ICSE 2024 EXAMINATION

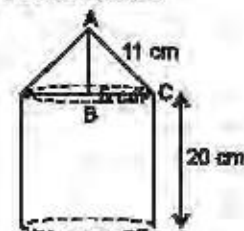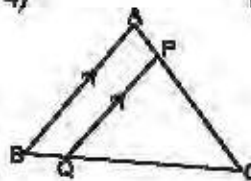## MATHEMATICS

SAMPLE PAPER - 8

SECTION - A (40 Marks)

(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: [15]

(i) A consumer bought a T.V. from a dealer at a discount of 20% on the marked price of ₹40,000. If the rate of GST is 18%, then the tax paid by the consumer is :

   (a) ₹5760   (b) ₹2880   (c) nil   (d) ₹7200

(ii) The discriminant of $2x^2 - 5x + 1 = 0$ is :

   (a) 25   (b) 33   (c) 17   (d) 15

(iii) If $(x - 2)$ is a factor of $x^2 + mx + 10$, then the value of m is :

   (a) 7   (b) –7   (c) 4   (d) –5

(iv) Which of the following gives the dividend of a share holder?

   (a) Face value × no. of shares × $\dfrac{\text{Rate of dividend}}{100}$ (b) Market value × no. of shares × $\dfrac{100}{\text{Rate of dividend}}$

   (c) Face value × $\dfrac{\text{Rate of dividend}}{\text{no. of shares}}$ (d) none of these

(v) The nth term of the AP 15, 10, 5, ... is :

   (a) $5n - 20$ (b) $20 - 5n$ (c) $20 + 5n$ (d) $10 + 5n$

(vi) The reflection of the point $P(1, -2)$ in the line $y = -1$ is :

   (a) $(-3, -2)$ (b) $(1, -4)$ (c) $(1, 4)$ (d) $(1, 0)$

(vii) In the figure, AB ‖ PQ, CQ = 6 cm, CB = 14 cm and AB = 7 cm. The length of PQ is :

   (a) 3 cm
   (b) 3.5 cm
   (c) 4 cm
   (d) 4.2 cm

(viii) If P(A) and P(not A) are complementary events and P(A) = 0.15, then P(not A) = ?

   (a) 0.35   (b) 0.85   (c) 0.3   (d) cannot be determined

(ix) The fourth term from the end of the GP 3, 6, 12, ... 3072 is :

   (a) 300   (b) 352   (c) 384   (d) 390

(x) The total surface area of the adjoining figure is:

   (a) 880 cm²
   (b) 110 cm²
   (c) 440 cm²
   (d) 210 cm²

(xi) If A is a matrix of order 3×2 and B is a matrix of order 2×3, then :
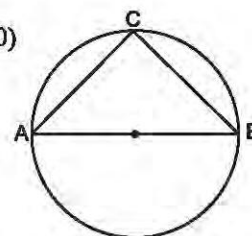
(a) A + B is possible   (b) B + A is possible

(c) AB is possible   (d) neither AB nor BA is possible

(xii) The coordinates of the centroid of $\triangle ABC$ with vertices A(–1, 0), B(5, –2) and C(8, 2) is:

(a) (12, 0)   (b) (6, 0)   (c) (0, 6)   (d) (4, 0)

(xiii) In the given figure, AB is a diameter of the circle. If AC = BC, then $\angle CAB$ is equal to :

(a) 30°   (b) 60°

(c) 90°   (d) 45°

(xiv) A container has a surface area 7,200 cm² and a capacity of 200 litres. If the capacity of a similar container is 5.4 litres, then its surface area is :

(a) 800 cm²   (b) 648 cm²   (c) 540 cm²   (d) none of these

(xv) Consider the following data:

Class	65–85	85–105	105–125	125–145	145–165	165–185	185–205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is:

(a) 0   (b) 19   (c) 20   (d) 38

Question 2 :

(i) Manisha has a recurring deposit account for 2 years at 10% p.a. If she receives ₹1900 as interest, find the value of the monthly instalment paid by her. [4]

(ii) A company with 10,000 shares of ₹50 each, declares an annual dividend of 5%. [4]

(a) What is the total amount of dividend paid by the company?

(b) What would be the annual income of a man who has 72 shares in the company?

(c) If he receives 4% on his investment, find the price he paid for each share.

(iii) The sum of first 3 terms of a GP is 16 and the sum of the next three terms is 128. Find the sum of first n terms of the GP. [4]

Question 3 :

(i) A model of a ship is made to scale of 1 : 200. [4]

(a) The length of the model is 4 m. Calculate the length of the ship.

(b) The area of the deck of the ship is 1,60,000 m². Find the area of the deck of the model.

(c) The volume of the model is 200 litres. Calculate the volume of the ship in m³.

(ii) Find the value of k for which the lines $kx - 5y + 4 = 0$ and $4x - 3y + 5 = 0$ are perpendicular to each other. [4]

(iii) Use graph paper for this question. [5]

Take 1 cm = 1 unit on both x and y axes.

(a) Plot the following points on your graph sheets :

A(– 4, 0), B(–3, 2), C(0, 4), D(4, 1) and E(7, 3)

(b) Reflect the points B, C, D and E on the x-axis and name them as B', C', D' and E' respectively.

(c) Join the points A, B, C, D, E, E', D', C', B' and A in order.

(d) Name the closed figure formed.

SECTION - B (40 Marks)

### (Attempt *any four* questions from this Section)

Question 4 :

(i) A dealer X in Rampur (U.P.) bought goods for ₹6,000 and sold the same goods to another dealer Y in Rampur (UP) for ₹6800. If the rate of GST is 12%, find the net CGST and SGST payable by the dealer X. [3]

(ii) Find the point which is equidistant from three non-collinear points. Give the point a specific name. [3]

(iii) Use graph paper for this question. Estimate the mode of the given distribution by plotting a histogram. [Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis] [4]

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	6	12	20	15	9

Question 5 :

(i) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 7 \\ 5 & -1 \end{bmatrix}$, show that AB = AC. [3]

(ii) In the figure, DE is a chord parallel to the diameter AC of a circle with centre O. If $\angle$CBD = 57°, calculate $\angle$CDE. [3]

(iii) Find the values of a and b if $(x - 1)$ and $(x - 3)$ are factors of $ax^3 - 3x^2 + bx + 3$. [4]
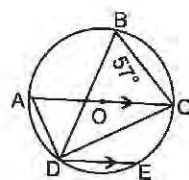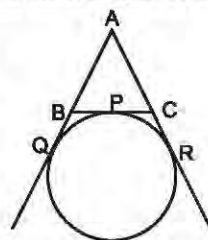
Question 6 :

(i) Find the equation of a line perpendicular to $3x + 8y = 12$ and passing through the point $(-1, -2)$. [3]

(ii) If $\operatorname{cosec}\theta + \cot\theta = x$, then prove that $\dfrac{x^2 - 1}{x^2 + 1} = \cos\theta$. [3]

(iii) In an AP first term is 8, nth term is 33 and the sum to first n terms is 123. Find n and the common difference of the AP. [4]

Question 7 :

(i) Two unbiased coins are tossed simultaneously. Find the probability of getting, one head. [3]

(ii) A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find the radius and slant height of the heap. [3]

(iii) In the given figure, a circle touches the side BC of a ΔABC at P and AB and AC produced at Q and R respectively. Prove that

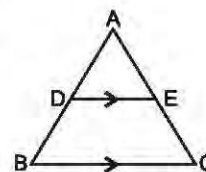$AQ = AR = \dfrac{1}{2}$ (perimeter of ΔABC) [4]

Question 8 :

(i) Find the smallest non-negative value of x which satisfies the inequation $x + \dfrac{7}{2} > \dfrac{5x}{3} + 3$, $x \in I$. [3]

(ii) Calculate the mean, the mode and the median of the following distribution. [3]

Number	0	1	2	3	4	5
Frequency	2	4	7	6	8	3

(iii) In the given figure, DE || BC. If AD = x cm, DB = $(x - 2)$ cm, AE = $(x + 2)$ cm and EC = $(x - 1)$ cm, find the value of x. [4]

Question 9 :

(i) The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. [4]

(ii) Draw an ogive for the following data and hence estimate the median. [6]

Wages (in ₹)	4000–4400	4400–4800	4800–5200	5200–5600	5600–6000	6000–6400
No. of workers	8	12	20	25	17	10

Question 10 :

(i) If $\dfrac{\sqrt{x+2} + \sqrt{x-3}}{\sqrt{x+2} - \sqrt{x-3}} = 5$, prove that $x = 7$. [3]

(ii) Construct a regular hexagon of side 2.4 cm. Circumscribe a circle to it. [3]

(iii) The shadow of a vertical tower AB on level ground is increased by 10 m, when the altitude of the sun changes from 45° to 30°. Find the height of the tower and give your answer correct to $\dfrac{1}{10}$ of a metre. [4]

ANSWERS

1. (i) (a)   (ii) (c)   (iii) (b)   (iv) (a)   (v) (b)   (vi) (d)   (vii) (a)   (viii) (b)   (ix) (c)
   (x) (a)   (xi) (c)   (xii) (d)   (xiii) (d)   (xiv) (b)   (xv) (c)

2. (i) ₹760   (ii) (a) ₹25,000 (b) ₹180 (c) ₹62.50 (iii) $\frac{16}{7}(2^n - 1)$

3. (i) (a) 800 m (b) 4 m^2 (c) 16,00,000 m^3 (ii) $\frac{-15}{4}$ (iii) Nonagon   4. (i) ₹48, ₹48   (iii) 56

5. (ii) 33°   (iii) $a = 1, b = -1$ 6. (i) $8x - 3y + 2 = 0$ (iii) $n = 6, d = 5$

7. (i) $\frac{1}{2}$ (ii) 36 cm, 43.26 cm   8. (i) 0   (ii) 2.76, 4, 3 (iii) 4

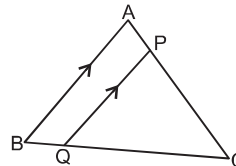9. (i) 27, 30 (ii) ₹4800   10. (iii)   13.7 m

SOLUTION

SECTION - A (40 Marks)

(Attempt *all* questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: [15]

(i)   A consumer bought a T.V. from a dealer at a discount of 20% on the marked price of ₹40,000. If the rate of GST is 18%, then the tax paid by the consumer is :

   (a) ₹5760   (b) ₹2880   (c) nil   (d) ₹7200

(ii) The discriminant of $2x^2 - 5x + 1 = 0$ is :

   (a) 25   (b) 33   (c) 17   (d) 15

(iii) If $(x - 2)$ is a factor of $x^2 + mx + 10$, then the value of m is :

   (a) 7   (b) –7   (c) 4   (d) –5

(iv) Which of the following gives the dividend of a share holder?

   (a) Face value × No. of shares × $\dfrac{\text{Rate of dividend}}{100}$ (b) Market value × no. of shares × $\dfrac{100}{\text{Rate of dividend}}$

   (c) Face value × $\dfrac{\text{Rate of dividend}}{\text{no. of shares}}$ (d) none of these

(v)   The nth term of the AP 15, 10, 5, ... is :

   (a) $5n - 20$ (b) $20 - 5n$ (c) $20 + 5n$ (d) $10 + 5n$

(vi) The reflection of the point P(1, –2) in the line $y = -1$ is :

   (a) (–3, –2)   (b) (1, –4)   (c) (1, 4)   (d) (1, 0)

(vii) In the figure, AB ∥ PQ, CQ = 6 cm, CB = 14 cm
      and AB = 7 cm. The length of PQ is :

   (a) 3 cm

   (b) 3.5 cm

   (c) 4 cm

   (d) 4.2 cm

(viii) If P(A) and P(not A) are complementary events and P(A) = 0.15, then P(not A) = ?

   (a) 0.35   (b) 0.85   (c) 0.3   (d) cannot be determind

(ix) The fourth term from the end of the GP 3, 6, 12, ... 3072 is :

   (a) 300   (b) 352   (c) 384   (d) 390

(x) The total surface area of the adjoining figure is:

(a) 880 cm²   (b) 110 cm²

(c) 440 cm²   (d) 210 cm²

(xi) If A is a matrix of order 3 × 2 and B is a matrix of order 2 × 3, then :

(a) A + B is possible   (b) B + A is possible

(c) AB is possible   (d) neither AB nor BA is possible

(xii) The coordinates of the centroid of △ABC with vertices A(–1, 0), B(5, –2) and C(8, 2) is:

(a) (12, 0)   (b) (6, 0)   (c) (0, 6)   (d) (4, 0)

(xiii) In the given figure, AB is a diameter of the circle. If AC = BC, then $\angle$CAB is equal to :

(a) 30°

(b) 60°

(c) 90°

(d) 45°

(xiv) A container has a surface area 7,200 cm² and a capacity of 200 litres. If the capacity of a similar container is 5.4 litres, then its surface area is :

(a) 800 cm²   (b) 648 cm²   (c) 540 cm²   (d) none of these

(xv) Assertion (A) : Look at the following frequency distribution table.

Class	65–85	85–105	105–125	125–145	145–165	165–185	185–205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is 10.

Reason (R) : For a data, median is always greater than the mode.

(a) A is true, R is false   (b) A is false, R is true   (c) both A and R are true   (d) both A and R are false
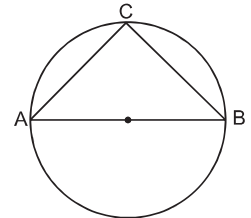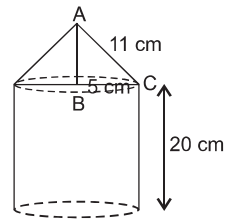
Solution :

(i) (a) Tax paid by the customer 18 % of (80% of ₹40,000)

$$= 18\% \text{ of } \left(₹\frac{40000 \times 80}{100}\right) = \frac{₹32,000 \times 18}{100} = ₹5760$$

(ii) (c) D = (–5)² – 4 × 2 × 1 = 25 – 8 = 17

(iii) (b) Since (x – 2) is a factor of $f(x) = x^2 + mx + 10$, so, $f(2) = 0$

$\Rightarrow 2^2 + 2 \times m + 10 = 0 \Rightarrow 2m = -14 \Rightarrow m = -7.$

(iv) (a) Dividend = $\dfrac{\text{Face value × No. of shares × Rate of dividend}}{100}$

(v) (b) $T_n = 15 + (n-1)(-5) = 15 - 5n + 5 = 20 - 5n$

(vi) (d) The image of the point (1, – 2) in the line $y = -1$ is (1, 2 + 2 × (–1)) i.e., (1, 0)

(vii) (a) △ABC~△PQR, so $\dfrac{AB}{PQ} = \dfrac{BC}{CQ} \Rightarrow \dfrac{7}{PQ} = \dfrac{14}{6} \Rightarrow PQ = 3$ cm

(viii) (b) P(not A) = 1 – P(A) = 1 – 0.15 = 0.85

(ix) (c) Required term = $\dfrac{3072}{2^3} = 384.$

(x) (a) Total surface area = $2\pi rh + \pi r^2 + \pi rl$

$= \pi r(2h + r + l) = \dfrac{22}{7} \times 5\ (2 \times 20 + 5 + 11)$ cm² = 880 cm²

(xi) (c)

A   B   So, AB is possible.

3 × 2   2 × 3

(xii) (d) Required coordinates are $\left(\dfrac{-1+5+8}{3}, \dfrac{0-2+2}{2}\right)$, i.e., (4, 0)

(xiii) (d) AC = BC $\Rightarrow \angle$CAB = $\angle$CBA

$\angle$ACB = 90°. So, $\angle$CAB = $\dfrac{90°}{2} = 45°$

(xiv) (b) $= \dfrac{200}{5.4} = \dfrac{x_1{}^3}{5.4} \Rightarrow \dfrac{x_1{}^3}{5.4} = \dfrac{10}{3}$

Now, $\left(\dfrac{x_1}{x_2}\right)^2 = \dfrac{7200}{S_2} \Rightarrow \dfrac{100}{9} = \dfrac{7200}{S_2} = S_2 = 648 \text{ cm}^2.$

(xv) (d) First we may prepare a table as below :

Class	65−85	85−105	105−125	125−145	145−165	165−185	185−205
Frequency	4	5	13	20	14	7	4
c.f.	4	9	22	42	56	63	67

$n = 67 \Rightarrow \dfrac{n}{2} = 33.5$

So, median class is 125 − 145.

Also, maximum frequency is 20, which falls in the class 125 − 145.

So, modal class is 125 − 145.

So, required difference = 145 − 125 = 20.

Question 2 :

(i) **Manisha has a recurring deposit account for 2 years at 10% p.a. If she receives ₹1900 as interest, find the value of the monthly instalment paid by her.** [4]

(ii) **A company with 10,000 shares of ₹50 each, declares an annual dividend of 5%.** [4]

   (a) **What is the total amount of dividend paid by the company?**

   (b) **What would be the annual income of a man who has 72 shares in the company?**

   (c) **If he receives 4% on his investment, find the price he paid for each share.**

(iii) **The sum of first 3 terms of a GP is 16 and the sum of the next three terms is 128. Find the sum of first n terms of the GP.** [4]

Solution :

(i) Here, $n = 2 \times 12 = 24$ months, $r = 10\%$ p.a, SI. = ₹1900

$\therefore \text{SI} = \text{P} \times \dfrac{n(n+1)}{2} \times \dfrac{\text{R}}{12} \times \dfrac{1}{100}$

$\Rightarrow 1900 = \text{P} \times \dfrac{24 \times 25}{2} \times \dfrac{10}{12} \times \dfrac{1}{100} \Rightarrow \text{P} = \dfrac{19000}{25} = ₹760. \quad \textbf{Ans.}$

(ii) (a) Amount of dividend paid by the company = 5% of $(10000 \times ₹50) = ₹25{,}000$ **Ans.**

   (b) Required income = 5% of $(72 \times ₹50) = ₹180$ **Ans.**

   (c) Amount invested on 72 shares $= \dfrac{₹180 \times 100}{4} = ₹4500$

      Market value of each share $= \dfrac{₹4500}{72} = ₹62.50$ **Ans.**

(iii) $a + ar + ar^2 = 16$

$ar^3 + ar^4 + ar^5 = 128 = r^3(a + ar + ar^2) = 128 \Rightarrow r^3 \times 16 = 128 \Rightarrow r = 2$

So, $a(1 + 2 + 4) = 16 \Rightarrow a = \dfrac{16}{7}$

$S_n = \dfrac{16}{7} \times \dfrac{(2^n - 1)}{2 - 1} = \dfrac{16}{7}(2^n - 1)$ **Ans.**

Question 3 :

(i) **A model of a ship is made to scale of 1 : 200.** [4]

   (a) **The length of the model is 4 m. Calculate the length of the ship.**

   (b) **The area of the deck of the ship is 1,60,000 m². Find the area of the deck of the model.**

   (c) **The volume of the model is 200 litres. Calculate the volume of the ship in m³.**

(ii) **Find the value of k for which the lines $kx - 5y + 4 = 0$ and $4x - 3y + 5 = 0$ are perpendicular to each other.** [4]

(iii) **Use graph paper for this question.** [5]

   Take 1 cm = 1 unit on both x and y axes.

(a) Plot the following points on your graph sheets :

A(– 4, 0), B(–3, 2), C(0, 4), D(4, 1) and E(7, 3)

(b) Reflect the points B, C, D and E on the *x*-axis and name them as B', C', D' and E' respectively.

(c) Join the points A, B, C, D, E, E', D', C', B' and A in order.

(d) Name the closed figure formed.

Solution :

(i) Here scale factor, $k = \dfrac{1}{200}$

(a) Length of the ship $= \dfrac{1}{k} \times$ length on the model $= 4 \times 200$ m $= 800$ m **Ans.**

(b) Area of the deck of the model

$= k^2 \times$ area of the ship $= \left(\dfrac{1}{200}\right)^2 \times 1,60,000$ m$^2 = 4$ m^2 **Ans.**

(c) Volume of ship $= \dfrac{1}{k^3} \times$ volume of the model $= (200)^3 \times 200$ litres

$= \dfrac{(200)^3 \times 200}{1000}$ m$^3 = 16,00,000$ m^3 **Ans.**

(ii) $kx - 5y + 4 = 0 \Rightarrow y = \dfrac{k}{5}x + \dfrac{4}{5}$

So, gradient of this line $= \dfrac{k}{5}$

Again, $4x - 3y + 5 = 0 \Rightarrow y = \dfrac{4}{3}x + \dfrac{5}{3}$
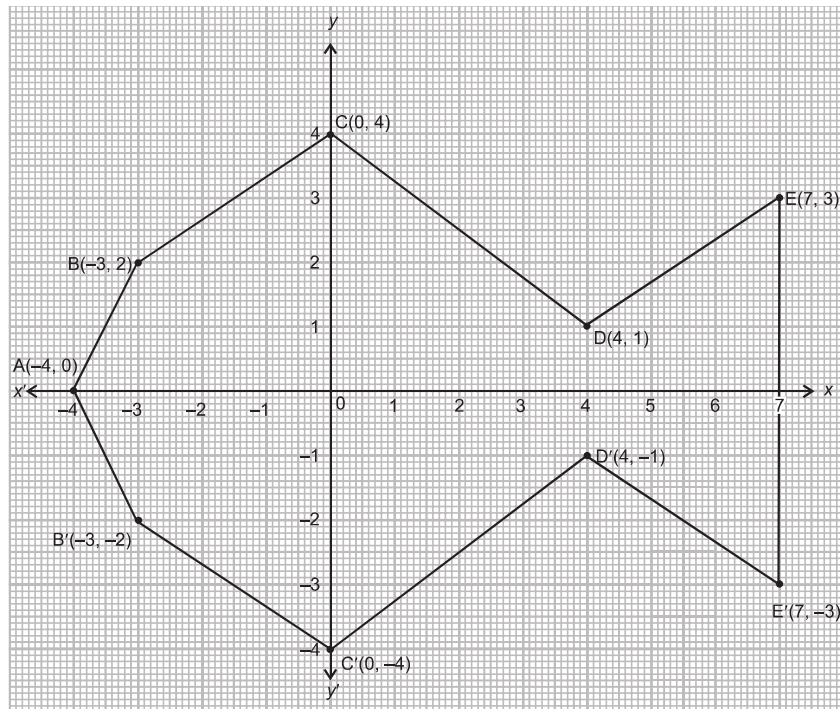
So, gradient of this line is $\dfrac{4}{3}$.

Since, the lines are perpendicular, so, $\dfrac{k}{5} \times \dfrac{4}{3} = -1 \Rightarrow k = \dfrac{-15}{4}$. **Ans.**

(iii) (a) The points A(– 4, 0), B(–3, 2), C(0, 4), D(4, 1) and E(7, 3) have been plotted as shown below. **Ans.**

(b) The points B, C, D, and E have been reflected on the *x*-axis as B′(–3, –2), C′(0, –4), D′(4, –1) and E′(7, –3) respectively as shown below. **Ans.**

(c) The points A, B, C, D, E, E′, D′, C′, B′ and A have been joined in order as shown below. **Ans.**

(d) The figure is a 9 sided polygon (or nonagon). **Ans.**

Question 4 :

(i) A dealer X in Rampur (U.P.) bought goods for ₹6,000 and sold the same goods to another dealer Y in Rampur (UP) for ₹6800. If the rate of GST is 12%, find the net CGST and SGST payable by the dealer X. [3]

(ii) Find the point which is equidistant from three non-collinear points. Give the point a specific name. [3]

(iii) Use graph paper for this question. Estimate the mode of the given distribution by plotting a histogram. [Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis] [4]

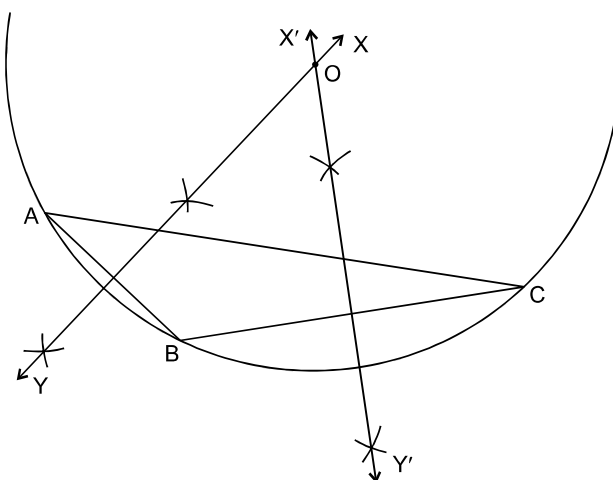| Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	6	12	20	15	9

Solution :

(i) Rate of GST = 12%

So, CGST paid by the dealer X = 6% of ₹(6800 − 6000) = $\dfrac{₹800 \times 6}{100}$ = ₹48 **Ans.**

SGST paid by the dealer X = 6% of ₹(6800 − 6000) = ₹48. **Ans.**

(ii) Let the three non-collinear points be A, B and C.

Join AB, AC and BC.

Draw the perpendicular bisectors of AB and BC, which intersect, at O. O is the required point. This point is the circumcentre of ΔABC.

(iii) The histogram for the given frequency distribution is shown. Upper corners of the highest rectangle are marked as A, B and the corners of the adjoining rectangles are marked as C and D. Joining AC and BD, we get the point of intersection as P. Draw PM ⊥ x-axis.

The abscissa of M is 56, which is the required mode. Hence, mode = 56. **Ans.**
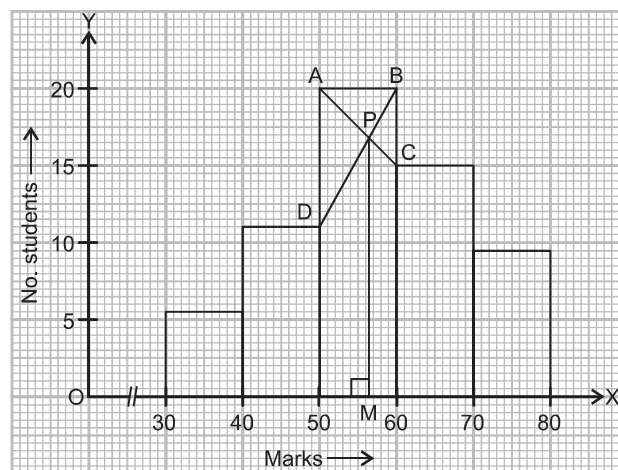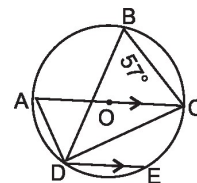
Question 5 :

(i) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 7 \\ 5 & -1 \end{bmatrix}$, show that $AB = AC$. [3]

(ii) In the figure, DE is a chord parallel to the diameter AC of a circle with centre O. If $\angle CBD = 57°$, calculate $\angle CDE$. [3]

(iii) Find the values of a and b, if $(x - 1)$ and $(x - 3)$ are factors of $ax^3 - 3x^2 + bx + 3$. [4]

Solution :

(i) $AB = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2+6 & 1+4 \\ 4+12 & 2+8 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 16 & 10 \end{bmatrix}$

$AC = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 7 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} -2+10 & 7-2 \\ -4+20 & 14-4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 16 & 10 \end{bmatrix}$
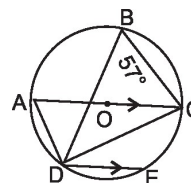
Hence, $AB = AC$ **Proved.**

(ii) $\angle CBD = \angle CAD = 57°$ [Angles in the same segment]

$\angle ADC = 90°$ [Angle in a semi-circle]

$\angle DCA = 90° - \angle CAD = 90° - 57° = 33°$

$\angle ACD = \angle CDE = 33°$ [Alternate angles] **Ans.**

(iii) Since, $(x - 1)$ is a factor of $f(x) = ax^3 - 3x^2 + bx + 3$, so, $f(1) = 0$

$\Rightarrow f(1) = a \times 1^3 - 3 \times 1^2 + b \times 1 + 3 = 0$

$\Rightarrow a + b = 0$ ...(1)

Again $(x - 3)$ is a factor of $f(x)$, so $f(3) = 0$

$\Rightarrow f(3) = a \times 3^3 - 3 \times 3^2 + b \times 3 + 3 = 0$

$\Rightarrow 27a + 3b = 24 \Rightarrow 9a + b = 8$ ...(2)

Solving (1) and (2), we get, $a = 1$, $b = -1$.

Question 6 :

(i) Find the equation of a line perpendicular to $3x + 8y = 12$ and passing through the point $(-1, -2)$. [3]

(ii) If $\operatorname{cosec}\theta + \cot\theta = x$, then prove that $\dfrac{x^2 - 1}{x^2 + 1} = \cos\theta$. [3]

(iii) In an AP first term is 8, nth term is 33 and the sum to first n terms is 123. Find n and the common difference of the AP. [4]

Solution :

(i) Gradient of the line $3x + 8y + 12 = 0$ $\left(y = \dfrac{-3}{8}x - \dfrac{12}{8}\right)$ is $-\dfrac{3}{8}$

So, gradient of the required line is $\dfrac{8}{3}$, as it is perpendicular to the above line.

$\therefore$ Also, required line passes through $(-1, -2)$.

So, equation of the required line is given by $y - (-2) = \dfrac{8}{3}(x + 1) \Rightarrow 3y + 6 = 8x + 8$

$\Rightarrow 8x - 3y + 2 = 0$. **Ans.**

(ii) LHS $= \dfrac{x^2 - 1}{x^2 + 1} = \dfrac{(\operatorname{cosec}\theta + \cot\theta)^2 - 1}{(\operatorname{cosec}\theta + \cot\theta)^2 + 1}$

$= \dfrac{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta + 1} = \dfrac{1 + \cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta - 1 + 2\operatorname{cosec}\theta\cot\theta + 1}$

$$= \frac{2(\cot^2\theta + \operatorname{cosec}\theta \cot\theta)}{2(\operatorname{cosec}^2\theta + \operatorname{cosec}\theta \cot\theta)} = \frac{\cot\theta(\cot\theta + \operatorname{cosec}\theta)}{\operatorname{cosec}\theta(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{\cos\theta}{\sin\theta} \times \sin\theta = \cos\theta = \text{RHS.} \quad \textbf{Proved.}$$

(iii) $T_n = a + (n-1)d$

$\Rightarrow 33 = 8 + (n-1)d \Rightarrow (n-1)d = 25$ ...(1)

Also, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\Rightarrow 123 = \dfrac{n}{2}[2 \times 8 + 25]$ [From (1)]

$\Rightarrow n = 3 \times 2 = 6$

From (1), $(6-1) \times d = 25 \Rightarrow d = 5$

So, $n = 6$ and $d = 5$.   **Ans.**

Question 7 :

(i) **The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers.**   **[5]**

(ii) **Draw an ogive for the following data and hence estimate the median.**   **[5]**

Wages (in ₹)	4000−4400	4400−4800	4800−5200	5200−5600	5600−6000	6000−6400
No. of workers	8	12	20	25	17	10

Solution :

(i) Let the two numbers be $3x$ and $3(x + 1)$.

Then, $3x \times 3(x + 1) = 810$

$\Rightarrow 9x^2 + 9x = 810 \Rightarrow x^2 + x - 90 = 0$

$\Rightarrow x^2 + 10x - 9x - 90 = 0 \Rightarrow x(x + 10) - 9(x + 10) = 0$

$\Rightarrow (x + 10)(x - 9) = 0 \Rightarrow x = -10 \quad \text{or} \quad x = 9$

$\Rightarrow x = 9$ [Rejecting $x = -10$, since x is a counting number]

$\therefore$ The numbers are 3×9 and $3 \times (9 + 1)$ or 27 and 30.   **Ans.**

(ii) The cumulative frequency table may be prepared as under :

Wages (in ₹)	No. of workers	Cumulative Frequency
4000 − 4400	8	8
4400 − 4800	12	20
4800 − 5200	20	40
5200 − 5600	25	65
5600 − 6000	17	82
6000 − 6400	10	92

Now take wages along x-axis and number of workers along y-axis. Then plot the points (4000, 0), (4400, 8), (4800, 20), (5200, 40), (5600, 65), (6000, 82), (6400, 92). Join these points by a free hand curve to get the required ogive.

Here, $n = 92 \Rightarrow \dfrac{n}{2} = 46$.

On the graph, take a point A on the y–axis representing 46. Through A, draw, horizontal line, meeting the ogive at B. From B, draw BC $\perp x$ – axis meeting the x-axis at C. The abscissa of C is 5300. So, median wage is ₹5300.   **Ans.**
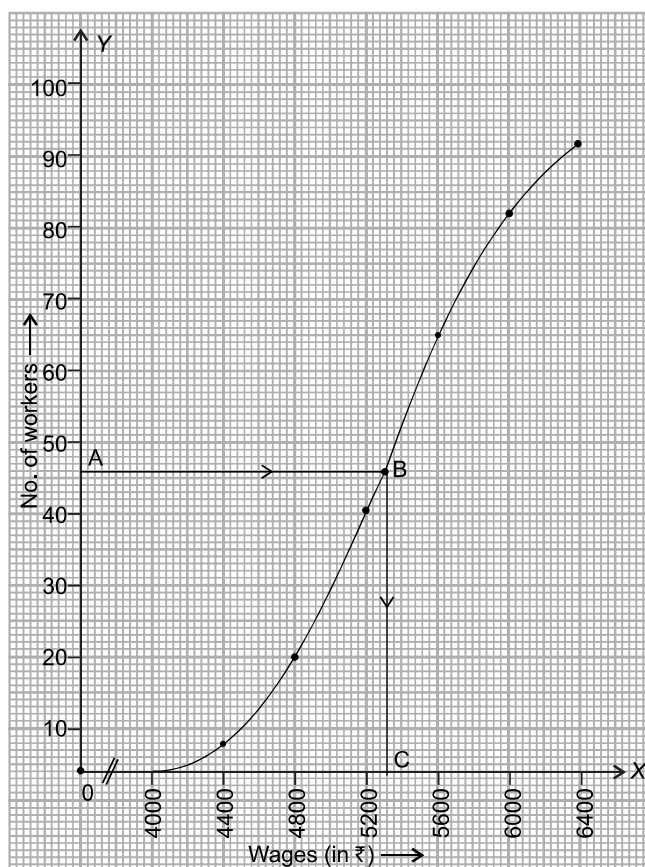
Question 8 :

   (i) **Two unbiased coins are tossed simultaneously. Find the probability of getting, one head.**   **[3]**

  (ii) **A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.**   **[3]**

 (iii) **In the given figure, a circle touches the side BC of a △ABC at P and AB and AC produced at Q and R respectively. Prove that**

     $AQ = AR = \dfrac{1}{2}$ **(perimeter of △ABC)**   **[4]**
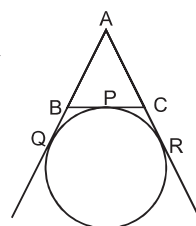
Solution :

 (i) Total number of outcomes = 4 (HH, HT, TH, TT)

    Favourable outcomes are TH, TH, i.e., 2.

    ∴ Required probability $= \dfrac{2}{4} = \dfrac{1}{2}$. **Ans.**

 (ii) Here, volume of cylindrical bucket = volume of conical heap

    $\Rightarrow \pi \times r^2 \times h = \dfrac{1}{3}\pi R^2 \times H \Rightarrow 18^2 \times 32 = \dfrac{1}{3} \times R^2 \times 24$

    $\Rightarrow R = \sqrt{18 \times 18 \times 4} = 36$ cm. **Ans.**

    ∴ Slant height of the conical heap $= \sqrt{R^2 + H^2} = \sqrt{36^2 + 24^2}$ cm $= \sqrt{1872}$ cm $= 43.26$ cm. **Ans.**

 (iii) $2AQ = AQ + AR$ [∵ AQ = AR, Tangent from an external point are equal]

      $= (AB + BQ) + (AC + CR) = AB + AC + (BQ + CR)$

$$= AB + AC + (BP + CP) \qquad [\because BQ = BP \text{ and } CR = CP]$$

$$= AB + AC + BC = (\text{Perimeter of } \Delta ABC)$$

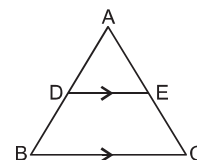$$\Rightarrow AQ = \frac{1}{2} (\text{Perimeter of } \Delta ABC). \textbf{ Proved.}$$

Question 9 :

(i) **Find the smallest non-negative value of x which satisfies the inequation** $x + \dfrac{7}{2} > \dfrac{5x}{3} + 3, x \in I.$ **[3]**

(ii) **Calculate the mean, the mode and the median of the following distribution.** **[3]**

Number	0	1	2	3	4	5
Frequency	2	4	7	6	8	3

(iii) **In the given figure, DE || BC. If AD = x cm, DB = $(x - 2)$ cm, AE = $(x + 2)$ cm and EC = $(x - 1)$ cm, find the value of x.** **[4]**

Solution :

(i) We have, $x + \dfrac{7}{2} > \dfrac{5x}{3} + 3, x \in I$

$$\Rightarrow \frac{7}{2} - 3 > \frac{5x}{3} - x, x \in I$$

$$\Rightarrow \frac{1}{2} > \frac{2x}{3}, x \in I \Rightarrow x < \frac{3}{4}, x \in I$$

So, solution set is $\{0, -1, -2, ...\}$

Hence, smallest non negative value of $x = 0$.

(ii) We may prepare a table as below :

Number (x)	Frequency f	fx	Cumulative frequency
0	2	0	2
1	4	4	6
2	7	14	13
3	6	18	19
4	8	32	27
5	3	15	30
	$\Sigma f = 30$	$\Sigma fx = 83$	

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{83}{30} = 2.76 \quad \textbf{Ans.}$$
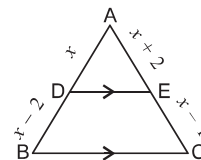
$$\text{Median} = \frac{\dfrac{n}{2} th \text{ value} + \left(\dfrac{n+1}{2}\right) th \text{ value}}{2}$$

$$\text{Median} = \frac{15 th \text{ value} + 16 th \text{ value}}{2} = \frac{3+3}{2} = 3$$

Since, the frequency of 4 is maximum 8, hence, mode is 4.

(iii) By Basic Proportionality Theorem, $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$$\Rightarrow \frac{x}{x-2} = = \frac{x+2}{x-1} \Rightarrow x^2 - x = x^2 - 4 \Rightarrow x = 4. \quad \textbf{Ans.}$$

Question 10 :

(i) If $\dfrac{\sqrt{x+2}+\sqrt{x-3}}{\sqrt{x+2}-\sqrt{x-3}} = 5$, prove that $x = 7$. [3]

(ii) Construct a regular hexagon of side 2.4 cm. Circumscribe a circle to it. [3]

(iii) The shadow of a vertical tower AB on level ground is increased by 10 m, when the altitude of the sun changes from 45° to 30°. Find the height of the tower and give your answer correct to $\dfrac{1}{10}$ of a metre. [4]

Solution :

(i) $\dfrac{\sqrt{x+2}+\sqrt{x-3}}{\sqrt{x+2}-\sqrt{x-3}} = 5$

$\Rightarrow \dfrac{\sqrt{x+2}+\sqrt{x-3}+\sqrt{x+2}-\sqrt{x-3}}{\sqrt{x+2}+\sqrt{x-3}-\sqrt{x+2}+\sqrt{x-3}} = \dfrac{5+1}{5-1}$ [Applying componendo and dividendo]

$\Rightarrow \dfrac{\sqrt{x+2}}{\sqrt{x-3}} = \dfrac{6}{4} \Rightarrow \dfrac{x+2}{x-3} = \dfrac{9}{4}$

$\Rightarrow 4x + 8 = 9x - 27$

$\Rightarrow 5x = 35 \Rightarrow x = 7.$ **Proved**

(ii) 1. Draw AB = 2.4 cm. Make ∠ABG = 120°

2. Along BG, set off BC = 2.4 cm.

3. Make ∠BCH = 120°.

4. Along CH, set off CD = 2.4 cm.

5. Make ∠CDI = 120°.

6. Along DI, set off DE = 2.4 cm.

7. Make ∠DEJ = 120°.

8. Along EJ, set off EF = 2.4 cm

9. Join AF. Then ABCDEF is the requried hexagon.

10. Draw the perpendicular bisectors PQ and RS of AB and BC respectively to intersect at a point O.

11. With O as centre and radius OA, draw a circle passing through all the vertices of the given hexagon.
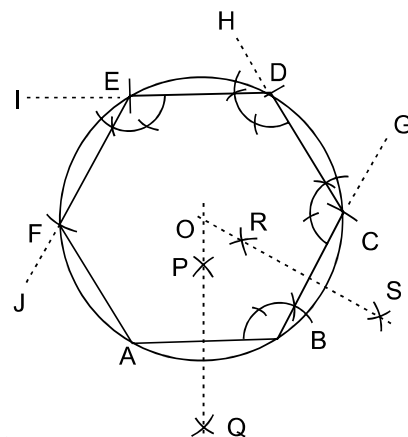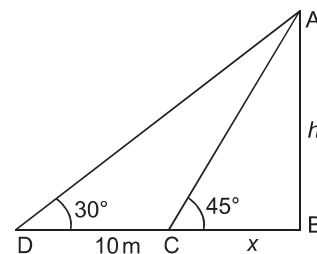
(iii) Let AB = h m be the tower,

BC = x and BD = $(x + 10)$ be the shadows of the tower.

Then, in ∆ABC,

$\tan 45° = \dfrac{h}{x} \Rightarrow h = x$ ... (1)

In ∆ABD, $\tan 30° = \dfrac{h}{x+10} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+10}$

$\Rightarrow h + 10 = h\sqrt{3} \Rightarrow h = \dfrac{10}{\sqrt{3}-1}$ [From (1)]

$\Rightarrow h = \dfrac{10(\sqrt{3}+1)}{3-1} = 5(\sqrt{3}+1) = 5(1.733+1) = 13.7$ m. **Ans.**

—— ❋ ❋ ❋ ——