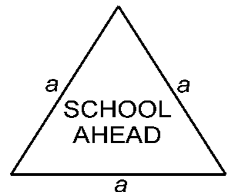
12 HERON'S FORMULA

EXERCISE 12.1

Q.1. *A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?*



Sol. Each side of the triangle = a

Perimeter of the triangle = $3a$

$\therefore\ s = \frac{3a}{2}$

$\therefore$ Area of the signal board (triangle) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{s(s-a)(s-a)(s-a)}$ $\quad [\because\ a = b = c]$

$= (s-a)\sqrt{s(s-a)} = \left(\frac{3a}{2} - a\right)\sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)}$

$= \frac{a}{2}\cdot\sqrt{\frac{3a^2}{4}} = \frac{a}{2}\cdot\frac{a}{2}\sqrt{3} = \frac{a^2}{4}\sqrt{3}$

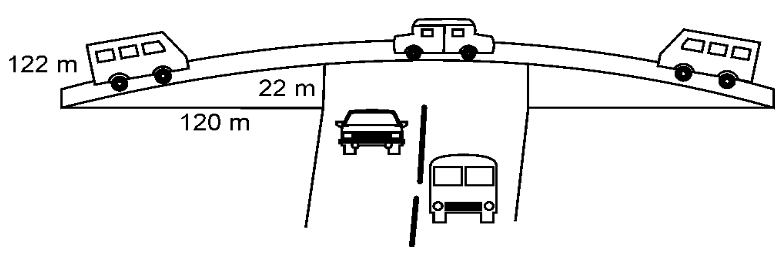
Hence, area of the signal board = $\mathbf{\frac{a^2}{4}\sqrt{3}}$ **sq units Ans.**

Now, perimeter = 180 cm

Each side of the triangle = $\frac{180}{3}$ cm = 60 cm

Area of the triangle = $\frac{(60)^2}{4} \times \sqrt{3}$ cm^2 = $\mathbf{900\sqrt{3}}$ **cm^2 Ans.**

Q.2. *The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig.). The advertisements yield an earning of Rs 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?*



Sol. Here, we first find the area of the triangular side walls.

a = 122 m, $\quad b$ = 120 m and $\quad c$ = 22 m

$\therefore\ s = \frac{122 + 120 + 22}{2}$ m = 132 m.

Area of the triangular side wall = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{132\,(132-122)\,(132-120)\,(132-22)}\ \text{m}^2$$

$$= \sqrt{132 \times 10 \times 12 \times 110}\ \text{m}^2 = 1320\ \text{m}^2$$

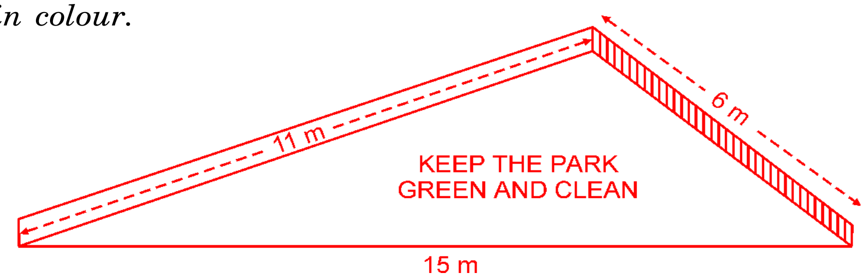
Rent of 1 m^2 of the wall for 1 year = Rs 5000

$\therefore$ Rent of 1 m^2 of the wall for 1 month = Rs $\frac{5000}{12}$

$\therefore$ Rent of the complete wall (1320 m^2) for 3 months

$$= \text{Rs}\ \frac{5000}{12} \times 1320 \times 3 = \textbf{Rs 16,50,000 Ans.}$$

Q.3. *There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig.). If the sides of the wall are* 15 *m,* 11 *m and* 6 *m, find the area painted in colour.*



Sol. Here a = 15 m, b = 11 m, c = 6 m

$$\therefore\ s = \frac{a+b+c}{2} = \frac{15+11+6}{2}\ \text{m} = 16\ \text{m}$$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16\,(16-15)\,(16-11)\,(16-6)}\ \text{m}^2$$

$$= \sqrt{16 \times 1 \times 5 \times 10}\ \text{m}^2 = 20\sqrt{2}\ \text{m}^2$$

Hence, the area painted in colour = $\mathbf{20\sqrt{2}\ m^2}$ **Ans.**

Q.4. *Find the area of a triangle two sides of which are* 18 *cm and* 10 *cm and the perimeter is* 42 *cm.*

Sol. Here a = 18 cm, b = 10 cm, c = ?

Perimeter of the triangle = 42 cm

$\Rightarrow a + b + c = 42$

$\Rightarrow 18 + 10 + c = 42$

$\Rightarrow c = 42 - 28 = 14$

Now, $s = \frac{a+b+c}{2} = \frac{42}{2}$ cm = 21 cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-18)\,(21-10)\,(21-14)}\ \text{cm}^2$$

$$= \sqrt{21 \times 3 \times 11 \times 7}\ \text{cm}^2 = \sqrt{7 \times 3 \times 3 \times 11 \times 7}\ \text{cm}^2$$

$$= 7 \times 3\ \sqrt{11}\ \text{cm}^2 = \mathbf{21\sqrt{11}\ cm^2}\ \textbf{Ans.}$$

Q.5. *Sides of a triangle are in the ratio of* 12 : 17 : 25 *and its perimeter is* 540 *cm. Find its area.*

Sol. Let the sides of the triangle be $12x$ cm $17x$ cm and $25x$ cm.

Perimeter of the triangle = 540 cm

$\therefore\ 12x + 17x + 25x = 540$

$\Rightarrow 54x = 540$

$$\Rightarrow \quad x = \frac{540}{54} = 10$$

$\therefore$ Sides of the triangle are (12 × 10) cm, (17 × 10) cm and (25 × 10) cm i.e., 120 cm, 170 cm and 250 cm.

Now, suppose a = 120 cm, b = 170 cm, c = 250 cm,

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{540}{2}\text{ cm} = 270 \text{ cm}$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{270\,(270-120)\,(270-170)\,(270-250)}\text{ cm}^2$$

$$= \sqrt{270 \times 150 \times 100 \times 20}\text{ cm}^2 = \mathbf{9000\ cm^2}\ \textbf{Ans.}$$

Q.6. *An isosceles triangle has perimeter* 30 *cm and each of the equal sides is* 12 *cm. Find the area of the tirangle.*

Sol. Here, $a = b = 12$ cm,

Also, $a + b + c = 30 \quad \Rightarrow 12 + 12 + c = 30 \quad \Rightarrow c = 30 - 24 = 6$

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{30}{2}\text{ cm} = 15 \text{ cm}$$

$$\therefore \quad \text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15\,(15-12)\,(15-12)\,(15-6)}\text{ cm}^2$$

$$= \sqrt{15 \times 3 \times 3 \times 9}\text{ cm}^2 = \mathbf{9\sqrt{15}\ cm^2}\ \textbf{Ans.}$$

12 HERON'S FORMULA

EXERCISE 12.2

Q.1. *A park, in the shape of a quadrilateral ABCD, has $\angle C = 90°$, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?*

Sol. ABCD is the park as shown in the figure.

Join BD.

In ΔDBC, we have

$DB^2 = BC^2 + CD^2$ [Pythagoras theorem]

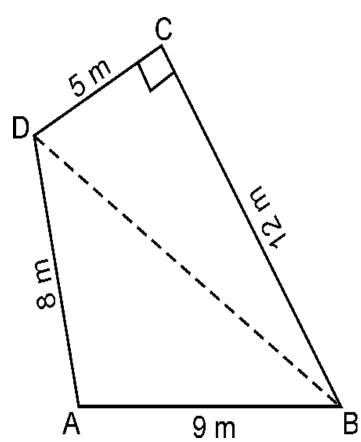
$\Rightarrow \quad DB^2 = (12)^2 + 5^2$

$\Rightarrow \quad DB = \sqrt{144 + 25} = \sqrt{169}$

$\Rightarrow \quad DB = 13$ m.

Area of $\Delta DBC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 12 \times 5 \text{ m}^2 = 30 \text{ m}^2$



In ΔABD, a = 9 m, b = 8 m, c = 13 m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{9+8+13}{2} \text{ m} = 15 \text{ m}$$

$$\therefore \quad \text{Area of } \Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15\,(15-9)\,(15-8)\,(15-13)} \text{ m}^2$$

$$= \sqrt{15 \times 6 \times 7 \times 2} \text{ m}^2$$

$$= \sqrt{1260} \text{ m}^2 = 35.5 \text{ m}^2 \text{ (approx.)}$$

$\therefore$ Area of the park = area of ΔDBC + area of ΔABD

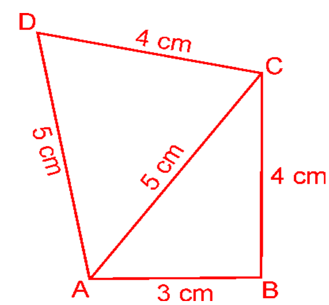
= (30 + 35.5) m^2 **= 65.5 m^2 Ans.**

Q.2. *Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.*

Sol. In ΔABC, we have

$AB^2 + BC^2 = 9 + 16 = 25$

$= AC^2$



Hence, ABC is a right triangle, right angled at B

[By converse of Pythagoras theorem]

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 3 \times 4 \text{ cm}^2 = 6 \text{ cm}^2.$$

In ΔACD, a = 5 cm, b = 4 cm, c = 5 cm.

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{5+4+5}{2} \text{ cm} = 7 \text{ cm}$$

$$\therefore \text{ Area of } \Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$$

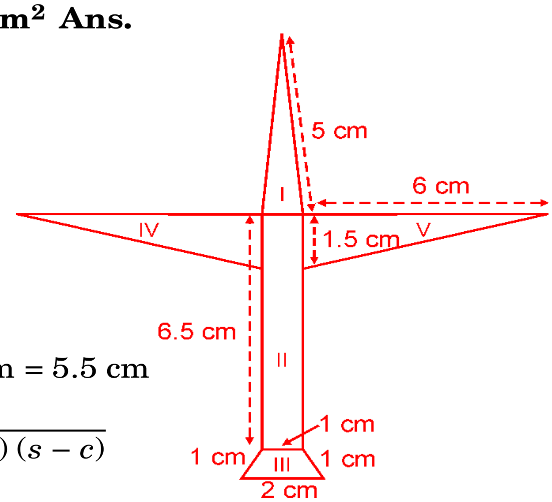
$$= \sqrt{7 \times (7-5)\,(7-4)\,(7-5)} \text{ cm}^2 = \sqrt{7 \times 2 \times 3 \times 2} \text{ cm}^2$$

$$= \sqrt{84} \text{ cm}^2 = 9.2 \text{ cm}^2 \text{ (approx.)}$$

$\therefore$ Area of the quadrilateral = area of ΔABC + area of ΔACD

= (6 + 9.2) cm^2 **= 15.2 cm^2 Ans.**

Q.3. *Radha made a picture of an aeroplane with coloured paper as shown in the figure. Find the total area of the paper used.*



Sol. For the triangle marked I :

a = 5 cm, b = 5 cm, c = 1 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{5+5+1}{2} \text{ cm} = \frac{11}{2} \text{ cm} = 5.5 \text{ cm}$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{5.5\,(5.5-5)\,(5.5-5)\,(5.5-1)}\ \text{cm}^2$$

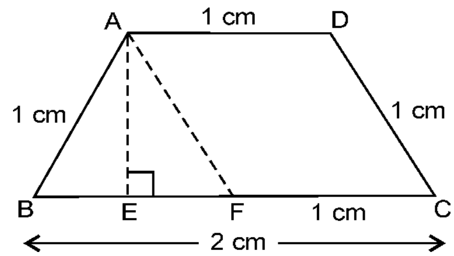
$$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5}\ \text{cm}^2 = \sqrt{6.1875}\ \text{cm}^2 = 2.5\ \text{cm}^2$$

For the rectangle marked II :

Length = 6.5 cm, Breadth = 1 cm

Area of the rectangle = $6.5 \times 1\ \text{cm}^2 = 6.5\ \text{cm}^2$

For the trapezium marked III :



Draw AF || DC and AE ⊥ BC.

AD = FC = 1 cm, DC = AF = 1 cm

∴ BF = BC – FC = (2 – 1) cm = 1 cm

Hence, ΔABF is equilateral.

Also, E is the mid-point of BF.

∴ BE = $\frac{1}{2}$ cm = 0.5 cm

Also, $AB^2 = AE^2 + BE^2$ [Pythagoras theorem]

$\Rightarrow AE^2 = 1^2 - (0.5)^2 = 0.75$

$\Rightarrow$ AE = 0.9 cm (approx.)

Area of the trapezium = $\frac{1}{2}$ (sum of the parallel sides) × distance between them.

$$= \frac{1}{2} \times (BC + AD) \times AE = \frac{1}{2} \times (2 + 1) \times 0.9\ \text{cm}^2 = 1.4\ \text{cm}^2.$$

For the triangle marked IV :

It is a right-triangle

∴ Area of the triangle = $\frac{1}{2}$ × base × height

$$= \frac{1}{2} \times 6 \times 1.5\ \text{cm cm}^2 = 4.5\ \text{cm}^2.$$

For the triangle marked V :

This triangle is congruent to the triangle marked IV.

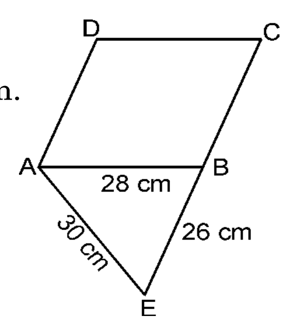
Hence, area of the triangle = $4.5\ \text{cm}^2$

Total area of the paper used = $(2.5 + 6.5 + 1.4 + 4.5 + 4.5)\ \text{cm}^2$

= 19.4 cm² Ans.

Q.4. *A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.*

Sol. In the figure, ABCD is a parallelogram and ABE is the triangle which stands on the base AB



For the triangle ABE, a = 30 cm, b = 28 cm, c = 26 cm.

$$\therefore\ s = \frac{a+b+c}{2} = \frac{30+28+26}{2}\ \text{cm} = 42\ \text{cm}$$

∴ Area of the ΔABE = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42\,(42-30)\,(42-28)\,(42-26)}\ \text{cm}^2$$

$= \sqrt{42 \times 12 \times 14 \times 16}$ cm^2 = $\sqrt{112896}$ cm^2

= 336 cm^2

Now, area of the parallelogram = base × height

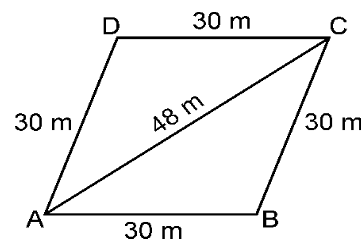
⇒ 336 = 28 × height [Given, area of the triangle = area of the parallelogram]

⇒ Height of the parallelogram = $\frac{336}{28}$ cm = **12 cm Ans.**

Q.5. *A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?*

Sol. Clearly, the diagonal AC of the rhombus divides it into two congruent triangles.

For triangle ABC, $a = b = 30$ m, $c = 48$ m.



$\therefore\ s = \frac{a+b+c}{2} = \frac{30+30+48}{2}$ m = 54 m

∴ Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54\,(54-30)\,(54-30)\,(54-48)}$ m^2

$= \sqrt{54 \times 24 \times 24 \times 6}$ m^2 = 432 m^2

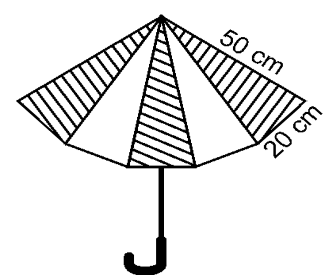
∴ Area of the rhombus = 2 × 432 m^2 = 864 m^2

Number of cows = 18

Hence, area of the grass field which each cow gets

$= \frac{864}{18}$ m^2 **= 48 m^2 Ans.**

Q.6. *An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.), each piece measuring 20 cm, 50 cm, and 50 cm. How much cloth of each colour is required for the umbrella?*



Sol. First we find the area of one triangular piece.

Here, $a = b = 50$ cm, $c = 20$ cm

$\therefore\ s = \frac{a+b+c}{2} = \frac{50+50+20}{2}$ cm = 60 m

∴ Area of one triangular piece = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60\,(60-50)\,(60-50)\,(60-20)}$ cm^2

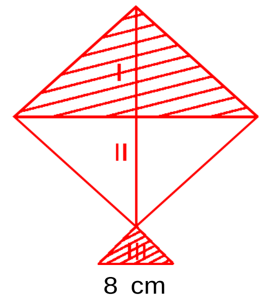
$= \sqrt{60 \times 10 \times 10 \times 40}$ cm^2 = $200\sqrt{6}$ cm^2

∴ Area of 10 such triangular pieces = 10 × $200\sqrt{6}$ cm^2

= $2000\sqrt{6}$ cm^2

Hence, cloth required for each colour = $\frac{2000\sqrt{6}}{2}$ cm^2 = **$1000\sqrt{6}$ cm^2 Ans.**

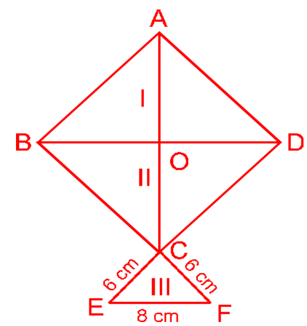
Q.7. *A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?*



Sol. ABCD is a square.

So, AO = OC = OB = OD

and $\angle AOB = 90°$ [Diagonals of a square bisect each other at right angles]



BD = 32 cm (Given) $\Rightarrow$ OA = $\frac{32}{2}$ cm = 16 cm.

ΔABD is a right triangle.

So, area of $\Delta ABD = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 32 \times 16 \text{ cm}^2 = 256 \text{ cm}^2$$

Thus, area of $\Delta BCD = 256 \text{ cm}^2$

For triangle CEF, $a = b = 6$ cm, $c = 8$ cm.

$$\therefore s = \frac{a+b+c}{2} = \frac{6+6+8}{2} \text{ cm} = 10 \text{ cm}$$

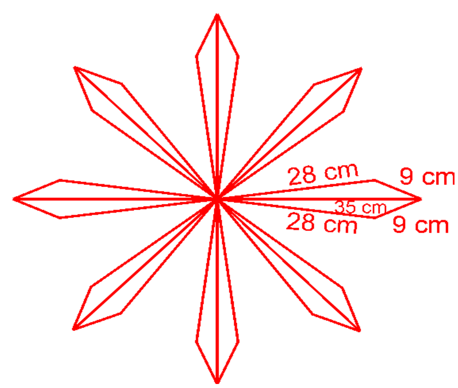
$$\therefore \text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{10(10-6)(10-6)(10-8)} \text{ cm}^2$$

$$= \sqrt{10 \times 4 \times 4 \times 2} \text{ cm}^2 = \sqrt{320} \text{ cm}^2 = 17.92 \text{ cm}^2$$

Hence, paper needed for shade I = **256 cm², for shade II = 256 cm² and for shade III = 17.92 cm² Ans.**

Q.8. *A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 p per cm².*



Sol. We have lengths of the sides of 1 triangular tile are $a = 35$ cm, $b = 28$ cm, $c = 9$ cm.

$$\therefore s = \frac{a+b+c}{2} = \frac{35+28+9}{2} \text{ cm} = 36 \text{ cm}$$

$$\therefore \text{Area of 1 triangular tile} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36(36-35)(36-28)(36-9)} \text{ cm}^2$$

$$= \sqrt{36 \times 1 \times 8 \times 27} \text{ cm}^2 = \sqrt{7776} \text{ cm}^2 = 88.2 \text{ cm}^2$$

$\therefore$ Area of 16 such tiles = $16 \times 88.2 \text{ cm}^2$

Cost of polishing 1 cm^2 = 50 p = Re 0.50

$\therefore$ Total cost of polishing the floral design = Rs 16 × 88.2 × 0.50

= Rs 705.60 Ans.

Q.9. *A field is in the shape of a trapezium whose parallel sides are* 25 *m and* 10 *m. The non-parallel sides are* 14 *m and* 13 *m. Find the area of the field.*

Sol. In the figure ABCD is the field. Draw CF || DA and CG ⊥ AB.

DC = AF = 10 m, AD = FC = 13 m

For ΔBCF, a = 15 m, b = 14 m, c = 13 m

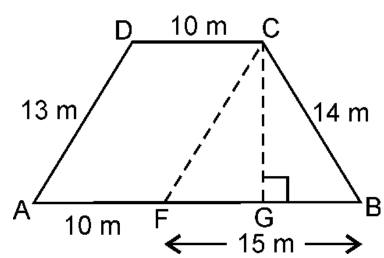
$$\therefore\ s = \frac{a+b+c}{2} = \frac{15+14+13}{2}\ \text{m} = 21\ \text{m}$$

$$\therefore\ \text{Area of } \Delta BCF = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21\,(21-15)\,(21-14)\,(21-13)}\ \text{m}^2$$

$$= \sqrt{21\times 6\times 7\times 8}\ \text{m}^2$$

$$= \sqrt{7056}\ \text{cm}^2 = 84\ \text{m}^2$$



$$\text{Also, area of } \Delta BCF = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times BF \times CG$$

$$\Rightarrow\quad 84 = \frac{1}{2} \times 15 \times CG$$

$$\Rightarrow\quad CG = \frac{84\times 2}{15}\ \text{m} = 11.2\ \text{m}$$

$$\therefore\ \text{Area of the trapezium} = \frac{1}{2} \times \text{sum of the parallel sides} \times \text{distance between them.}$$

$$= \frac{1}{2} \times (25 + 10) \times 11.2\ \text{m}^2$$

$$= 196\ \text{m}^2$$

Hence, area of the field **= 196 m^2 Ans.**