

# ELLIPSE

## SYNOPSIS

- A conic is said to be an ellipse if its eccentricity is less than 1
- Standard equation of the ellipse is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- The general second degree equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents an ellipse if  $h^2 - ab < 0$  and  $\Delta \neq 0$
- If  $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c$  and  $S = 0$  represents an ellipse then to find the center of the ellipse, solve the equations  $\frac{\partial S}{\partial x} = 0; \frac{\partial S}{\partial y} = 0$
- Equation of the ellipse of the type  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ :
  - Centre  $c(0,0)$
  - Eccentricity  $e = \sqrt{\frac{a^2 - b^2}{a^2}}$  or  $b^2 = a^2(1 - e^2)$
  - Foci  $s(ae, 0); s'(-ae, 0)$
  - Vertices  $A(a, 0); A'(-a, 0)$
  - Length of the latus rectum  $\frac{2b^2}{a}$
  - Length of the major axis  $2a$
  - Length of the minor axis  $2b$
  - Equations of the directrices  $x = \pm \frac{a}{e}$
  - Equations of the latus recta  $x = \pm ae$
  - Equation of the major axis  $y = 0$
  - Equation of the minor axis  $x = 0$
  - Feet of the Directrices  $Z\left(\frac{a}{e}, 0\right); Z^1\left(-\frac{a}{e}, 0\right)$
  - Ends of minor axis  $B(0,b) B'(0, -b)$
  - 14) Ends of latusrecta are  $\left(\pm ae, \pm \frac{b^2}{a}\right)$ 
    - Let 'P' be any point on the ellipse & S, S' are foci then  $SP + S'P = 2a$ . where SP, S'P are called focal distances of P i.e sum of the focal distances is equal to length of the major axis.
    - CS : SA = e : 1-e
    - CA : AZ = e : 1-e

- CS : SZ =  $e^2 : 1 - e^2$
- SZ' : SZ =  $1 + e^2 : 1 - e^2$
- AZ' : AZ =  $1 + e : 1 - e$
- S'Z' : SS' =  $1 - e^2 : 2e^2$
- AS' : AZ =  $e(1+e) : 1 - e$
- Equation of the ellipse of the type  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 (a > b)$ 
  - Centre  $(0,0) = (x-h, y-k) \Rightarrow (h,k) = (x,y)$
  - Eccentricity  $e = \sqrt{\frac{a^2 - b^2}{a^2}}$  or  $b^2 = a^2(1 - e^2)$
  - Foci  $(\pm ae, 0) = (x-h, y-k) \Rightarrow (x,y) = (h \pm ae, k)$
  - Vertices  $(\pm a, 0) = (x-h, y-k) \Rightarrow (x,y) = (h \pm a, k)$
  - Length of the latus rectum  $\frac{2b^2}{a}$
  - Length of the Major axis is  $2a$
  - Length of the minor axis is  $2b$
  - Equations of directrices  $x - h = \pm \frac{a}{e} \Rightarrow x = h \pm \frac{a}{e}$
  - Equations of the latus recta are:  $x - h = \pm ae \Rightarrow x = h \pm ae$
  - 10) Equation of the major axis  $y - k = 0$
  - 11) Equation of the minor axis  $x - h = 0$
  - 12) Feet of the directrices  $\left(\pm \frac{a}{e}, 0\right) = (x-h, y-k) \Rightarrow (x,y) = \left(h \pm \frac{a}{e}, k\right)$
  - 13) Ends of minor axis  $(0, \pm b) = (x-h, y-k) \Rightarrow (x,y) = (h, k \pm b)$
  - 14) Ends of the latus recta are  $\left(\pm ae, \pm \frac{b^2}{a}\right) = (x-h, y-k)$
- Equation of the ellipse of the type  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (b > a)$ 
  - Centre  $c(0,0)$
  - Eccentricity  $e = \sqrt{\frac{b^2 - a^2}{b^2}}$  (or)  $a^2 = b^2(1 - e^2)$
  - Foci  $(0, \pm be)$
  - Vertices  $(0, \pm b)$
  - Length of the latus rectum  $\frac{2a^2}{b}$
  - Length of the major axis  $2b$
  - Length of the minor axis  $2a$

- Equations of the directrices  $y = \pm \frac{b}{e}$

- Equations of the latus recta  $y = \pm be$

- Equation of the major axis  $x = 0$

- Equation of the minor axis  $y = 0$

- Feet of the directrices  $\left(0, \pm \frac{b}{e}\right)$

- Ends of minor axis  $(\pm a, 0)$

- Ends of latusrectum are  $\left(\pm \frac{a^2}{b}, \pm be\right)$

- $SP + S'P = 2b$

- Equation of the ellipse of the type

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 (b > a)$$

- Centre  $(0,0) = (x-h, y-k) \Rightarrow (x,y) = (h,k)$

- Eccentricity  $e = \sqrt{\frac{b^2 - a^2}{b^2}}$  or  $a^2 = b^2(1 - e^2)$

- Foci  $(0, \pm be) = (x-h, y-k) \Rightarrow (x,y) = (h, k \pm be)$

- Vertices  $(0, \pm b) = (x-h, y-k) \Rightarrow (x,y)$

$$= (h, k \pm b)$$

- Length of the latus rectum is  $\frac{2a^2}{b}$

- Length of the major axis  $2b$

- Length of the minor axis  $2a$

- Equations of the directrices  $y - k = \pm \frac{b}{e}$

- Equation of the Major axis  $x - h = 0$

- Equation of the minor axis  $y - k = 0$

- Equations of the latus recta are  $y - k = \pm be$

- Feet of the Directrices  $\left(0, \pm \frac{b}{e}\right) = (x-h, y-k)$

- Ends of minor axis  $(\pm a, 0) = (x-h, y-k)$

14) Ends of latus recta are  $\left(\pm \frac{a^2}{b}, \pm be\right)$

$$= (x-h, y-k)$$

- NOTATIONS:  $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$

$$S_1 \equiv \frac{x x_1}{a^2} + \frac{y y_1}{b^2} - 1$$

$$S_{12} \equiv \frac{x_1 x_2}{a^2} + \frac{y_1 y_2}{b^2} - 1$$

$$S_{11} \equiv \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$$

- Equation of the tangent to the ellipse  $S = 0$  at  $(x_1, y_1)$

$$\text{is } \frac{x x_1}{a^2} + \frac{y y_1}{b^2} = 1 \quad (\text{i.e. } S_1 = 0)$$

- If  $y = mx + c$  is a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\text{then the condition is } c^2 = a^2 m^2 + b^2 \text{ and point of}$$

$$\text{contact is } \left(-\frac{a^2 m}{c}, \frac{b^2}{c}\right)$$

- If  $lx + my + n = 0$  is a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ then the condition is } a^2 l^2 + b^2 m^2 = n^2$$

$$\text{and point of contact is } \left(-\frac{a^2 l}{n}, -\frac{b^2 m}{n}\right)$$

- Equation of the normal to the ellipse  $S = 0$  having the slope  $m$  is

$$y = mx \pm \frac{m(a^2 - b^2)}{\sqrt{a^2 + b^2 m^2}} \text{ is called slope form of the}$$

normal

- Equation of normal at  $(x_1, y_1)$  to the ellipse  $S = 0$  is

$$0 \text{ is } \frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$$

- The condition that the line  $lx + my + n = 0$  may be normal to the ellipse  $S = 0$  is

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

- Equation of any tangent to the ellipse  $S = 0$  is

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

- If  $m_1$  &  $m_2$  are the slopes of tangents to the ellipse  $S = 0$  drawn from  $(x_1, y_1)$  then  $m_1$  &  $m_2$  are satisfying the equation

$$(x_1^2 - a^2)m^2 - 2x_1 y_1 m + (y_1^2 - b^2) = 0 \text{ and}$$

- $m_1 + m_2 = \frac{2x_1 y_1}{x_1^2 - a^2}$  •  $m_1 m_2 = \frac{y_1^2 - b^2}{x_1^2 - a^2}$

- $m_1 - m_2 = \frac{2ab\sqrt{S_{11}}}{x_1^2 - a^2}$

- If  $\theta$  is the acute angle between the tangents, drawn from  $(x_1, y_1)$  to the ellipse  $S = 0$ , then

$$\tan \theta = \left| \frac{2ab\sqrt{S_{11}}}{x_1^2 + y_1^2 - a^2 - b^2} \right|$$

- Equation of the chord of contact of  $(x_1, y_1)$  to the ellipse  $S=0$  is  $S_1 = 0$
- If  $lx + my + n = 0$  is the chord of contact of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  then the point of intersection of tangents drawn at the ends of chord of contact is  $\left(-\frac{a^2 l}{n}, -\frac{b^2 m}{n}\right)$  (Pole of  $lx + my + n = 0$ )
- The locus of points of intersection of tangents, which are drawn at the ends of the chords passing through the fixed point, is called polar. And fixed point is called pole.
- Polar of  $(x_1, y_1)$  w.r.t the ellipse  $S = 0$  is  $S_1 = 0$
- If  $lx + my + n = 0$  is the polar w.r.t the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  then its pole is  $\left(-\frac{a^2 l}{n}, -\frac{b^2 m}{n}\right)$
- If 'P' is an external point of the ellipse  $S = 0$ , then the polar of P meets the ellipse in two points and the polar becomes the chord of contact of P.
- If P lies on the ellipse  $S = 0$ , then the polar of P to the ellipse  $S = 0$  becomes the tangent at P.
- If P is an internal point of the ellipse  $S = 0$ , then the polar of P does not meet the ellipse  $S = 0$
- If the polar of P with respect to the ellipse  $S = 0$  passes through Q and vice versa then P & Q are called conjugate points.
- The condition for the points  $P(x_1, y_1)$  &  $Q(x_2, y_2)$  to be conjugate with respect to the ellipse  $S = 0$  is  $S_{12} = 0$
- If the pole of the line  $L_1 = 0$  with respect to the ellipse  $S = 0$  lies on the line  $L_2 = 0$  then the pole of  $L_2 = 0$  with respect to  $S = 0$  lies on  $L_1 = 0$
- Two lines  $L_1 = 0$ ;  $L_2 = 0$  are said to be conjugate lines with respect to the ellipse  $S = 0$ , if the pole of  $L_1 = 0$  lies on  $L_2 = 0$
- The condition for the lines  $l_1 x + m_1 y + n_1 = 0$ ;  $l_2 x + m_2 y + n_2 = 0$  to be conjugate with respect to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $a^2 l_1 l_2 + b^2 m_1 m_2 = n_1 n_2$
- Equation of the chord of the ellipse  $S = 0$  having  $P(x_1, y_1)$  as its mid point is  $S_1 = S_{11}$  and its slope is  $-\frac{b^2 x_1}{a^2 y_1}$

- Let P (x,y) be a point on the ellipse with centre C. let N be the foot of the perpendicular of P on the major axis. Let NP meets the auxiliary circle at  $P^1$ . Then  $\angle NCP^1$  is called eccentric angle of P. The point  $P^1$  is called corresponding point of P and its range is  $[0^\circ, 360^\circ)$
- Equation of the Director circle of the ellipse  $S = 0$  is  $x^2 + y^2 = a^2 + b^2$
- Equation of the auxiliary circle of the ellipse  $S = 0$  is i)  $x^2 + y^2 = a^2$  ( $a > b$ ) ii)  $x^2 + y^2 = b^2$  ( $a < b$ )
- $x = a \cos \theta$ ;  $y = b \sin \theta$  are called parametric equations of the ellipse  $S = 0$ . ' $\theta$ ' is called parameter and  $\theta \in [0^\circ, 360^\circ)$
- Any point on the ellipse  $S = 0$  is  $(a \cos \theta, b \sin \theta)$  and it is called point  $\theta$ .
- Equation of the tangent at  $\theta$  to the ellipse  $S = 0$  is  $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$
- The co-ordinates of the point of intersection of tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at  $\theta_1$  &  $\theta_2$  is  $\left[ \frac{a \cos\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)}, \frac{b \sin\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)} \right]$
- Equation of the normal at ' $\theta$ ' to the ellipse  $S = 0$  is  $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$
- Equation of the chord joining the points ' $\alpha$ ' and ' $\beta$ ' on the ellipse  $S = 0$  is  $\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right)$
- If  $\alpha, \beta$  are the eccentric angles of the extremities of a focal chord (through  $S(ae, 0)$ ) of an ellipse  $S = 0$  ( $a > b$ ), then
  - $\cos\left(\frac{\alpha - \beta}{2}\right) = e \cos\left(\frac{\alpha + \beta}{2}\right)$
  - $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{e-1}{e+1}$
- If the chord joining the points  $\alpha$  and  $\beta$  on the ellipse  $S = 0$  cuts the major axis at a distance 'd' units from the center then  $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{d-a}{d+a}$

- If  $PSP^1$  is the focal chord of the ellipse and SL be the semi latusrectum then  $\frac{1}{SP} + \frac{1}{SP^1} = \frac{2}{SL}$
- $S, S^1$  are the foci of an ellipse, then the tangent at any point P on the ellipse is the external angle bisector of  $\angle S^1PS$
- $S, S^1$  are the foci of an ellipse, then the normal at any point P on the ellipse is the internal angle bisector of  $\angle S^1PS$
- If a circle cuts an ellipse in four distinct points then the sum of their eccentric angles is an even multiple of  $\pi$  radians, i.e.,  

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 = 2n\pi$$
- The sum of the eccentric angles of the feet of the normals to an ellipse through a point is an odd multiple of  $\pi$  radians, i.e.,  

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 = (2n+1)\pi$$
- If the line  $lx + my + n = 0$  is the chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  then the mid point of the chord is  $\left( \frac{-a^2ln}{a^2l^2 + b^2m^2}, \frac{-b^2mn}{a^2l^2 + b^2m^2} \right)$
- Let  $P(x_1, y_1)$  be a point and  $S = 0$  is an ellipse . If
  - P lies on the ellipse then  $S_{11} = 0$
  - P lies inside the ellipse then  $S_{11} < 0$
  - P lies outside the ellipse then  $S_{11} > 0$
- If  $PSP^1$  be the focal chord of an ellipse of semi-latusrectum SL then  $\frac{1}{SP} + \frac{1}{SP^1} = \frac{2}{SL}$
- The product of the perpendicular drawn from the foci of any tangent of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $b^2$
- Equation of the normal drawn at  $\left( \frac{ae^3}{e}, \frac{b^2}{a} \right)$  is  $x - ey = ae^3$
- The tangents at the ends of the focal chord meet on the directrix.
- The equation of the diameter bisecting the parallel chords of slope  $m$  of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $y = \frac{-b^2x}{a^2m}$

## CONCEPTUAL QUESTIONS

- Let  $S, S^1$  be the foci of an ellipse. If  $\angle BSS^1 = q$  Then its eccentricity is  
 1)  $\tan q$     2)  $\sin q$     3)  $\cos q$     4)  $\cot q$
- If the major axis is "n" times the minor axis of the ellipse, then eccentricity is  
 1)  $\frac{\sqrt{n-1}}{n}$     2)  $\frac{\sqrt{n-1}}{n^2}$   
 3)  $\frac{\sqrt{n^2-1}}{n^2}$     4)  $\frac{\sqrt{n^2-1}}{n}$
- If  $\alpha$  and  $\beta$  are the eccentric angles of the extremities of a focal chord of an ellipse, then the eccentricity of the ellipse is  
 1)  $\frac{\cos \alpha + \cos \beta}{\cos(\alpha - \beta)}$     2)  $\frac{\sin \alpha - \sin \beta}{\sin(\alpha - \beta)}$   
 3)  $\frac{\cos \alpha - \cos \beta}{\cos(\alpha - \beta)}$     4)  $\frac{\sin \alpha + \sin \beta}{\sin(\alpha + \beta)}$
- The sum of the squares of perpendicular on any tangent of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  from two points on the minor axis each one at a distance of  $\sqrt{a^2 - b^2}$  units from the centre is  
 1)  $a^2$     2)  $b^2$     3)  $2a^2$     4)  $2b^2$

## Key

1.3    2.4    3.4    4.3

## LEVEL - I

- The eccentricity of the Ellipse  $9x^2 + 16y^2 = 576$  is  
 1)  $\frac{\sqrt{7}}{2}$     2)  $\frac{\sqrt{5}}{4}$     3)  $\frac{7}{12}$     4)  $\frac{\sqrt{7}}{4}$
- The ends of major axis of an Ellipse are (5,0) (-5,0) and one of the foci lies on  $3x - 5y - 9 = 0$ . Then the 'e' of the Ellipse is  
 1)  $\frac{2}{3}$     2)  $\frac{3}{5}$     3)  $\frac{4}{5}$     4)  $\frac{1}{3}$
- The eccentricity of the Ellipse whose major axis is double the minor axis  
 1)  $\frac{1}{2}$     2)  $\frac{1}{\sqrt{2}}$     3)  $\frac{1}{3}$     4)  $\frac{\sqrt{3}}{2}$
- If the major axis of an ellipse is thrice the minor axis of an Ellipse then its eccentricity is  
 1)  $\frac{1}{\sqrt{2}}$     2)  $\frac{2\sqrt{2}}{3}$     3)  $\frac{2}{3}$     4)  $\frac{1}{3}$

5. In an Ellipse distance between the foci is 6 and the length of minor axis is 8. Its eccentricity is  
 1)  $\frac{2}{5}$       2)  $\frac{4}{5}$       3)  $\frac{3}{5}$       4)  $\frac{1}{3}$
6. In the ellipse distance between the foci is equal to the distance between a focus and one end of minor axis then its eccentricity is  
 1)  $\frac{1}{2}$       2)  $\frac{1}{4}$       3)  $\frac{1}{3}$       4)  $\frac{1}{5}$
7. If the length of latusrectum of an Ellipse be equal to one half its minor axis then its eccentricity is  
 1)  $\frac{1}{2\sqrt{2}}$       2)  $\frac{\sqrt{3}}{2}$       3)  $\frac{1}{\sqrt{2}}$       4)  $\frac{2\sqrt{2}}{3}$
8. The distance between the foci of an Ellipse is equal to the length of latusrectum. Its eccentricity is  
 1)  $\frac{1}{2\sqrt{2}}$       2)  $\frac{2\sqrt{2}}{3}$       3)  $\frac{\sqrt{5}-1}{2}$       4)  $\frac{\sqrt{3}-1}{2}$
9. If the length of minor axis of an Ellipse is equal to the distance between the foci then its eccentricity is  
 1)  $\frac{\sqrt{3}}{2}$       2)  $\frac{1}{2}$       3)  $\frac{1}{2\sqrt{2}}$       4)  $\frac{1}{\sqrt{2}}$
10. If the angle between the lines joining the foci to an extremity of minor axis of an Ellipse is  $90^\circ$  its eccentricity is  
 1)  $\frac{1}{2}$       2)  $\frac{\sqrt{3}}{2}$       3)  $\frac{1}{\sqrt{3}}$       4)  $\frac{1}{\sqrt{2}}$
11. The Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  cuts x axis at A and  $A'$ , y axis at B and  $B'$ . The line joining the focus S and B makes an angle  $\frac{3\pi}{4}$  with x-axis. Then the eccentricity of the Ellipse is  
 1)  $\frac{1}{\sqrt{2}}$       2)  $\frac{1}{2}$       3)  $\frac{\sqrt{3}}{2}$       4)  $\frac{1}{3}$
12. In an Ellipse the distance between the foci is one third of the distance between the directrices then its e is  
 1)  $\frac{1}{2}$       2)  $\frac{1}{\sqrt{3}}$       3)  $\frac{2\sqrt{2}}{3}$       4)  $\frac{1}{3}$
13. One focus of an Ellipse is (1,0) with centre(0,0). If the length of major axis is 6 its e =  
 1)  $\frac{1}{4}$       2)  $\frac{2}{3}$       3)  $\frac{1}{3}$       4)  $\frac{1}{2}$
14. In an Ellipse the major and minor axes are in the ratio 5 : 3. The eccentricity of the Ellipse is  
 1)  $\frac{3}{5}$       2)  $\frac{1}{5}$       3)  $\frac{2}{3}$       4)  $\frac{4}{5}$
15. S and T are the foci of an Ellipse and B is one end of minor axis. If STB is an equilateral triangle then the eccentricity of the Ellipse is  
 1)  $\frac{1}{4}$       2)  $\frac{1}{3}$       3)  $\frac{1}{2}$       4)  $\frac{2}{3}$
16. S and  $S'$  are the foci and B is an end of minor axis. If  $\angle SBS' = 120^\circ$  then its eccentricity  
 1)  $\frac{\sqrt{5}}{2}$       2)  $\frac{\sqrt{3}}{2}$       3)  $\frac{1}{\sqrt{2}}$       4)  $\frac{1}{\sqrt{3}}$
17. If P is a point on the Ellipse of eccentricity e and A,  $A'$  are the vertices and S,  $S'$  are the foci then  $\Delta SPS' : \Delta APA'$  is  
 1)  $e^3 : 1$       2)  $e^2 : 1$       3)  $e : 1$       4)  $2e : 1$
18. The eccentricity of the Ellipse  $4x^2 + y^2 - 8x - 2y + 1 = 0$  is  
 1)  $\frac{\sqrt{3}}{2}$       2)  $\frac{1}{2}$       3)  $\frac{1}{\sqrt{3}}$       4)  $\frac{2}{\sqrt{3}}$
19. An Ellipse with foci  $(\pm 3, 0)$  passes through (4,1) then its eccentricity is  
 1)  $\frac{1}{2}$       2)  $\frac{1}{3}$       3)  $\frac{2}{3}$       4)  $\frac{1}{\sqrt{2}}$
20. The eccentricity of the Ellipse  $25x^2 + 9y^2 - 150x - 90y + 225 = 0$  is  
 1)  $\frac{3}{5}$       2)  $\frac{4}{5}$       3)  $\frac{2}{3}$       4)  $\frac{1}{3}$
21. If the minor axis of an ellipse forms an equilateral triangle with one vertex of the ellipse then e =  
 1)  $\sqrt{\frac{1}{2}}$       2)  $\sqrt{\frac{2}{3}}$       3)  $\sqrt{\frac{3}{4}}$       4)  $\sqrt{\frac{4}{5}}$
22. The eccentricity of the conic represented by  $\sqrt{(x+2)^2 + y^2} + \sqrt{(x-2)^2 + y^2} = 8$  is  
 1)  $1/3$       2)  $1/2$       3)  $1/4$       4)  $1/5$
23. If the eccentricity of an ellipse tends to zero, then the ellipse becomes  
 1. a closed figure      2. a quadrilateral  
 3. a circle      4. a hexagon
24. Foci of the Ellipse  $25x^2 + 9y^2 - 150x - 90y + 225 = 0$  are  
 1) (3,9) (3,1)      2) (4,9) (4,1)  
 3) (5,7) (4,7)      4) (6,9) (5,1)
25. If  $x = 2(\cos t - \sin t)$ ;  $y = 3(\cos t + \sin t)$  represents a Conic, its foci are  
 1)  $(\pm\sqrt{10}, 0)$       2)  $(\pm\sqrt{13}, 0)$   
 3)  $(0, \pm\sqrt{13})$       4)  $(0, \pm\sqrt{10})$
26. Foci of the ellipse  $3x^2 + 4y^2 - 12x - 8y + 4 = 0$  are  
 1) (2,1) (4,1)      2) (1,1) (3,1)  
 3) (3,1) (5,1)      4) (-4,1) (5,1)

27. S and S' are the foci of the Ellipse  $25x^2 + 16y^2 = 1600$ . Then the sum of the distances from S and S' to the point  $(4\sqrt{3}, 5)$  is  
 1) 20      2) 15      3) 40      4) 30
28. Distance between the foci of the Ellipse  $25x^2 + 16y^2 + 100x - 64y - 236 = 0$  is  
 1) 8      2) 12      3) 9      4) 6
29. The distance between one focus to one end of minor axis of the Ellipse  $16x^2 + 25y^2 - 50y - 375 = 0$   
 1) 4      2) 5      3) 6      4) 7
30. The distance between the foci of the Ellipse  $x = 5\cos\theta, y = 4\sin\theta$  is  
 1) 8      2) 10      3) 7      4) 6
31. Equations of the Latusrecta of the Ellipse  $\frac{(x-2)^2}{36} + \frac{(y-3)^2}{11} = 1$  are  
 1)  $x = 7, x = -3$       2)  $x = 7, x = -4$   
 3)  $x = 8, x = -5$       4)  $x = 9, x = -6$
32. The vertices of the Ellipse  $\frac{(x+1)^2}{25} + \frac{(y-3)^2}{16} = 1$  are  
 1) (3,4) (-6,3)      2) (4,3) (-6,3)  
 3) (5,3) (-7,3)      4) (6,3) (-8,3)
33. The vertices of the Ellipse  $\frac{(x+2)^2}{16} + \frac{(y-2)^2}{25} = 1$  are  
 1) (-2,7) (-2,-3)      2) (4,3) (-6,3)  
 3) (5,3) (-7,3)      4) (6,3) (-8,3)
34. Area of the rectangle formed by the ends of latusrecta of the Ellipse  $4x^2 + 9y^2 = 144$  is  
 1)  $\frac{32\sqrt{5}}{3}$       2)  $\frac{64\sqrt{5}}{3}$       3)  $\frac{16\sqrt{5}}{3}$       4)  $\frac{32\sqrt{3}}{5}$
35. Area of the rectangle formed by the ends of latusrecta of the Ellipse  $25x^2 + 4y^2 = 100$  is  
 1)  $\frac{16\sqrt{21}}{5}$       2)  $\frac{32\sqrt{21}}{5}$       3)  $\frac{8\sqrt{21}}{5}$       4)  $\frac{7\sqrt{21}}{5}$
36. Centre of the ellipse  $4(x-2y+1)^2 + 9(2x+y+2)^2 = 5$  is  
 1) (-2, 2)      2) (1,5)      3) (-5,2)      4) (-1, 0)
37. Centre of the Ellipse  $5x^2 - 6xy + 5y^2 + 22x - 26y + 29 = 0$  is  
 1) (1,-2)      2) (2,-1)      3) (1,-1)      4) (-1,2)
38.  $P\left(\frac{\pi}{6}\right)$  is a point on the ellipse  $\frac{x^2}{36} + \frac{y^2}{9} = 1$ , and S and S' are the foci of the ellipse. Then  $SP + S'P =$   
 1) 6      2) 12      3)  $6\sin 60^\circ$       4)  $6\cos 60^\circ$

39. PSQ is a focal chord of the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ , then  $\frac{1}{SP} + \frac{1}{SQ} =$   
 1)  $\frac{2}{3}$       2)  $\frac{3}{2}$       3)  $\frac{4}{3}$       4)  $\frac{4}{9}$
40. The distance of the point ' $\theta$ ' on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  from a focus is  
 1)  $a(e + \cos\theta)$       2)  $a(e - \cos\theta)$   
 3)  $a(1 + e \cos\theta)$       4)  $a(1 + 2e \cos\theta)$
41. The directrices of the Ellipse  $3x^2 + 4y^2 + 12x - 8y - 32 = 0$  are  
 1)  $x = 6, x = -10$       2)  $x = 5, x = -11$   
 3)  $x = 4, x = 12$       4)  $x = 3, x = 9$
42. Distance between the directrices of the ellipse  $9x^2 + 5y^2 - 30y = 0$  is  
 1)  $\frac{9}{2}$       2) 18      3) 9      4) 12
43. A focal chord perpendicular to major axis of the Ellipse  $9x^2 + 5y^2 = 45$  cuts the curve at P and Q then length of PQ is  
 1)  $\frac{10}{3}$       2)  $\frac{18}{\sqrt{5}}$       3) 6      4)  $2\sqrt{5}$
44. A point moves so that its distance from the point (2,0) is always  $\frac{1}{3}$  of its distance from the line  $x - 18 = 0$ . If the locus of the point is a conic, its length of latusrectum is  
 1)  $16/3$       2)  $32/3$       3)  $8/3$       4)  $15/4$
45. The eccentricity of an ellipse is  $\frac{\sqrt{3}}{2}$ , its length of latusrectum is  
 1)  $\frac{1}{2}$  (Length of major axis)  
 2)  $\frac{1}{3}$  (Length of major axis)  
 3)  $\frac{1}{4}$  (length of major axis)  
 4)  $\frac{2}{3}$  (length of major axis)
46. A focal chord perpendicular to major axis of the Ellipse  $9x^2 + 16y^2 = 144$  cuts the curve at P and Q then the length of PQ is  
 1)  $\frac{9}{2}$       2)  $\frac{18}{\sqrt{5}}$       3) 6      4)  $2\sqrt{5}$
47. Given two fixed points A and B and  $AB = 6$ . Then simplest form of the equation to the locus of P such that  $PA + PB = 8$  is  
 1)  $\frac{x^2}{16} + \frac{y^2}{7} = 1$       2)  $\frac{x^2}{16} + \frac{y^2}{9} = 1$   
 3)  $\frac{x^2}{9} + \frac{y^2}{16} = 1$       4)  $\frac{x^2}{12} + \frac{y^2}{21} = 1$

48. Equation to the locus of the point which moves such that the sum of the distances from the points (3,9) (3,1) is 10 is

1)  $\frac{(x-5)^2}{9} + \frac{(y-3)^2}{25} = 1$

2)  $\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$

3)  $\frac{x^2}{9} + \frac{y^2}{25} = 1$

4)  $\frac{(x-3)^2}{36} + \frac{(y-5)^2}{49} = 1$

49. Equation of the ellipse with focus (3,-2), eccentricity  $\frac{3}{4}$  and directrix  $2x-y+3=0$  is

1)  $44x^2+36xy+71y^2-374x-528y+756=0$

2)  $44x^2+36xy+71y^2-588x-374y+959=0$

3)  $44x^2+36xy+71y^2-125x-274y+659=0$

4)  $44x^2+36xy+71y^2-135x-47xy+859=0$

50. Equation of the ellipse with focus (0,0), directrix  $x+6=0$  and  $e = 1/2$  is

1)  $3x^2+4y^2-14x-32=0$

2)  $3x^2+4y^2-16x-42=0$

3)  $3x^2+4y^2-12x-36=0$

4)  $3x^2+4y^2-12x+32=0$

51. Equation of the ellipse with focus (2,0), directrix  $x=8$  and  $e = 1/2$  is

1)  $4x^2+3y^2=48$

2)  $3x^2+4y^2=48$

3)  $3x^2+4y^2=12$

4)  $4x^2+3y^2=12$

52. Equation of the ellipse with foci  $(2 \pm \sqrt{7}, 3)$  and the lengths of major and minor axes are 8, 6 respectively is

1)  $9(x-2)^2 + 16(y-3)^2 = 144$

2)  $16(x+2)^2 + 9(y+3)^2 = 144$

3)  $9(x-2)^2 + 36(y+3)^2 = 144$

4)  $9(x+2)^2 + 36(y-3)^2 = 144$

53. Equation of the ellipse with foci  $(\pm 4, 0)$  and length of latusrectum  $\frac{20}{3}$  is

1)  $\frac{x^2}{20} + \frac{y^2}{36} = 1$

2)  $\frac{x^2}{20} + \frac{y^2}{4} = 1$

3)  $\frac{x^2}{36} + \frac{y^2}{20} = 1$

4)  $\frac{x^2}{24} + \frac{y^2}{8} = 1$

54. Equation of the ellipse with foci  $(\pm 5, 0)$  and length of major axis 26 is

1)  $\frac{x^2}{144} + \frac{y^2}{169} = 1$

2)  $\frac{x^2}{29} + \frac{y^2}{4} = 1$

3)  $\frac{x^2}{39} + \frac{y^2}{14} = 1$

4)  $\frac{x^2}{169} + \frac{y^2}{144} = 1$

55. Equation of the ellipse with foci  $(\pm\sqrt{2}, 0)$  and  $e = 1/4$  is

1)  $\frac{x^2}{16} + \frac{y^2}{14} = 1$

2)  $\frac{x^2}{18} + \frac{y^2}{16} = 1$

3)  $\frac{x^2}{20} + \frac{y^2}{18} = 1$

4)  $\frac{x^2}{32} + \frac{y^2}{30} = 1$

56. Equation of the ellipse with vertices  $(\pm 5, 0)$  foci  $(\pm 4, 0)$  is

1)  $\frac{x^2}{25} + \frac{y^2}{9} = 1$

2)  $\frac{x^2}{32} + \frac{y^2}{16} = 1$

3)  $\frac{x^2}{25} + \frac{y^2}{7} = 7$

4)  $\frac{x^2}{25} + \frac{y^2}{12} = 1$

57. Equation of the ellipse with vertices  $(0, \pm 17)$  foci  $(0, \pm 8)$  is

1)  $\frac{x^2}{289} + \frac{y^2}{225} = 1$

2)  $\frac{x^2}{225} + \frac{y^2}{289} = 1$

3)  $\frac{x^2}{132} + \frac{y^2}{289} = 1$

4)  $\frac{x^2}{196} + \frac{y^2}{289} = 1$

58. Equation of the ellipse with vertices  $(-4, 3)$   $(8, 3)$  and  $e = 5/6$  is

1)  $\frac{(x-2)^2}{36} + \frac{(y-3)^2}{11} = 1$

2)  $\frac{x^2}{36} + \frac{y^2}{11} = 1$

3)  $\frac{(x-2)^2}{11} + \frac{(y-1)^2}{36} = 1$

4)  $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{16} = 1$

59. Equation of the ellipse with foci  $(0, \pm 4)$  and  $e = 4/5$  is

1)  $9x^2 + 25y^2 = 225$

2)  $25x^2 + 9y^2 = 225$

3)  $\frac{x^2}{36} + \frac{y^2}{100} = 1$

4)  $\frac{x^2}{100} + \frac{y^2}{36} = 1$

60. Axes are coordinate axes, the ellipse passes through the points where the straight line  $\frac{x}{4} + \frac{y}{3} = 1$  meets the coordinate axes. Then equation of the ellipse is

1)  $\frac{x^2}{16} + \frac{y^2}{9} = 1$

2)  $\frac{x^2}{64} + \frac{y^2}{36} = 1$

3)  $\frac{x^2}{4} + \frac{y^2}{3} = 1$

4)  $\frac{x^2}{8} + \frac{y^2}{6} = 1$

61. Axes are coordinate axes. A and B are the ends of major axis and minor axis respectively. Area of

$\Delta OAB$  is 16 sq.units,  $e = \frac{\sqrt{3}}{2}$ , then equation of the ellipse is

1)  $\frac{x^2}{32} + \frac{y^2}{8} = 1$       2)  $\frac{x^2}{64} + \frac{y^2}{16} = 1$

3)  $\frac{x^2}{64} + \frac{y^2}{8} = 1$       4)  $\frac{x^2}{64} + \frac{y^2}{32} = 1$

62. An ellipse with foci  $(-1,1)$   $(1,1)$  passes through  $(0,0)$  then its equation is

1)  $x^2 + 2y^2 - 8y = 0$       2)  $x^2 + 2y^2 + 4y = 0$

3)  $x^2 + 2y^2 + 8y = 0$       4)  $x^2 + 2y^2 - 4y = 0$

63. Equation of the Ellipse with foci  $(\pm 5,0)$  and directrix  $x = \frac{36}{5}$  is

1)  $\frac{x^2}{11} + \frac{y^2}{36} = 1$       2)  $\frac{x^2}{25} + \frac{y^2}{11} = 1$

3)  $\frac{x^2}{36} + \frac{y^2}{11} = 1$       4)  $\frac{x^2}{16} + \frac{y^2}{10} = 1$

64. An ellipse with centre at  $(0,0)$  cuts x axis at  $(3,0)$  and  $(-3,0)$ . If its  $e = \frac{1}{2}$  then the length of the semiminor axis is

1)  $2\sqrt{3}$       2)  $\sqrt{5}$       3)  $3\sqrt{2}$       4)  $\frac{3\sqrt{3}}{2}$

65. An ellipse with centre  $(0,0)$  cuts y axis at  $(0,6)$  and  $(0,-6)$ . If its  $e = \frac{\sqrt{3}}{2}$  then the length of major axis is

1) 18      2) 36      3) 20      4) 24

66. C is the centre of the Ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  and S is one focus. Then the ratio of CS to semi major axis is

1) 4:5      2) 2:3      3) 3:5      4) 2:5

67. The ratio of the lengths of the major and minor axes of the ellipse  $9x^2 + 16y^2 = 144$  is

1) 5:3      2) 3:2      3) 6:5      4) 4:3

68. A man running round a race course notes that the sum of the distances of two flag posts from him is 8 meters. The area of the path he encloses in square meters if the distance between flag posts is 4 is

1)  $15\sqrt{3}\pi$       2)  $12\sqrt{3}\pi$       3)  $18\sqrt{3}\pi$       4)  $8\sqrt{3}\pi$

## RADII

69. Circles are described on the major axis and the line joining the foci of the ellipse  $3x^2 + 2y^2 = 6$  as diameters. Then the radii of the circles are in the ratio

1)  $\sqrt{2}:1$       2)  $\sqrt{3}:1$       3)  $3:2$       4)  $5:4$

70. The radius of the circle passing through the foci of the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  and having its centre at  $(0,3)$  is

1) 3      2) 4      3)  $5/2$       4)  $7/2$

## STANDARD EQUATION OF ELLIPSE

71. The equation  $\frac{x^2}{10-a} + \frac{y^2}{4-a} = 1$  represents an ellipse if

1)  $a < 4$       2)  $a > 4$       3)  $4 < a < 10$       4)  $a > 10$

72. If A and B are two fixed points and if the point P moves such that  $PA + PB = \text{constant}$ , then the locus of P is

1) Circle      2) Parabola  
3) Ellipse      4) Hyperbola

## POLE & POLAR, CONJUGATE POINTS AND CONJUGATE LINES

73. If  $\left(-\frac{1}{5}, k\right)$   $\left(-\frac{10}{3}, \frac{1}{3}\right)$  are conjugate points w.r

to the ellipse  $3x^2 + 5y^2 = 7$  then the value of k is

1) 2      2) 5      3) 4      4) 3

74. The conjugate point of  $(-4/3, 2)$  w.r. to  $\frac{x^2}{4} + \frac{y^2}{3} = 1$  is

1)  $(-6, -3/2)$       2)  $(4, 1)$       3)  $(1, 7)$       4)  $(5, 3)$

75. If  $3x - 2y + 4 = 0$  and  $2x + 5y + k = 0$  are conjugate lines w.r to the Ellipse  $9x^2 + 16y^2 = 144$  then the value of k is

1)  $5/2$       2)  $-5/2$       3)  $7/2$       4)  $3/2$

76. The angle between the conjugate lines through a focus of an Ellipse is

1)  $\frac{\pi}{4}$       2)  $\frac{\pi}{3}$       3)  $\frac{\pi}{6}$       4)  $\frac{\pi}{2}$

77. The polar of a point w.r to the Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

touch the parabola  $y^2 = 4px$ . Then the locus of its pole is

1)  $pa^4y^2 + b^2x = 0$       2)  $pa^2y^2 + b^4x = 0$   
3)  $pay^2 + b^4x = 0$       4)  $p^2a^4y^2 + b^2x = 0$



78. The locus of poles of tangents to the Ellipse  $S = 0$  w.r to the circle  $x^2 + y^2 = a^2$  is  
 1)  $a^4x^2 + b^4y^2 = a^2$       2)  $a^2x^2 + b^4y^2 = a^6$   
 3)  $a^2x^2 + b^2y^2 = a^4$       4)  $a^4x^2 + b^4y^2 = a^4$
79. The locus of poles of the tangents of  $x^2 + y^2 = d^2$  w.r to the Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   
 1)  $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{d^2}$       2)  $\frac{x^2}{b^4} + \frac{y^2}{a^4} = \frac{1}{a^2}$   
 3)  $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{4}{d^2}$       4)  $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{2}{d^2}$
80. Pole of the line  $3x + 8y - 24 = 0$  w.r to the Ellipse  $9x^2 + 16y^2 = 144$  is  
 1)  $(-2, 3)$       2)  $(2, -3)$       3)  $(2, 3)$       4)  $(-2, -3)$
81. Equation of the line through the point  $(1, 4)$  and conjugate to the line  $9x + 2y = 1$  w.r to the Ellipse  $3x^2 + 2y^2 = 1$  is  
 1)  $3x + 2y - 11 = 0$       2)  $2x + y - 6 = 0$   
 3)  $x + 3y - 13 = 0$       4)  $2x - 3y + 10 = 0$
82. The distance between the polars of the foci of the Ellipse  $\frac{x^2}{25} + \frac{y^2}{9} = 1$  w.r to itself is  
 1)  $25/2$       2)  $25/9$       3)  $25/8$       4)  $25/3$
83. Equation of the tangent at  $(\sqrt{3}, 2)$  to the Ellipse  $4x^2 + 3y^2 = 24$  is  
 1)  $4x + \sqrt{3}y = 2\sqrt{3}$       2)  $2x + \sqrt{3}y = 4\sqrt{3}$   
 3)  $4x + 3\sqrt{3}y = 7\sqrt{3}$       4)  $2x - \sqrt{3}y = 6\sqrt{3}$
84. Equation of the tangent to  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a^2 > b^2$ ) at the end of latusrectum in the first quadrant is  
 1)  $ax + ey - a = 0$       2)  $ex + y - a = 0$   
 3)  $x - ey + a = 0$       4)  $ex - 2y + a = 0$
85. The point of intersection of the two tangents to the ellipse  $2x^2 + 3y^2 = 6$  at the ends of latusrectum is  
 1)  $(3, 0)$       2)  $(7/2, 0)$   
 3)  $(9/2, 0)$       4)  $(4, 0)$
86. Tangents are drawn to the Ellipse  $\frac{x^2}{a} + \frac{y^2}{b} = a + b$  at the points where it is cut by the line  $\frac{x}{a} \cos \theta - \frac{y}{b} \sin \theta = 1$ , then the point of intersection of the tangents is  
 1)  $1) \{(a+b) \cos \theta, -(a+b) \sin \theta\}$   
 2)  $2) \{(a-b) \cos \theta, -(a-b) \sin \theta\}$   
 3)  $3) \{(a+b) \sin \theta, -(a+b) \cos \theta\}$   
 4)  $4) \{(a+b) \cos \theta, -(a-b) \sin \theta\}$
87. If the line  $y = x + c$  touches the Ellipse  $2x^2 + 3y^2 = 1$  then  $c =$   
 1)  $\pm \sqrt{\frac{5}{6}}$       2)  $\pm \sqrt{\frac{3}{2}}$       3)  $\pm \sqrt{\frac{2}{3}}$       4)  $\pm \sqrt{\frac{6}{5}}$
88. Equation of the tangent of  $3x^2 + 4y^2 = 12$  parallel to  $x - 2y + 1 = 0$  is  
 1)  $x - 2y + 7 = 0$       2)  $x - 2y + 4 = 0$   
 3)  $x - 2y + 5 = 0$       4)  $x - 2y + 9 = 0$
89. If the line  $x + ky - 5 = 0$  is a tangent to  $4x^2 + 9y^2 = 20$  then the value of  $K$  is  
 1)  $\pm 5$       2)  $\pm 4$       3)  $\pm 3$       4)  $\pm 2$
90. A tangent  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  meets the axes at  $A$  and  $B$ . Then the locus of mid point of  $AB$  is  
 1)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 2$       2)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$   
 3)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$       4)  $\frac{a^2}{x^2} + \frac{b^2}{y^2} = \frac{1}{2}$
91. Equation of the tangent of  $\frac{x^2}{28} + \frac{y^2}{16} = 1$  making an angle  $60^\circ$  with  $x$  - axis is  
 1)  $y = \sqrt{3}x + 5$       2)  $y = \sqrt{3}x + 10$   
 3)  $y = \sqrt{3}x + 7$       4)  $y = \sqrt{3}x + 4$
92. A tangent to  $3x^2 + 4y^2 = 12$  is equally inclined with the coordinate axes. Then the perpendicular distance from the centre of the Ellipse to this tangent is  
 1)  $\sqrt{\frac{7}{2}}$       2)  $\sqrt{\frac{5}{2}}$       3)  $\sqrt{\frac{9}{2}}$       4)  $\sqrt{\frac{11}{2}}$
93. A tangent having slope  $-4/3$  to the Ellipse  $\frac{x^2}{18} + \frac{y^2}{32} = 1$  meets the major and minor axes at  $A$  and  $B$ . If  $O$  is the centre of the Ellipse then the area of  $\Delta OAB$  is  
 1) 16 Sq units      2) 20 Sq. units  
 3) 24 Sq.Units      4) 22 Sq.Units
94. The equation to the locus of point of intersection of the lines  $y - mx = \sqrt{4m^2 + 3}$ ,  $my + x = \sqrt{4 + 3m^2}$  is  
 1)  $x^2 + y^2 = 12$       2)  $x^2 + y^2 = 7$   
 3)  $x^2 + y^2 = 1$       4)  $x^2 + y^2 = 4$
95. The locus of point of intersection of the lines  $\frac{tx}{a} - \frac{y}{b} + t = 0$ ,  $\frac{x}{a} + \frac{ty}{b} - 1 = 0$  is ( $t$  is a parameter)  
 1) Parabola      2) Circle  
 3) Hyperbola      4) Ellipse

96. Tangents are drawn from any point on the circle

$x^2 + y^2 = 41$  to the Ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  then the angle

between the two tangents is

- 1)  $\frac{\pi}{4}$       2)  $\frac{\pi}{3}$       3)  $\frac{\pi}{6}$       4)  $\frac{\pi}{2}$

97. Product of the perpendicular distances from the foci of the Ellipse  $x^2 + 4y^2 = 25$  on any tangent to it is

- 1) 25      2) 25/4      3) 15      4) 12

98. If the polar of the point  $(-4, 1)$  w.r to the parabola  $y^2 = 2x$  touches the Ellipse  $13x^2 + 3y^2 = 39$  then the point of contact is

- 1)  $\left(\frac{-3}{4}, \frac{13}{4}\right)$       2)  $\left(\frac{3}{4}, \frac{-13}{4}\right)$

- 3)  $\left(\frac{-3}{4}, \frac{-13}{4}\right)$       4)  $\left(\frac{3}{4}, \frac{13}{4}\right)$

99. If  $m_1, m_2$  be the slopes of the two tangents drawn from  $(1, 2)$  to the ellipse  $2x^2 + 3y^2 = 6$  then  $m_1 + m_2 =$

- 1) 3      2) -1      3) -2      4) 5

100. If  $\theta$  is the angle between the two tangents from  $(4, 1)$  to the Ellipse  $x^2 + 2y^2 = 6$  then  $\tan \theta$

- 1)  $3/4$       2)  $3/5$       3)  $1/2$       4)  $3/2$

101. Tangents to the ellipse  $b^2x^2 + a^2y^2 = a^2b^2$  makes angles

$\theta_1$  and  $\theta_2$  with major axis such that  $\cot \theta_1 + \cot \theta_2 = k$ . Then the locus of the point of intersection is

- 1)  $xy = 2k(y^2 + b^2)$       2)  $2xy = k(y^2 - b^2)$   
3)  $4xy = k(y^2 - b^2)$       4)  $8xy = k(y^2 - b^2)$

102. Tangents to the ellipse  $b^2x^2 + a^2y^2 = a^2b^2$  make complimentary angles with the major axis. Then the locus of their point of intersection is

- 1)  $x^2 + y^2 = a^2 - b^2$       2)  $x^2 - y^2 = a^2 + b^2$   
3)  $x^2 - y^2 = a^2 - b^2$       4)  $x^2 + y^2 = a^2 + b^2$

103. The locus of point of Intersection of orthogonal

tangents to the ellipse  $\frac{(x-1)^2}{16} + \frac{(y-2)^2}{9} = 1$  is

- 1)  $(x-1)^2 + (y-2)^2 = 25$       2)  $(x-1)^2 + (y-2)^2 = 7$   
3)  $(x+1)^2 + (y+2)^2 = 25$       4)  $(x+1)^2 + (y+2)^2 = 7$

104. The nature of the intercepts made on the axes by

the tangent at the point  $\left(\frac{16}{5}, \frac{9}{5}\right)$  to the el-

lipse  $9x^2 + 16y^2 = 144$  are

1. equal      2. unequal  
3. equal in magnitude but opposite in sign  
4. intercepts in the ratio 1 : 2

105. Tangents one to each of the ellipses  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and  $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$  are drawn. If the tangents

meet at right angles then the locus of their point of intersection is

1.  $x^2 + y^2 = a^2 + \lambda$   
2.  $x^2 + y^2 = b^2 + \lambda$   
3.  $x^2 + y^2 = a^2 + b^2 + \lambda$   
4.  $x^2 + y^2 = a^2x^2 - b^2 + \lambda$

106. Equation of the pair of tangents from  $(3, 4)$  to the

ellipse  $\frac{x^2}{9} + \frac{y^2}{16} = 1$

1.  $(x+3)(y+4) = 0$       2.  $(x-3)(y-4) = 0$   
3.  $(x+4)(y+3) = 0$       4.  $(x-4)(y-3) = 0$

107. Area of the triangle formed by the x axis, the tangent

and normal at  $(3, 2)$  to the Ellipse  $\frac{x^2}{18} + \frac{y^2}{8} = 1$  is

- 1) 5      2)  $\frac{13}{3}$       3)  $\frac{15}{2}$       4)  $\frac{9}{2}$

108. The minimum area of the triangle formed by any

tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the

coordinate axes is

- 1)  $ab$       2)  $2ab$       3)  $4ab$       4)  $a^2b^2$

109. Equation of the two tangents drawn from  $(2, -1)$  to  $x^2 + 3y^2 = 3$  are

- 1)  $x = 2, 4x + 3y - 7 = 0$       2)  $y = -1, 4x + y - 7 = 0$   
3)  $x + y - 1 = 0, 4x + y - 7 = 0$       4)  $x + y + 1 = 0, 4x + y - 7 = 0$

110. Equation to the pair of tangents drawn from  $(2, -1)$  to the ellipse  $x^2 + 3y^2 = 3$  is

- 1)  $y^2 + 4xy + 4x - 6y - 7 = 0$       2)  $y^2 - 4xy - 8x + 5y + 9 = 0$   
3)  $y^2 - 4xy - 6x - 8y + 5 = 0$       4)  $y^2 + 4xy + 4x + 6y + 9 = 0$

111. Product of the perpendicular distances from

$(\pm\sqrt{7}, 0)$  to the line  $\frac{x}{4} \cos \theta + \frac{y}{3} \sin \theta = 1$  is

- 1) 12      2) 7      3) 9      4) 8

112. If  $x + y\sqrt{2} = 2\sqrt{2}$  is a tangent to the ellipse  $x^2 + 2y^2 = 4$  then the eccentric angle of the point of contact is

- 1)  $\frac{\pi}{6}$       2)  $\frac{\pi}{4}$       3)  $\frac{\pi}{3}$       4)  $\frac{\pi}{2}$

113.  $l_1$  is the tangent to  $2x^2 + 3y^2 = 35$  at  $(4, -1)$  &  $l_2$  is the tangent to  $4x^2 + y^2 = 25$  at  $(2, -3)$ . The distance between  $l_1$  &  $l_2$  is

- 1)  $\frac{10}{\sqrt{73}}$       2)  $\frac{60}{\sqrt{73}}$       3) 0      4)  $\frac{5}{3\sqrt{2}}$

114. The tangent at any point P on the ellipse meets the tangents at the vertices A & A' of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ at L and M respectively. Then}$$

AL .A'M =

- 1)  $a^2$       2)  $b^2$       3)  $a^2+b^2$       4)  $ab$
115. The number of tangents that can be drawn to an ellipse perpendicular to a given straight line is  
1) 0      2) 1      3) 2      4) 3
116. The locus of the foot of the perpendicular drawn from the foci of the ellipse  $S = 0$  to any tangent to it is  
1) a circle      2) an ellipse  
3) a hyperbola      4) not a conic
117. If the normal at one end of latusrectum of the ellipse with eccentricity 'e', passes through one end of minor axis then  
1)  $e^4+2e^2-1=0$       2)  $e^4-2e^2+1=0$   
3)  $e^4+e^2-1=0$       4)  $e^4+2e^2+2=0$
118. Number of normals that can be drawn at the point (-2,3) to the ellipse  $3x^2+2y^2=30$  are  
1) 2      2) 3      3) 4      4) 1
119. Number of normals that can be drawn from the point (0,0) to  $3x^2+2y^2=30$  are  
1) 2      2) 4      3) 1      4) 3
120. C is the centre of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , the normal at the end of latusrectum to the ellipse meets the major axis in G. Then CG =  
1)  $ae^4$       2)  $ae^3$       3)  $ae^2$       4)  $ae$
121. Equation of the normal to the ellipse  $x^2+4y^2 = 25$  at the point whose ordinate is 2 is  
1)  $8x-3y-16=0$       2)  $3x-7y+15=0$   
3)  $8x-3y-18=0$       4)  $3x-8y-17=0$
122. The points on the ellipse  $3x^2 + y^2 = 37$ , where the normals to it are perpendicular to  $6x + 5y - 2 = 0$  are  
1)  $(3,5);(-3,-5)$       2)  $(2,5);(-2,-5)$   
3)  $(5,3);(-5,-3)$       4)  $(-5,3);(5,-3)$
123. The normals at a point P on the ellipse having A,A' as vertices and S, S' as foci, bisects the angle  
1)  $\angle A^1PA$     2.  $\angle A^1PS$     3.  $\angle S^1PS$     4.  $\angle S^1PA$
124. The point  $P\left(\frac{\pi}{4}\right)$  lie on the ellipse  $\frac{x^2}{4} + \frac{y^2}{2} = 1$  whose foci are S and S'. The equation of the external angular bisector of  $\angle SPS^1$  of  $\Delta SPS^1$  is  
1)  $x + \sqrt{2}y = 2\sqrt{2}$       2)  $2x + 3\sqrt{3}y = 12$   
3)  $3x - 4y + 12\sqrt{2} = 0$       4)  $x + y + 12\sqrt{2} = 0$

125. Equation of the auxiliary circle of the ellipse

$$\frac{x^2}{12} + \frac{y^2}{18} = 1 \text{ is}$$

- 1)  $x^2+y^2=9$       2)  $x^2+y^2=18$   
3)  $x^2+y^2=12$       4)  $x^2+y^2=30$
126. The radius of the director circle of the ellipse  $9x^2 + 25y^2 - 18x - 100y - 116 = 0$  is  
1)  $\sqrt{34}$       2)  $\sqrt{29}$       3) 5      4) 8
127. If  $\pi + \theta$  is the eccentric angle of a point on the Ellipse  $16x^2 + 25y^2 = 400$  then the corresponding point on the auxiliary circle is  
1)  $(-4 \cos \theta, -4 \sin \theta)$     2)  $(-5 \cos \theta, -5 \sin \theta)$   
3)  $(4 \cos \theta, 4 \sin \theta)$       4)  $(5 \cos \theta, 5 \sin \theta)$
128. The area of the ellipse  $9x^2 + 25y^2 - 18x - 100y - 116 = 0$  is  
1)  $9\pi$  sq. units      2)  $25\pi$  sq. units  
3)  $15\pi$  sq. units      4)  $20\pi$  sq. units
129. If the chord joining the points ' $\alpha$ ' ' $\beta$ ' on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  subtends a right angle at its centre then  $\tan \alpha \tan \beta =$   
1)  $-\frac{a^2}{b^2}$     2)  $\frac{a^2}{b^2}$     3)  $-\frac{b^2}{a^2}$     4)  $\frac{b^2}{a^2}$
130. If the equation of the chord joining the points P( $\theta$ ) and D  $\left(\theta + \frac{\pi}{2}\right)$  on  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $x \cos \alpha + y \sin \alpha = p$  then  $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha =$   
1)  $4p^2$       2)  $p^2$       3)  $\frac{p^2}{2}$       4)  $2p^2$
131. P( $\theta$ ),  $D\left(\theta + \frac{\pi}{2}\right)$  are two points on the Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Then the locus of mid point of chord PD is  
1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$       2)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$   
3)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{4}$       4)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$

132.  $P(\theta)$ ,  $D\left(\theta + \frac{\pi}{2}\right)$  are two points on the

Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  Then the locus of point of intersection of the two tangents at P and D to the ellipse is

1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{4}$       2)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$

3)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$       4)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$

133. Equation of the chord joining the points with eccentric

angles  $\frac{\pi}{3}, \frac{\pi}{6}$  on the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  is

1)  $3x+2y = 3(1+\sqrt{3})$       2)  $2x+3y = 2(1+\sqrt{3})$

3)  $2x+3y = 3(1+\sqrt{3})$       4)  $2x+3y = 4(1+\sqrt{3})$

134.  $\alpha$  and  $\beta$  are the extremities of a focal chord of

the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  then

$\cos^2 \frac{\alpha - \beta}{2} / \cos^2 \frac{\alpha + \beta}{2} =$

1)  $\frac{a^2 + b^2}{a^2}$       2)  $\frac{a^2}{a^2 + b^2}$

3)  $\frac{a^2}{a^2 - b^2}$       4)  $\frac{a^2 - b^2}{a^2}$

135. The line  $2x+5y = 12$  cuts the ellipse

$4x^2 + 5y^2 = 20$  in A and B. Then the mid point of the chord is

1) (2, 1)      2) (-2, -1)      3) (-1, -2)      4) (1, 2)

136. (2,1) is the mid point of chord AB of the ellipse

$x^2 + 4y^2 = 36$ . Then the point of

intersection of the two tangents to the ellipse at A and B is

1)  $\left(\frac{9}{2}, 9\right)$       2)  $\left(9, \frac{9}{2}\right)$       3)  $\left(7, \frac{7}{2}\right)$       4)  $\left(\frac{7}{2}, 7\right)$

137. The locus of middle points of the chords of the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  which passes through the positive end of the major axis is

1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{y}{2b} = 0$       2)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{a}$

3)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{x}{a} = 0$       4)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{2x}{a} = 0$

138. The locus of mid points of the chords of the ellipse  $2x^2 + 3y^2 = 4$  each of which makes an angle  $45^\circ$  with the x- axis is

1)  $3x+4y = 0$       2)  $4x+3y = 0$   
2)  $3x+2y = 0$       4)  $2x+3y = 0$

139. Locus of mid points of focal chords of the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ex}{a}$       2)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{ex}{a}$

3)  $x^2 + y^2 = a^2 + b^2$       4)  $x^2 + y^2 = a^2 - b^2$

140. The length of the chord intercepted by the ellipse  $4x^2 + 9y^2 = 1$  on the line  $9y = 1$  is

1)  $2\sqrt{2}$       2)  $\frac{2\sqrt{2}}{3}$       3)  $\sqrt{2}$       4)  $3\sqrt{2}$

141. The distance of a point on the ellipse  $x^2 + 3y^2 = 6$  from its centre is  $\sqrt{2}$ . Then the eccentric angle of the point is

1)  $\frac{\pi}{2}$       2)  $\frac{\pi}{4}$       3)  $\frac{\pi}{6}$       4)  $\frac{\pi}{3}$

142. The distance of a point P on the ellipse  $x^2 + 3y^2 = 6$  from the centre is 2, the eccentric angle of P is

1)  $\frac{\pi}{2}$       2)  $\frac{\pi}{6}$       3)  $\frac{\pi}{4}$       4)  $\frac{\pi}{3}$

143. The points on the ellipse  $\frac{x^2}{25} + \frac{y^2}{9} = 1$  whose eccentric angles differ by a right angle are

1)  $(5 \cos \theta, 3 \sin \theta), (5 \sin \theta, 3 \cos \theta)$

2)  $(5 \cos \theta, 3 \sin \theta), (-5 \sin \theta, 3 \cos \theta)$

3)  $(5 \cos \theta, -3 \sin \theta), (5 \sin \theta, 3 \cos \theta)$

4)  $(5 \cos \theta, -3 \sin \theta), (5 \sin \theta, 3 \cos \theta)$

144. The locus of the midpoints of parallel chords of an ellipse is a straight line

- 1) parallel to the major axis
- 2) parallel to the minor axis
- 3) passing through the focus
- 4) passing through the centre

145. Let 'E' be the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and C be the circle  $x^2 + y^2 = 9$ . let P and Q be two points (1,2) and (2,1) respectively. Then

- 1) Q lies inside 'C' but outside 'E'
- 2) Q lies outside of both C and E
- 3) P lies inside of both C and E
- 4) P lies inside C but outside E

146. The length of the double ordinate which is conjugate

to the directrix of the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  is

- 1) 32/3      2) 32/9      3) 32/25      4) 32/5

147. The chords of the ellipse  $S = 0$  passes through the pole of the directrix then the locus of mid points of chords is

1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = e$       2)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{e}{x}$

3)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ex}{a}$       4)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ae}{x}$

148. The circle drawn on the minor axis as diameter passes through the foci of the ellipse  $S = 0$  then its eccentricity  $e =$

- 1)  $\sin 18^\circ$       2)  $\sin 30^\circ$   
3)  $\cos 45^\circ$       4)  $\cos 30^\circ$

### KEY

- |        |        |        |        |        |
|--------|--------|--------|--------|--------|
| 1) 4   | 2) 2   | 3) 4   | 4) 2   | 5) 3   |
| 6) 1   | 7) 2   | 8) 3   | 9) 4   | 10) 4  |
| 11) 1  | 12) 2  | 13) 3  | 14) 4  | 15) 3  |
| 16) 2  | 17) 3  | 18) 1  | 19) 4  | 20) 2  |
| 21) 2  | 22) 2  | 23) 3  | 24) 1  | 25) 4  |
| 26) 2  | 27) 1  | 28) 4  | 29) 2  | 30) 4  |
| 31) 1  | 32) 2  | 33) 1  | 34) 2  | 35) 1  |
| 36) 4  | 37) 4  | 38) 2  | 39) 2  | 40) 3  |
| 41) 1  | 42) 3  | 43) 1  | 44) 2  | 45) 3  |
| 46) 1  | 47) 1  | 48) 2  | 49) 2  | 50) 3  |
| 51) 2  | 52) 1  | 53) 3  | 54) 4  | 55) 4  |
| 56) 1  | 57) 2  | 58) 1  | 59) 2  | 60) 1  |
| 61) 2  | 62) 4  | 63) 3  | 64) 4  | 65) 4  |
| 66) 3  | 67) 4  | 68) 4  | 69) 2  | 70) 2  |
| 71) 1  | 72) 3  | 73) 4  | 74) 1  | 75) 4  |
| 76) 4  | 77) 2  | 78) 3  | 79) 1  | 80) 3  |
| 81) 1  | 82) 1  | 83) 2  | 84) 2  | 85) 1  |
| 86) 1  | 86) 1  | 88) 2  | 89) 3  | 90) 2  |
| 91) 2  | 92) 1  | 93) 3  | 94) 2  | 95) 4  |
| 96) 4  | 97) 2  | 98) 2  | 99) 3  | 100) 4 |
| 101) 2 | 102) 3 | 103) 1 | 104) 1 | 105) 3 |
| 106) 2 | 107) 2 | 108) 1 | 109) 2 | 110) 1 |
| 111) 3 | 112) 2 | 113) 1 | 114) 2 | 115) 3 |
| 116) 1 | 117) 3 | 118) 4 | 119) 1 | 120) 2 |
| 121) 3 | 122) 2 | 123) 3 | 124) 1 | 125) 2 |
| 126) 1 | 127) 2 | 128) 3 | 129) 1 | 130) 4 |
| 131) 4 | 132) 3 | 133) 3 | 134) 4 | 135) 4 |
| 136) 2 | 137) 2 | 138) 4 | 139) 1 | 140) 2 |
| 141) 1 | 142) 3 | 143) 2 | 144) 4 | 145) 4 |
| 146) 4 | 147) 3 | 148) 3 |        |        |

### HINTS

5.  $2ae = 6, 2b = 8$

$b^2 = a^2 - a^2 e^2, 16 = a^2 - 9 \Rightarrow a^2 = 25$

$e = \frac{6}{10} = \frac{3}{5}$

7.  $\frac{2b^2}{a} = \frac{1}{2} \cdot 2b \Rightarrow 2b = a$

$e = \sqrt{\frac{4b^2 - b^2}{4b^2}} = \frac{\sqrt{3}}{2}$

8.  $2ae = \frac{2b^2}{a} \Rightarrow a^2 e = a^2 (1 - e^2)$

$\Rightarrow e^2 + e - 1 = 0$

$e = \frac{-1 \pm \sqrt{1+4}}{2}$

10.  $\frac{1}{\sqrt{2}} = e$

14.  $2a : 2b = 5 : 3$

$\frac{a}{b} = \frac{5}{3}$

$25a^2(1 - e^2) = 9a^2$

$\therefore e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

16.  $\frac{\sqrt{3}}{2} = e$

18.  $e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

20.  $e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

22.  $(-2, 0), (2, 0) \quad a = 4$

$ae = 2 \quad e = \frac{1}{2}$

24.  $25(x-3)^2 + 9(y-5)^2 = 225$

$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$

$\left(0, \pm 5 \times \frac{4}{5}\right) = (x-3, y-5)$

$(3, 1) (3, 9)$

26.  $3(x^2 - 4x) + 4(y^2 - 2y) = -4$

$3(x-2)^2 + 4(y-1)^2 = 12$

$\frac{(x-2)^2}{4} + \frac{(y-1)^2}{3} = 1$

$$\left(\pm 2, \frac{1}{2}, 0\right) = (x-2, y-1)$$

$$(3,1)(1,1)$$

$$27. \quad SP + S^1P = 2b = 2 \times 10 = 20$$

$$30. \quad 2ae = 2 \times 5 \times \sqrt{\frac{25-16}{25}} = 2 \times 3 = 6$$

$$31. \quad x = 2 \pm 6\sqrt{\frac{25}{36}} = 7, -3$$

$$32. \quad (-1 \pm 5, 3)$$

$$(4, 3), (-6, 3)$$

$$34. \quad \text{Use } 4b^2e$$

$$38. \quad 2a$$

$$39. \quad \text{Use } l = \frac{a^2}{b}$$

$$40. \quad a + ex_1$$

$$42. \quad x - h = \pm \frac{a}{e}$$

$$43. \quad 9x^2 + 5(y^2 - 6y) = 0$$

$$9x^2 + 5(y-3)^2 = 45$$

$$\frac{x^2}{5} + \frac{(y-3)^2}{9} = 1, \quad e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$$

$$\frac{2b}{e} = \frac{2 \times 3 \times 3}{2} = 9$$

$$45. \quad (x-2)^2 + y^2 = \frac{1}{9}(x-18)^2$$

$$8x^2 + 9y^2 = 288$$

$$\frac{x^2}{36} + \frac{y^2}{32} = 1 \Rightarrow LLR = \frac{32}{3}$$

$$48. \quad 2ae = 6; \quad 2a = 8$$

$$e = \frac{6}{8} = \frac{3}{4}; \quad b^2 = 16 - 9 = 7$$

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

$$49. \quad 2b = 10 \text{ \& } 2be = 8$$

$$52. \quad (x-2)^2 + y^2 = \frac{1}{4}(x-8)^2$$

$$3x^2 + 4y^2 = 48$$

$$54. \quad \frac{2 \cdot 20}{6} \text{ verify}$$

$$55. \quad \text{Verify the major axis} = 26$$

$$58. \quad \text{Verify the foci.}$$

$$59. \quad \text{Verify the foci.}$$

$$62. \quad \text{Verify the } a^2 \text{ \& } b^2$$

$$64. \quad ae = 5; \quad x = \frac{a}{e}$$

$$e = \sqrt{1 - \frac{11}{36}} = \frac{5}{6}$$

$$65. \quad a = 3; \quad e = \frac{1}{2}; \quad b^2 = a^2(1 - e^2) = 9\left(1 - \frac{1}{4}\right)$$

$$b = \frac{3\sqrt{3}}{2}$$

$$67. \quad CS : a = e : 1$$

$$69. \quad SP + S^1P = 8; SS^1 = 4$$

$$70. \quad b : be = 1 : e = 1 : \frac{1}{\sqrt{3}} = \sqrt{3} : 1$$

$$71. \quad a = 4$$

$$79. \quad xx_1 + yy_1 - a^2 = 0$$

$$yy_1 = -xx_1 + a^2; \quad \frac{a^4}{y_1^2} = a^2 \frac{x_1^2}{y_1^2} + b^2$$

$$a^4 = a^2 x^2 + b^2 y^2$$

$$80. \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$$

$$d = \frac{1}{\sqrt{\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4}}}$$

$$89. \quad \frac{x^2}{4} + \frac{y^2}{3} = 1 \quad \frac{-b^2 x_1}{a^2 y_1} = \frac{1}{2}$$

$$y = mx \pm \sqrt{a^2 m^2 + b^2} \quad \frac{-3x_1}{4y_1} = \frac{1}{2}$$

$$y = \frac{1}{2}x \pm \sqrt{\frac{4}{4} + 3} \quad 2y = x \pm 4$$

$$y = \frac{x}{2} \pm 2; \quad x - 2y \pm 4 = 0$$

$$92. \quad y = \sqrt{3}x \pm \sqrt{28 \times 3 + 16}$$

$$y = \sqrt{3}x \pm 10$$

$$96. \quad t\left(\frac{x}{a} + 1\right) = \frac{y}{b} \quad \frac{x}{a} - 1 = \frac{-ty}{b}$$

$$t\left(\frac{x^2}{a^2} - 1\right) = -t\frac{y^2}{b^2} \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$100. \quad \text{Use } m_1 + m_2 = \frac{2x_1y_1}{x_1^2 - a^2}$$

$$103. \quad \text{Use } m_1m_2 = 1$$

$$106. \quad \text{Tgts. are } y - mx = \sqrt{a^2m^2 + b^2} \text{ \& }$$

$$my + x = \sqrt{(a^2 + \lambda) + (b^2 + \lambda)m^2}$$

square and add.

$$109. \quad \text{Any tgt. is } \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

$$\text{Area is } \frac{ab}{2 \cos \theta \sin \theta}$$

$$\Rightarrow \text{max. area} = ab \text{ (Q max. value of } \sin 2\theta = 1)$$

$$112. \quad b^2 = 3^2 = 9$$

$$114. \quad \text{Slope of the tgts are } \frac{8}{3} \text{ each.}$$

$$\therefore \text{tgts are } 8x - 3y = 35 \text{ \& } 8x - 3y = 25$$

$$\text{Distance} = \frac{10}{\sqrt{73}}$$

$$117. \quad \text{Def. of the auxiliary circle.}$$

$$\therefore \text{circle}$$

$$119. \quad S_{11} = 0 \therefore \text{one normal.}$$

$$120. \quad 2 \text{ normals i.e. axes.}$$

$$122. \quad y = 2 \Rightarrow x = \pm 3 \text{ slope of the normal at } (3, 2) \text{ is}$$

$$\frac{8}{3} \Rightarrow \text{its equation is } 8x - 3y = 18$$

$$123. \quad \text{Substitute the options in the ellipse.}$$

$$124. \quad \text{Normal is the angular bisector of } \angle S^1PS$$

$$125. \quad \text{External angular bisector is the tangent at that point}$$

$$P\left(\frac{\pi}{4}\right) = (\sqrt{2}, 1) \Rightarrow SLT = \frac{-2\sqrt{2}}{4 \times 1} = -\frac{1}{\sqrt{2}}$$

$$\therefore \text{E.T. is } x + y\sqrt{2} = 2\sqrt{2}$$

$$127. \quad x^2 + y^2 = 9 + 25 = 34 \Rightarrow \text{radius} = \sqrt{34}$$

$$129. \quad \pi ab = \pi \times 5 \times 3 = 15\pi$$

$$130. \quad (a \cos \alpha, b \sin \alpha)(a \cos \beta, b \sin \beta),$$

$$c(0,0), (\Theta x_1x_2 + y_1y_2 = 0)$$

$$a^2 \cos \alpha \cos \beta + b^2 \sin \alpha \sin \beta = 0$$

$$\therefore \tan \alpha \tan \beta = \frac{-a^2}{b^2}$$

$$131. \quad \text{Substitute the points } P(\theta) \text{ \& } D\left(\theta + \frac{\pi}{2}\right)$$

$$\text{in the chord.}$$

$$\therefore a \cos \theta \cos \alpha + b \sin \theta \sin \alpha = p \rightarrow (1)$$

$$-a \sin \theta \cos \alpha + b \cos \theta \sin \alpha = p \rightarrow (2)$$

$$(1)^2 + (2)^2 \Rightarrow a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = 2p^2$$

$$132. \quad \text{The tangents at P and Q are}$$

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \rightarrow (1)$$

$$-\frac{x}{a} \sin \theta + \frac{y}{b} \cos \theta = 1 \rightarrow (2) \text{ eliminate } \theta$$

$$135. \quad \text{Substitute } (ae, 0) \text{ in the chord.}$$

$$\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right) \text{ then}$$

$$\frac{\cos^2\left(\frac{\alpha - \beta}{2}\right)}{\cos^2\left(\frac{\alpha + \beta}{2}\right)} = e^2 = \frac{a^2 - b^2}{a^2}$$

$$136. \quad \text{Slope of the chord is } -\frac{4x_1}{5y_1} = -\frac{2}{5} \Rightarrow y_1 = 2x_1$$

$$\text{Substitute in the given line}$$

$$2x_1 + 10x_1 = 12 \Rightarrow x_1 = 1, y_1 = 2$$

$$\text{mid point} = (1, 2)$$

$$139. \quad -\frac{\frac{4}{3}x_1}{\frac{4}{2}y_1} = \tan 45^\circ \left( Q - \frac{b^2x_1}{a^2y_1} \right)$$

$$2x + 3y = 0$$

$$140. \quad \text{Substitute } (ae, o) \text{ in } \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

$$142. \quad C(0,0), \text{ let } P(\sqrt{6} \cos \theta, \sqrt{2} \sin \theta) \text{ and } CP = \sqrt{2}$$

$$6 \cos^2 \theta + 2 \sin^2 \theta = 2 \Rightarrow \theta = \frac{\pi}{2} \text{ satisfied.}$$

143.  $C(0,0)$ , let  $P(\sqrt{6} \cos \theta, \sqrt{2} \sin \theta)$  and  $CP = 2$
- $$6 \cos^2 \theta + 2 \sin^2 \theta = 4 \Rightarrow \theta = \frac{\pi}{4} \text{ satisfied.}$$

### LEVEL - II

- "e" is the eccentricity of the ellipse  $4x^2 + 9y^2 = 36$  and C is the centre and S is the focus and A is the vertex then CS. SA =  
 1)  $3 - \sqrt{5} : \sqrt{5}$       2)  $\sqrt{5} : 3 - \sqrt{5}$   
 3)  $3 + \sqrt{5} : \sqrt{5}$       4)  $\sqrt{5} : 3 + \sqrt{5}$
- $(-4, 1), (6, 1)$  are the vertices of an Ellipse and one of the foci lies on  $x - 2y = 2$  then the eccentricity is  
 1)  $\frac{3}{5}$       2)  $\frac{4}{5}$       3)  $\frac{2}{5}$       4)  $\frac{1}{5}$
- The latus rectum subtends a right angle at the centre of the ellipse then its eccentricity is  
 1)  $2 \sin 18^\circ$       2)  $2 \cos 18^\circ$   
 3)  $2 \sin 54^\circ$       4)  $2 \cos 54^\circ$
- The angle of inclination of the chord joining the ends of major axis and minor axis of an ellipse is  $\sin^{-1} \frac{1}{\sqrt{5}}$  then e =  
 1)  $\frac{1}{\sqrt{2}}$       2)  $\frac{\sqrt{3}}{2}$       3)  $\sqrt{\frac{2}{3}}$       4) 4
- If the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $b > a$ ) and the parabola  $y^2 = 4ax$  cut at right angles, then eccentricity of the ellipse is  
 1)  $\frac{3}{5}$       2)  $\frac{2}{3}$       3)  $\frac{1}{\sqrt{2}}$       4)  $\frac{1}{2}$
- $S(3,4)$  and  $S^1(9,12)$  are the foci of an ellipse and the foot of the perpendicular from S to a tangent to the ellipse is  $(1,-4)$ . Then the eccentricity of the ellipse is  
 1)  $\frac{3}{13}$       2)  $\frac{4}{13}$       3)  $\frac{5}{13}$       4)  $\frac{7}{13}$

- P is a variable point on the ellipse  $9x^2 + 16y^2 = 144$  with foci S and  $S^1$ . If K is the area of the triangle  $SS^1P$  then the maximum value of K is  
 1)  $7\sqrt{3}$       2)  $3\sqrt{5}$       3)  $7\sqrt{5}$       4)  $3\sqrt{7}$
- The major axis and minor axis of an ellipse are respectively  $x - 2y - 5 = 0$  and  $2x + y + 10 = 0$ , one end of latusrectum is  $(3,4)$ , then the foci are  
 1)  $(5,0); (-3,-4)$       2)  $(5,0); (-6,-4)$   
 3)  $(5,0); (-11,-8)$       4)  $(5,0); (11,-4)$
- The abscissae of the points on the ellipse  $9x^2 + 25y^2 - 18x - 100y - 116 = 0$  lie between  
 1) 3, -5      2) -4, 6      3) -5, 7      4) 2, 5
- The ordinates of the points on the ellipse  $x^2 + 4y^2 - 8x + 8y + 4 = 0$  lie between  
 1) 3, 5      2) 7, 9      3) -3, 1      4) 4, 7
- One focus and the corresponding directrix of an ellipse are  $(1,2)$  and  $x - y = 5$ , its eccentricity is  $\frac{1}{2}$  then centre is  
 1)  $(3,0)$       2)  $(0,3)$       3)  $(3,-3)$       4)  $(0,-3)$
- $S$  and  $S^1$  are the foci of the ellipse  $25x^2 + 16y^2 = 1600$ . Then the area of the triangle formed by the foci S and  $S^1$  with the point  $(4\sqrt{3}, 5)$  is  
 1)  $24\sqrt{3}$       2)  $25\sqrt{3}$   
 3)  $40\sqrt{3}$       4)  $30\sqrt{3}$
- $SPS^1$  is focal chord of the ellipse  $4x^2 + 9y^2 = 36$ . If  $SP=4$  then  $SP^1$   
 1)  $\frac{2}{3}$       2)  $\frac{3}{5}$       3)  $\frac{4}{3}$       4)  $\frac{4}{5}$
- The distances from the foci to a points  $P(x_1, y_1)$  on the ellipse  $\frac{x^2}{9} + \frac{y^2}{25} = 1$  are  
 1)  $4 \pm \frac{2}{3}y_1$       2)  $5 \pm \frac{4}{5}y_1$   
 3)  $5 \pm \frac{4}{5}x_1$       4)  $4 \pm \frac{2}{3}x_1$



15. Ratio of the greatest and least focal distances of a point on the ellipse  $4x^2 + 9y^2 = 36$  is

- 1)  $4 + \sqrt{5} : 4 - \sqrt{5}$
- 2)  $5 + \sqrt{5} : 4 - \sqrt{5}$
- 3)  $3 + \sqrt{5} : 3 - \sqrt{5}$
- 4)  $\sqrt{7} : 2$

16. Equation of the directrices of the ellipse  $4x^2 + y^2 - 8x + 2y + 1 = 0$  are

- 1)  $\sqrt{3}y + \sqrt{3} \pm 5 = 0$
- 2)  $\sqrt{3}y + \sqrt{3} \pm 4 = 0$
- 3)  $\sqrt{3}y + \sqrt{3} \pm 7 = 0$
- 4)  $\sqrt{3}y + \sqrt{3} \pm 8 = 0$

17. The foci of an ellipse are  $S(-1, -1)$ ,  $S^1(0, -2)$ , its  $e=1/2$ , then the equation of the directrix corresponding to the focus S is

- 1)  $x - y + 3 = 0$
- 2)  $x - y + 7 = 0$
- 3)  $x - y + 5 = 0$
- 4)  $x - y + 4 = 0$

18. The foci of an ellipse are  $S(-2, -3)$ ,  $S^1(0, 1)$  its  $e = \frac{1}{\sqrt{2}}$  then the directrix corresponding to the focus  $S^1$

- 1)  $x + 2y - 5 = 0$
- 2)  $x + 2y - 9 = 0$
- 3)  $x + 2y - 11 = 0$
- 4)  $x + 2y - 7 = 0$

19. An ellipse with foci  $(2, 2)$ ,  $(3, -5)$  passes through  $(6, -1)$  then its semi-latus rectum is

- 1)  $\frac{7}{2}$
- 2)  $\frac{5}{2}$
- 3)  $\frac{9}{2}$
- 4)  $\frac{11}{2}$

20. Equation to the locus of the point which moves such that the sum of its distances from  $(-4, 3)$  and  $(4, 3)$  is 12 is

- 1)  $\frac{x^2}{36} + \frac{(y-3)^2}{20} = 1$
- 2)  $\frac{x^2}{20} + \frac{(y-3)^2}{36} = 1$

$$3) \frac{(x-3)^2}{36} + \frac{y^2}{20} = 1$$

$$4) \frac{(x-1)^2}{36} + \frac{(y-3)^2}{20} = 1$$

21. Equation of the ellipse with length of latus rectum 10 and distance between the foci is equal to length of minor axis is

- 1)  $2x^2 + y^2 = 100$
- 2)  $3x^2 + y^2 = 50$
- 3)  $x^2 + 2y^2 = 100$
- 4)  $x^2 + 3y^2 = 50$

22. P is a point on the ellipse having  $(3, 4)$  and  $(3, -2)$  as the ends of minor axis. If the sum of the focal distances of P be equal to 10 then its equation is

$$1) \frac{(x-3)^2}{36} + \frac{(y-1)^2}{12} = 1$$

$$2) \frac{(x-3)^2}{36} + \frac{(y-1)^2}{25} = 1$$

$$3) \frac{(x-3)^2}{25} + \frac{(y-1)^2}{9} = 1$$

$$4) \frac{(x-3)^2}{16} + \frac{(y-1)^2}{7} = 1$$

23. Equation of the ellipse with centre  $(1, 2)$ , one focus at  $(6, 2)$  and passing through  $(4, 6)$  is

$$1) \frac{(x-1)^2}{45} + \frac{(y-2)^2}{20} = 1$$

$$2) \frac{(x-1)^2}{35} + \frac{(y-2)^2}{15} = 1$$

$$3) \frac{(x-1)^2}{3} + \frac{(y-2)^2}{25} = 1$$

$$4) \frac{(x-1)^2}{25} + \frac{(y-2)^2}{15} = 1$$

24. Equation of the ellipse with centre at origin, passing through the point  $(-3,1)$  and  $e = \frac{2}{\sqrt{5}}$  is
- $3x^2 + 5y^2 = 32$
  - $3x^2 + 7y^2 = 34$
  - $5x^2 + 3y^2 = 32$
  - $3x^2 + 7y^2 = 36$
25. Axes are coordinate axes, the area of the max. rectangle that can be inscribed in the ellipse is 16 Sq. units,  $e = \frac{\sqrt{3}}{2}$  then equation of the ellipse is
- $\frac{x^2}{16} + \frac{y^2}{4} = 1$
  - $\frac{x^2}{16} + \frac{y^2}{8} = 1$
  - $\frac{x^2}{64} + \frac{y^2}{32} = 1$
  - $\frac{x^2}{20} + \frac{y^2}{16} = 1$
26. Axes are coordinate axes. A and L are the ends of major axis and latusrectum respectively. Area of  $DOAL = 8 \text{ sq. units}$ ,  $e = \frac{1}{\sqrt{2}}$ , then equation of the ellipse is
- $\frac{x^2}{16} + \frac{y^2}{8} = 1$
  - $\frac{x^2}{32} + \frac{y^2}{16} = 1$
  - $\frac{x^2}{64} + \frac{y^2}{32} = 1$
  - $\frac{x^2}{8} + \frac{y^2}{4} = 1$
27. In an ellipse the length of major axis is 10 and the distance between the foci is 8. Then the length of minor axis is
- 5
  - 7
  - 4
  - 6
28. An ellipse with centre at origin passes through the points  $(2,2)$ ,  $(1,4)$ . Then the length of its major axis is
- $2\sqrt{5}$
  - $3\sqrt{5}$
  - $5\sqrt{5}$
  - $4\sqrt{5}$
29. In an ellipse the length of minor axis is equal to the distance between the foci, the length of latusrectum is 10 and  $e = \frac{1}{\sqrt{2}}$ . Then the length of major axis is
- 16
  - 18
  - 20
  - 22
30. If N is the foot of the perpendicular drawn from any point P on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) to its major axis  $AA^1$  then  $\frac{PN^2}{AN \cdot A^1N} =$
- $\frac{a^2}{b^2}$
  - $\frac{b^2}{a^2}$
  - $\frac{2a^2}{b^2}$
  - $\frac{2b^2}{a^2}$
31. The locus of poles of chords of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  which subtend a right angle at the centre of the ellipse is
- $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$
  - $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{2}{a^2} + \frac{2}{b^2}$
  - $\frac{x^2}{a^4} - \frac{y^2}{b^4} = \frac{1}{a^2} - \frac{1}{b^2}$
  - $\frac{x^2}{a^4} - \frac{y^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$
32. The locus of poles w.r. to  $x^2 + y^2 = a^2 - b^2$  of normal chords of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is
- $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$
  - $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$
  - $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 2$
  - $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 6$
33. If the polar of the point  $(-4,1)$  w.r. to the parabola  $y^2 = 2x$  touches the ellipse  $x^2 + 3y^2 = 12$  then the point of contact is
- $(2, -3)$
  - $(3, -1)$
  - $(-3, 1)$
  - $(-2, 3)$

34. C is the centre of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . If a tangent of this ellipse meets the major and minor axes at T and t respectively, then  $\frac{a^2}{CT^2} + \frac{b^2}{Ct^2} =$   
 1) 4      2) 3      3) 2      4) 1
35. The tangent at P to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  cuts the major axis in T and PN is the perpendicular to x axis. If C is the centre of the ellipse then CT.CN =  
 1) a      2) b      3)  $b^2$       4)  $a^2$
36. The locus of the foot of the perpendicular from the centre of the ellipse  $x^2 + 3y^2 = 3$  on any tangent to it is  
 1)  $(x^2 + y^2)^2 = 5x^2 + 7y^2$   
 2)  $(x^2 + y^2)^2 = 7x^2 + 5y^2$   
 3)  $(x^2 + y^2)^2 = x^2 + 3y^2$   
 4)  $(x^2 + y^2)^2 = 3x^2 + y^2$
37. A line touches the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the circle  $x^2 + y^2 = r^2$ . Then the slope m of the common tangent is given by  $m^2 =$   
 1)  $\frac{a^2 - r^2}{b^2 - r^2}$       2)  $\frac{r^2 - b^2}{a^2 - r^2}$   
 3)  $\frac{r^2 + b^2}{a^2 - r^2}$       4)  $\frac{r^2 - 2b^2}{a^2 - 2r^2}$
38. Slope of the common tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $\frac{x^2}{a^2} + \frac{y^2}{a^2 + b^2} = 1$  is  
 1)  $\frac{2a}{b}$       2)  $\frac{2b}{a}$       3)  $\frac{a}{b}$       4)  $\frac{b}{a}$
39. A tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  having slope m meets the auxiliary circle in P and Q if angle PCQ is  $90^\circ$  where C is the centre of the ellipse, then  $m^2 =$

- 1)  $3e^2 - 2$       2)  $3e^2 - 1$   
 3)  $3e^2 + 2$       4)  $2e^2 - 1$
40. The locus of point of intersection of the two tangents to the ellipse  $b^2x^2 + a^2y^2 = a^2b^2$  which make an angle  $60^\circ$  with one another is  
 1)  $4(x^2 + y^2 - a^2 - b^2)^2 = 3(b^2x^2 + a^2y^2 - a^2b^2)$   
 2)  $3(x^2 + y^2 - a^2 - b^2)^2 = 4(b^2x^2 + a^2y^2 - a^2b^2)$   
 3)  $3(x^2 + y^2 - a^2 - b^2)^2 = 2(b^2x^2 + a^2y^2 - a^2b^2)$   
 4)  $3(x^2 + y^2 - a^2 - b^2)^2 = (b^2x^2 + a^2y^2 - a^2b^2)$
41. The sum of the eccentric angles of two points of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $2a$  (constant) then the locus of point of intersection of the two tangents at these points is  
 1)  $ay = bx \tan a$       2)  $ax = by \tan a$   
 3)  $ay = bx \cot a$       4)  $ax = by \cot a$
42. A tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  cuts the axes in M and N. Then the least length of MN is  
 1)  $a + b$       2)  $a - b$   
 3)  $a^2 + b^2$       4)  $a^2 - b^2$
43. Tangents are drawn through the points  $(4, \sqrt{3})$  to the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ . The points at which these tangents touch the ellipse are  
 1)  $(4, 2)$       2)  $(2, 0)$   
 3)  $(4, 0)$       4)  $(0, 4)$

44. The point in the first quadrant of the ellipse  $\frac{x^2}{25} + \frac{y^2}{144} = 1$  at which the tangent makes equal angles with the axes is

- 1)  $\left(\frac{25}{13}, \frac{144}{13}\right)$  2)  $\left(\frac{25}{13}, \frac{144}{13}\right)$   
 3)  $\left(\frac{25}{13}, \frac{144}{13}\right)$  4)  $\left(\frac{25}{13}, -\frac{144}{13}\right)$

45. The points on the ellipse  $x^2 + 4y^2 = 2$ , where the tangents are parallel to the line  $x - 2y - 6 = 0$  are

- 1)  $\left(-1, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right)$  2)  $\left(\frac{1}{2}, -1\right), \left(\frac{1}{2}, 1\right)$   
 3)  $\left(1, \frac{1}{2}\right), \left(-1, \frac{1}{2}\right)$  4)  $(-1, -1), (1, 1)$

46. Perpendiculars are drawn from the points  $(0, \pm ae)$  on any tangent to  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then the sum of their squares is

- 1)  $2b^2$  2)  $2a^2$  3)  $b^2$  4)  $4a^2$

47. The locus of point of intersection of tangents drawn at  $a, b$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  such that

$$a - b = \frac{p}{3} \text{ is}$$

- 1)  $3\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 4$   
 2)  $4\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 3$   
 3)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$   
 4)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$

48. The total number of real tangents that can be drawn to the ellipse  $3x^2 + 5y^2 = 32$  and  $25x^2 + 9y^2 = 450$  passing through (3,5) is  
 1) 0 2) 2 3) 3 4) 4

49. The tangents from which of the following points to the ellipse  $5x^2 + 4y^2 = 20$  are perpendicular

- 1)  $(\sqrt{5}, 2\sqrt{2})$  2)  $(2\sqrt{2}, 1)$   
 3)  $(\sqrt{5}, -1)$  4)  $(\sqrt{5}, 1)$

50. If any tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  makes equal intercepts of length "l" on the axes then  $l =$

- 1)  $a^2 + b^2$  2)  $\sqrt{a^2 + b^2}$   
 3)  $(a^2 + b^2)^2$  4)  $a + b$

51. A normal to  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  meets the axes in L and M. The perpendiculars to the axes through L and M intersect at P. Then the equation to the locus of P is

- 1)  $a^2x^2 - b^2y^2 = (a^2 + b^2)^2$   
 2)  $a^2x^2 + b^2y^2 = (a^2 + b^2)^2$   
 3)  $b^2x^2 - a^2y^2 = (a^2 - b^2)^2$   
 4)  $a^2x^2 + b^2y^2 = (a^2 - b^2)^2$

52. The tangent and the normal to the ellipse  $4x^2 + 9y^2 = 36$  at the point P meet the major axis in Q and R respectively. If QR=4 then the eccentric angle of P is given by

- 1)  $\cos^{-1} \frac{2}{3}$  2)  $\cos^{-1} \frac{2}{5}$   
 3)  $\cos^{-1} \frac{3}{5}$  4)  $\cos^{-1} \frac{1}{3}$

53. If the normal at the point  $q$  on the ellipse  $5x^2 + 14y^2 = 70$  intersect it again at the point  $2q$  then  $\cos q =$   
 1)  $-3/5$  2)  $-3/4$  3)  $-2/3$  4)  $-3/7$

54. The normal at  $P(2 \cos t, \sin t)$  of an ellipse meets x - axis at Q and y axis at R. A point S is taken on QP produced such that  $QR=QS$ . If the locus of S is a circle, then its radius is

- 1) 4      2) 5      3) 6      4) 3

55.  $P$  is a point on the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  and  $Q$  is its corresponding point on the auxiliary circle. If the locus of the point of intersection of the normal at  $P$  and  $Q$  to the respective curves is circle, then its radius is

- 1) 5      2) 7      3) 6      4) 8

56. The maximum distance of any normal to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  from the centre is

- 1)  $a + b$       2)  $a - b$   
3)  $a^2 + b^2$       4)  $a^2 - b^2$

57. The radius of the circle passing through the foci of the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  and having its centre at  $(0,3)$  is

- 1) 3      2) 4      3)  $\sqrt{7}$       4)  $\sqrt{12}$

58. If the chord joining the points whose eccentric angles are  $a$  and  $b$  on the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  meets the major axis at a distance

“d” from the centre, then  $\tan \frac{a}{2} \tan \frac{b}{2}$

- 1)  $\frac{d + 2a}{d - 2a}$       2)  $\frac{d - 2a}{d + 2a}$   
3)  $\frac{d - a}{d + a}$       4)  $\frac{d + a}{d - a}$

59. The line  $lx + my = 1$  meets the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  in the points P and Q. The mid point of chord PQ is

- 1)  $\frac{a^2 l}{k}, \frac{b^2 m}{k}$       2)  $\frac{a^2 l}{k}, -\frac{b^2 m}{k}$   
3)  $\frac{b^2 l}{k}, \frac{a^2 m}{k}$       4)  $\frac{b^2 l}{k}, -\frac{a^2 m}{k}$

Where  $k = a^2 l^2 + b^2 m^2$

60. The locus of mid points of normal chords of the

ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

1)  $\frac{a^6}{x^2} + \frac{b^2}{y^2} = (a^2 - b^2)^2$

2)  $\frac{a^6}{x^2} - \frac{b^2}{y^2} = (a^2 - b^2)^2$

3)  $\frac{a^6}{x^2} + \frac{b^2}{y^2} = (a^2 - b^2)^2$

4)  $\frac{a^6}{x^2} - \frac{b^2}{y^2} = (a^2 - b^2)^2$

61. The locus of mid points of chords of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  which passes through the foot of the directrix is

1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ex}{a^2}$       2)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{ae}$

3)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{a^2 e}$       4)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{ae^2}$

62. The locus of mid points of the chords of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  whose poles lie on the auxiliary circle is

1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2 + y^2}{a^2 + b^2}$

2)  $\frac{x^2 + y^2}{a^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

3)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2 - y^2}{a^2 - b^2}$

4)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2 - y^2}{a^2 - b^2}$

63. The length of the chord intercepted by the ellipse  $x^2 + 2y^2 = 4$  on the normal at the point  $(\sqrt{2}, 1)$  is

1)  $\frac{3\sqrt{6}}{5}$  2)  $\frac{2\sqrt{6}}{5}$  3)  $\frac{7\sqrt{6}}{5}$  4)  $\frac{6\sqrt{6}}{5}$

64. Length of the chord intercepted by the ellipse  $x^2 + 4y^2 = 16$  on the line  $y = x\sqrt{2} + 2$  is

1)  $\frac{16\sqrt{5}}{3}$  2)  $\frac{16\sqrt{6}}{9}$  3)  $\frac{12\sqrt{3}}{5}$  4)  $\frac{14\sqrt{3}}{5}$

65. The area of the parallelogram formed by the tangents at the points whose eccentric angles are  $q, q + \frac{p}{2}, q + p, q + \frac{3p}{2}$  on the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

1)  $ab$  2)  $4ab$  3)  $3ab$  4)  $2ab$

66. The eccentric angles of the ends of latusrectum of

the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

1)  $\tan^{-1} \frac{ae}{b}$  2)  $\sin^{-1} \frac{ae}{b}$

3)  $\cos^{-1} \frac{ae}{b}$  4)  $\sec^{-1} \frac{ae}{b}$

67. If  $A, A^1$  are the vertices,  $S, S^1$  are the foci and  $Z, Z^1$  are the feet of the directrices of an ellipse with centre  $C$  then  $CS, CA, CZ$  are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

68.  $S$  and  $S^1$  are the foci of an ellipse whose eccentricity is  $\frac{1}{\sqrt{2}}$ .  $B$  and  $B^1$  are the ends of minor axis

then  $SBS^1B^1$  is

1) Parallelogram 2) Rhombus  
3) Square 4) Rectangle

### KEY

1-10: 2 1 1 2 3  
3 4 3 2 3

11-20:	2	1	4	2	3
	2	1	4	2	1
21-30:	3	3	1	1	1
	2	4	4	3	2
31-40:	1	2	2	4	4
	4	2	3	4	2
41-50:	1	1	3	2	1
	2	1	3	2	2
51-60:	4	3	3	4	2
	2	2	3	1	1
61-68:	2	2	4	2	2
	1	2	3		

### LEVEL - III

1. An ellipse passing through the point  $(2\sqrt{13}, 4)$  has its foci at  $(-4, 1)$  and  $(4, 1)$ . Then its eccentricity is

1)  $\frac{2}{3}$  2)  $\frac{1}{3}$  3)  $\frac{1}{4}$  4)  $\frac{1}{2}$

2. The length of subnormal at  $(4, 2)$  to an ellipse is 3. Then its eccentricity is

1)  $\frac{1}{3}$  2)  $\frac{1}{4}$  3)  $\frac{2}{3}$  4)  $\frac{1}{2}$

3. The length of subtangent corresponding to the point  $(\frac{12}{5}, \frac{4}{5})$  on the ellipse is  $16/3$ . Then the eccentricity is

1)  $\frac{4}{5}$  2)  $\frac{2}{3}$  3)  $\frac{1}{5}$  4)  $\frac{3}{5}$

4. The area of the ellipse is  $8p$  sq. units, distance between the foci is  $4\sqrt{3}$ , then  $e =$

1)  $\sin 30^\circ$  2)  $\sin 45^\circ$   
3)  $\sin 60^\circ$  4)  $\sin 75^\circ$

5. Area of the quadrilateral formed by the ends of major axis and minor axis is  $8\sqrt{3}$ . The distance between the foci is  $4\sqrt{2}$ , then the eccentricity of the ellipse is

1)  $\frac{1}{\sqrt{3}}$  2)  $\frac{1}{3}$

3)  $\sqrt{\frac{2}{3}}$  4) 4

6. The tangent drawn to the ellipse at the parametric point  $q$ , where  $q = \tan^{-1} 2$  meets the auxiliary circle at P and Q and PQ subtends a right angle at the centre of the ellipse, then eccentricity is

1)  $\frac{1}{\sqrt{3}}$     2)  $\frac{1}{3}$     3)  $\sqrt{\frac{2}{3}}$     4)  $\frac{\sqrt{5}}{3}$

7. An ellipse is inscribed in a rectangle and the angle between the diagonals of the rectangle is  $\tan^{-1}(2\sqrt{2})$  then the eccentricity of the ellipse is

1)  $\cot 15^\circ$     2)  $\cos 45^\circ$   
3)  $\cot 60^\circ$     4)  $\cot 75^\circ$

8. A bar of length 20 units moves with its ends on two fixed lines which are at right angles. A point is marked at a dist. of 8 units from one end. If the focus of the point is an ellipse, then its eccentricity is

1)  $\frac{2}{3}$     2)  $\frac{4}{9}$     3)  $\frac{\sqrt{5}}{3}$     4)  $\frac{2}{5}$

9. At some point P on the ellipse, the segment  $SS^1$  subtends a right angle, then its eccentricity is

1)  $e = \frac{\sqrt{2}}{2}$     2)  $e < \frac{1}{\sqrt{2}}$

3)  $e > \frac{1}{\sqrt{2}}$     4)  $\frac{\sqrt{3}}{2}$

10. The circle on  $SS^1$  as diameter intersects the ellipse in real points then its eccentricity is

1)  $e = \frac{1}{\sqrt{2}}$     2)  $e < \frac{1}{\sqrt{2}}$

3)  $e > \frac{1}{\sqrt{2}}$     4)  $\frac{\sqrt{3}}{2}$

11. Let S and  $S^1$  be the foci of an ellipse. At any point P on the ellipse if  $\angle SPS^1 < 90^\circ$  then the eccentricity

1)  $e > \frac{1}{\sqrt{2}}$     2)  $e < \frac{1}{\sqrt{2}}$

3)  $e = \frac{1}{\sqrt{2}}$     4)  $e = \frac{1}{2}$

12. Axes are coordinate axes, S and  $S^1$  are foci, B and  $B^1$  are the ends of minor axis,

$\angle SBS^1 = \sin^{-1} \frac{4}{5}$ . Area of  $SBS^1B^1$  is 20

sq. units., then the equation of the ellipse is

1)  $\frac{x^2}{20} + \frac{y^2}{16} = 1$     2)  $\frac{x^2}{25} + \frac{y^2}{16} = 1$

3)  $\frac{x^2}{25} + \frac{y^2}{9} = 1$     4)  $\frac{x^2}{25} + \frac{y^2}{20} = 1$

13. An archway is in the form of semi ellipse. The major axis of this coincides with the Road level. The breadth of the road is 50ft. A rod of just 16ft. length touches the top when it was 10ft. from one end of the road. Then the maximum height of the Archway is

1) 10    2) 12    3) 15    4) 20

14. A bridge is in the shape of a semi ellipse. It is 400 mts, long and has a maximum height 10mts. At the middle point. The height of the bridge at a point distant 80 mts. From one end is

1) 4mts    2) 2mts    3) 8mts    4) 6mts.

15. S is one focus of an ellipse and P is any point on the ellipse. If the maximum and minimum values of SP are m and n respectively, then the length of semi major axis is

1) AM of m,n    2) G.M. of m,n  
3) HM of m,n    4) AGP of m,n

16. If the variable lines  $l_1(x - a) + y = 0$  and  $l_2(x + a) + y = 0$  are conjugate lines with re-

spect to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then the locus of their point of intersection is

1)  $\frac{2x^2}{a^2} + \frac{y^2}{b^2} = 1$     2)  $\frac{x^2}{a^2} + \frac{2y^2}{b^2} = 1$

3)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$     4)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$

17. Tangents are drawn at right angles to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Then the locus of mid points of chords of contact is

1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2}{a^2} + \frac{y^2}{b^2} = 1$

2)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2}{a^2} + \frac{y^2}{b^2} = 1$





6. Then ordinates of 4 points on the ellipse  $\frac{x^2}{16} + \frac{y^2}{4} = 1$  are  $\sqrt{2}, \sqrt{3}, 1, 2$ . The eccentric angles corresponding to the points arranged in the increasing order is

A.  $\sqrt{2}$  B.  $\sqrt{3}$  C. 1 D. 2

1) D,B,A,C 2) C,A,B,D

3) A,B,C,D 4) C,D,A,B

7. The arrangement of the ellipse in ascending order of their eccentricities when  $|SBS^1|$  is given, Where S &  $S^1$  are foci & B is one end of the minor axis

A.  $|SBS^1| = 20^\circ$  B.  $|SBS^1| = 60^\circ$

C.  $|SBS^1| = 30^\circ$  D.  $|SBS^1| = 90^\circ$

1) A,C,B,D 2) D,B,C,A

3) B,D,C,A 4) A,C,D,B

8. Observe the following lists  
In List-I there are ellipses and in List-II their foci are given

**List-I**

**List-II**

A.  $\frac{x^2}{16} + \frac{y^2}{12} = 1$

1.  $(0, \pm 5)$

B.  $\frac{x^2}{11} + \frac{y^2}{36} = 1$

2.  $(\pm 2, 0)$

C.  $\frac{x^2}{7} + \frac{y^2}{16} = 1$

3.  $(0, \pm\sqrt{5})$

D.  $\frac{x^2}{9} + \frac{y^2}{4} = 1$

4.  $(0, \pm 3)$

5.  $(\pm\sqrt{5}, 0)$

**The correct match for List-I from List-II is**

	A	B	C	D
1)	2	1	5	4
2)	5	4	1	2
3)	3	1	4	5
4)	2	1	4	5

9. Observe the following lists  
In List-I there are ellipse and in list-II their length of Latusrectum are given

**List-I**

**List-II**

A.  $\frac{(x-2)^2}{36} + \frac{(y-3)^2}{11} = 1$

1. 2

B.  $\frac{(x+1)^2}{16} + \frac{(y+1)^2}{25} = 1$

2. 18/5

C.  $\frac{(x-2y+1)^2}{49} + \frac{(2x+y+2)^2}{7} = 1$

3. 11/3

D.  $x = 3 \cos \theta, y = 5 \sin \theta$

4. 1

5. 32/5

**The correct match for List-I from List-II is**

	A	B	C	D
1)	2	1	5	4
2)	3	5	1	2
3)	3	5	2	1
4)	2	1	5	3

10. Assertion (A): Product of the perpendicular distances

from  $(\pm\sqrt{7}, 0)$  to the line  $\frac{x}{4} \cos \theta + \frac{y}{3} \sin \theta = 1$  is

9.

Reason (R): Given line is tangent to the ellipse

$\frac{x^2}{16} + \frac{y^2}{9} = 1$  and  $(\pm\sqrt{7}, 0)$  are its foci

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is not the correct explanation of A

3) A is true but R is false

4) A is false but R is true

11. Assertion (A) : If  $\alpha, \beta$  are the eccentric angles of the extremities of a focal chord of an ellipse of eccentricity 'e' then

$$\cos\left(\frac{\alpha - \beta}{2}\right) = e \sin\left(\frac{\alpha + \beta}{2}\right)$$

Reason (R): The equation of the chord joining two points with eccentric angles  $\alpha$  and  $\beta$  on the el-

lipse is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$  is

$$\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right)$$

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true but R is false.

4) A is false but R is true

12. Assertion (A): The point (5,-2) lies outside the ellipse  $6x^2 + 7y^2 = 12$ .

Reason (R): If the point  $(x_1, y_1)$  lie outside the ellipse  $S = 0$  then  $S_{11} > 0$

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true.

13. Observe the following lists for the ellipse

$$16x^2 + 9y^2 = 144$$

**List-I**

A. Eccentricity

B. Foci

C. Latus rectum

D. Equation of  
directrices

**List-II**

1.  $(0, \pm\sqrt{7})$

2.  $y = \pm \frac{16}{\sqrt{7}}$

3.  $(\pm\sqrt{7}, 0)$

4.  $\frac{\sqrt{7}}{4}$

5.  $x = \pm \frac{16}{\sqrt{7}}$

6.  $\frac{9}{2}$

**The correct match for List-I from List-II is**

	A	B	C	D
1)	4	5	1	6
2)	4	1	6	2
3)	4	3	6	5
4)	4	1	3	5

14. For the ellipse  $4x^2 + 5y^2 = 20$ , observe the following lists

**List-I**

A. Equation of  
auxiliary circle

B. Equation of  
director circle

C. Equation of  
tangent at (1,1)

D. Equation of tangent  
with slope 1

**List-II**

1.  $4x + 5y = 20$

2.  $x^2 + y^2 = 5$

3.  $y - x + 9 = 0$

4.  $x^2 + y^2 = 9$

5.  $5x + 4y = 20$

6.  $y = x + 3$

**The correct match for List-I from List-II is**

	A	B	C	D
1)	2	4	1	6
2)	4	2	6	5
3)	4	3	1	6
4)	4	2	1	3

15. Assertion (A): In an ellipse the distance between the foci is 6 and length of minor axis is 8.

Then its eccentricity is  $\frac{3}{5}$

Reason (R): The distance between the foci of the ellipse  $x = 5 \cos \theta$ ,  $y = 4 \sin \theta$  is 6

- Both A and R are true but R is not the correct explanation of A
- Both A and R are true and R is the correct explanation of A
- A is true but R is false
- A is false but R is True.

16. Assertion (A): Equation of the normal to the ellipse

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \text{ at } P\left(\frac{\pi}{4}\right) \text{ is } 5x - 3y - 8\sqrt{2} = 0$$

Reason (R): Equation of the normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ at } P(x_1, y_1) \text{ is}$$

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

- Both A and R are true but R is not the correct explanation of A
- Both A and R are true and R is the correct explanation of A
- A is true but R is false
- A is false but R is True

17. Assertion (A): If  $l_1x + m_1y + n_1 = 0$  and  $l_2x + m_2y + n_2 = 0$  are conjugate lines with respect

$$\text{to } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ then } a^2l_1l_2 + b^2m_1m_2 = n_1n_2$$

Reason (R): The lines  $2x + y + 1 = 0$  and  $x - 3y - 6 = 0$  are conjugate lines with respect to

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

- Both A and R are true but R is not the correct explanation of A
- Both A and R are true and R is the correct explanation of A
- A is true but R false
- A is false but R is True.

18. Arrange the following ellipses in the ascending order of their lengths of major axis>

A:  $x^2 + 2y^2 - 4x + 12y + 14 = 0$

B:  $\frac{(x-1)^2}{9} + \frac{(y-1)^2}{16} = 1$

C:  $4x^2 + 9y^2 = 1$

D:  $x = 3 + 6 \cos \theta$ ,  $y = 5 + 7 \sin \theta$

- C, A, B, D
- C, A, D, B
- A, B, C, D
- C, D, A, B

19. A: Pole of the line  $21x - 6y = 12$  with respect to the ellipse  $3x^2 + 4y^2 = 12$

B: The positive vertex of  $x^2 + 3y^2 = 12$

C: Centre of the ellipse  $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{25} = 1$

D: End point of Latus rectum of the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1. \text{ (In the first quadrant)}$$

Write the points of the above statements in the descending order of their abscissae.

1) A,B,D,C                      2) A,D,B,C

3) A,C,D,B                      4) B,C,D,A

20. Indicate which of the following statements are true or false with 'T' or 'F' in the order of the given statements

A:  $x + 2y - 5 = 0$  is the tangent of

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

B:  $x + y + 2 = 0$  is the normal to the ellipse

$$2x^2 + y^2 = 8$$

C: If the normal at one end of the Latus rectum of the ellipse passes through one end of the minor axis then  $e^4 + e^2 = 1$

D:  $x + y - 1 = 0$  is the polar of  $(9, 5)$  with respect to  $5x^2 + 9y^2 = 45$

1) T,F,F,T                      2) T,T,T,T

3) T,F,T,F                      4) T,F,T,T

21. Arrange the eccentricities of the following ellipse in the ascending order.

A:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  in which whose Latus rectum is half of its major axis.

B: In the ellipse distance between the foci is equal to the distance between a focus and one end of minor axis

C: In the ellipse, whose major axis is double the minor axis

D: In an ellipse, distance between the foci 6 and the length of minor axis is 8.

1) B,A,D,C                      2) B,D,A,C

3) B,C,A,D                      4) A,B,C,D

22. Write the centres of the ellipse in the descending order of ordinates.

$$A: 6(x+2)^2 + 9(y-3)^2 = 18$$

$$B: \frac{(x+y-2)^2}{9} + \frac{(x-y)^2}{16} = 1$$

$$C: 4x^2 + 9y^2 - 24x + 36y - 72 = 0$$

$$D: \frac{(x-2)^2}{9} + \frac{(y+3)^2}{4} = 1$$

1) A,B,C,D

2) D,C,B,A

3) A,B,D,C

4) A,D,C,B

23. Which of the following statements is correct.

I. Equation of the director circle with respect to the ellipse  $16x^2 + 25y^2 = 400$  is

$$x^2 + y^2 = 41$$

II. Equation of the tangent at  $(3, 2)$  on the ellipse

$$x^2 + 4y^2 = 25 \text{ is } 3x + 8y - 25 = 0$$

1) Only I

2) Only II

3) Both I and II

4) Neither I nor II

24. I. Equation of the ellipse whose foci  $(\pm 5, 0)$  and

$$\text{eccentricity } \frac{5}{8} \text{ is } \frac{x^2}{64} + \frac{y^2}{39} = 1$$

II. The eccentricity of the ellipse

$$\frac{x^2}{16} + \frac{y^2}{25} = 1 \text{ is } \frac{5}{3}$$

Which of the above statement is correct.

1) Only I

2) Only II

3) Both I and II

4) Neither I nor II

25. Which of the following statement is correct:

I: If  $x \cos \alpha + y \sin \alpha = P$  is the tangent with respect to standard ellipse then

$$P = \sqrt{a^2 \cos^2 \alpha - b^2 \sin^2 \alpha}$$

II. The point of contact of the tangent line

$$4x + y - 7 = 0 \text{ with the ellipse } x^2 + 3y^2 = 3 \text{ is}$$

$$\left( \frac{12}{7}, \frac{1}{7} \right)$$

1) Only

2) Only II

3) Both I and II

4) Neither I nor II

### Key

1.3	2.1	3.1	4.2	5.1
6.2	7.1	8.4	9.2	10.1
11.4	12.1	13.2	14.1	15.1
16.2	17.4	18.2	19.1	20.4
21.2	22.1	23.3	24.1	25.2

### PREVIOUS EAMCET QUESTIONS

#### 2005

In this year questions were not asked in this lesson.

**2004**

1. The eccentricity of the conic  $36x^2 + 144y^2 - 36x - 96y - 119 = 0$  is
- 1)  $\frac{\sqrt{3}}{2}$       2)  $\frac{1}{2}$       3)  $\frac{\sqrt{3}}{4}$       4)  $\frac{1}{\sqrt{3}}$

**2003**

In this year questions were not asked in this lesson.

**2002**

2. The pole of the straight line  $x+4y=4$  with respect to the ellipse  $x^2 + 4y^2 = 4$  is  
 1) (1,4)      2) (4,1)      3) (4,4)      4) (1,1)
3. If  $e$  and  $e'$  are the eccentricities of the ellipse  $5x^2+9y^2 = 45$  and the hyperbola  $5x^2-4y^2 = 45$  respectively then  $e \cdot e'$   
 1) 9      2) 5      3) 4      4) 1

**2001**

4. The eccentricity of the ellipse  $\frac{x^2}{9} + \frac{y^2}{16} = 1$  is
- 1)  $\frac{7}{16}$       2)  $\frac{5}{4}$       3)  $\frac{\sqrt{7}}{4}$       4)  $\frac{\sqrt{7}}{2}$

**2000**

5. The eccentricity of the ellipse  $5x^2+9y^2=1$  is  
 1)  $\frac{2}{3}$       2)  $\frac{3}{4}$       3)  $\frac{4}{5}$       4)  $\frac{1}{2}$
6. The product of the perpendiculars from the foci on any tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  
 1)  $a$       2)  $a^2-b^2$       3)  $b^2$       4)  $\sqrt{a^2+b^2}$
7. The curve represented by  $x = 2(\cos t + \sin t)$  and  $y = 5(\cos t - \sin t)$  is  
 1) a circle      2) a parabola  
 3) an ellipse      4) hyperbola

**1999**

8. The pole of the line  $x = \frac{a}{e}$  with respect to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  
 1)  $\left(\frac{a}{e}, 0\right)$       2)  $\left(\frac{-a}{e}, 0\right)$   
 3)  $(ae, 0)$       4)  $(-ae, 0)$

**1998**

9. The eccentricity of the ellipse  $9x^2+4y^2=36$  is  
 1)  $\sqrt{\frac{5}{3}}$       2)  $\sqrt{\frac{3}{5}}$       3)  $\frac{\sqrt{3}}{5}$       4)  $\frac{\sqrt{5}}{3}$

**1997**

10. The eccentricity of the ellipse  $9x^2 + 16y^2 = 144$  is  
 1)  $\frac{4}{\sqrt{7}}$       2)  $\frac{2}{\sqrt{7}}$       3)  $\frac{\sqrt{7}}{4}$       4)  $\frac{\sqrt{7}}{3}$
11. An ellipse has the co ordinate axes as its axes. One of its foci is at (4,0) and its eccentricity is  $\frac{4}{5}$ . Then the equation of the ellipse is

- 1)  $\frac{x^2}{25} + \frac{y^2}{9} = 1$       2)  $\frac{x^2}{9} + \frac{y^2}{25} = 1$   
 3)  $\frac{x^2}{5} + \frac{y^2}{4} = 25$       4)  $\frac{x^2}{4} + \frac{y^2}{5} = 25$

**1996**

12. Equation of the line conjugate to  $3x + 8y - 24 = 0$  with the respect to the ellipse  $9x^2 + 16y^2 = 144$  is  
 1)  $3x - 4y + 2 = 0$       2)  $4x - 3y + 1 = 0$   
 3)  $2x + 3y - 4 = 0$       4)  $x + 2y - 6 = 0$

**1996 - RE - EXAM**

13. The eccentricity of the ellipse  $9x^2 + 5y^2 - 30y = 0$  is  
 1)  $\frac{1}{3}$       2)  $\frac{2}{3}$       3)  $\frac{3}{4}$       4)  $\frac{1}{2}$
14. The length of the major axis of the ellipse is three times the length of the minor axis, then its eccentricity is  
 1)  $\frac{1}{3}$       2)  $\frac{1}{\sqrt{3}}$       3)  $\frac{1}{\sqrt{2}}$       4)  $\frac{2\sqrt{2}}{3}$
15. The number of normals that can be drawn from a point to the ellipse is  
 1) 1      2) 2      3) 3      4) 4

**1996**

16. The equation of the Ellipse with one of the foci at (4, 0) and eccentricity  $\frac{4}{5}$  is  
 1)  $\frac{x^2}{25} + \frac{y^2}{9} = 1$       2)  $\frac{x^2}{9} + \frac{y^2}{25} = 1$   
 3)  $\frac{x^2}{16} + \frac{y^2}{9} = 1$       4)  $\frac{x^2}{9} + \frac{y^2}{36} = 1$
17. A circle is described with minor axis of the ellipse as diameter. If the foci lie on the circle, then the eccentricity of the ellipse is  
 1)  $\frac{1}{\sqrt{3}}$       2)  $\frac{1}{\sqrt{2}}$       3)  $\frac{1}{\sqrt{7}}$       4)  $\frac{1}{\sqrt{5}}$

1995

18. If the polar with respect to  $y^2 = 4ax$  touches the ellipse  $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$ , then the locus of its pole is
- 1)  $\frac{x^2}{\alpha^2} - \frac{y^2}{(4\alpha^2/\beta^2)} = 1$       2)  $\alpha^2 x^2 + \beta^2 y^2 = 1$
- 3)  $\frac{x^2}{\alpha^2} + \frac{y^2}{(4\alpha^2/\beta^2)} = 1$       4)  $\beta^2 x^2 + \alpha^2 y^2 = 1$
19. The locus of mid-point of a focal chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is
- 1)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ex}{a}$       2)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{ex}{a}$
- 3)  $x^2 + y^2 = a^2 + b^2$       4)  $x^2 + y^2 = a^2 - b^2$
20. If  $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$  touches the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  then the eccentric angle  $\theta$  of the point of contact is equal to
- 1)  $0^\circ$       2)  $90^\circ$       3)  $45^\circ$       4)  $60^\circ$

1994

21. The centre of the ellipse  $\frac{(x+y-2)^2}{9} + \frac{(x-y)^2}{16} = 1$  is
- 1) (0, 0)      2) (1, 1)      3) (1, 0)      4) (0, 1)
22. For an ellipse the distance between its foci is 6 and its minor axis is 8, then its eccentricity is
- 1)  $4/5$       2)  $\frac{1}{\sqrt{52}}$       3)  $3/5$       4)  $1/2$

1993

23. Pole of the line  $2x + 3y + 4 = 0$  with respect to the ellipse  $\frac{x^2}{2} + \frac{y^2}{4} = 1$  is
- 1) (-1, -3)      2) (-3, -1)      3) (-3, 1)      4) (3, -1)

1992

24. S and T are the foci of an ellipse and B is an end of the minor axis. If STB is an equilateral triangle then e is
- 1)  $1/4$       2)  $1/3$       3)  $1/2$       4)  $2/3$
25. Tangents are drawn through the point  $(4, \sqrt{3})$  to the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , the points at which these tangents touch the ellipse are
- 1) (4, 0)      2) (0, 4)      3) (-4, 0)      4) (0, -4)

1991

26. The length of the latusrectum of an ellipse is equal to one-half of its minor axis. Then the eccentricity of the ellipse is
- 1)  $\frac{\sqrt{3}}{2}$       2)  $\frac{\sqrt{2}}{3}$       3)  $\frac{1}{\sqrt{3}}$       4)  $\frac{1}{\sqrt{2}}$
27. The locus of the point of intersection of the perpendicular tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is
- 1)  $x^2 + y^2 = a^2 + b^2$       2)  $x^2 + y^2 = a^2 - b^2$
- 3)  $x^2 + y^2 = a^2$       4)  $x^2 + y^2 = b^2$

1990

28. If the length of the major axis of an ellipse is three times the length of its minor axis, then its eccentricity is
- 1)  $1/3$       2)  $\frac{1}{\sqrt{3}}$       3)  $\frac{1}{\sqrt{2}}$       4)  $\frac{2\sqrt{2}}{3}$
29. The latusrectum of an ellipse is equal to one-half of its minor axis. Then the eccentricity of the ellipse is
- 1)  $\frac{\sqrt{3}}{5}$       2)  $\frac{\sqrt{3}}{2}$       3)  $\frac{2}{\sqrt{3}}$       4)  $\frac{2}{\sqrt{5}}$

1989

30. The condition that the chord of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  whose middle point is  $(x_1, y_1)$ , subtends a right angle at the centre of the ellipse is
- 1)  $\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4} = \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}\right)^2$
- 2)  $\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right) \left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}\right)^2$
- 3)  $\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4} = \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \left(\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}\right)^2$
- 4)  $\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right) \left(\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}\right)^2$

1988

31. The curve with parametric equations  $x = 3(\cos t + \sin t)$  and  $y = 4(\cos t - \sin t)$  is
- 1) An Ellipse      2) A Parabola
- 3) Hyperbola      4) Circle

32. A conic passing through origin has its foci at (5, 12) and (24, 7). Then its eccentricity is

1)  $\frac{\sqrt{386}}{38}$  2)  $\frac{\sqrt{386}}{39}$  3)  $\frac{\sqrt{386}}{47}$  4)  $\frac{\sqrt{386}}{51}$

**1987**

33. If P is a point on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) and

its foci are S and S', then  $SP + S'P =$

1) 2a 2) 2b 3) a 4) b

**KEY**

1) 1	2) 4	3) 4	4) 3	5) 1
6) 3	7) 3	8) 3	9) 4	10) 3
11) 1	12) 2	13) 2	14) 4	15) 4
16) 1	17) 2	18) 1	19) 1	20) 3
21) 2	22) 3	23) 1	24) 3	25) 1
26) 1	27) 1	28) 4	29) 2	30) 1
31) 1	32) 1	33) 1		

### PREVIOUS QUESTIONS FROM DIFFERENT ENTRANCE TESTS

**IIT 1994**

34. Let 'P' be a variable point on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with foci  $F_1$  and  $F_2$ . A is the area of the triangle  $PF_1F_2$ , then the maximum value of A is

1)  $b\sqrt{a^2 - b^2}$  2)  $b\sqrt{a^2 + b^2}$   
3)  $a\sqrt{a^2 + b^2}$  4)  $a\sqrt{a^2 - b^2}$

**IIT 1998**

- 34.(A): If  $P(x, y)$ ,  $F_1(3, 0)$ ,  $F_2(-3, 0)$  and

$16x^2 + 25y^2 = 400$ , then  $PF_1 + PF_2 =$

1) 8 2) 6 3) 10 4) 12

**ROORKEE 1970**

35. The points of intersection of the line  $2x + y = 3$  and the ellipse  $4x^2 + y^2 = 5$  are

1)  $\left(\frac{1}{2}, 2\right), (1, 1)$ ; 2)  $\left(\frac{1}{2}, 2\right), (-1, 1)$ ;  
3)  $\left(-\frac{1}{2}, 2\right), (-1, 1)$ ; 4)  $\left(-\frac{1}{2}, 2\right), (1, 1)$ ;

**MNREC 1984**

36. The angle between the pair of tangents drawn from the point (1, 2) to the ellipse  $3x^2 + 2y^2 = 5$  is

1)  $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$  2)  $\tan^{-1}\left(\frac{\sqrt{5}}{12}\right)$   
3)  $\tan^{-1}\left(\frac{12}{5}\right)$  4)  $\tan^{-1}\left(\frac{5}{12}\right)$

**MNCRE 1990**

37. The distance of a point on the ellipse  $x^2 + 3y^2 = 6$  from the centre is 2 units. Then the eccentric angle of the point is

1)  $\frac{7\pi}{4}$  2)  $\frac{3\pi}{5}$  3)  $\frac{11\pi}{4}$  4)  $\frac{13\pi}{4}$

**BITS RANCHI 1997**

38. If the line containing a focal chord of the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  intersects the auxiliary circle in Q and

$Q'$  then  $SQ \cdot SQ' =$

1)  $a^2$  2)  $b^2$  3)  $a^4$  4)  $b^4$

**MNREC 1997**

39. The locus of poles with respect to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  of any tangent to the auxiliary circle

is the curve  $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{k}$ , then  $k$  is

1)  $a^2$  2)  $b^2$  3)  $a^4$  4)  $b^4$

**KARNATAKA CET-2002**

40. Which of the statements in the following is false with respect to the conic

$x^2 - 3y^2 - 4x - 6y - 11 = 0$

1) Length of the latusrectum is  $\frac{4}{\sqrt{3}}$   
2) asymptotes intersect at right angles  
3) The eccentricity of the conic is  $\frac{2}{\sqrt{3}}$   
4) centre of the conic is (2, -1)

**WEST BENGAL JEE-2002**

41. Is the point (5, -2) w.r.to the ellipse  $6x^2 + 7y^2 = 12$  lying  
1) inside 2) outside  
3) on the ellipse 4) on the hyperbola

**NDA-2002**

42. The equation of the ellipse with foci at  $(\pm 5, 0)$  and

$x = \frac{36}{5}$  as one directrix is

- 1)  $\frac{x^2}{3} + \frac{y^2}{5} = 1$       2)  $\frac{x^2}{36} + \frac{y^2}{11} = 1$   
 3)  $\frac{x^2}{36} + \frac{y^2}{9} = 1$       4)  $\frac{x^2}{11} + \frac{y^2}{36} = 1$

**PUNJAB COMMON ENTRANCE EXAM-2001**

43. The eccentricity of the ellipse satisfying the conditions  $SP + S'P = 8$ ,  $CP = 2$  and  $S, S'$  are the foci and  $P$  is extreme point of the minor axis, is

- 1)  $\frac{\sqrt{3}}{2}$       2)  $\frac{1}{\sqrt{7}}$       3)  $\frac{1}{\sqrt{3}}$       4)  $\frac{1}{2}$

**NDA-2001**

44. The eccentricity of the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  is

- 1)  $\frac{7}{16}$       2)  $\frac{9}{16}$       3)  $\frac{57}{2}$       4)  $\frac{\sqrt{7}}{4}$

**IIIT KOLKATA-2001**

45. The product of the perpendiculars from the two foci

of the ellipse  $\frac{x^2}{9} + \frac{y^2}{25} = 1$  on the tangent at

any point on the ellipse is

- 1) 8      2) 16      3) 12      4) 9

**JAMIA MILIA ENTERENCE EXAM-2001**

46. When the eccentricity tends to zero, then the equation of ellipse becomes

- 1) a parabola      2) a circle  
 3) an ellipse      4) a hyperbola

**KERALA.PET-2001**

47. The sum of the distances of any point on the ellipse  $3x^2 + 4y^2 = 24$  from its foci is

- 1)  $8\sqrt{2}$       2) 8      3)  $16\sqrt{2}$       4)  $4\sqrt{2}$

48. If  $e_1$  and  $e_2$  are the eccentricities of two conics with

$e_1^2 + e_2^2 = 3$ , then the conics are

- 1) ellipse      2) parabola  
 3) hyperbola      4) circle

49. If the major axis of an ellipse is thrice the minor axis, then its eccentricity is equal to

- 1)  $\frac{1}{3}$       2)  $\frac{1}{\sqrt{3}}$       3)  $\frac{1}{\sqrt{2}}$       4)  $\frac{2\sqrt{2}}{3}$

**IIT SCREENING TEST -1999**

50. On the ellipse  $4x^2 + 9y^2 = 1$ , the points at which the tangents are parallel to the line  $8x = 9y$  are

- 1)  $\left(\frac{2}{5}, \frac{1}{5}\right)$       2)  $\left(\frac{-2}{5}, \frac{1}{5}\right)$       3)  $\left(\frac{-2}{5}, \frac{-1}{5}\right)$       4)  $\left(\frac{2}{5}, \frac{-3}{5}\right)$

**AIEEE -2004**

51. The eccentricity of an ellipse, with its centre at origin, is  $1/2$ . If one of the directrices is  $x = 4$ , then the equation of the ellipse is

- 1)  $3x^2 + 4y^2 = 1$       2)  $4x^2 + 3y^2 = 1$   
 3)  $4x^2 + 3y^2 = 12$       4)  $3x^2 + 4y^2 = 12$

**Eamcet-2007**

52. The value of 'k' if  $(1, 2)$ ,  $(k, -1)$  are conjugate points with respect to the ellipse  $2x^2 + 3y^2 = 6$  is

**E-2007**

- 1) 2      2) 4      3) 6      4) 8

**KEY**

- |       |          |       |       |
|-------|----------|-------|-------|
| 34) 1 | 34(A): 3 | 35) 1 | 36) 1 |
| 37) 1 | 38) 2    | 39) 1 | 40) 2 |
| 41) 2 | 42) 2    | 43) 1 | 44) 4 |
| 45) 4 | 46) 2    | 47) 4 | 48) 3 |
| 49) 4 | 50) 2    | 51) 4 | 52) 3 |