?

(i) There are special sets of numbers a, b and c that makes the Pythagorean relationship true and these sets of special numbers are called Pythagorean triplets. Example: (3, 4, 5) is a Pythagorean triplet.

(ii) Let **k** be any positive integer greater than 1 and (a, b, c) be a Pythagorean triplet, then (ka, kb, kc) is also a Pythagorean triplet.

Examples:

k	**(3,4,5)**	**(5,12,13)**
2k	(6,8,10)	(10,24,26)
3k	(9,12,15)	(15,36,39)
4k	(12,16,20)	(20,48,52)

So, it is clear that we can have infinitely many Pythagorean triplets just by multiplying any Pythagorean triplet by **k.**

We shall now see a few examples on the use of Pythagoras theorem in problems.

Example 5.11

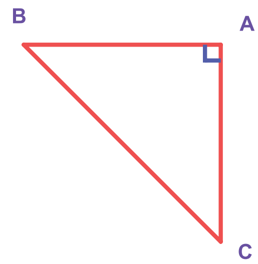


Fig. 5.25

In the figure, $AB \perp AC$

a) What type of ∅ is ABC?

b) What are AB and AC of the ΔABC ?

c) What is CB called as ?

d) If $AC = AB$ then, what is the measure of $\angle B$ and $\angle C$?

Solution:

a) ΔABC is right angled as $AB \perp AC$ at A.

b) AB and AC are legs of ΔABC .

c) CB is called as the hypotenuse.

d) $\angle B + \angle C = 90^\circ$ and equal angles are opposite to equal sides. Hence, $\angle B = \angle C = \dfrac{90^\circ}{2} = 45^\circ$

Example 5.12

Can a right triangle have sides that measure 5*cm*, 12*cm* and 13*cm*?

Solution:

Take $a = 5$, $b = 12$ and $c = 13$

Now, $a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169 = 13^2 = c^2$

By the converse of Pythagoras theorem, the triangle with given measures is a right angled triangle.

Example 5.13

A 20-feet ladder leans against a wall at height of 16 feet from the ground. How far is the base of the ladder from the wall?

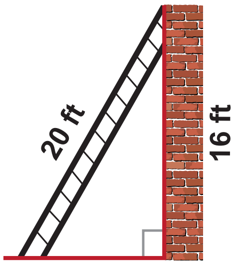


Fig. 5.26

Solution:

The ladder, wall and the ground form a right triangle with the ladder as the hypotenuse. From the figure, by Pythagoras theorem,

$$20^2 = 16^2 + x^2$$

$$\Rightarrow 400 = 256 + x^2$$

$$\Rightarrow \quad x^2 = 400 - 256 = 144 = 12^2 \Rightarrow \quad x = 12 \text{ feet}$$

Therefore, the base (foot) of the ladder is 12 feet away from the wall.

Activity

1. We can construct sets of Pythagorean triplets as follows.
   Let **m** and **n** be any two positive integers (m > n):
   (a, b, c) is a Pythagorean triplet if $a = m^2 - n^2, b = 2mn$ and $c = m^2 + n^2$ (Think, why?)

Complete the table.

m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$	Pythagorean triplet
2	1	___	___	___	______
3	2	___	___	___	______
4	1	15	8	17	(15, 8, 17)
7	2	45	28	53	(45, 28, 53)

2. Find all integer-sided right angled triangles with hypotenuse 85.

Example 5.14

Find LM, MN, LN and also the area of ΔLON.

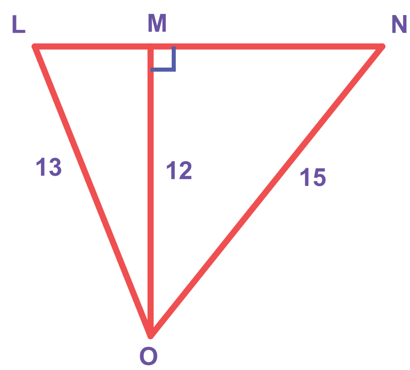


Fig. 5.27

Solution:

From ΔLMO, by Pythagoras theorem,

$$LM^2 = OL^2 - OM^2$$

$$\Rightarrow LM^2 = 13^2 - 12^2 = 169 - 144 = 25 = 5^2$$

$$\therefore \quad LM = 5 \text{ units}$$

From ΔNMO, by Pythagoras theorem,

$$MN^2 = ON^2 - OM^2$$

$$= 15^2 - 12^2 = 225 - 144 = 81 = 9^2$$

$$\therefore \quad MN = 9 \text{ units}$$

Hence, $LN = LM + MN = 5 + 9 = 14$ *units*

$$\text{Area of } \Delta LON = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times LN \times OM$$

$$= \frac{1}{2} \times 14 \times 12 = 84 \text{ square units.}$$

Example 5.15

ΔABC is equilateral and CD of the right triangle BCD is 8 *cm*. Find the side of the equilateral ΔABC and also BD.

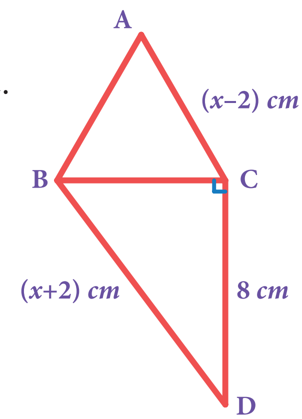


Fig. 5.28

Solution:

As ΔABC is equilateral from the figure, AB=BC=AC= (*x*–2) *cm*.

$\therefore$ From ΔBCD, by Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

$$\Rightarrow (x+2)^2 = (x-2)^2 + 8^2$$

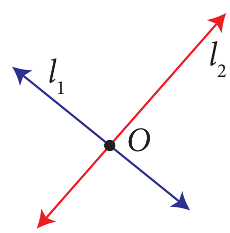
$$x^2 + 4x + 4 = x^2 - 4x + 4 + 8^2$$

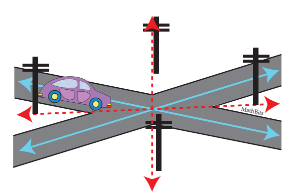
$$\Rightarrow 8x = 8^2$$

$$\Rightarrow x = 8\,cm$$

$\therefore$ The side of the equilateral $\Delta ABC = 6$ *cm* and BD =10 *cm*.

5.5 Point of Concurrency

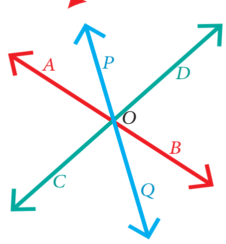


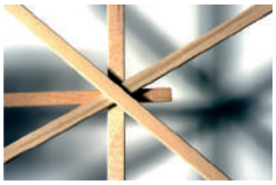


Intersecting Roads

When two lines in a plane cross each other, they are called *intersecting lines*. Here, lines l_1 and l_2 intersect at point O and hence it is called the *point of intersection* of l_1 and l_2.

Three or more lines in a plane are said to be concurrent, if all of them pass through the same point.





Concurring wooden pieces.

In this figure, $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{PQ}$ are concurrent lines and *O* is the *point of concurrency*.

5.6 Medians of a Triangle

A median of a triangle is a line segment from a vertex to the midpoint of the side opposite that vertex.

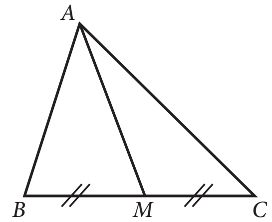


Fig. 5.29

In the figure $\overline{AM}$ is a median of ΔABC.

Are there any more medians for ΔABC ? Yes, since there are three vertices in a triangle, one can identify three medians in a triangle.

Example 5.16

In the figure, ABC is a triangle and AM is one of its medians. If BM = 3.5 cm, find the length of the side BC.

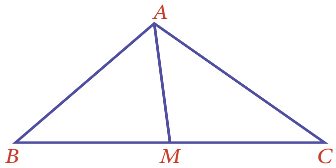


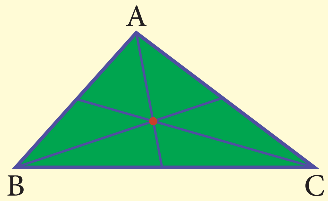
Fig. 5.30

Solution:

AM is median $\Rightarrow$ *M* is the midpoint of *BC*.

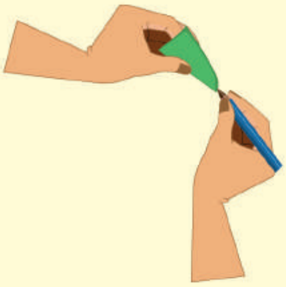
Given that, *BM* = 3.5 cm, hence *BC* = twice the length *BM* = 2 × 3.5 cm = 7 cm.

Activity

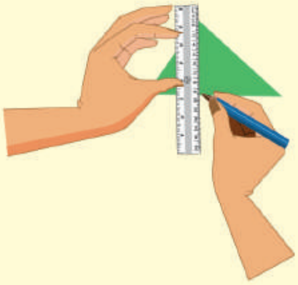


1. Consider a paper cut-out of a triangle. (Let us have an acute-angled triangle, to start with). Name it, say ABC.

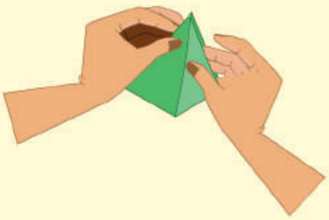
2. Fold the paper along the line that passes through the point A and meets the line BC such that point B falls on C. Make a crease and unfold the sheet.



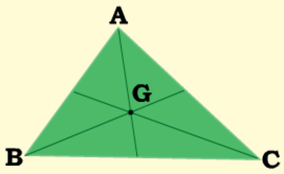
3. Mark the mid point M of BC.



4. You can now draw the median AM, if you want to see it clearly. (Or you can leave it as a fold).



5. In the same way, fold and draw the other two medians.



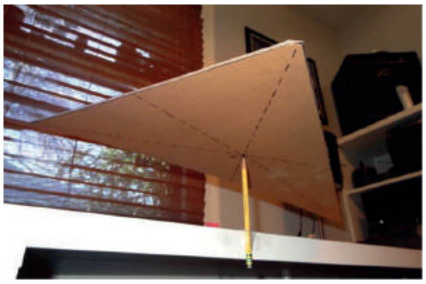
6. Do the medians pass through the same point?

Now you can repeat this activity for an obtuse-angled triangle and a right triangle. What is the conclusion? We see that,

The three medians of *any* triangle are concurrent.

5.6.1 Centroid

The point of concurrence of the three medians in a triangle is called its **Centroid**, denoted by the letter **G**. Interestingly, it happens to be the centre of mass of the triangle. One can easily verify this fact. Take a stiff cut out of triangle of paper. It can be balanced horizontally at this point on a finger tip or a pencil tip.



Should you fold all the three medians to find the centroid? Now you can explore among yourself the following questions:

(i) How can you find the centroid of a triangle?
(ii) Is the centroid equidistant from the vertices?
(iii) Is the centroid of a triangle always in its interior?
(iv) Is there anything special about the medians of an
(a) Isosceles triangle? (b) Equilateral triangle?

Properties of the centroid of a triangle

The location of the centroid of a triangle exhibits some nice properties.

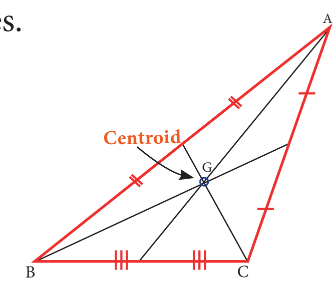


Fig. 5.31

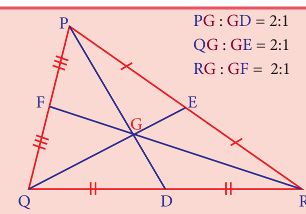
- It is always *located inside the triangle.*
- We have already seen that it *serves as the Centre of gravity* for any triangular lamina.
- Observe the figure given. The lines drawn from each vertex to G form the three triangles ΔABG, ΔBCG, and ΔCAG. Surprisingly, the areas of these triangles are equal.

That is, the medians of a triangle divide it into three smaller triangles of equal area!

DO YOU KNOW?

The centroid of a triangle splits each of the medians in two segments, the one closer to the vertex being twice as long as the other one.

This means the centroid divides each median in a ratio of 2:1. (For example, GD is ⅓ of PD). *(Try to verify this by paper folding).*



Example 5.17

In the figure G is the centroid of the triangle XYZ.

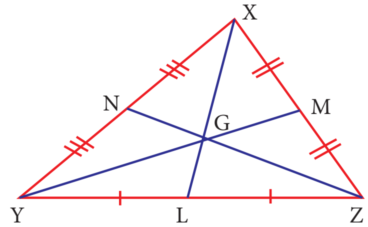
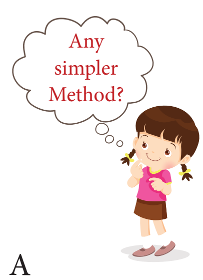


Fig. 5.32

(i) If GL = 2.5 cm, find the length XL.

(ii) If YM = 9.3 cm, find the length GM.

Solution:



(i) Since G is the centroid, XG : GL = 2 : 1 which gives XG : 2.5 = 2 : 1.
Therefore, we get $1 \times (XG) = 2 \times (2.5) \Rightarrow XG = 5$ cm.
Hence, length XL = XG + GL = 5 + 2.5 = 7.5 cm.

(ii) If YG is of 2 parts then GM will be 1 part. (Why?)
This means YM has 3 parts.
3 parts is 9.3 cm long. So GM (made of 1 part) must be $9.3 \div 3 = 3.1$ cm.

Example 5.18

ABC is a triangle and G is its centroid. If AD=12 cm, BC=8 cm and BE=9 cm,find the perimeter of ΔBDG.

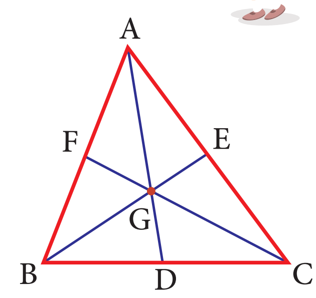


Fig. 5.33

Solution:

ABC is a triangle and G is its centroid. If,

$$AD = 12\text{ cm} \Rightarrow GD = \frac{1}{3}\text{ of } AD = \frac{1}{3}(12) = 4\text{ cm} \text{ and } BE = 9\text{ cm} \Rightarrow BG = \frac{2}{3}\text{ of } BE = \frac{2}{3}(9) = 6\text{cm}.$$

Also D is a midpoint of BC $\Rightarrow BD = \frac{1}{2}\text{ of } BC = \frac{1}{2}(8) = 4\text{cm}$.

$\therefore$ The perimeter of $\Delta BDG = BD+GD+BG = 4+4+6 = 14$ cm

5.7 Altitude of a Triangle

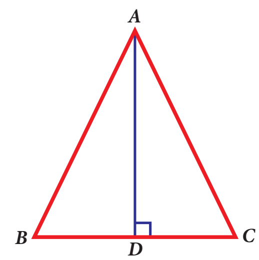
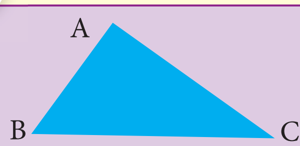
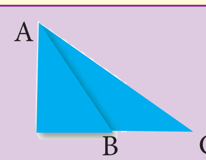
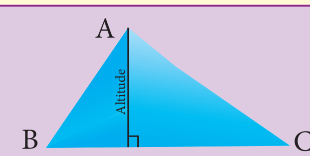


Fig. 5.34

Altitude of a triangle also known as the height of the triangle, is the perpendicular drawn from the vertex of the triangle to the opposite side. The altitude makes a right angle with the base of a triangle. Here, in ΔABC, AD is one of the altitudes as $AD \perp BC$.

Activity

 A B C	 A B C	 A B C Altitude
1. Consider a paper cut-out of an acute angled triangle. Name it, say ABC.	2. Fold the triangle so that a side overlaps itself and the fold contains the vertex opposite to that side.	3. You can now draw the altitude AM, if you want to see it clearly.

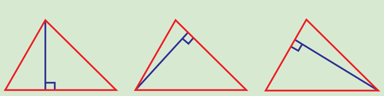
In the same way, you find altitudes of other two sides. Also, with the help of your teacher, you find altitudes of right angled triangle and obtuse angled triangle. Do the altitudes of triangle pass through the same point? What is your conclusion? We see that,

The three altitudes of *any* triangle are concurrent.

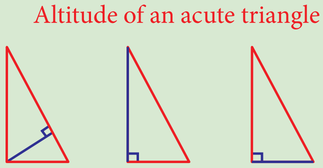
The point of concurrence is known as its Orthocentre, denoted by the letter H.

Think

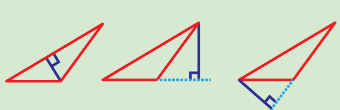
1. **In any acute angled triangle,** all three altitudes are inside the triangle. Where will be the orthocentre? In the interior of the triangle or in its exterior?


Altitude of an acute triangle

2. **In any right angled triangle,** the altitude perpendicular to the hypotenuse is inside the triangle; the other two altitudes are the legs of the triangle. Can you identify the orthocentre in this case?

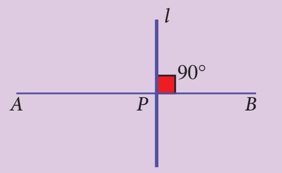
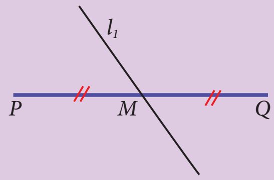
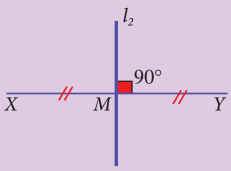

Altitude of a right triangle

3. **In any obtuse angled triangle,** the altitude connected to the obtuse vertex is inside the triangle, and the two altitudes connected to the acute vertices are outside the triangle. Can you identify the orthocentre in this case?

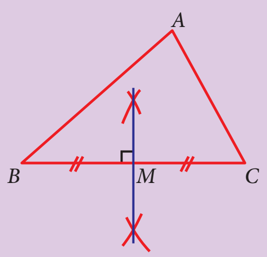
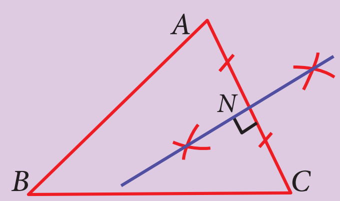
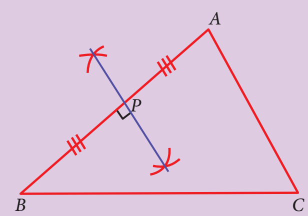

Altitude of an obtuse triangle

5.8 Perpendicular Bisectors of a Triangle

Let us first recall the following ideas.

Perpendicular	Bisector	Perpendicular bisector
		
$\overline{AB}$ is a line segment. l is perpendicular to $\overline{AB}$. P is the foot of the $\perp^r$. Note that $\overline{AP} \uparrow \overline{PB}$ here.	$\overline{PQ}$ is a line segment. l_1 is a bisector to $\overline{PQ}$. M is the midpoint of $\overline{PQ}$. l_1 need not be $\perp^r$ to $\overline{PQ}$	$\overline{XY}$ is a line segment. l_2 is a bisector to $\overline{XY}$. l_2 is also $\perp^r$ to $\overline{XY}$. M is the midpoint of $\overline{XY}$.

Consider a triangle ABC. It has three sides. For each side you can have a perpendicular bisector as follows:

		
Perpendicular bisector of side BC	Perpendicular bisector of side AC	Perpendicular bisector of side AB
M is the midpoint of $\overline{BC}$ We have a perpendicular at M.	N is the midpoint of $\overline{AC}$ We have a perpendicular at N.	P is the midpoint of $\overline{AB}$ We have a perpendicular at P.

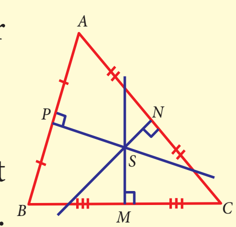
Surprisingly, all the three perpendicular bisectors of the sides of a triangle are concurrent at a point!

Activity

One can visualize the point of concurrence of the perpendicular bisectors, through simple paper folding. Try and see that,

The perpendicular bisectors of the sides of *any* triangle are concurrent.

As done in the earlier activity on Centroid, you can repeat the experiment for various types of triangle, acute, obtuse, right, isosceles and equilateral. Do you find anything special with the equilateral triangle in this case?



5.8.1 Circumcentre

The point of concurrence of the three perpendicular bisectors of a triangle is called as its **Circumcentre,** denoted by the letter **S**.

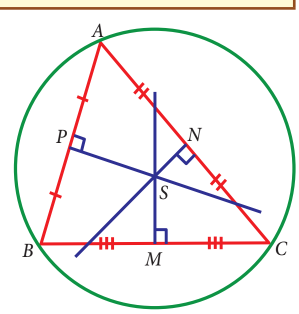


Fig. 5.35

Why should it be called so? Because one can draw a circle exactly passing through the three vertices of the triangle, with centre at the point of concurrence of the perpendicular bisectors of sides. Thus, the circumcentre is equidistant from the vertices of the triangle.

Activity

Check if the following are true by paper-folding:

1. The circumcentre of an acute angled triangle lies in the interior of the triangle.
2. The circumcentre of an obtuse angled triangle lies in the exterior of the triangle.
3. The circumcentre of a right triangle lies at the midpoint of its hypotenuse.

Example 5.19

In ΔABC, S is the circumcentre, BC = 72 cm and DS = 15 cm. Find the radius of its circumcircle.

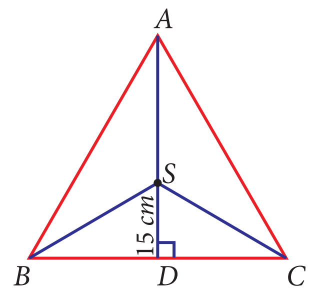


Fig. 5.36

Solution:

As S is the circumcentre of ΔABC, it is equidistant from A,B and C. So AS=BS=CS=radius of its circumcircle. As AD is the perpendicular bisector of BC, $BD = \frac{1}{2} \times BC = \frac{1}{2} \times 72 = 36$ cm

In right angled triangle BDS, by Pythagoras theorem,

$$BS^2 = BD^2 + SD^2 = 36^2 + 15^2 = 1521 = 39^2 \Rightarrow BS = 39 \text{ cm}.$$

$\therefore$ The radius of the circumcircle of ΔABC is 39 cm.

5.9 Angle Bisectors of a Triangle

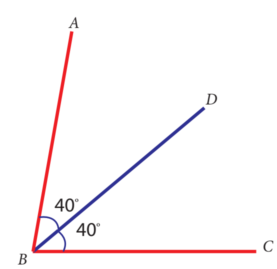
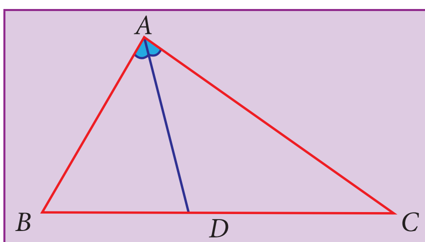
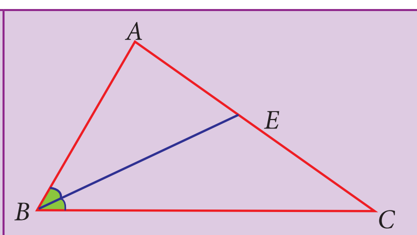
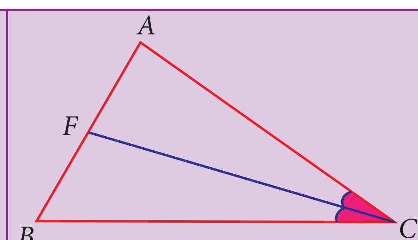


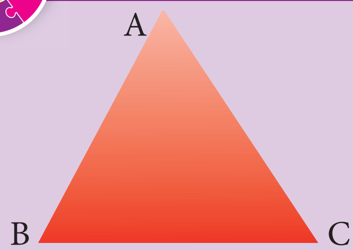
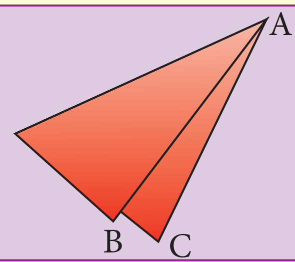
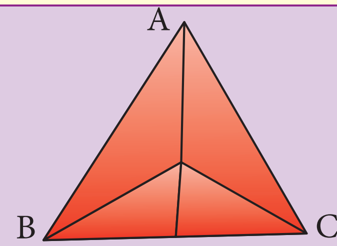
Fig. 5.37

We have learnt about angle bisectors in the previous class. An **angle bisector** is a line or ray that divides an angle into two congruent angles. In the figure, $\angle ABC$ is bisected by the line BD such that $\angle ABD = \angle CBD$.

Consider a triangle ABC. How many angles does a triangle have ? 3 angles. For each angle you can have an angle bisector as follows:

		
AD bisects $\angle$A into two congruent angles. Hence it is an angle bisector of $\angle$A.	BE bisects $\angle$B into two congruent angles. Hence it is an angle bisector of $\angle$B.	CF bisects $\angle$C into two congruent angles. Hence it is an angle bisector of $\angle$C.

Activity

		
1. Consider a paper cut-out of a triangle. Name it, say ABC.	2. Fold the triangle so that the opposite sides meet and contain the vertex. Repeat the same to find angle bisectors of other two angles also.	3. Trace all of the folds. Do the angle bisectors pass through the same point?

Now you can repeat this activity for an obtuse-angled triangle and a right angled triangle. What is the conclusion? Do the angle bisectors pass through the same point in all the cases? Yes, we see that,

The three angle bisectors of any triangle are concurrent.

5.9.1 Incentre

The point of concurrence of the three angle bisectors of a triangle is called as its **incentre,** denoted by the letter **I**.

Why should it be called so? Because one can draw a circle inside of the triangle so that it touches all three sides internally, with centre at the point of concurrence of the angle bisectors. The lengths of a perpendicular line drawn from incentre to each side is found to be same. Thus, the incentre is equidistant from the sides of the triangle.

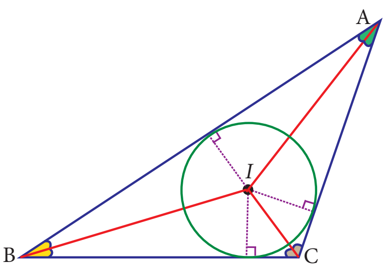


Fig. 5.38

Example 5.20

Identify the incentre of the triangle PQR.

Solution:

Incentre is the point of intersection of angle bisectors.

Here, PM and QN are angle bisectors of ∠P and ∠Q respectively, intersecting at B.

So, the incentre of the triangle PQR is B.

(Can A and C be the incentre of ΔPQR? Why?)

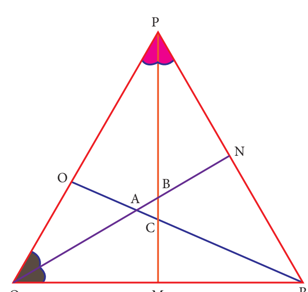


Fig. 5.39

DO YOU KNOW?

The position of the Centroid, Orthocentre, Circumcentre and Incentre differs depending on the type of triangles given. The following points will help us in locating and remembering these.

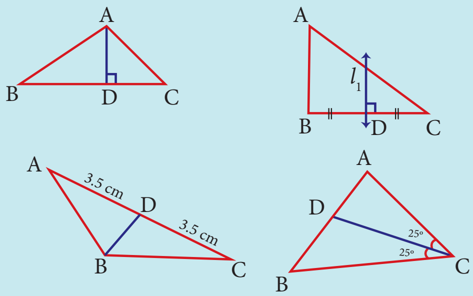
(i) For all types of triangles, Centroid (G) and Incentre (I) will be inside the triangle.

(ii) The Orthocentre (H) will be inside in an acute angled triangle, outside in an obtuse angled triangle and on the vertex containing 90° in a right angled triangle.

(iii) The Circumcentre (S) will be inside in an acute angled triangle, outside in an obtuse angled triangle and on the hypotenuse in a right-angled triangle.

Try these

Identify the type of segment required in each triangle:
(median, altitude, perpendicular bisector, angle bisector)



(i) AD =__________ (ii) l_1 =__________

(iii) BD =__________ (iv) CD =__________

Activity

1. By paper folding, find the centroid, orthocentre, circumcentre and incentre of an equilateral triangle. Do they coinside?
2. By paper folding, find the centroid (G), orthocentre(H), circumcentre (S) and incentre (I) of a triangle. Join G,H,S and I. Are they collinear?

Exercise 5.2

1. **Fill in the blanks:**

(i) If in a ΔPQR, $PR^2 = PQ^2 + QR^2$, then the right angle of ΔPQR is at the vertex ________.

(ii) If 'l' and 'm' are the legs and 'n' is the hypotenuse of a right angled triangle then, l^2 = ________.

(iii) If the sides of a triangle are in the ratio 5:12:13 then, it is ________.

(iv) The medians of a triangle cross each other at ___________.

(v) The centroid of a triangle divides each medians in the ratio ___________.

2. **Say True or False.**

(i) 8, 15, 17 is a Pythagorean triplet.

(ii) In a right angled triangle, the hypotenuse is the greatest side.

(iii) In any triangle the centroid and the incentre are located inside the triangle.

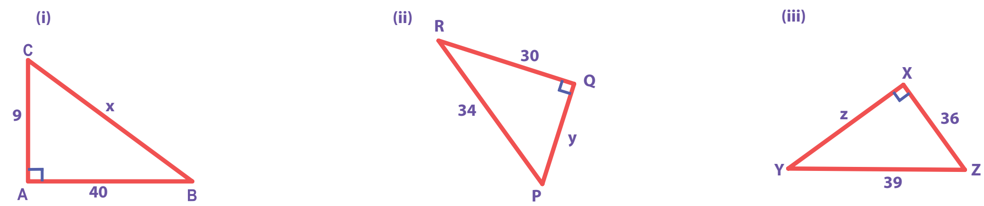
(iv) The centroid, orthocentre, and incentre of a triangle are collinear.

(v) The incentre is equidistant from all the vertices of a triangle.

3. Check whether given sides are the sides of right-angled triangles, using Pythagoras theorem.

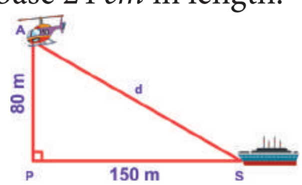
(i) 8,15,17 (ii) 12,13,15 (iii) 30,40,50 (iv) 9,40,41 (v) 24,45,51

4. Find the unknown side in the following triangles.

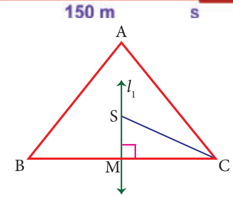


5. An isosceles triangle has equal sides each 13 *cm* and a base 24 *cm* in length. Find its height.

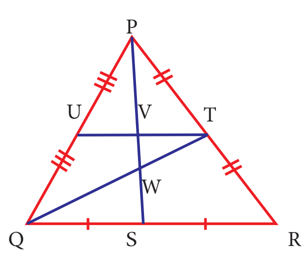
6. Find the distance between the helicopter and the ship.



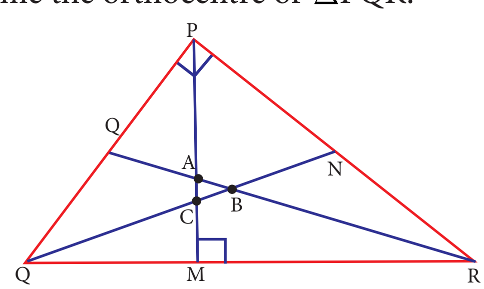
7. In triangle ABC, line l_1 is a perpendicular bisector of BC. If BC=12 cm, SM=8 cm, find CS.



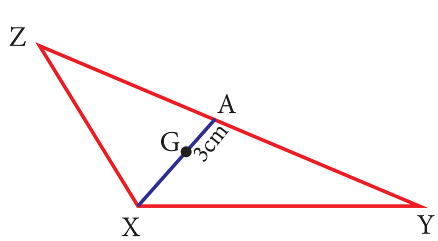
8. Identify the centroid of ΔPQR.



9. Name the orthocentre of ΔPQR.

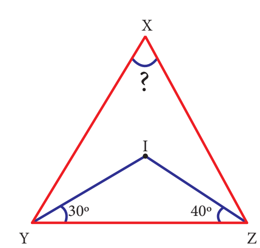


10. In the given figure, A is the midpoint of YZ and G is the centroid of the triangle XYZ. If the length of GA is 3 cm, find XA.



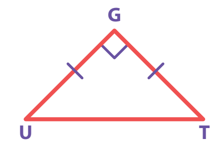
11. If I is the incentre of ΔXYZ, $\angle IYZ = 30°$ and $\angle IZY = 40°$, find $\angle YXZ$.

M5M9T7



Objective Type Questions

12. If ΔGUT is isosceles and right angled, then $\angle TUG$ is ________.



(A) 30° (B) 40° (C) 45° (D) 55°

13. The hypotenuse of a right angled triangle of sides 12*cm* and 16*cm* is __________.
(A) 28 *cm* (B) 20 *cm* (C) 24 *cm* (D) 21 *cm*

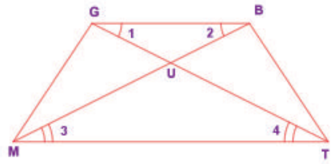
14. The area of a rectangle of length 21*cm* and diagonal 29*cm* is __________.
(A) 609 cm^2 (B) 580 cm^2 (C) 420 cm^2 (D) 210 cm^2

15. The sides of a right angled triangle are in the ratio 5:12:13 and its perimeter is 120 units then, the sides are ______________.
(A) 25, 36, 59 (B) 10,24,26 (C) 36, 39, 45 (D) 20,48,52

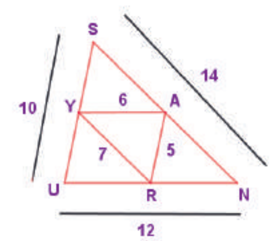
Exercise 5.3

Miscellaneous Practice Problems

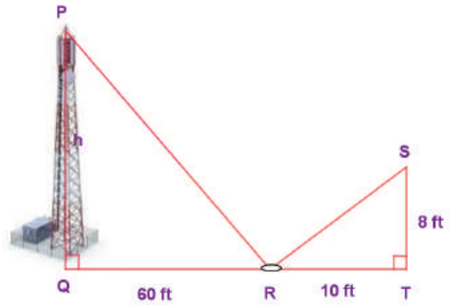
1. In the figure, given that $\angle 1 \equiv \angle 2$ and $\angle 3 \equiv \angle 4$. Prove that $\Delta\, MUG \equiv \Delta\, TUB$.



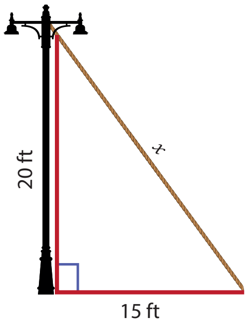
2. From the figure, prove that $\Delta SUN \sim \Delta\, RAY$.



3. The height of a tower is measured by a mirror on the ground at R by which the top of the tower's reflection is seen. Find the height of the tower. If $\Delta PQR \sim \Delta STR$



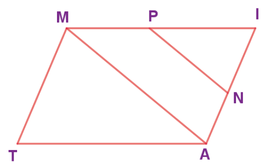
4. Find the length of the support cable required to support the tower with the floor.



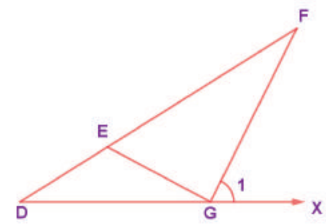
5. Rithika buys an LED TV which has a 25 inches screen. If its height is 7 inches, how wide is the screen? Her TV cabinet is 20 inches wide. Will the TV fit into the cabinet? Give reason.

Challenging problems

6. In the figure, $\angle TMA \equiv \angle IAM$ and $\angle TAM \equiv \angle IMA$. P is the midpoint of MI and N is the midpoint of AI. Prove that $\Delta\ PIN \sim \Delta\, ATM$.

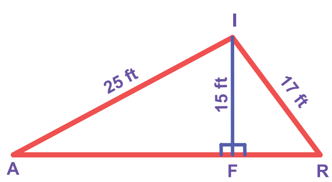


7. In the figure, if $\angle FEG \equiv \angle 1$ then, prove that $DG^2 = DE.DF$.



8. The diagonals of the rhombus is 12 *cm* and 16 *cm*. Find its perimeter. (Hint: the diagonals of rhombus bisect each other at right angles).

9. In the figure, find AR.



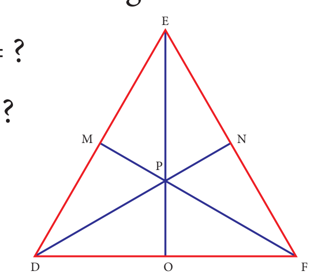
10. In ΔDEF, DN, EO, FM are medians and point P is the centroid. Find the following.

(i) IF DE = 44, then DM = ?

(ii) IF PD = 12, then PN = ?

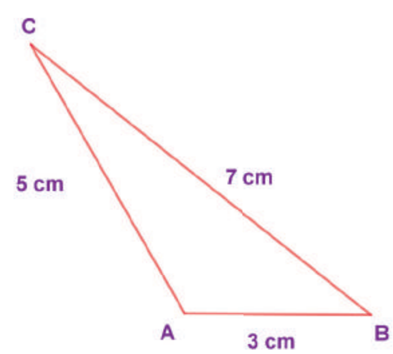
(iii) If DO = 8, then FD = ?

(iv) IF OE = 36 then EP = ?

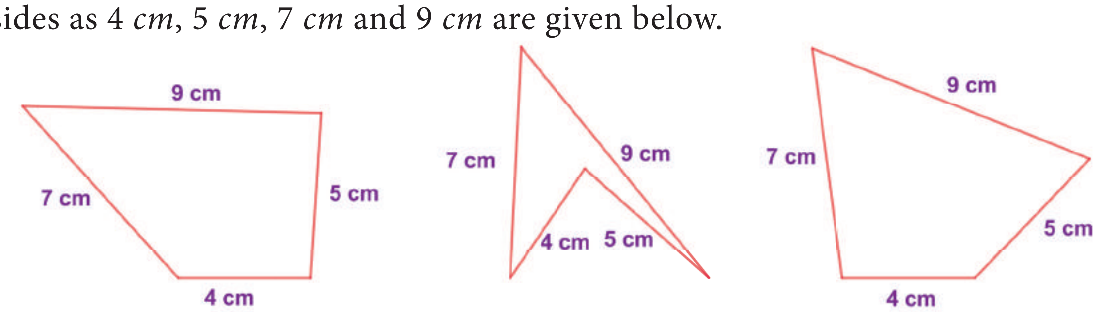


5.10 Construction of Quadrilaterals

We have already learnt how to draw triangles in the earlier classes. A polygon that has got 3 sides is a triangle. To draw a triangle, we need 3 independent measures. Also, there is only one way to construct a triangle, given its 3 sides. For example, to construct a triangle with sides 3*cm*, 5*cm* and 7*cm*, there is only one way to do it.



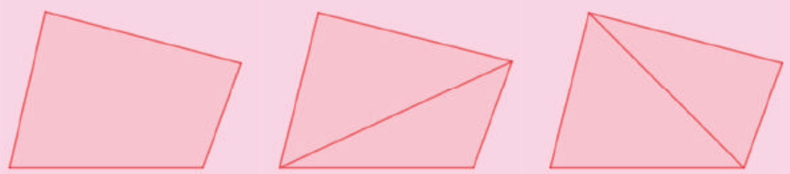
Now, let us move on to quadrilaterals. A polygon that is formed by 4 sides is called a quadrilateral. Isn't it? But, a quadrilateral can be of different shapes. They need not look like the same for the given 4 measures. For example, some of the quadrilaterals having their sides as 4 *cm*, 5 *cm*, 7 *cm* and 9 *cm* are given below.



So, to construct a particular quadrilateral, we need a 5^{th} measure. That can be its diagonal or an angle measure. Moreover, even if 2 or 3 sides are given, using the measures of the diagonals and angles, we can construct quadrilaterals.

Note

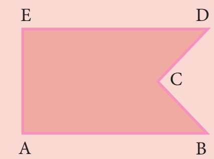
We can split any quadrilateral into two triangles by drawing a diagonal.



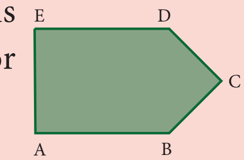
In the above figures, a quadrilateral is split in two ways by its diagonals. So, if a diagonal is given, first draw the lower triangle with two sides and one diagonal. Then, draw the upper triangle with other two measures.

DO YOU KNOW?

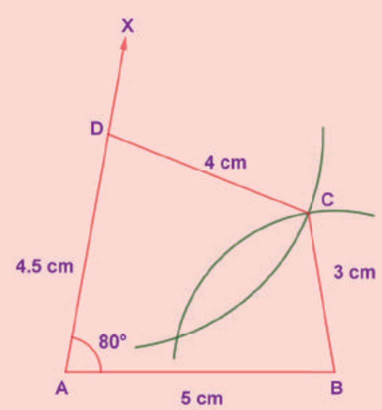
(i) • A polygon in which atleast one interior angle is more than 180°, is called a concave polygon. In the given polygon, interior angle at C is more than 180°.



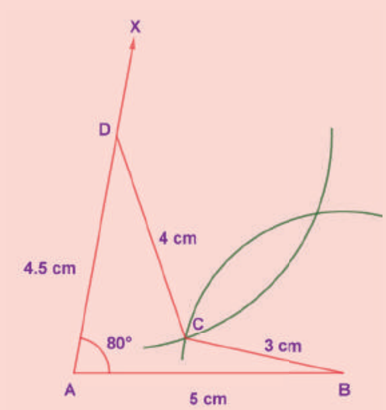
• A polygon in which each interior angle is less than 180°, is called a convex polygon. In the given polygon, all interior angles are less than 180°.



(ii) Look at the following quadrilaterals.



Convex quadrilateral



Concave quadrilateral

Though, we can construct a quadrilateral in two ways as shown above, we do not take into account the concave quadrilaterals in this chapter. Hence, the construction of only convex quadrilaterals are treated here.

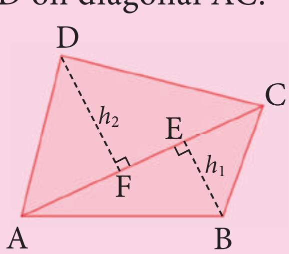
Note

Consider the given Quadrilateral ABCD. In which AC is a diagonal (d), BE (h_1) and DF (h_2) are the perpendiculars drawn from the vertices B and D on diagonal AC.

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

$$= \frac{1}{2} \times AC \times BE + \frac{1}{2} \times AC \times FD$$

$$= \frac{1}{2} \times d \times h_1 + \frac{1}{2} \times d \times h_2 = \frac{1}{2} \times d \times (h_1 + h_2) \text{ sq.units.}$$



We shall now see a few types on constructing these quadrilaterals when its:

(i) 4 sides and a diagonal are given.	(ii) 3 sides and 2 diagonals are given.
(iii) 4 sides and an angle are given.	(iv) 3 sides and 2 angles are given.
(v) 2 sides and 3 angles are given.	

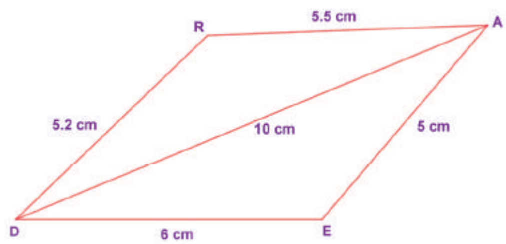
5.10.1 Constructing a quadrilateral when its 4 sides and a diagonal are given

Example 5.21

Construct a quadrilateral *DEAR* with *DE*=6 *cm*, *EA* = 5 *cm*, *AR* = 5.5cm, *RD* = 5.2 *cm* and *DA* = 10 *cm*. Also find its area.

Solution:

Given:
DE = 6 *cm*, *EA* = 5 *cm*, *AR* = 5.5 *cm*, *RD* = 5.2 *cm* and a diagonal *DA* = 10 *cm*



Rough diagram

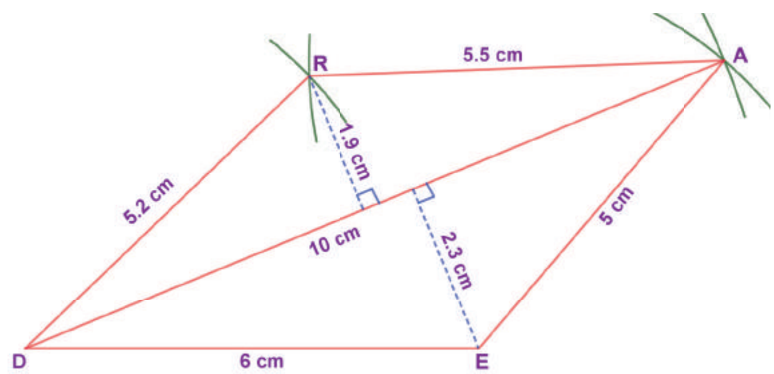


Fig. 5.40

Steps:

4. Draw a line segment *DE* = 6 *cm*.
5. With *D* and *E* as centres, draw arcs of radii 10 *cm* and 5 *cm* respectively and let them cut at *A*.
6. Join *DA* and *EA*.
7. With *D* and *A* as centres, draw arcs of radii 5.2 *cm* and 5.5 *cm* respectively and let them cut at *R*.
8. Join *DR* and *AR* .
9. *DEAR* is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral *DEAR* $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 10 \times (1.9 + 2.3) = 5 \times 4.2 = 21\ cm^2$

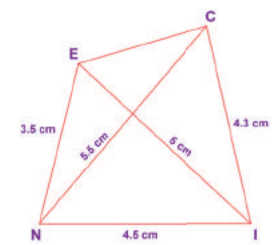
5.10.2 Construct a quadrilateral when its 3 sides and 2 diagonals are given

Example 5.22

Construct a quadrilateral *NICE* with *NI*=4.5 *cm*, *IC*= 4.3 *cm*, *NE*= 3.5 *cm*, *NC*= 5.5 *cm* and *IE* = 5 *cm*. Also find its area.

Solution:

Given:
NI = 4.5 *cm*, *IC*= 4.3 *cm*,
NE = 3.5 *cm* and two diagonals,
NC = 5.5 *cm* and *IE* = 5 *cm*



Rough diagram

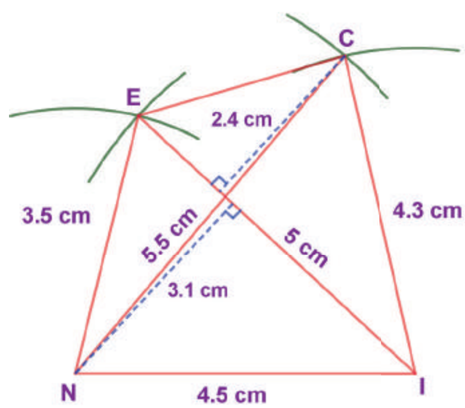


Fig. 5.41

Steps:

1. Draw a line segment *NI* = 4.5 *cm*.
2. With *N* and *I* as centres, draw arcs of radii 5.5 *cm* and 4.3 *cm* respectively and let them cut at *C*.
3. Join *NC* and *IC*.
4. With *N* and *I* as centres, draw arcs of radii 3.5 *cm* and 5 *cm* respectively and let them cut at *E*.
5. Join *NE*, *IE* and *CE* .
6. *NICE* is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral *NICE* $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 5 \times (2.4 + 3.1)$

$= 2.5 \times 5.5 = 13.75\ cm^2$

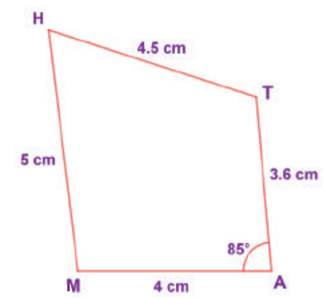
5.10.3 Construct a quadrilateral when its 4 sides and one angle are given

Example 5.23

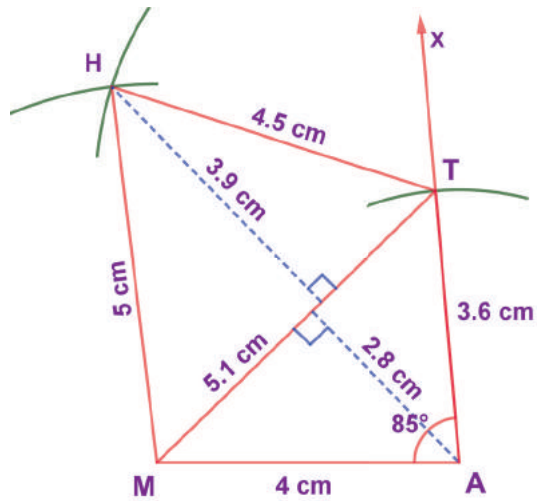
Construct a quadrilateral *MATH* with *MA*=4 *cm*, *AT*= 3.6 *cm*, *TH* = 4.5 *cm*, *MH*= 5 *cm* and $\angle A = 85°$. Also find its area.

Solution:

Given:
MA=4 *cm*, *AT*= 3.6 *cm*,
TH = 4.5 *cm*, *MH*= 5 *cm* and $\angle A = 85°$



Rough diagram



T7L2U7

Fig. 5.42

Steps:

1. Draw a line segment *MA* = 4 *cm*.
2. Make $\angle A = 85°$.
3. With *A* as centre, draw an arc of radius 3.6 *cm*. Let it cut the ray *AX* at *T*.
4. With *M* and *T* as centres, draw arcs of radii 5 *cm* and 4.5 *cm* respectively and let them cut at *H*.
5. Join *MH* and *TH*.
6. *MATH* is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral *MATH* $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

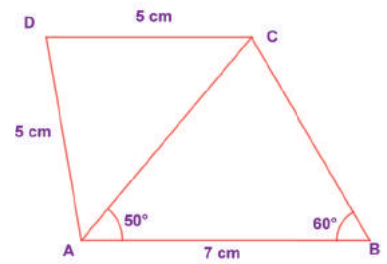
$= \frac{1}{2} \times 5.1 \times (3.9 + 2.8)$

$= 2.55 \times 6.7 = 17.09\ cm^2$

5.10.4 Construct a quadrilateral when its 3 sides and 2 angles are given

Example 5.24

Construct a quadrilateral $ABCD$ with $AB=7\ cm$, $AD=5\ cm$, $CD=5\ cm$, $\angle BAC=50^\circ$ and $\angle ABC=60^\circ$. Also find its area.



Rough diagram

Solution:

Given:

$AB=7\ cm$, $AD=5\ cm$, $CD=5\ cm$ and two angles $\angle BAC=50^\circ$ and $\angle ABC=60^\circ$

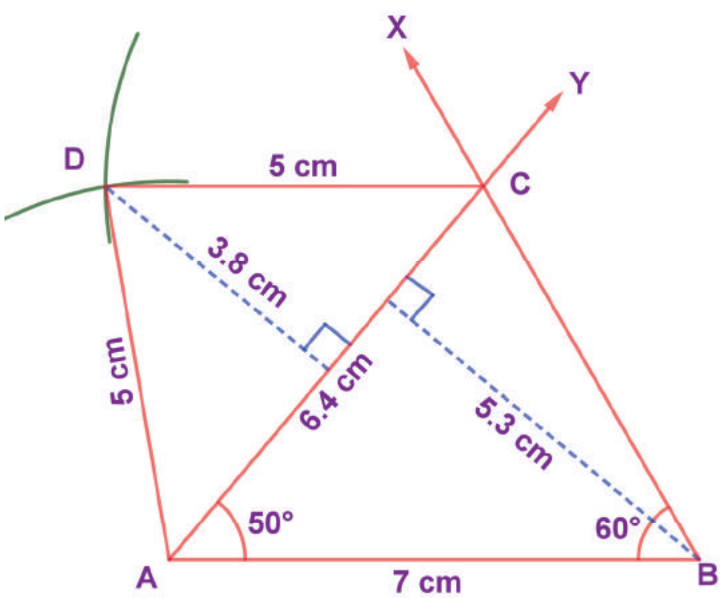


Fig. 5.43

Steps:

1. Draw a line segment $AB = 7\ cm$.
2. At A on AB, make $\angle BAY = 50^\circ$ and at B on AB, make $\angle ABX = 60^\circ$. Let them intersect at C.
3. With A and C as centres, draw arcs of radius 5 cm each. Let them intersect at D.
4. Join AD and CD.
5. $ABCD$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $ABCD$ $= \frac{1}{2}\times d\times(h_1+h_2)$ *sq. units*

$= \frac{1}{2}\times 6.4\times(3.8+5.3)$

$= 3.2 \times 9.1 = 29.12\ cm^2$

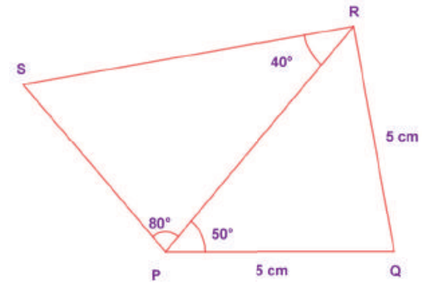
5.10.5 Construct a quadrilateral when its 2 sides and 3 angles are given

Example 5.25

Construct a quadrilateral $PQRS$ with $PQ=QR=5$ cm, $\angle QPR = 50^\circ$, $\angle PRS = 40^\circ$ and $\angle RPS = 80^\circ$. Also find its area.

Solution:

Given:
$PQ = 5$ cm, $QR=5$ cm, $\angle QPR = 50^\circ$,
$\angle PRS = 40^\circ$ and $\angle RPS = 80^\circ$



Rough diagram

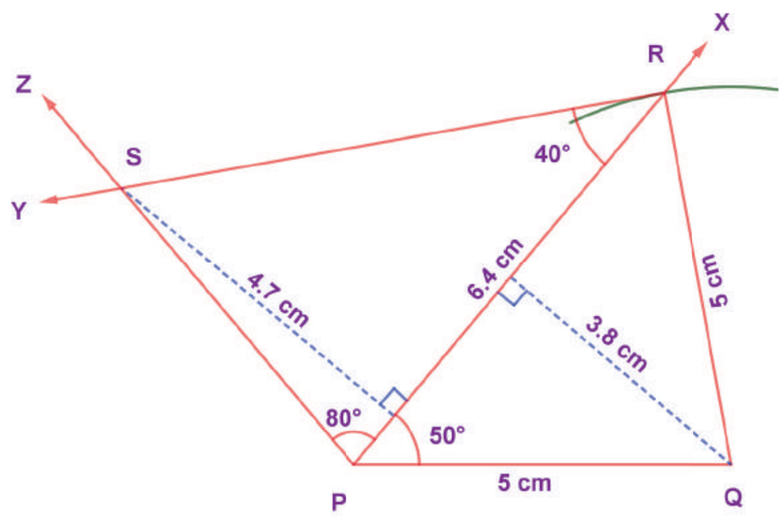


Fig. 5.44

Steps:

1. Draw a line segment PQ = 5 cm.
2. At P on PQ, make $\angle QPX = 50^\circ$.
3. With Q as centre, draw an arc of radius 5 cm. Let it cut PX at R.
4. At R on PR, make $\angle PRS = 40^\circ$ and at P on PR, make $\angle RPS = 80^\circ$. Let them intersect at S.
5. $PQRS$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $PQRS$ $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 6.4 \times (4.7 + 3.8)$

$= 3.2 \times 8.5 = 27.2$ cm^2

Think

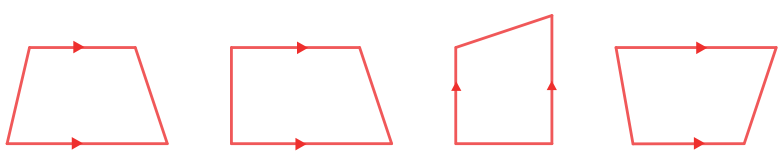
Is it possible to construct a quadrilateral $PQRS$ with $PQ = 5$ cm, $QR = 3$ cm, $RS = 6$ cm, $PS = 7$ cm and $PR = 10$ cm. If not, why?

5.11 Construction of Trapeziums

In the first term, we have learnt how to construct the quadrilaterals. To draw a quadrilateral, how many measurements do you need? 5 measurements. Isn't it? Let us see the special quadrilaterals which need less than 5 measurements. Based on the nature of sides and angles of a quadrilateral, it gets special names like trapezium, parallelogram, rhombus, rectangle, square and kite.

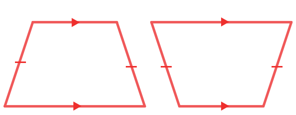
Now, you will learn how to construct trapeziums.

Trapezium is a quadrilateral in which a pair of opposite sides are parallel. To construct a trapezium, draw one of the parallel sides as a base and on that base construct a triangle with the 2 more measurements. Now, through the vertex of that triangle, construct the parallel line opposite to the base so that the triangle lies between the parallel sides. As the fourth vertex lies on this parallel line, mark it with the remaining measure. Hence, we need four independent measures to construct a trapezium. The given shapes are examples of trapeziums.



Note: The arrow marks in the above shapes represent parallel sides.

If the non-parallel sides of a trapezium are equal in length and form equal angles at one of its bases, then it is called an **isosceles trapezium**.



Try these

1. The area of the trapezium is __________.
2. The distance between the parallel sides of a trapezium is called as __________.
3. If the height and parallel sides of a trapezium are $5cm$, $7cm$ and $5cm$ respectively, then its area is __________.
4. In an isosceles trapezium, the non-parallel sides are __________ in length.
5. To construct a trapezium, __________ measurements are enough.
6. If the area and sum of the parallel sides are $60\ cm^2$ and $12\ cm$, its height is __________.

Let us construct a trapezium with the given measurements

1. Three sides and one diagonal.
2. Three sides and one angle.
3. Two sides and two angles.
4. Four sides.

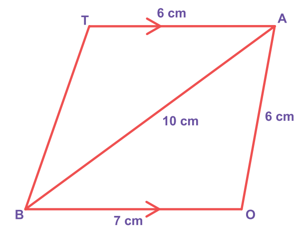
5.11.1 Constructing a trapezium when its three sides and one diagonal are given

Example 5.26

Construct a trapezium **BOAT** in which $\overline{BO}$ is parallel to $\overline{TA}$, BO=7*cm*, OA=6*cm*, BA=10*cm* and TA=6*cm*. Also find its area.

Solution:

Given:
BO=7*cm*, OA=6*cm*, BA=10*cm* ,
TA=6*cm* and $\overline{BO} \parallel \overline{TA}$



Rough diagram

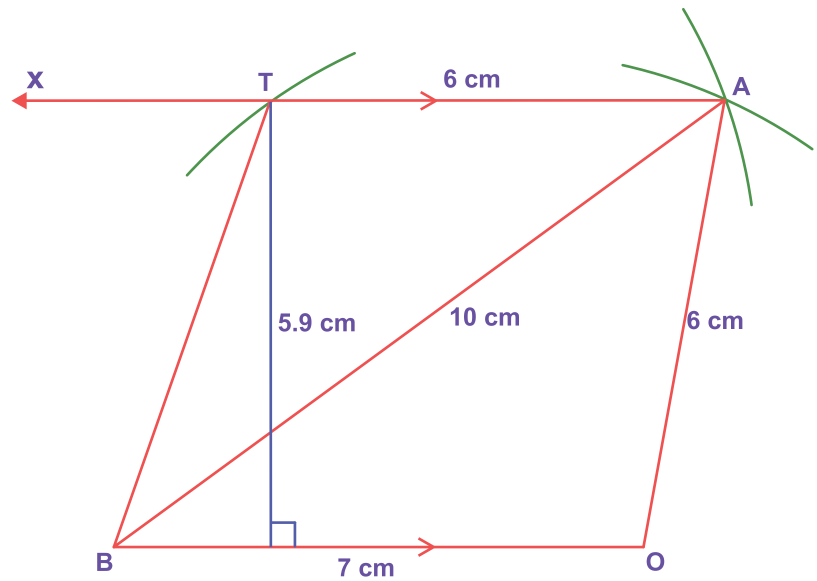


Fig. 5.45

Steps:

1. Draw a line segment BO = 7*cm*.
2. With B and O as centres, draw arcs of radii 10*cm* and 6*cm* respectively and let them cut at A.
3. Join BA and OA.
4. Draw AX parallel to BO
5. With A as centre, draw an arc of radius 6*cm* cutting AX at T.
6. Join BT. BOAT is the required trapezium.

Calculation of Area:

Area of the trapezium BOAT $= \frac{1}{2} \times h \times (a+b)\ sq.units$

$= \frac{1}{2} \times 5.9 \times (7+6) = 38.35\ sq.cm$

5.11.2 Constructing a trapezium when its three sides and one angle are given

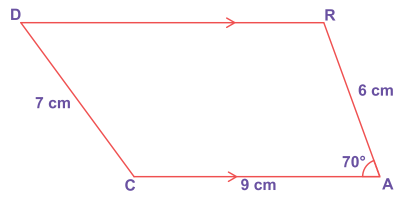
Example 5.27

Construct a trapezium **CARD** in which $\overline{CA}$ is parallel to $\overline{DR}$, CA=9*cm*, $\angle CAR = 70°$, AR=6*cm* and CD=7*cm*. Also find its area.

Solution:

Given:

CA=9*cm*, $\angle CAR = 70°$ AR=6*cm*, and CD=7*cm* and $\overline{CA} \parallel \overline{DR}$



Rough diagram

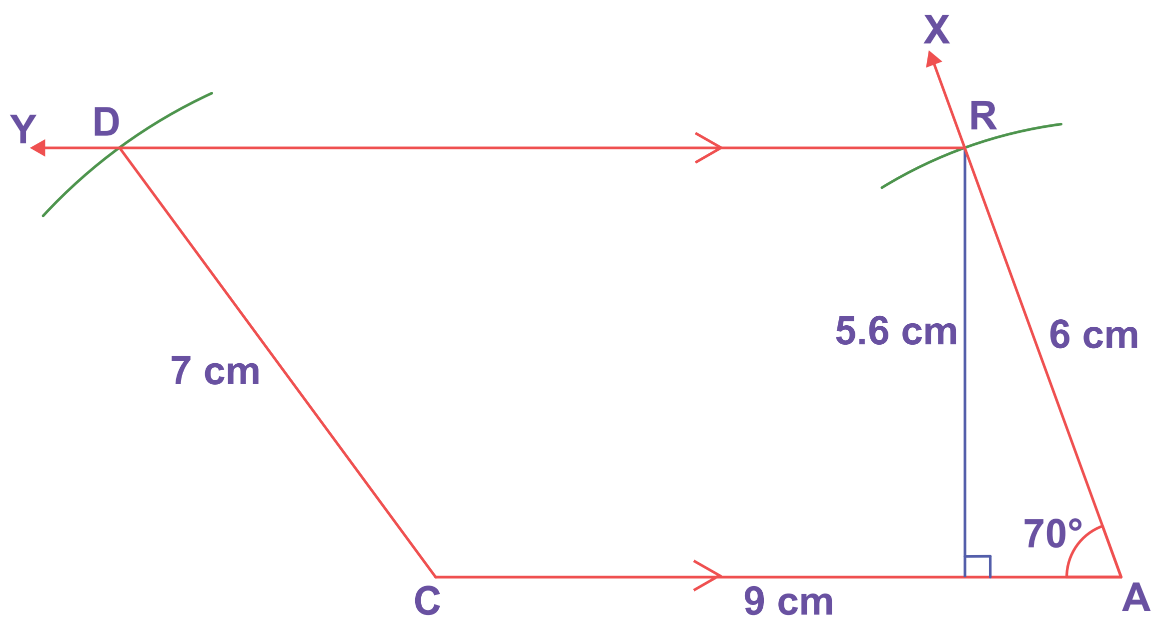


Fig. 5.46

Steps:

1. Draw a line segment CA= 9*cm*.
2. Construct an angle $\angle CAX = 70°$ at A.
3. With A as centre, draw an arc of radius 6*cm* cutting AX at R.
4. Draw RY parallel to CA.
5. With C as centre, draw an arc of radius 7*cm* cutting RY at D.
6. Join CD. CARD is the required trapezium.

Calculation of Area:

Area of the trapezium CARD $= \frac{1}{2} \times h \times (a+b)\ sq.units$

$= \frac{1}{2} \times 5.6 \times (9+11) = 56\ sq.cm$

5.11.3 Constructing a trapezium when its two sides and two angles are given

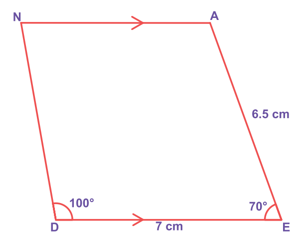
Example 5.28

Construct a trapezium DEAN in which $\overline{DE}$ is parallel to $\overline{NA}$, DE=7cm, EA=6.5cm $\angle EDN = 100°$ and $\angle DEA = 70°$. Also find its area.

Solution:

Given:

DE=7cm, EA=6.5cm $\angle EDN = 100°$ and $\angle DEA = 70°$ and $\overline{DE} \,||\, \overline{NA}$



Rough diagram

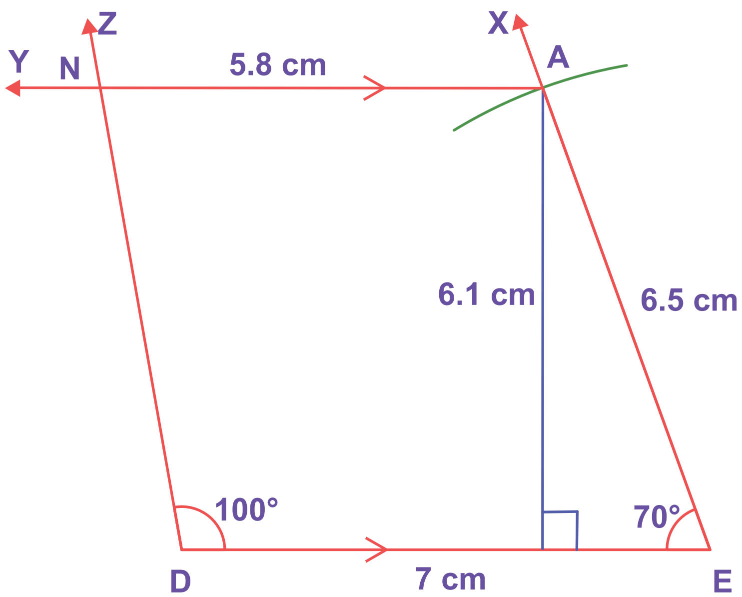


Fig. 5.47

Steps:

1. Draw a line segment DE= 7cm.
2. Construct an angle $\angle DEX = 70°$ at E.
3. With E as centre draw an arc of radius 6.5cm cutting EX at A.
4. Draw AY parallel to DE.
5. Construct an angle $\angle EDZ = 100°$ at D cutting AY at N.
6. DEAN is the required trapezium.

Calculation of Area:

Area of the trapezium DEAN $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 6.1 \times (7+5.8) = 39.04\, sq.cm$

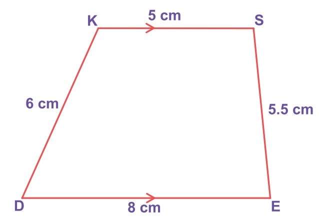
5.11.4 Constructing a trapezium when its four sides are given

Example 5.29

Construct a trapezium DESK in which $\overline{DE}$ is parallel to $\overline{KS}$, DE=8*cm*, ES=5.5*cm*, KS =5*cm* and KD=6*cm*. Find also its area.

Solution:

Given:
DE=8*cm*, ES=5.5*cm*, KS =5*cm*,
KD=6*cm* and $\overline{DE} \parallel \overline{KS}$



Rough diagram

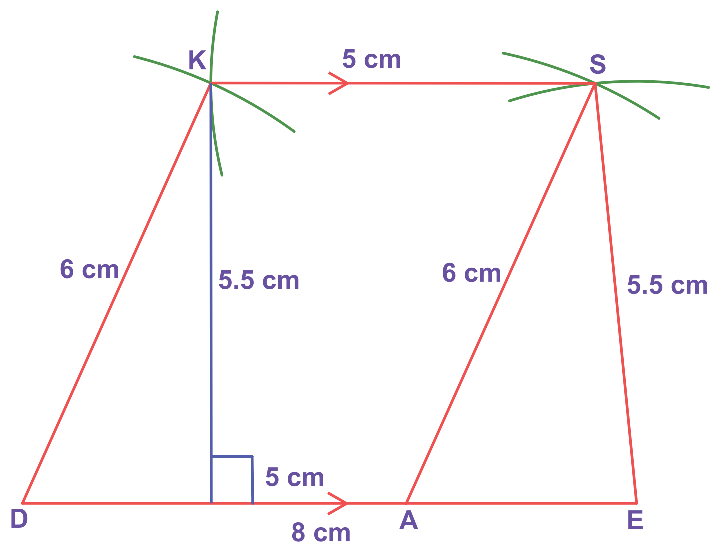


Fig. 5.48

Steps:

1. Draw a line segment DE= 8*cm*.
2. Mark the point A on DE such that DA=5*cm*.
3. With A and E as centres, draw arcs of radii 6*cm* and 5.5*cm* respectively. Let them cut at S. Join AS and ES.
4. With D and S as centres, draw arcs of radii 6*cm* and 5*cm* respectively. Let them cut at K. Join DK and KS.
5. DESK is the required trapezium.

Calculation of Area:

Area of the trapezium DESK $= \frac{1}{2} \times h \times (a+b)\ sq.units$

$= \frac{1}{2} \times 5.5 \times (8+5) = 35.75\ sq.cm$

Exercise 5.4

I. Construct the following quadrilaterals with the given measurements and also find their area.

1. ABCD, AB = 5 cm, BC = 4.5 cm, CD = 3.8 cm, DA = 4.4 cm and AC= 6.2 cm.
2. PLAY, PL= 7 cm, LA = 6 cm, AY= 6 cm, PA = 8 cm and LY = 7 cm.
3. PQRS, PQ=QR= 3.5 cm, RS= 5.2 cm, SP = 5.3 cm and $\angle Q = 120°$.
4. MIND, MI =3.6 cm, ND = 4 cm, MD= 4 cm, $\angle M = 50°$ and $\angle D = 100°$.
5. AGRI, AG= 4.5 cm, GR = 3.8 cm, $\angle A = 60°$, $\angle G = 110°$ and $\angle R = 90°$.

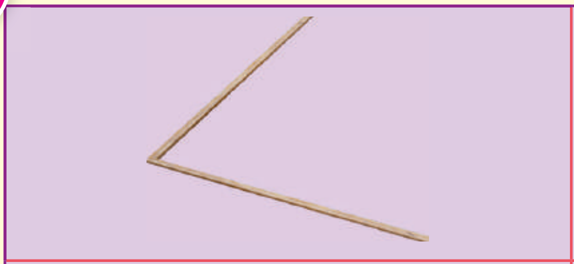
II. Construct the following trapeziums with the given measures and also find their area.

1. AIMS with $\overline{AI} \parallel \overline{SM}$, AI=6*cm*, IM=5*cm*, AM=9*cm* and MS=6.5*cm*.
2. CUTE with $\overline{CU} \parallel \overline{ET}$, CU=7*cm*, $\angle UCE = 80°$ CE=6*cm* and TE=5*cm*.
3. ARMY with $\overline{AR} \parallel \overline{YM}$, AR=7*cm*, RM=6.5*cm* $\angle RAY = 100°$ and $\angle ARM = 60°$
4. CITY with $\overline{CI} \parallel \overline{YT}$, CI=7*cm*, IT=5.5*cm*, TY=4*cm* and YC=6*cm*.

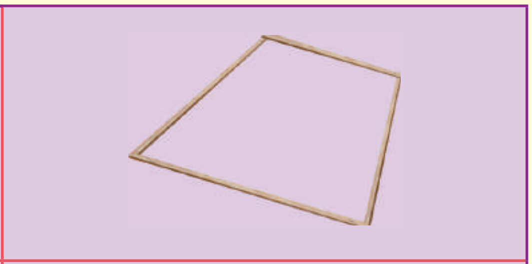
5.12 Construction of Special Quadrilaterals

Before we begin to learn constructing certain quadrilaterals, it is essential to recall their basic properties that would help us during the process. We will try to do this by performing some activities and then sum them up.

Activity



1. Place a pair of *unequal* sticks (say pieces of broomstick) such that they have their end points joined at one end.



2. Now place another such pair meeting the free ends of the first pair.

What is the figure enclosed? It is a quadrilateral. Name it as ABCD. How many sides are there? What are its diagonals? Are the diagonals equal? Are the angles equal?

In the above activity can you get a quadrilateral in which

(i) All the four angles are acute.

(ii) One of the angles is obtuse.

(iii) Two of the angles are obtuse.

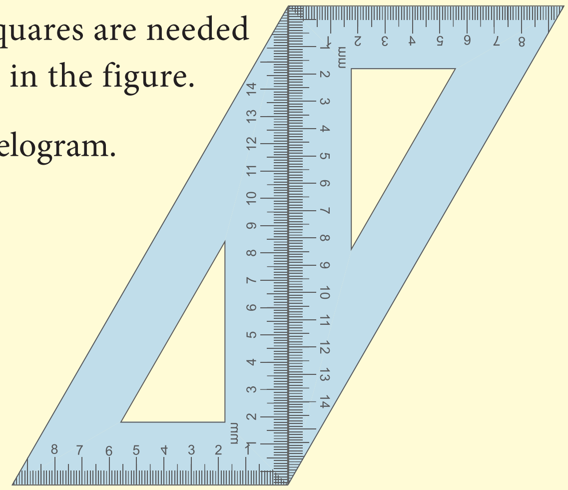
(iv) One of the angles is a right angle.

(v) Two of the angles are right angles.

(vi) The diagonals are mutually $\perp^r$.

Activity

1. A pair of identical 30º–60º–90º set-squares are needed for this activity. Place them as shown in the figure.



   (i) What is the shape we get? It is a parallelogram.

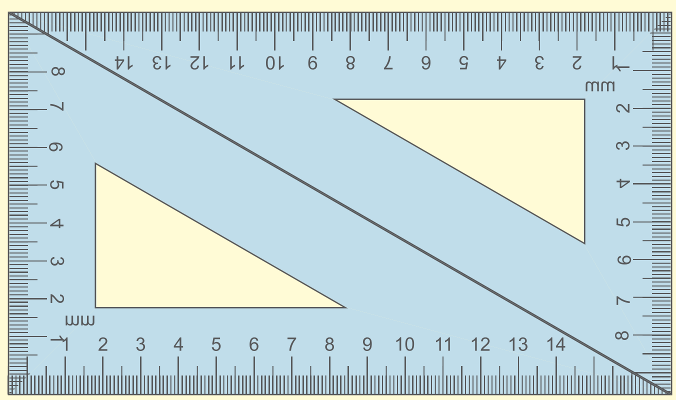
   (ii) Are the opposite sides parallel?

   (iii) Are the opposite sides equal?

   (iv) Are the diagonals equal?

   (v) Can you get this shape by using any other pair of identical set-squares?

2. We need a pair of 30º–60º–90º set- squares for this activity. Place them as shown in the figure.



   (i) What is the shape we get?

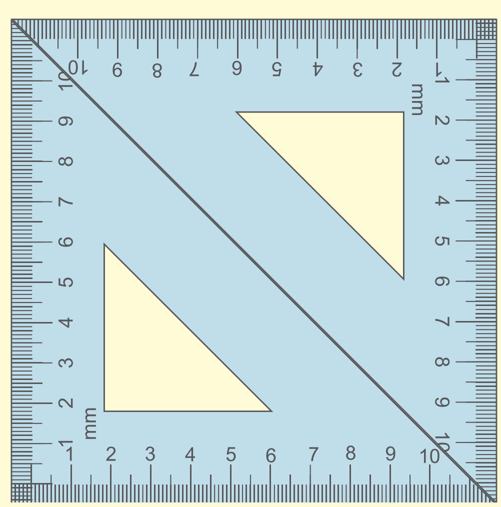
   (ii) Is it a parallelogram?

   It is a quadrilateral; infact it is a rectangle. (How?)

   (iii) What can we say about its lengths of sides, angles and diagonals?

   Discuss and list them out.

3. Repeat the above activity, this time with a pair of 45º–45º–90º set-squares.



   (i) How does the figure change now? Is it a parallelogram? It becomes a square! (How did it happen?)

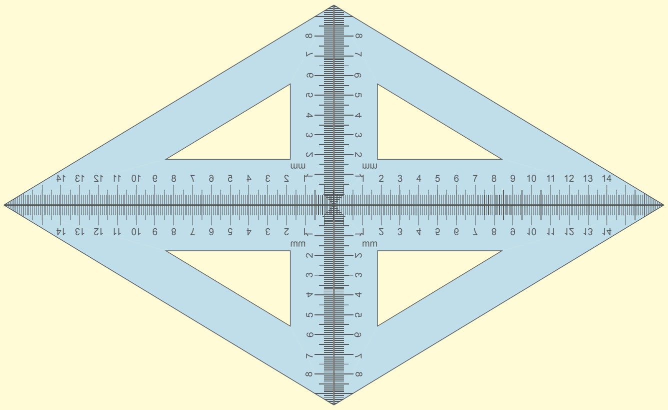
   (ii) What can we say about its lengths of sides, angles and diagonals?

   Discuss and list them out.

   (iii) How does it differ from the list we prepared for the rectangle?

Activity

4. We again use four identical 30º–60º–90º set- squares for this activity. Note carefully how they are placed touching one another.



(i) Do we get a parallelogram now?

(ii) What can we say about its lengths of sides, angles and diagonals?

(iii) What is special about their diagonals?

Based on the outcome of the above activities, we can list out the various properties of the above quadrilaterals, all of which happen to be parallelograms!

Special Quadrilaterals	All sides	All angles	Opposite Sides		All angles	Opposite angles	Diagonals	
	Equal	Equal	Equal	Parallel	90º	Supple-mentary	Bisect each other	Cut at rt.angles
Parallelogram	Sometimes	Sometimes	Always	Always	Sometimes	Sometimes	Always	Sometimes
Rhombus	Always	Sometimes	Always	Always	Sometimes	Sometimes	Always	Always
Rectangle	Sometimes	Always	Always	Always	Always	Always	Always	Sometimes
Square	Always	Always	Always	Always	Always	Always	Always	Always

Try these

1. Say True or False:

(a) A square is a special rectangle.

(b) A square is a parallelogram.

(c) A square is a special rhombus.

(d) A rectangle is a parallelogram

2. Name the quadrilaterals

(a) which have diagonals bisecting each other.

(b) In which the diagonals are perpendicular bisectors of each other.

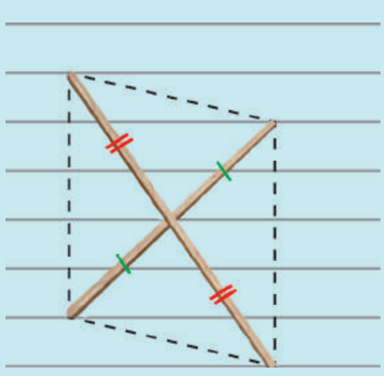
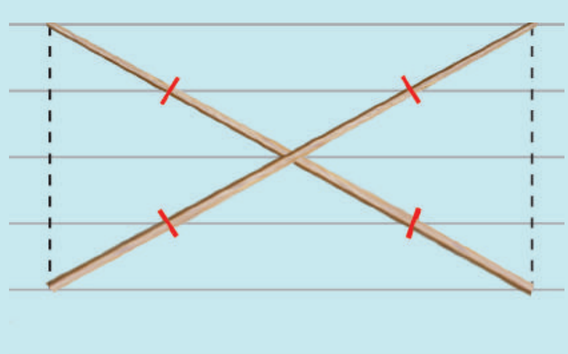
(c) Which have diagonals of different lengths.

(d) Which have equal diagonals.

(e) Which have parallel opposite sides.

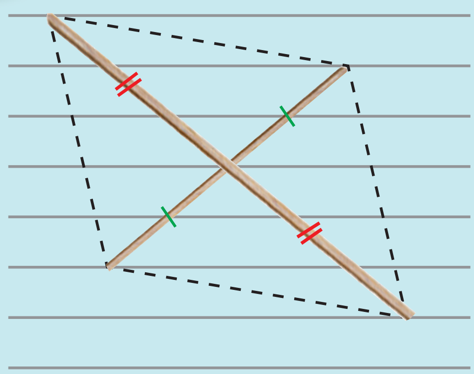
(f) In which opposite angles are equal.

3. Two sticks are placed on a ruled sheet as shown. What figure is formed if the four corners of the sticks are joined?

(a)	 Two unequal sticks. Placed such that their midpoints coincide.	(b)	 Two equal sticks. Placed such that their midpoints coincide.

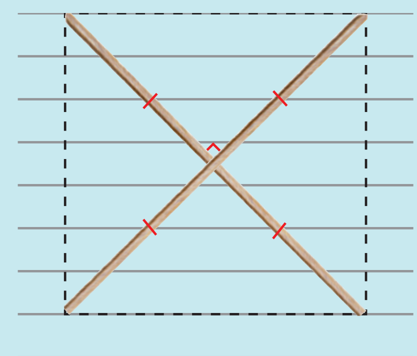
Try these

(c)



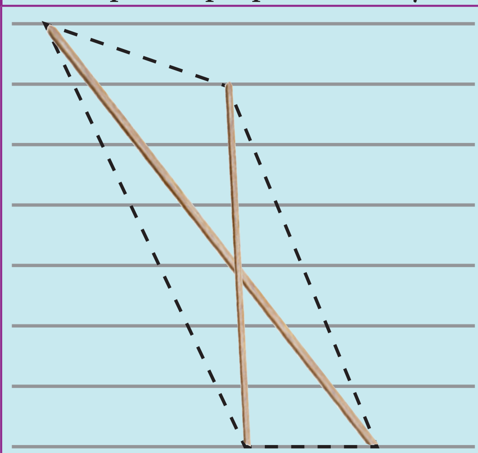
Two unequal sticks. Placed intersecting at mid points perpendicularly.

(d)



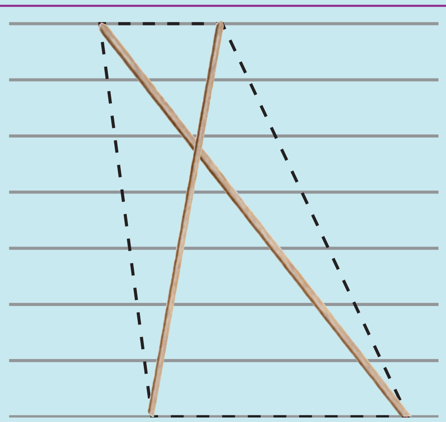
Two equal sticks. Placed intersecting at mid points perpendicularly.

(e)



Two unequal sticks. Tops are not on the same ruling. Bottoms on the same ruling. Not cutting at the mid point of either.

(f)



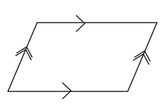
Two unequal sticks. Tops on the same ruling. Bottoms on the same ruling. Not necessarily cutting at the mid point of either.

5.13 Construction of a Parallelogram

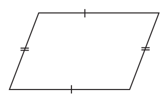
Let us construct a parallelogram with the given measurements

1. Two adjacent sides and one angle.	2. Two adjacent sides and one diagonal.
3. Two diagonals and one included angle.	4. One side, one diagonal and one angle.

Note



Similar arrows indicates the parallel sides.



Similar lines indicates the congruent sides.

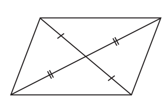


Figure shows that the diagonals bisect each other.

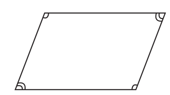


Figure shows that the opposite angles are congruent.

5.13.1 Constructing a parallelogram when its two adjacent sides and one angle are given

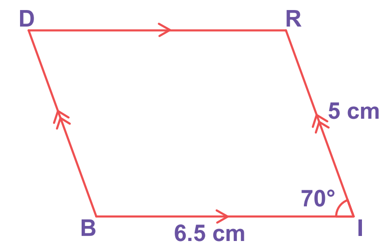
Example 5.30

Construct a parallelogram BIRD with BI=6.5*cm*, IR=5*cm* and $\angle BIR = 70°$. Also find its area.

Solution:

Given:

BI=6.5*cm*, IR=5*cm* and $\angle BIR = 70°$



Rough diagram

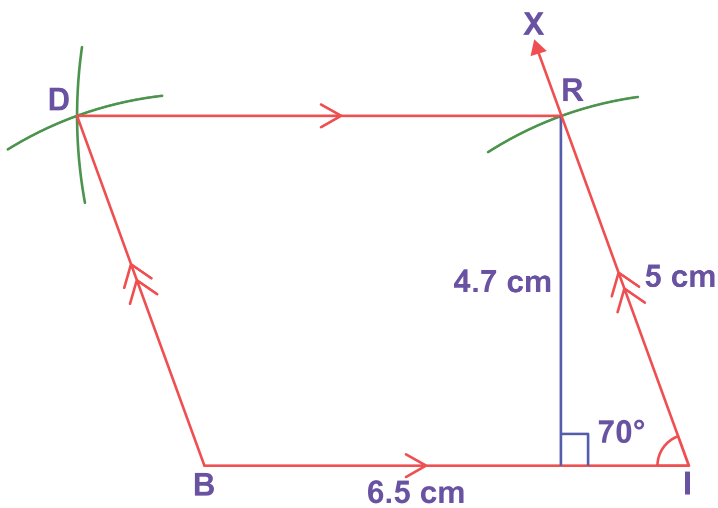


Fig. 5.49

Steps:

1. Draw a line segment BI=6.5*cm*.
2. Make an angle $\angle BIX = 70°$ at I on $\overline{BI}$.
3. With I as centre, draw an arc of radius 5*cm* cutting IX at R.
4. With B and R as centres, draw arcs of radii 5*cm* and 6.5*cm* respectively. Let them cut at D.
5. Join BD and RD.
6. BIRD is the required parallelogram.

Calculation of Area:

Area of the parallelogram BIRD $= bh$ sq.units

$= 6.5 \times 4.7 = 30.55\, sq.cm$

5.13.2 Constructing a parallelogram when its two adjacent sides and one diagonal are given

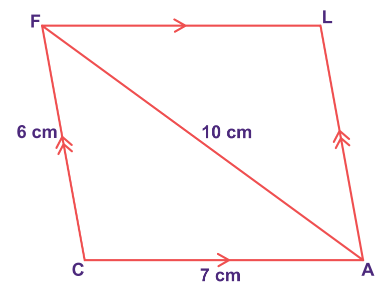
Example 5.31

Construct a parallelogram CALF with CA=7cm, CF=6cm and AF=10cm. Also find its area.

Solution:

Given:

CA=7cm, CF=6cm and AF=10cm



Rough diagram

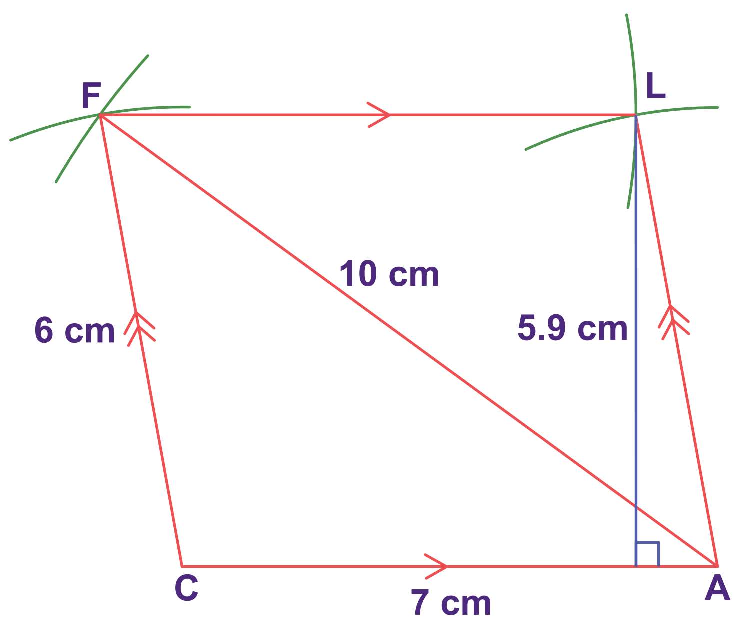


Fig. 5.50

Steps:

1. Draw a line segment CA=7cm.
2. With C and A as centres, draw arcs of radii 6cm and 10cm respectively. Let them cut at F.
3. Join CF and AF.
4. With A and F as centres, draw arcs of radii 6cm and 7cm respectively. Let them cut at L.
5. Join AL and FL.
6. CALF is the required parallelogram.

Calculation of Area:

Area of the parallelogram CALF = bh sq.units

$= 7 \times 5.9 = 41.3\, sq.cm$

5.13.3 Constructing a parallelogram when its two diagonals and one included angle are given

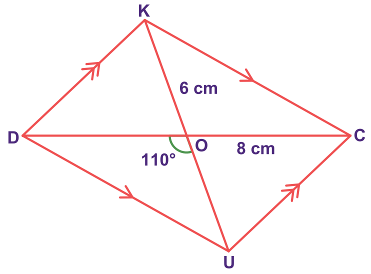
Example 5.32

Construct a parallelogram DUCK with DC=8*cm*, UK=6*cm* and $\angle DOU = 110°$. Also find its area.

Solution:

Given:

DC=8*cm*, UK=6*cm* and $\angle DOU = 110°$



Rough diagram

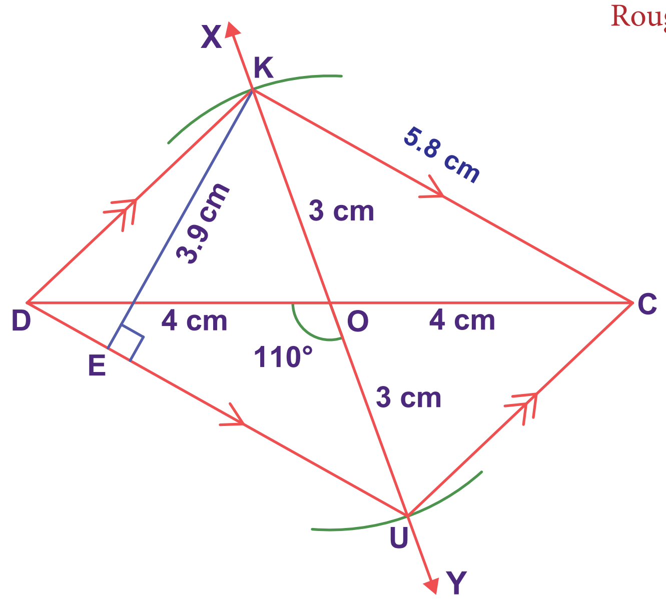


Fig. 5.51

Steps:

1. Draw a line segment DC=8*cm*.
2. Mark O the midpoint of $\overline{DC}$.
3. Draw a line $\overleftrightarrow{XY}$ through O which makes $\angle DOY = 110°$.
4. With O as centre and 3*cm* as radius draw two arcs on $\overleftrightarrow{XY}$ on either sides of $\overline{DC}$. Let the arcs cut $\overrightarrow{OX}$ at K and $\overrightarrow{OY}$ at U
5. Join $\overline{DU}$, $\overline{UC}$, $\overline{CK}$ and $\overline{KD}$.
6. DUCK is the required parallelogram.

Calculation of Area:

Area of the parallelogram DUCK = bh sq.units

$= 5.8 \times 3.9 = 22.62\, sq.cm$

5.13.4 Constructing a parallelogram when its one side, one diagonal and one angle are given

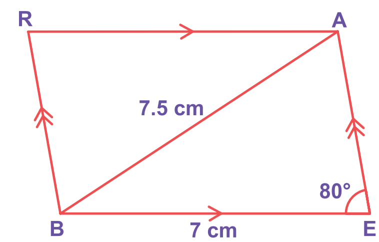
Example 5.33

Construct a parallelogram BEAR with BE=7cm, BA=7.5cm and $\angle BEA = 80°$. Also find its area.

Solution:

Given:

BE=7cm, BA=7.5cm and $\angle BEA = 80°$



Rough diagram

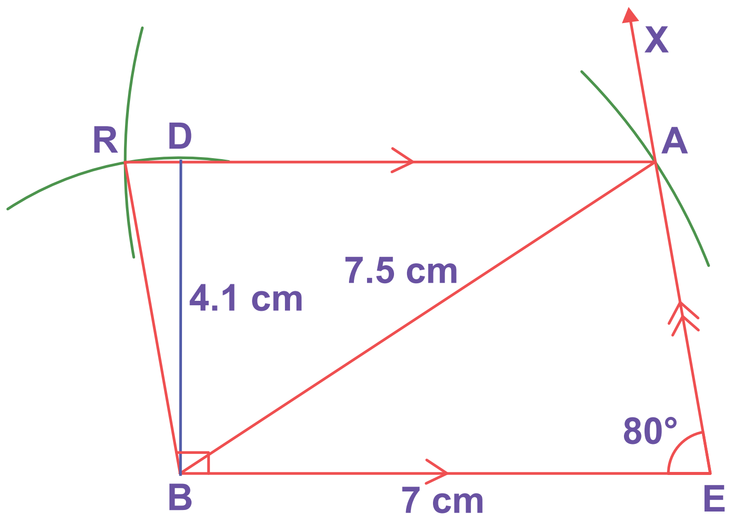


Fig. 5.52

Steps:

1. Draw a line segment BE=7cm.
2. Make an angle $\angle BEX = 80°$ at E on $\overline{BE}$.
3. With B as centre, draw an arc of radius 7.5cm cutting EX at A and Join BA.
4. With B as centre, draw an arc of radius equal to the length of $\overline{AE}$.
5. With A as centre, draw an arc of radius 7cm. Let both arcs cut at R.
6. Join BR and AR.
7. BEAR is the required parallelogram.

Calculation of Area:

Area of the parallelogram BEAR = bh sq.units

= $7 \times 4.1 = 28.7$ $sq.cm$

5.14 Construction of a Rhombus

Let us now construct a rhombus with the given measurements

(i) One side and one diagonal

(ii) One side and one angle

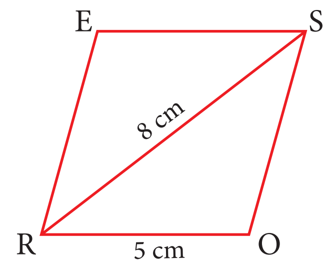
(iii) Two diagonals

(iv) One diagonal and one angle

5.14.1 Construction of a rhombus when one side and one diagonal are given

Example 5.34

Construct a rhombus ROSE with RO = 5 cm and RS = 8 cm. Also find its area.



Rough diagram

Solution:

Given: RO = 5 cm and RS = 8 cm

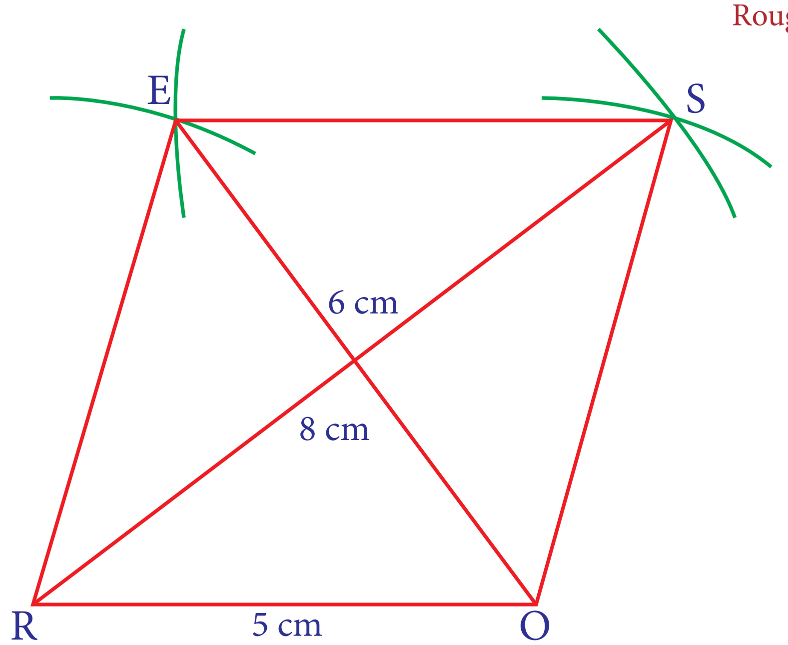


Fig. 5.53

Steps:

1. Draw a line segment RO = 5 cm.
2. With R and O as centres, draw arcs of radii 8 cm and 5 cm respectively and let them cut at S.
3. Join RS and OS.
4. With R and S as centres, draw arcs of radius 5 cm each and let them cut at E.
5. Join RE and SE.
6. ROSE is the required rhombus.

Calculation of Area:

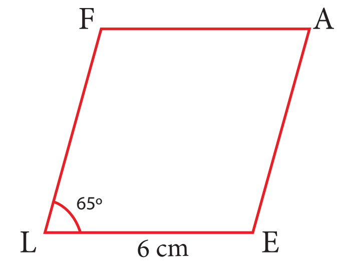
Area of rhombus ROSE $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 8 \times 6 = 24 sq.cm$

5.14.2 Construction of a rhombus when one side and one angle are given

Example 5.35

Construct a rhombus LEAF with LE = 6 cm and $\angle L = 65°$. Also find its area.



Rough diagram

Solution:

Given: LE = 6 cm and $\angle L = 65°$

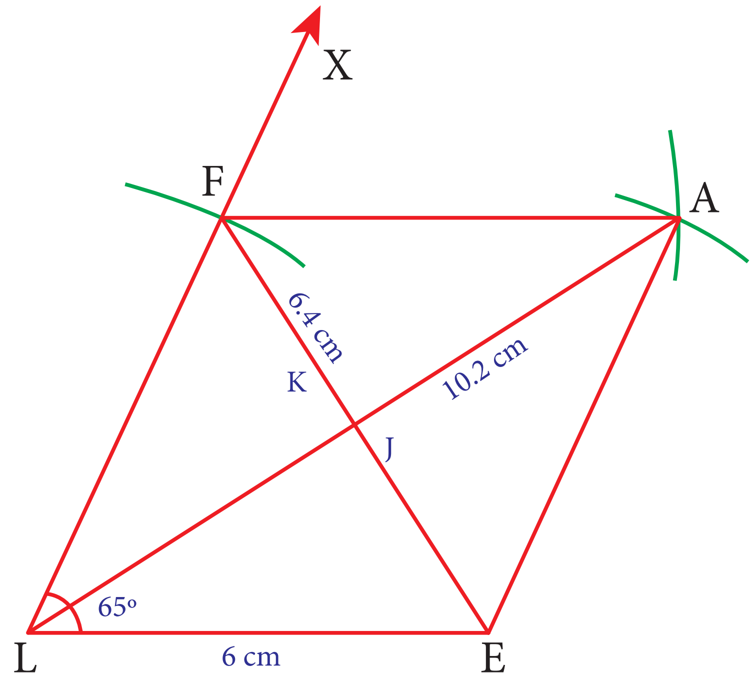


Fig. 5.54

Steps:

1. Draw a line segment LE = 6 cm.
2. At L on LE, make $\angle ELX = 65°$.
3. With L as centre draw an arc of radius 6 cm. Let it cut LX at F.
4. With E and F as centres, draw arcs of radius 6 cm each and let them cut at A.
5. Join EA and AF.
6. LEAF is the required rhombus.

Calculation of Area:

Area of rhombus LEAF $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 6.4 \times 10.2 = 32.64 \; sq.cm$

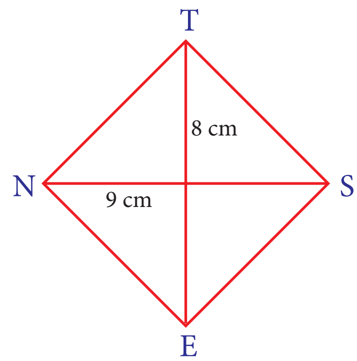
5.14.3 Construction of a rhombus when two diagonals are given

Example 5.36

Construct a rhombus NEST with NS = 9 cm and ET = 8 cm. Also find its area.

Solution:

Given: NS = 9 cm and ET = 8 cm



Rough diagram

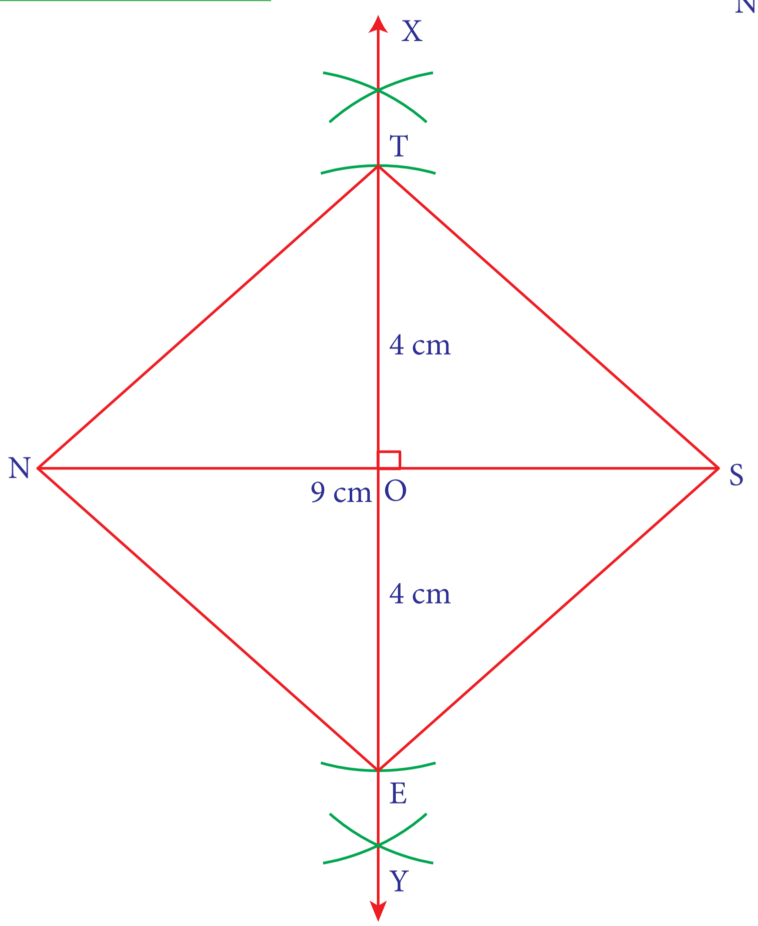


Fig. 5.55

Steps:

1. Draw a line segment NS = 9 cm.
2. Draw the perpendicular bisector XY to NS. Let it cut NS at O.
3. With O as centre, draw arcs of radius 4 cm on either side of O which cut OX at T and OY at E.
4. Join NE, ES, ST and TN.
5. NEST is the required rhombus.

Calculation of Area:

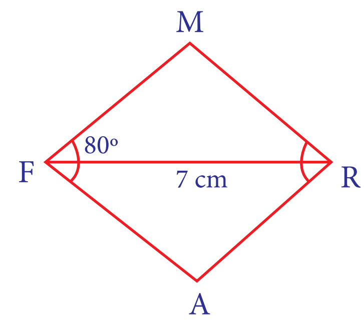
Area of rhombus NEST $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 9 \times 8 = 36 \; sq.cm$

5.14.4 Construction of a rhombus when one diagonal and one angle are given

Example 5.37

Construct a rhombus FARM with FR = 7 cm and $\angle F = 80°$. Also find its area.



Rough diagram

Solution:

Given: FR = 7 cm and $\angle F = 80°$

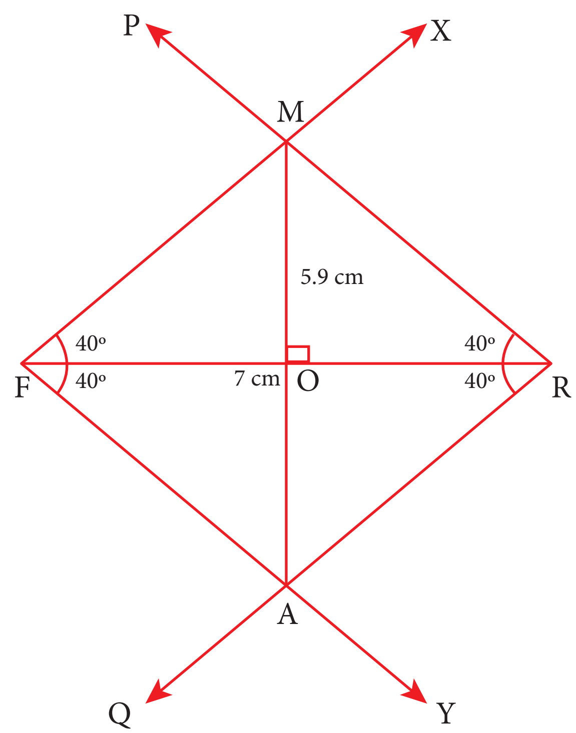


Fig. 5.56

Steps:

1. Draw a line segment FR = 7 cm.
2. At F, make $\angle RFX = \angle RFY = 40°$ on either side of FR.
3. At R, make $\angle FRP = \angle FRQ = 40°$ on either side of FR.
4. Let FX and RP cut at M and FY and RQ cut at A.
5. FARM is the required rhombus.

Calculation of Area:

Area of rhombus FARM $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 7 \times 5.9 = 20.65 \; sq.cm$

5.15 Construction of a Rectangle

Let us now construct a rectangle with the given measurements

(i) length and breadth

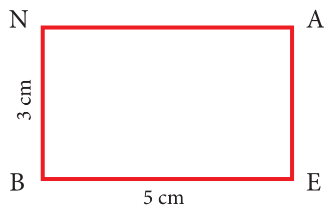
(ii) a side and a diagonal

5.15.1 Construction of a rectangle when its length and breadth are given

Example 5.38

Construct a rectangle BEAN with BE = 5 cm and BN = 3 cm. Also find its area.

Solution:



Rough diagram

Given: BE = 5 cm and BN = 3 cm

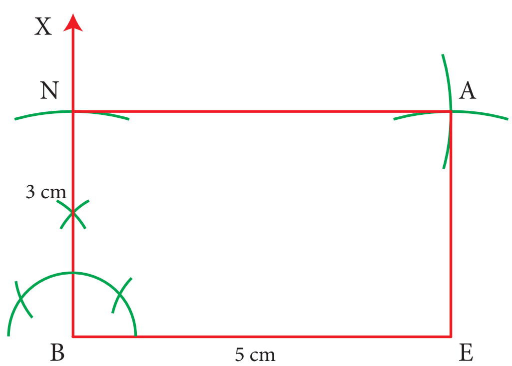


Fig. 5.57

Steps:

1. Draw a line segment BE = 5 cm.
2. At B, construct $BX \perp BE$.
3. With B as centre, draw an arc of radius 3 cm and let it cut BX at N.
4. With E and N as centres, draw arcs of radii 3 cm and 5 cm respectively and let them cut at A.
5. Join EA and NA.
6. BEAN is the required rectangle.

Calculation of Area:

Area of rectangle BEAN $= l \times b \; sq.units$

$= 5 \times 3 = 15 \; sq.cm$

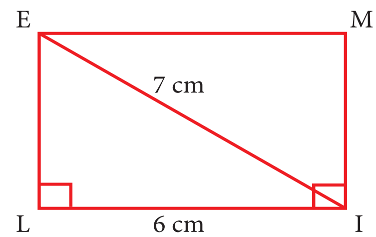
5.15.2 Construction of a rectangle when a side and a diagonal are given

Example 5.39

Construct a rectangle LIME with LI = 6 cm and IE = 7 cm. Also find its area.

Solution:

Given: LI = 6 cm and IE = 7 cm



Rough diagram

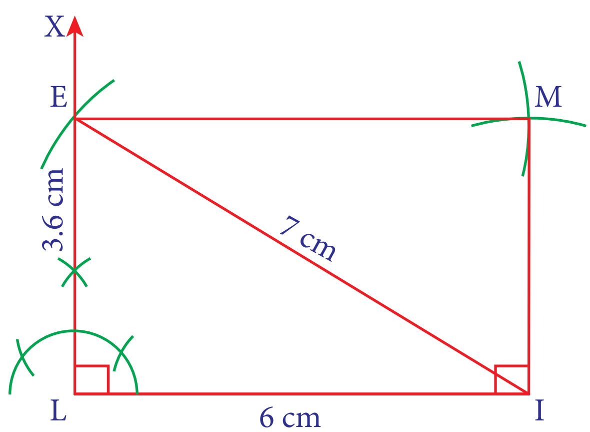


Fig. 5.58

Steps:

1. Draw a line segment LI = 6 cm.
2. At L, construct $LX \perp LI$.
3. With I as centre, draw an arc of radius 7 cm and let it cut LX at E.
4. With I as centre and LE as radius draw an arc. Also, with E as centre and LI as radius draw an another arc. Let them cut at M.
5. Join IM and EM.
6. LIME is the required rectangle.

Calculation of Area:

Area of rectangle LIME $= l \times b\ sq.units$

$= 6 \times 3.6 = 21.6\ sq.cm$

5.16 Construction of a Square

Let us now construct a square when (i) its side is given and (ii) its diagonal is given

5.16.1 Construction of a square when its side is given

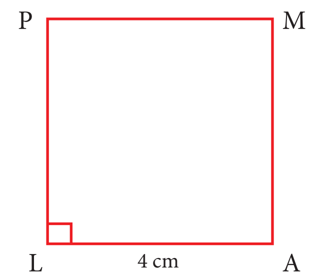
E5Y1R6

Example 5.40

Construct a square LAMP of side 4 cm. Also find its area.

Solution:

Given: side = 4 cm



Rough diagram

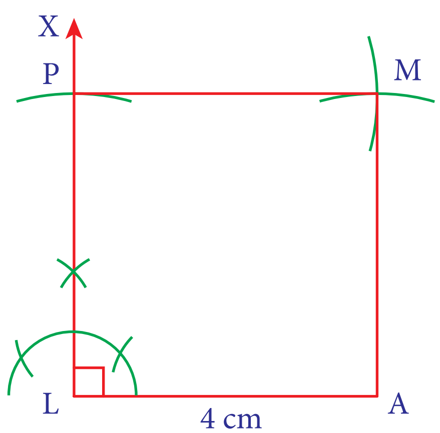


Fig. 5.59

Steps:

1. Draw a line segment LA = 4 cm.
2. At L, construct $LX \perp LA$.
3. With L as centre, draw an arc of radius 4 cm and let it cut LX at P.
4. With A and P as centres, draw arcs of radius 4 cm each and let them cut at M.
5. Join AM and PM. LAMP is the required square.

Calculation of Area:

Area of square LAMP $= a^2\ sq.units$

$= 4 \times 4 = 16\ sq.cm$

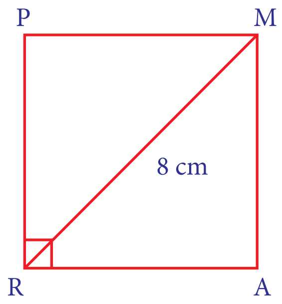
5.16.2 Construction of a square when its diagonal is given

Example 5.41

Construct a square RAMP of a diagonal 8 cm. Also find its area.

Solution:

Given: diagonal = 8 cm



Rough diagram

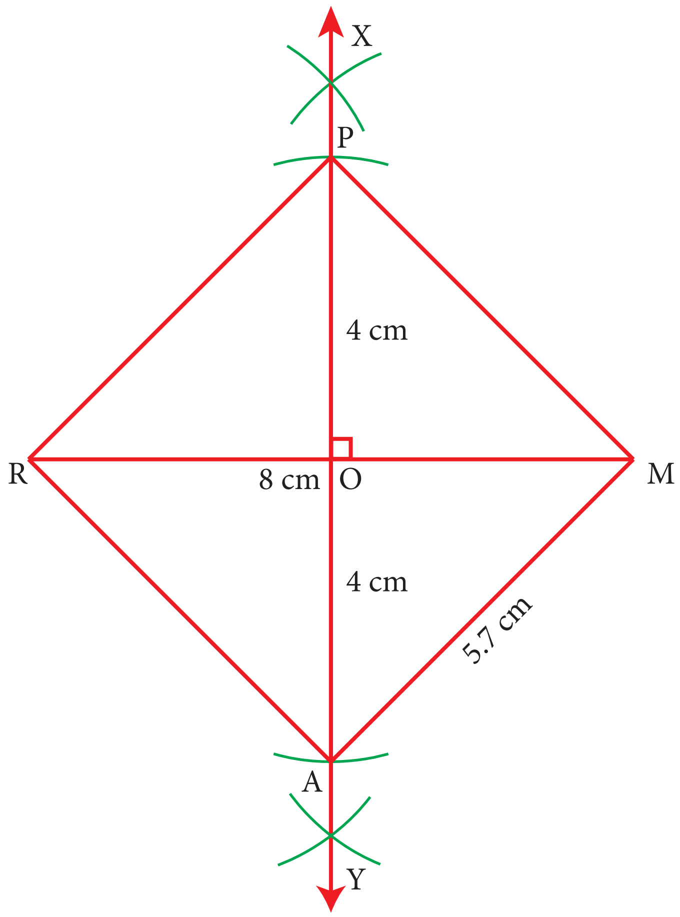


Fig. 5.60

Steps:

1. Draw a line segment RM = 8 cm.
2. Draw the perpendicular bisector XY to RM. Let it bisect RM at O.
3. With O as centre, draw arcs of radius 4 cm on either side of O which cut OX at P and OY at A.
4. Join RA, AM, MP and PR.
5. RAMP is the required square.

Calculation of Area:

Area of square RAMP $= a^2$ *sq.units*

$= 5.7 \times 5.7 = 32.49$ *sq.cm*

Exercise 5.5

I. Construct the following parallelograms with the given measurements and find their area.

1. ARTS, AR=6*cm*, RT=5*cm* and $\angle ART = 70°$.
2. CAMP, CA=6*cm*, AP=8*cm* and CP=5.5*cm*.
3. EARN, ER=10*cm*, AN=7*cm* and $\angle EOA = 110°$ where $\overline{ER}$ and $\overline{AN}$ intersect at O.
4. GAIN, GA=7.5*cm*, GI=9*cm* and $\angle GAI = 100°$.

II. Construct the following rhombuses with the given measurements and also find their area.

(i) FACE, FA= 6 cm and FC = 8 cm

(ii) CAKE, CA=5 cm and ∠A = 65°

(iii) LUCK, LC = 7.8 cm and UK = 6 cm

(iv) PARK, PR = 9 cm and ∠P =70°

III. Construct the following rectangles with the given measurements and also find their area.

(i) HAND, HA = 7 cm and AN = 4 cm

(ii) LAND, LA = 8 cm and AD = 10 cm

IV. Construct the following squares with the given measurements and also find their area.

(i) EAST, EA = 6.5 cm

(ii) WEST, WS = 7.5 cm

SUMMARY

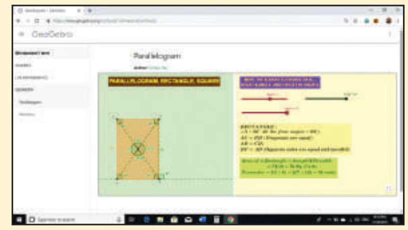
- Congruent figures are exactly the same in shape and size.
- Similar figures have the same shape but different sizes.
- In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. This is known as Pythagoras theorem.
- The three medians of any triangle are concurrent. The point of concurrence of the three medians in a triangle is called its Centroid, denoted by the letter G.
- The three altitudes of any triangle are concurrent. The point of concurrence of the three altitudes of a triangle is called as its Orthocentre, denoted by the letter H.
- The three perpendicular bisectors of the sides of any triangle are concurrent. The point of concurrence of the three perpendicular bisectors of a triangle is called as its Circumcentre, denoted by the letter S.
- The three angle bisectors of any triangle are concurrent. The point of concurrence of the three angle bisectors of a triangle is called as its Incentre, denoted by the letter I.
- A trapezium is a quadrilateral in which a pair of opposite sides are parallel.
- A parallelogram is a quadrilateral in which the opposite sides are parallel.
- Rhombus is a parallelogram in which all its sides are congruent.
- Rectangle is a parallelogram whose all its angles are right angles.
- Square is a parallelogram in which all its sides and angles are equal.

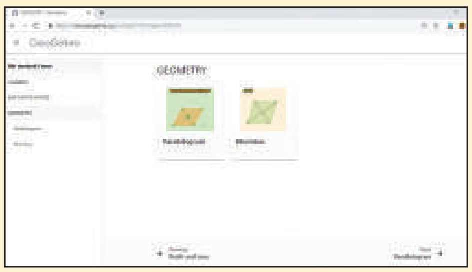
ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "GEOMETRY" will open. Click on the worksheet named "Parallelogram".

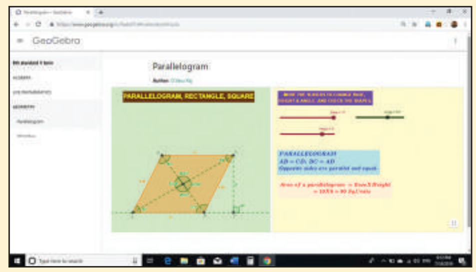
Step-2 In the given worksheet you can move the sliders Base, Height and the angle. Check for what value(s) the parallelogram becomes rectangle and square. Study the properties.

Expected Outcome





Step 1



Step 2

B358_8_MATHS_EM

Browse in the link

Geometry:

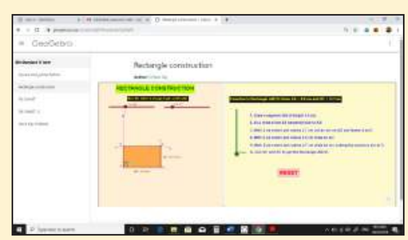
https://www.geogebra.org/m/fqxbd7rz#chapter/409576 or Scan the QR Code.

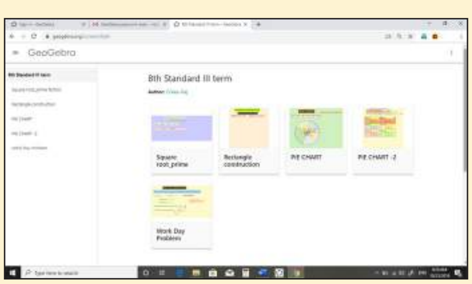
ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Rectangle Construction".

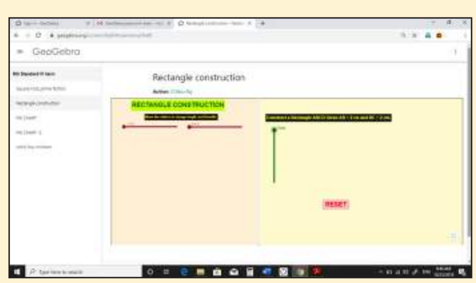
Step-2 Move the sliders on left side to change the length and breadth of the rectangle. Drag the slider step by step on right side to see the steps for construction.

Expected Outcome





Step 1



Step 2

B358_8_MATHS_EM

Browse in the link

Geometry:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.