Chapter
Three Dimensional Geometry

Topic-1: Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1 MCQs with One Correct Answer

1. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals **[2007 (S), 2010]**
(a) 45° (b) 60° (c) 75° (d) 30°

2. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals **[2004]**
(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Topic-2: Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1 MCQs with One Correct Answer

1. Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \in \{0,1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, let d(ℓ_1, ℓ_2) denote the shortest distance between them. Then the maximum value of d(ℓ_1, ℓ_2), as ℓ_1 varies over F and ℓ_2 varies over S, is **[Adv. 2023]**
(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{\sqrt{8}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{12}}$

2. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is **[2004S]**
(a) 3/2 (b) 9/2 (c) −2/9 (d) −3/2

6 MCQs with One or More than One Correct Answer

3. Three lines $L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R$
$L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in R$ and
$L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$
are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear? **[Adv. 2019]**
(a) $\hat{k} - \frac{1}{2}\hat{j}$ (b) $\hat{k}$ (c) $\hat{k} + \hat{j}$ (d) $\hat{k} + \frac{1}{2}\hat{j}$

4. Let L_1 and L_2 denote the lines
$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$
and $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$
respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ? **[Adv. 2019]**
(a) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
(b) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
(c) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
(d) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

5. From a point $P(\lambda, \lambda, \lambda)$, perpendicular PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/(are) [Adv. 2014]

(a) $\sqrt{2}$ (b) 1 (c) -1 (d) $-\sqrt{2}$

6. A line l passing through the origin is perpendicular to the lines $l_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}, \quad -\infty < t < \infty$

$l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, \quad -\infty < s < \infty$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are) [Adv. 2013]

(a) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (b) $(-1, -1, 0)$

(c) $(1, 1, 1)$ (d) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

7 Match the Following

7. Let $\gamma \in \mathbb{R}$ be such that the lines

$L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$

intersect. Let R_1 be the point of intersection of L_1 and L_2. Let $O = (0, 0, 0)$, and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2. Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) γ equals	(1) $-\hat{i} - \hat{j} + \hat{k}$
(Q) A possible choice for $\hat{n}$ is	(2) $\sqrt{\frac{3}{2}}$
(R) $\overrightarrow{OR_1}$ equals	(3) 1
(S) A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4) $\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
	(5) $\sqrt{\frac{2}{3}}$

The correct option is [Adv. 2024]

(a) (P)→(3) (Q)→(4) (R)→(1) (S)→(2)
(b) (P)→(5) (Q)→(4) (R)→(1) (S)→(2)
(c) (P)→(3) (Q)→(4) (R)→(1) (S)→(5)
(d) (P)→(3) (Q)→(1) (R)→(4) (S)→(5)

8. Match the statement in **Column-I** with the values in **Column-II** [2010]

Column – I	Column – II
(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(p) -4
(B) The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0

(C) Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b} = 0$. $(\vec{b}-\vec{a}).(\vec{b}+\vec{c}) = 0$ and $2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$.

If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are — (r) 4

(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$

and $f(x) = \sin\left(\frac{9x}{2}\right) \Big/ \sin\left(\frac{x}{2}\right)$ for $x \neq 0$ — (s) 5

The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is — (t) 6

Topic-3: Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1 MCQs with One Correct Answer

1. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is **[2012]**

(a) $5x - 11y + z = 17$ (b) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(c) $x + y + z = \sqrt{3}$ (d) $x - \sqrt{2}y = 1 - \sqrt{2}$

2. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is **[2012]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

3. If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is **[2010]**

(a) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (b) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (d) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

4. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is **[2010]**

(a) $x + 2y - 2z = 0$ (b) $3x + 2y - 2z = 0$

(c) $x - 2y + z = 0$ (d) $5x + 2y - 4z = 0$

5. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals **[2009]**

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

6. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is **[2009]**

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

7. A plane which is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, passes through $(1, -2, 1)$. The distance of the plane from the point $(1, 2, 2)$ is **[2006 - 3M, –1]**

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

8. A variable plane at a distance of the one unit from the origin cuts the coordinates axes at A, B and C. If the centroid $D(x, y, z)$ of triangle ABC satisfies the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then the value k is **[2005S]**

(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 9

9. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is **[2003S]**

(a) 7 (b) –7

(c) no real value (d) 4

2 Integer Value Answer/ Non-Negative Integer

10. Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $a, b \in \mathbb{R} - \{0\}$ and O denotes the origin. If $(\overrightarrow{OP} \times \overrightarrow{OQ}).\overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$ then the value of l is ________. **[Adv. 2024]**

11. Let P be the plane $\sqrt{3}x+2y+3z=16$ and let $S=\left\{\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}:\alpha^2+\beta^2+\gamma^2=1 \text{ and the distance of } (\alpha,\beta,\gamma) \text{ from the plane P is } \frac{7}{2}\right\}$. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is **[Adv. 2023]**

12. Let P be a point in the first octant, whose image Q in the plane $x+y=3$ (that is, the line segment PQ is perpendicular to the plane $x+y=3$ and the mid-point of PQ lies in the plane $x+y=3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is ______ . **[Adv. 2018]**

13. If the distance between the plane $Ax-2y+z=d$ and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then find $|d|$. **[2010]**

3 Numeric/ New Stem Based Questions

14. Three lines are given by $\vec{r}=\lambda\hat{i}, \lambda\in R$; $\vec{r}=\mu(\hat{i}+\hat{j}), \mu\in R$ and $\vec{r}=v(\hat{i}+\hat{j}+\hat{k}), v\in R$. Let the lines cut the plane $x+y+z=1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals ____. **[Adv. 2019]**

4 Fill in the Blanks

15. A nonzero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is .... **[1996 - 2 Marks]**

16. The unit vector perpendicular to the plane determined by $P(1,-1,2)$, $Q(2,0,-1)$ and $R(0,2,1)$ is .... **[1983 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

17. Let $\mathbb{R}$ denote the three-dimensional space. Take two points $P=(1,2,3)$ and $Q=(4,2,7)$. Let dist (X, Y) denote the distance between two points X and Y in $\mathbb{R}^3$. Let $S=\{X\in\mathbb{R}^3:(\text{dist}(X,P))^2-(\text{dist}(X,Q))^2=50\}$ and $T=\{Y\in\mathbb{R}^3:(\text{dist}(Y,Q))^2-(\text{dist}(Y,P))^2=50\}$. Then which of the following statements is (are) TRUE? **[Adv. 2024]**
    (a) There is a triangle whose area is 1 and all of whose vertices are from S.
    (b) There are two distinct points L and M in T such that each point on the line segment LM is also in T.
    (c) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.
    (d) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

18. A straight line drawn from the point P (1, 3, 2), parallel to the line $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$, intersects the plane $L_1: x-y+3z=6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2: 2x-y+z=-4$ at the point R. Then which of the following statements is (are) TRUE? **[Adv. 2024]**
    (a) The length of the line segment PQ is $\sqrt{6}$
    (b) The coordinates of R are (1, 6, 3)
    (c) The centroid of the triangle PQR is $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$
    (d) The perimeter of the triangle PQR is $\sqrt{2}+\sqrt{6}+\sqrt{11}$

19. Let P_1 and P_2 be two planes given by **[Adv. 2022]**
    $P_1: 10x+15y+12z-60=0$,
    $P_2: -2x+5y+4z-20=0$.
    Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2?
    (a) $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}$ (b) $\frac{x-6}{-5}=\frac{y}{2}=\frac{z}{3}$
    (c) $\frac{x}{-2}=\frac{y-4}{5}=\frac{z}{4}$ (d) $\frac{x}{1}=\frac{y-4}{-2}=\frac{z}{3}$

20. Let S be the reflection of a point Q with respect to the plane given by $\vec{r}=-(t+p)\hat{i}+t\hat{j}+(1+p)\hat{k}$ where t, p are real parameters and $\hat{i},\hat{j},\hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are TRUE ? **[Adv. 2022]**
    (a) $3(\alpha+\beta)=-101$ (b) $3(\beta+\gamma)=-71$
    (c) $3(\gamma+\alpha)=-86$ (d) $3(\alpha+\beta+\gamma)=-121$

21. Let L_1 and L_2 be the following straight line.
    $L_1: \frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3}$ and $L_2: \frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}$.
    Suppose the straight line $L: \frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$ lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE? **[Adv. 2020]**
    (a) $\alpha-\gamma=3$ (b) $l+m=2$
    (c) $\alpha-\gamma=1$ (d) $l+m=0$

22. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0$ and $\alpha+\gamma=1$. Suppose the point (3, 2, –1) is the mirror image of the point (1, 0, –1) with respect to the plane $\alpha x+\beta y+\gamma z=\delta$.
Then which of the following statements is/are TRUE?
(a) $\alpha+\beta=2$ (b) $\delta-\gamma=3$ [Adv. 2020]
(c) $\delta+\beta=4$ (d) $\delta+\beta+\gamma=\delta$

23. Let $P_1: 2x+y-z=3$ and $P_2: x+2y+z=2$ be two planes. Then, which of the following statement(s) is (are) TRUE? [Adv. 2018]
(a) The line of intersection of P_1 and P_2 has direction ratios 1, 2, – 1
(b) The line $\frac{3x-4}{9}=\frac{1-3y}{9}=\frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
(c) The acute angle between P_1 and P_2 is 60°.
(d) If P_3 is the plane passing through the point (4, 2, – 2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

24. Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, and OP and OR along the x–axis and the y–axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point, T of diagonal OQ such that TS = 3. Then [Adv. 2016]
(a) the acute angle between OQ and OS is $\frac{\pi}{3}$
(b) the equation of the plane containing the triangle OQS is $x-y=0$
(c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

25. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M? [Adv. 2015]
(a) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$
(c) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ (d) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

26. In R^3, consider the planes $P_1: y=0$ and $P_2: x+z=1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true? [Adv. 2015]
(a) $2\alpha+\beta+2\gamma+2=0$ (b) $2\alpha-\beta+2\gamma+4=0$
(c) $2\alpha+\beta-2\gamma-10=0$ (d) $2\alpha-\beta+2\gamma-8=0$

27. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) [Adv. 2013]
(a) 1 (b) 2 (c) 3 (d) 4

28. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane (s) containing these two lines is (are) [2012]
(a) $y+2z=-1$ (b) $y+z=-1$
(c) $y-z=-1$ (d) $y-2z=-1$

29. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and that P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and a given vector $2\hat{i}+\hat{j}-2\hat{k}$ is [2006 - 5M, –1]
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

7 Match the Following

30. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1=\lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2=(\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.
Match each entry in **List-I** to the correct entries in **List-II.**

List-I	List-II
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$

(R) The distance of origin from H_0 is (3) 0

(S) The distance of origin from the point of intersection of planes $y = z$, $x = 1$ and H_0 is (4) $\sqrt{2}$

(5) $\frac{1}{\sqrt{2}}$

The correct option is: **[Adv. 2023]**

(a) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)
(b) (P) → (5), (Q) → (4), (R) → (3), (S) → (1)
(c) (P) → (2), (Q) → (1), (R) → (3), (S) → (2)
(d) (P) → (5), (Q) → (1), (R) → (4), (S) → (2)

31. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 = 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists : **[Adv. 2013]**

List I	List II
P. $a =$	1. 13
Q. $b =$	2. -3
R. $c =$	3. 1
S. $d =$	4. -2

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

32. Match the statements/expressions given in **Column-I** with the values given in **Column-II**. **[2009]**

Column-I	Column-II
(A) The number of solutions of the equation $x\,e^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution(s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$, then value(s) of y (1n 2)	(s) 4
	(t) 5

33. Consider the following linear equations

$ax + by + cz = 0; \quad bx + cy + az = 0; \quad cx + ay + bz = 0$

Match the conditions/expressions in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the *ORS*. **[2007]**

Column I	Column II
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at asingle point
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

34. Match the following : [2006 - 6M]

Column I	Column II
(A) Two rays $x+y=\|a\|$ and $ax-y=1$ intersects each other in the first quadrant in the interval $a \in (a_0, \infty)$, the value of a_0 is	(p) 2
(B) Point (α, β, γ) lies on the plane $x+y+z=2$. Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\hat{k} \times (\hat{k} \times \vec{a}) = 0$, then $\gamma =$	(q) $\frac{4}{3}$
(C) $\left\|\int_0^1 (1-y^2)\,dy\right\| + \left\|\int_1^0 (y^2-1)\,dy\right\|$	(r) $\left\|\int_0^1 \sqrt{1-x}\,dx\right\| + \left\|\int_{-1}^0 \sqrt{1+x}\,dx\right\|$
(D) If $\sin A \sin B \sin C + \cos A \cos B = 1$, then the value of $\sin C =$	(s) 1

8 Comprehension/Passage Based Questions

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \quad L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

[2008]

35. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

36. The shortest distance between L_1 and L_2 is

(a) 0 (b) $\frac{17}{\sqrt{3}}$ (c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$

37. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(a) $\frac{2}{\sqrt{75}}$ (b) $\frac{7}{\sqrt{75}}$ (c) $\frac{13}{\sqrt{75}}$ (d) $\frac{23}{\sqrt{75}}$

9 Assertion and Reason/Statement Type Questions

38. Consider three planes

$P_1 : x-y+z=1$ $P_2 : x+y-z=-1$

$P_3 : x-3y+3z=2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

STATEMENT - 1 : At least two of the lines L_1, L_2 and L_3 are non-parallel and

STATEMENT - 2 : The three planes doe not have a common point. [2008]

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

39. Consider the planes $3x-6y-2z=15$ and $2x+y-2z=5$.

STATEMENT-1 : The parametric equations of the line of intersection of the given planes are $x = 3+14t, y = 1+2t, z = 15t$. **because**

STATEMENT-2 : The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of given planes. [2007 -3 marks]

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

10 Subjective Problems

40. Find the equation of the plane containing the line $2x-y+z-3=0$, $3x+y+z=5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1). [2005 - 2 Marks]

41. P_1 and P_2 are planes passing through origin. L_1 and L_2 are two line on P_1 and P_2 respectively such that their intersection is origin. Show that there exists points A, B, C, whose permutation A', B', C' can be chosen such that (i) A is on L_1, B on P_1 but not on L_1 and C not on P_1 (ii) A' is on L_2, B' on P_2 but not on L_2 and C' not on P_2. [2004 - 4 Marks]

42. A parallelepiped 'S' has base points A, B, C and D and upper face points A', B', C' and D'. This parallelepiped is compressed by upper face $A'B'C'D'$ to form a new parallelepiped 'T' having upper face points A'', B'', C'' and D''. Volume of parallelepiped T is 90 percent of the volume of parallelepiped S. Prove that the locus of 'A''', is a plane. [2004 - 2 Marks]

43. Find the equation of plane passing through (1, 1, 1) & parallel to the lines L_1, L_2 having direction ratios (1,0,–1), (1,–1,0). Find the volume of tetrahedron formed by origin and the points where these planes intersect the coordinate axes. [2004 - 2 Marks]

44. (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. [2003 - 4 Marks]

Answer Key

Topic-1 : Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1. (b) **2.** (c)

Topic-2 : Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1. (a) **2.** (b) **3.** (a,d) **4.** (a,b,d) **5.** (c) **6.** (b,d) **7.** (c) **8.** (A)-t; (B)-p, r; (C)-q, s; (D)-r

Topic-3 : Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1. (a) **2.** (a) **3.** (a) **4.** (c) **5.** (c) **6.** (a) **7.** (d) **8.** (d) **9.** (a) **10.** (5)

11. (45) **12.** (8) **13.** (6) **14.** (0.75) **15.** $\frac{\pi}{4}, \frac{3\pi}{4}$ **16.** $\pm\frac{(2\hat{i}+\hat{j}+\hat{k})}{\sqrt{6}}$ **17.** (a,b, c) **18.** (a,c) **19.** (a,b)

20. (a,b,c) **21.** (a,b) **22.** (a,b,c) **23.** (c,d) **24.** (b,c,d) **25.** (a,b) **26.** (b,d) **27.** (a,d) **28.** (b,c) **29.** (b,d)

30. (b) **31.** (a) **32.** (A)-p; (B)-q, s; (C)-q, r, s, t; (D)-t **33.** (A)-r; (B)-q; (C)-p; (D)-s

34. (A)-s; (B)-p; (C)-q, r; (D)-s **35.** (b) **36.** (d) **37.** (c) **38.** (d) **39.** (d)

Hints & Solutions

Topic-1: Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1. **(b)** As per question, direction cosines of the line :

$$\ell = \cos 45° = \frac{1}{\sqrt{2}}, \quad m = \cos 120° = \frac{-1}{2}, \quad n = \cos\theta$$

where θ is the angle, which line makes with positive z-axis.

We know that, $\ell^2 + m^2 + n^2 = 1$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \quad (\theta \text{ being acute})$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

2. **(c)** As per question the direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$

$$\therefore \quad \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$$

$$\because \quad 2\cos^2\theta = 1 - \cos^2\theta$$

$$\Rightarrow \quad 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$$

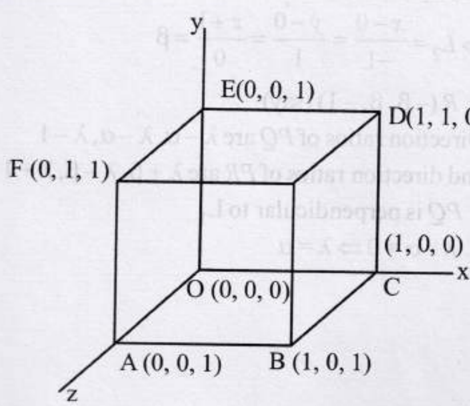
(given)

$$\Rightarrow \quad 2\cos^2\theta = 3 - 3\cos^2\theta$$

$$\therefore \quad \cos^2\theta = \frac{3}{5}$$

Topic-2: Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1. **(a)**



Equation of face diagonal OD line is

$$l_1 : \vec{r} = \lambda\left(\hat{i} + \hat{j}\right)$$

Equation of main diagonal BE is

$$l_2 : \vec{r} = \hat{j} + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$$

$$\text{Shortest distance} = \left|\frac{\hat{j}.\left(\hat{i}+\hat{j}\right)\times\left(\hat{i}-\hat{j}+\hat{k}\right)}{\left|\left(\hat{i}+\hat{j}\right)\times\left(\hat{i}-\hat{j}+\hat{k}\right)\right|}\right|$$

$$= \left|\frac{\hat{j}.\left(\hat{i}-\hat{j}-2\hat{k}\right)}{\left|\hat{i}-\hat{j}-2\hat{k}\right|}\right| = \frac{1}{\sqrt{6}}$$

In other case S.D is zero.

2. **(b)** Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

$$\Rightarrow \quad x = 2\lambda + 1, y = 3\lambda - 1 \text{ and } z = 4\lambda + 1$$

and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$

$$\Rightarrow \quad x = 3 + \mu, y = k + 2\mu \text{ and } z = \mu$$

Since given lines intersect each other

$$\Rightarrow \quad 2\lambda + 1 = 3 + \mu \qquad \text{...(i)}$$

$$3\lambda - 1 = 2\mu + k \qquad \text{...(ii)}$$

$$\mu = 4\lambda + 1 \qquad \text{...(iii)}$$

Solving (i) and (iii) and putting the value of λ and μ in (ii) we get, $k = \frac{9}{2}$

3. **(a, d)** Let any point

$P(\lambda, 0, 0)$ on L_1, $Q(0, \mu, 1)$ on L_2 and $R(1, 1, v)$ on L_3

$\because$ P, Q, R are collinear, $\therefore \overline{PQ} \parallel \overline{PR}$

$$\Rightarrow \frac{\lambda}{\lambda - 1} = \frac{-\mu}{-1} = \frac{-1}{-v}$$

$$\Rightarrow \mu = \frac{\lambda}{\lambda - 1}, v = \frac{\lambda - 1}{\lambda}$$

Clearly from above that $\lambda \neq 0, 1$

$$\therefore Q\left(0, \frac{\lambda}{\lambda - 1}, 1\right)$$

(a) For $Q = \hat{k} - \frac{1}{2}\hat{j}$

$$\frac{\lambda}{\lambda-1}=-\frac{1}{2}\Rightarrow 3\lambda=+1, \text{ which is possible.}$$

(b) For $Q=\hat{k}$

$$\frac{\lambda}{\lambda-1}=0\Rightarrow \lambda=0, \text{ not possible}$$

(c) For $Q=\hat{k}+\hat{j}$

$$\frac{\lambda}{\lambda-1}=1\Rightarrow \lambda=\lambda-1, \text{ not possible}$$

(d) For $Q=\hat{k}+\frac{1}{2}\hat{j}$

$$\frac{\lambda}{\lambda-1}=\frac{1}{2}\Rightarrow 2\lambda=\lambda-1\Rightarrow \lambda=-1,$$

which is possible

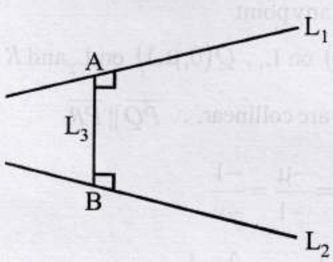
Hence options (a) and (d) are correct and options (b) and (c) are incorrect.

4. **(a, b, d)** $L_1:\vec{r}=\hat{i}+\lambda(-i+2j+2\hat{k})$ $L_2:\vec{r}=\mu(2i-j+2\hat{k})$

Since L_3 being perpendicular to both L_1 and L_2, is the shortest distance line between L_1 & L_2.

$\therefore$ Direction vector of line L_3 : $(-\hat{i}+2\hat{j}+2\hat{k})\times(2\hat{i}-\hat{j}+2\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 2\\ 2 & -1 & 2\end{vmatrix}=6\hat{i}+6\hat{j}-3\hat{k}$$



$\because$ L_1 and L_2 are skew lines

Let any point on L_1 and L_2 be $A(1-\lambda, 2\lambda, 2\lambda)$ and $B(2\mu, -\mu, 2\mu)$.

$\therefore$ dr's of AB $=2\mu+\lambda-1, -\mu-2\lambda, 2\mu-2\lambda$

$\because$ AB and L_3 are representing the same line

$$\therefore\quad \frac{2\mu+\lambda-1}{6}=\frac{-\mu-2\lambda}{6}=\frac{2\mu-2\lambda}{-3}$$

$$\Rightarrow\quad 3\lambda+3\mu=1 \qquad ...(i)$$

$$6\lambda-3\mu=0 \qquad ...(ii)$$

Solving (i) and (ii) we get : $\lambda=\frac{1}{9}, \mu=\frac{2}{9}$

$$\therefore\quad A\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right) \text{ and } B\left(\frac{4}{9},\frac{-2}{9},\frac{4}{9}\right)$$

$\therefore$ Equation of L_3 is given by

$$\vec{r}=\frac{2}{9}(4i+\hat{j}+\hat{k})+t(2i+2j-\hat{k})$$

$\therefore$ (a) is correct.

$$\text{or}\quad \vec{r}=\frac{2}{9}(2\hat{i}-\hat{j}+2\hat{k})+t(2i+2j-\hat{k})$$

$\therefore$ (b) is correct

Also mid-point of AB is $\left(\frac{2}{3},0,\frac{1}{3}\right)$

$\therefore$ L_3 can also be written as

$$\vec{r}=\frac{1}{3}(2\hat{i}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k}), \text{ where } t\in R$$

$\therefore$ (d) is correct.

Clearly (0, 0, 0) does not lie on

$$\vec{r}=\frac{2}{9}(4\hat{i}+\hat{j}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k})$$

$\therefore$ $\vec{r}=t(2\hat{i}+2\hat{j}-\hat{k})$ can not describe the line L_3.

$\therefore$ (c) is incorrect.

5. **(c)** Given that lines are $x=y, z=1$

$$\Rightarrow L_1=\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-1}{0}=\alpha \qquad ...(i)$$

$\therefore Q(\alpha, \alpha, 1)$

and $y=-x, z=-1$

$$\Rightarrow L_2=\frac{x-0}{-1}=\frac{y-0}{1}=\frac{z+1}{0}=\beta \qquad ...(ii)$$

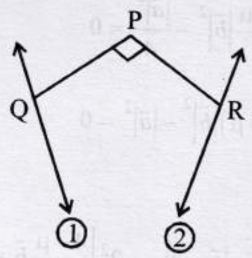
$\therefore R(-\beta, \beta, -1)$ (say)

Direction ratios of PQ are $\lambda-\alpha, \lambda-\alpha, \lambda-1$

and direction ratios of PR are $\lambda+\beta, \lambda-\beta, \lambda+1$

$\because PQ$ is perpendicular to L_1

$\therefore \lambda-\alpha=0\Rightarrow\lambda=\alpha \qquad ...(iii)$



$\because$ PR is perpendicular to L_2

$\therefore -(\lambda+\beta)+\lambda-\beta=0 \Rightarrow \beta=0$

$\therefore$ dr's of PQ are $0, 0, \lambda-1$

and dr's of PR are $\lambda, \lambda, \lambda+1$

$\because \angle QPR = 90° \Rightarrow (\lambda-1)(\lambda+1)=0 \Rightarrow \lambda = 1$ or -1

But for $\lambda = 1$, we get point Q itself

$\therefore$ we take $\lambda = -1$

6. **(b, d)** The given lines are

$\ell_1 : \left(3\hat{i}-\hat{j}+4\hat{k}\right)+t\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

$\ell_2 : \left(3\hat{i}+3\hat{j}+2\hat{k}\right)+s\left(2\hat{i}+2\hat{j}+\hat{k}\right)$

Direction vector perpendicular to bo ℓ_1 and ℓ_2

$$\vec{b}=\ell_1\times\ell_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 2 & 2 & 1\end{vmatrix}=-2\hat{i}+3\hat{j}-2\hat{k}$$

$\therefore \quad \ell : \dfrac{x}{2}=\dfrac{y}{-3}=\dfrac{z}{2}=\lambda$

Any point on ℓ_1 is $(t+3, 2t-1, 2t+4)$ and any point on ℓ is $(2\lambda, -3\lambda, 2\lambda)$

$\therefore$ Let intersection point of ℓ and ℓ_1 be P.

$t+3=2\lambda,\ 2t-1=-3\lambda,\ 2t+4=2\lambda$

$\Rightarrow \quad t=-1,\ \lambda=1 \qquad \therefore P(2,-3,2)$

Any point Q on ℓ_2 is $(2s+3, 2s+3, s+2)$

According to question $PQ = \sqrt{17}$

$\Rightarrow \quad (2s+1)^2+(2s+6)^2+s^2=17$

$\Rightarrow \quad 9s^2+28s+20=0 \Rightarrow s=-2, \dfrac{-10}{9}$

$\therefore$ Point Q can be $(-1,-1,0)$ and $\left(\dfrac{7}{9},\dfrac{7}{9},\dfrac{8}{9}\right)$

7. **(c)** Let $L_1 : \dfrac{x+11}{1}=\dfrac{y+21}{2}=\dfrac{z+29}{3}=\lambda$

and $L_2 : \dfrac{x+16}{3}=\dfrac{y+11}{2}=\dfrac{z+4}{\gamma}=\mu$

$x=\lambda-11=3\mu-16 \Rightarrow \lambda-3\mu=-5$ ...(i)

$y=2\lambda-21=2\mu-11 \Rightarrow 2\lambda-2\mu=10$ ...(ii)

$z=3\lambda-29=\mu\gamma-4 \Rightarrow 3\lambda-\mu\gamma=25$ ...(iii)

from (i) & (ii)

$\lambda=10, \mu=5$

Now from (iii)

$3(10)-5\gamma=25 \qquad \therefore \gamma=1$

So, $R_1 \equiv (-1,-1,1)$

Now, $OR_1 = -\hat{i}-\hat{j}+\hat{k}$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 3 & 2 & 1\end{vmatrix}=-4\hat{i}-(-8)\hat{j}-4\hat{k}$$

$$\vec{n}=-4\hat{i}+8\hat{j}+4\hat{k}=-4(\hat{i}-2\hat{j}+\hat{k})$$

$$\hat{n}=\pm\frac{4(\hat{i}-2\hat{j}+\hat{k})}{4\sqrt{6}}=\pm\frac{(\hat{i}-2\hat{j}+\hat{k})}{\sqrt{6}}$$

$$\overline{OR}.\hat{n}=\pm(-\hat{i}-\hat{j}+\hat{k})\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

$$=\pm\frac{2}{\sqrt{6}}=\pm\sqrt{\frac{4}{6}}=\pm\sqrt{\frac{2}{3}}$$

8. **(A) → t ; (B) → p, r ; (C) → q, s ; (D) → r**

Let the line through origin be $L : \dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}$ .... (i)

since line L intersects

$L_1 : \dfrac{x-2}{1}=\dfrac{y-1}{-2}=\dfrac{z+1}{1}$ .....(ii)

and $L_2 : \dfrac{x-8/3}{2}=\dfrac{y+3}{-1}=\dfrac{z-1}{1}$ ....(iii)

at P and Q,

$\therefore$ line L and L_1 coplaner.

$$\therefore \text{Using} \begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$$

$$\text{we get} \begin{vmatrix}2 & 1 & -1\\ a & b & c\\ 1 & -2 & 1\end{vmatrix}=0 \Rightarrow a+3b+5c=0 \quad \text{...(iv)}$$

Also L and L_2 coplaner

and $\begin{vmatrix} 8/3 & -3 & 1 \\ a & b & c \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow 3a+b-5c=0$ ...(v)

On solving (iv) and (v), we get

$$\frac{a}{-15-5}=\frac{b}{15+5}=\frac{c}{1-9} \text{ or } \frac{a}{5}=\frac{b}{-5}=\frac{c}{2}$$

Hence equation (i) becomes $\frac{x}{5}=\frac{y}{-5}=\frac{z}{2}=\lambda$

Any point on L, $P(5\lambda, -5\lambda, 2\lambda)$
which lies on (ii) also

$$\therefore \frac{5\lambda-2}{1}=\frac{-5\lambda-1}{-2}=\frac{2\lambda+1}{1} \Rightarrow \lambda=1$$

$\therefore$ P (5, –5, 2)

Also Any point on L, $Q(5\lambda, -5\lambda, 2\lambda)$
which lies on (iii) also

$$\therefore \frac{5\lambda-8/3}{3}=\frac{-5\lambda+3}{-1}=\frac{2\lambda-1}{1} \Rightarrow \lambda = 2/3$$

$$\therefore Q\left(\frac{10}{3}, \frac{-10}{3}, \frac{4}{3}\right)$$

Hence $d^2 = PQ^2 = \left(\frac{25}{9}+\frac{25}{9}+\frac{4}{9}\right)=6$

(B) $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\tan^{-1}\frac{3}{4}$

$\left[\because \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}\right]$

$$\Rightarrow \tan^{-1}\left(\frac{x+3-x+3}{1+x^2-9}\right)=\tan^{-1}\left(\frac{3}{4}\right), x^2-9>-1$$

$$\Rightarrow \frac{6}{x^2-8}=\frac{3}{4} \Rightarrow x^2=16 \text{ or } x=4,-4$$

(C) $\because \vec{a}=\mu\vec{b}+4\vec{c} \Rightarrow \vec{c}=\frac{\vec{a}-\mu\vec{b}}{4}$

Then $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$

$$\Rightarrow (\vec{b}-\vec{a}).\left(\vec{b}+\frac{\vec{a}-\mu\vec{b}}{4}\right)=0$$

$$\Rightarrow (\vec{b}-\vec{a}).\left(\frac{4-\mu}{4}\vec{b}+\frac{\vec{a}}{4}\right)=0$$

$$\Rightarrow \frac{4-\mu}{4}|\vec{b}|^2-\frac{|\vec{a}|^2}{4}=0$$

$$\Rightarrow (4-\mu)|\vec{b}|^2-|\vec{a}|^2=0 \quad \text{...(i)}$$

Also,

$$2|\vec{b}+\vec{c}|=|\vec{b}-\vec{a}| \Rightarrow 2^2\left|\frac{4-\mu}{4}\vec{b}+\frac{\vec{a}}{4}\right|^2=|\vec{b}-\vec{a}|^2$$

$$\Rightarrow (4-\mu)^2|\vec{b}|^2+|\vec{a}|^2=4|\vec{b}|^2+4|\vec{a}|^2 \left[\because \vec{a}.\vec{b}=0\right]$$

$$\Rightarrow \left[(4-\mu)^2-4\right]|\vec{b}|^2=3|\vec{a}|^2 \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{(4-\mu)^2-4}{4-\mu}=\frac{3}{1}$$

$$\Rightarrow \mu^2-8\mu+12=12-3\mu \Rightarrow \mu^2-5\mu=0$$

$$\Rightarrow \mu=0 \text{ or } 5$$

(D) $I=\frac{2}{\pi}\int_{-\pi}^{\pi}\frac{\sin(9x/2)}{\sin(x/2)}dx=\frac{2}{\pi}\times 2\int_0^{\pi}\frac{\sin 9x/2}{\sin x/2}dx$

$[\because f(x) \text{ is even function}]$

Let $\frac{x}{2}=\theta \Rightarrow dx=2d\theta$

Also at $x=0, \theta=0$ and at $x=\pi, \theta=\pi/2$

$$\therefore I=\frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 9\theta}{\sin\theta}d\theta$$

$$I=\frac{8}{\pi}\int_0^{\pi/2}\left[\frac{\sin 9\theta-\sin 7\theta}{\sin\theta}+\frac{(\sin 7\theta-\sin 5\theta)}{\sin\theta}+\frac{(\sin 5\theta-\sin 3\theta)}{\sin\theta}+\frac{(\sin 3\theta-\sin\theta)}{\sin\theta}+\frac{\sin\theta}{\sin\theta}\right]d\theta$$

$$=\frac{16}{\pi}\int_0^{\pi/2}(\cos 8\theta+\cos 6\theta+\cos 4\theta+\cos 2\theta)d\theta+\frac{8}{\pi}\int_0^{\pi/2}d\theta$$

$$=\frac{16}{\pi}\left[\frac{\sin 8\theta}{8}+\frac{\sin 6\theta}{6}+\frac{\sin 4\theta}{4}+\frac{\sin 2\theta}{2}\right]_0^{\pi/2}+\frac{8}{\pi}(\theta)_0^{\pi/2}$$

$$=0+\frac{8}{\pi}\left(\frac{\pi}{2}-0\right)=4$$

Topic-3: Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1. **(a)** Equation of the plane passing through the intersection line of given planes is

$$(x+2y+3z-2)+\lambda(x-y+z-3)=0$$

or $(1+\lambda)x+(2-\lambda)y+(3+\lambda)z+(-2-3\lambda)=0$

$\because$ Its distance from the point $(3, 1, -1)$ is $\dfrac{2}{\sqrt{3}}$

$$\therefore \left|\frac{3(1+\lambda)+1(2-\lambda)-1(3+\lambda)+(-2-3\lambda)}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}\right|=\frac{2}{\sqrt{3}}$$

$$\Rightarrow \left|\frac{-2\lambda}{\sqrt{3\lambda^2+4\lambda+14}}\right|=\frac{2}{\sqrt{3}}$$

$$\Rightarrow 3\lambda^2+4\lambda+14=3\lambda^2 \Rightarrow \lambda=-\frac{7}{2}$$

$\therefore$ Required equation of plane is

$$(x+2y+3z-2)-\frac{7}{2}(x-y+z-3)=0$$

or $5x-11y+z=17$

2. **(a)** Equation of st. line joining $Q(2, 3, 5)$ and $R(1, -1, 4)$ is

$$\frac{x-2}{-1}=\frac{y-3}{-4}=\frac{z-5}{1}=\lambda$$

Let $P(-\lambda+2, -4\lambda+3, -\lambda+5)$

Since P also lies on $5x-4y-z=1$

$\therefore -5\lambda+10+16\lambda-12+\lambda-5=1$

$$\Rightarrow 12\lambda=8 \Rightarrow \lambda=\frac{2}{3} \qquad \therefore\ P=\left(\frac{4}{3},\frac{1}{3},\frac{13}{3}\right)$$

Now let another point S on QR be

$(-\mu+2, -4\mu+3, -\mu+5)$

Since S is the foot of perpendicular drawn from $T(2, 1, 4)$ to QR, where dr's of ST are $\mu, 4\mu-2, \mu-1$ and dr's of QR are $-1, -4, -1$

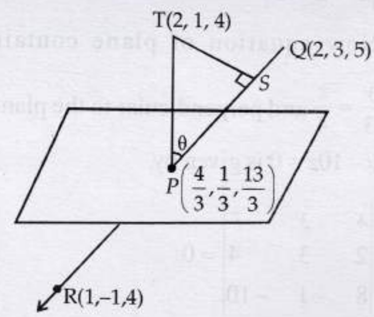
$$\therefore\ -\mu-16\mu+8-\mu+1=0 \Rightarrow 18\mu=9 \Rightarrow \mu=\frac{1}{2}$$

$$\therefore\ S=\left(\frac{3}{2},1,\frac{9}{2}\right)$$

$\therefore$ Distance between P and S

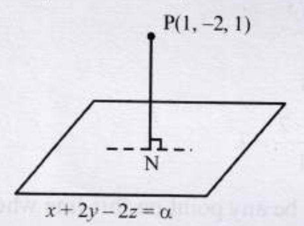
$$=\sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2+\left(\frac{1}{3}-1\right)^2+\left(\frac{13}{3}-\frac{9}{2}\right)^2}$$

$$=\sqrt{\frac{1}{36}+\frac{4}{9}+\frac{1}{36}}=\frac{1}{\sqrt{2}}$$



3. **(a)** Since perpendicular distance of $x+2y-2z-\alpha=0$ from the point $(1, -2, 1)$ is 5

$$\therefore \left|\frac{1-4-2-\alpha}{3}\right|=5$$



$$\Rightarrow \frac{-5-\alpha}{3}=5 \text{ or } -5$$

$\Rightarrow \alpha=-20$ or 10

But $\alpha>0 \Rightarrow \alpha=10$

$\therefore$ Equation of plane : $x+2y-2z-10=0$

We know that foot of perpendicular from point (x, y, z) to the plane $ax+by+cz+d=0$ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-(ax_1+by_1+cz_1+d)}{(a^2+b^2+c^2)}$$

$$\therefore\ \frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=\frac{-(1-4-2-10)}{9}=\frac{5}{3}$$

$$\Rightarrow x=\frac{8}{3}, y=\frac{4}{3}, z=-\frac{7}{3}$$

$$\therefore\ \text{Foot of } \perp^r \equiv \left(\frac{8}{3},\frac{4}{3},\frac{-7}{3}\right)$$

4. **(c)** Equation of plane containing two lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is given by

$$\begin{vmatrix} x & y & z \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix}=0 \Rightarrow 8x-y-10z=0$$

Now equation of plane containing the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane $8x-y-10z=0$ is given by,

$$\begin{vmatrix} x & y & z \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix}=0$$

$$\Rightarrow -26x+52y-26z=0 \quad \text{or} \quad x-2y+z=0$$

5. **(c)** Since line makes equal angle with coordinate axes and which has positive direction cosines

$\therefore$ D·c's $=\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

$\Rightarrow$ D·r's $=1,1,1$

$\therefore$ Equation of line is

$$\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-2}{1}=\lambda$$

$\therefore Q(\lambda+2,\lambda-1,\lambda+2)$ be any point on this line where it meets the plane $2x+y+z=9$

$\Rightarrow \quad 2(\lambda+2)+\lambda-1+\lambda+2=9 \Rightarrow \lambda=1$

$\therefore$ Q has coordintes $(3, 0, 3)$

$\therefore \quad PQ=\sqrt{(2-3)^2+(-1-0)^2+(2-3)^2}=\sqrt{3}$

6. **(a)** $\because \vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$

$=(1-3\mu)\hat{i}+(-1+\mu)\hat{j}+(2+5\mu)\hat{k}$

Let coordinates of Q be $(-3\mu+1,\mu-1,5\mu+2)$

$\therefore$ d.r's of $\overrightarrow{PQ}=-3\mu-2,\ \mu-3,\ 5\mu-4$

Given that $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$

$\therefore \quad 1.(-3\mu-2)-4.(\mu-3)+3.(5\mu-4)=0$

$\Rightarrow \quad 8\mu=2$ or $\mu=\frac{1}{4}$

7. **(d)** We know that the equation of plane through the point $(1,-2,1)$ and perpendicular to the planes $2x-2y+z=0$ and $x-y+2z=4$ is

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}=0 \Rightarrow x+y+1=0$$

It's distance from the point $(1, 2, 2)$ is

$$\left|\frac{1+2+1}{\sqrt{2}}\right|=2\sqrt{2}.$$

8. **(d)** Let $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ be the eq^n of variable plane which meets the axes at $A(a,0,0), B(0,b,0)$ and $C(0.0,c)$.

$\therefore$ Centroid of ΔABC is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

$\Rightarrow x=\frac{a}{3}, y=\frac{b}{3}, z=\frac{c}{3}$

putting these values in

$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k \Rightarrow \frac{9}{a^2}+\frac{9}{b^2}+\frac{9}{c^2}=k$

$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{z^2}=\frac{k}{9}$ ...(i)

Also given that the distance of plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ from $(0,0,0)$ is 1 unit.

$\Rightarrow \quad \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1 \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1$

From (i) we get $\frac{k}{9}=1$ i.e. $k=9$

9. **(a)** Since the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, then the point $(4,2,k)$ on line also lie on the given plane and hence

$2\times4-4\times2+k=7 \Rightarrow k=7$

10. **(5)** Given that $(\overrightarrow{OP}\times\overrightarrow{OQ}).\overrightarrow{OR}=0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix}=0$$

$$\Rightarrow \frac{\alpha-1}{\alpha}\left(\frac{\beta-1}{2\beta}-1\right)-\left(\frac{1}{2}-1\right)+1\left(1-\frac{\beta-1}{\beta}\right)=0$$

$$\frac{\alpha-1}{\alpha}\left(\frac{-\beta-1}{2\beta}\right)+\frac{1}{2}+\frac{1}{\beta}=0$$

$$\Rightarrow \frac{\beta+2}{2\beta}$$

$$=\frac{\alpha\beta+\alpha-\beta-1}{2\alpha\beta}$$

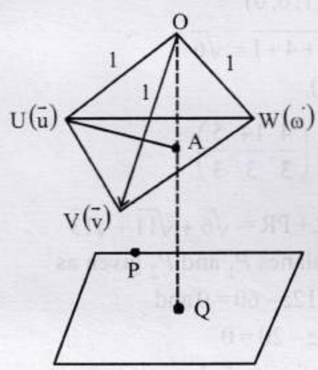
$$\Rightarrow \alpha\beta+2\alpha=\alpha\beta+\alpha-\beta-1 \Rightarrow \alpha+\beta+1=0 \quad ...(i)$$

Since, $(\alpha, \beta, 2)$ lies on plane $3x+3y-z+l=0$

$$\Rightarrow 3(\alpha+\beta)-2+l=0 \quad ...(ii)$$

$$\Rightarrow -3-2+l=0 \Rightarrow l=5$$

11. (45)



Given, $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$

So, ΔUVW is one equilateral triangle

Given that distances of points U, V, W from plane $P=\frac{7}{2} \Rightarrow AQ=\frac{7}{2}$

Distance of plane P from origin

$$=\left|\frac{0+0+0-16}{\sqrt{3+4+9}}\right|=4=OQ$$

$$\therefore OA=OQ-AQ=4-\frac{7}{2}=\frac{1}{2}$$

In ΔOAU, $UA=\sqrt{OV^2-OA^2}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}=R$

In ΔUVW, is circumcenter

$US=R\cos 30^\circ \Rightarrow UV=2R\cos 30^\circ=\frac{3}{2}$

$$\therefore \text{Ar } \Delta UVW=\frac{\sqrt{3}}{4}\left(\frac{3}{2}\right)^2=\frac{9\sqrt{3}}{16}$$

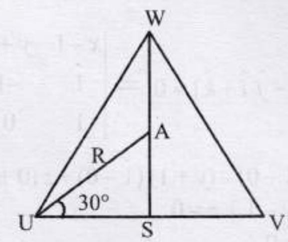
Volume of tetrahedron with coteminous edges

$\vec{u}, \vec{v}, \vec{w}=\frac{1}{3}$ (Ar ΔUVW) $\times$ OA

$$=\frac{1}{3}\times\frac{9\sqrt{3}}{16}\times\frac{1}{2}=\frac{3\sqrt{3}}{32}$$

$\therefore$ Volume of parallelopiped:

$$V=6\times\text{volume of tetrahedron}=\frac{6\times3\sqrt{3}}{32}=\frac{9\sqrt{3}}{16}$$



Now, $\frac{80}{\sqrt{3}}V=\frac{80}{\sqrt{3}}\times\frac{9\sqrt{3}}{16}=45$

12. (8) Let coordinates of P are (a, b, c).

So, coordinates of Q are (0, 0, c) and coordinates of R are (a, b, –c).

Given that, PQ is perpendicular to the plane $x+y=3$.

So, PQ is parallel to the normal of given plane

i.e. $(a\hat{i}+b\hat{j})$ is parallel to $(\hat{i}+\hat{j})$ on comparing

$\Rightarrow \quad a=b$

As mid-point of PQ lies in the plane $x+y=3$, so

$$\frac{a}{2}+\frac{b}{2}=3$$

$\Rightarrow \quad a+b=6 \Rightarrow a=3=b$

Therefore, distance of P from the x-axis

$$=\sqrt{b^2+c^2}=5 \quad \text{(given)}$$

$\Rightarrow \quad b^2+c^2=25$

$\Rightarrow \quad c^2=25-9=16$

$\Rightarrow \quad c=\pm 4$

Hence, $PR=|2c|=8$

13. (6) The equation of plane containing the given lines:

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}=0 \Rightarrow x-2y+z=0$$

$\therefore$ Distance between $x-2y+z=0$ and $Ax-2y+z=d$

= Perpendicular distance between parallel planes $(\because A=1)$

$$=\frac{|d|}{\sqrt{6}}=\sqrt{6} \Rightarrow |d|=6.$$

14. (0.75)

15. Equation of plane containing vectors $\hat{i}$ and $\hat{i}+\hat{j}$ is

$$\left[\vec{r}-\hat{i} \;\; \hat{i} \;\; \hat{i}+\hat{j}\right]=0 \Rightarrow \begin{vmatrix} x-1 & y & z \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix}=0$$

$$\Rightarrow z=0 \quad ...(i)$$

Similarly, equation of plane containing vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ is

$$[\hat{r}-(\hat{i}-\hat{j})\ \hat{i}-\hat{j}\ \hat{i}+\hat{k}]=0 \Rightarrow \begin{vmatrix} x-1 & y+1 & z \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix}=0$$

$\Rightarrow (x-1)(-1-0)-(y+1)(1-0)+z(0+1)=0$

$\Rightarrow -x+1-y-1+z=0$

$\Rightarrow x-y+z=0$ ....(ii)

Let $\vec{a}=a\hat{i}+b\hat{j}+c\hat{k}$

Since $\vec{a}$ is parallel to (i) and (ii)

$\therefore c=0$ and $a+b-c=0 \Rightarrow a=-b$

$\therefore$ a vector in direction of $\vec{a}$ is $\hat{i}-\hat{j}$

Let θ is the angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ then

$$\cos\theta=\pm\frac{1.1+(-1)(-2)}{\sqrt{1+1}\sqrt{1+4+4}}=\pm\frac{3}{\sqrt{2}.3}$$

$$\Rightarrow \cos\theta=\pm\frac{1}{\sqrt{2}}$$

$\Rightarrow \theta=\pi/4$ or $3\pi/4$

16. Unit vector perpendicular to plane, $\hat{n}=\pm\frac{\overline{PQ}\times\overline{PR}}{|\overline{PQ}\times\overline{PR}|}$

$\overline{PQ}=\hat{i}+\hat{j}-3\hat{k}$; $\overline{PR}=-\hat{i}+3\hat{j}-\hat{k}$

$$\therefore \overline{PQ}\times\overline{PR}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$$=(-1+9)\hat{i}-(-1-3)\hat{j}+(3+1)\hat{k}=8\hat{i}+4\hat{j}+4\hat{k}$$

$$|\overline{PQ}\times\overline{PR}|=\sqrt{64+16+16}=\sqrt{96}=4\sqrt{6}$$

$$\hat{n}=\pm\left(\frac{8\hat{i}+4\hat{j}+4\hat{k}}{4\sqrt{6}}\right)=\pm\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

17. **(a, b, c)** Let X = (x, y) S: $\{((x-1)^2+(y-2)^2+(z-3)^2)-((x-4)^2+(y-2)^2+(z-7)^2)=50\}$

$\Rightarrow$ S : $\{6x+8z-105=0\}$

Similarly T = $\{6x+8z-5=0\}$

Both S and T represents the equation of plane and parallel to each other.

Other Distance between plane = $\left|\frac{105-5}{\sqrt{36+64}}\right|=10$ unit

So. S will contain a triangle of area 1. So (a) is correct. Hence (b) and (c) are correct but (d) is incorrect.

18. **(a, c)** Equation of line parallel to $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$ through P(1, 3, 2) is $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-2}{1}=\lambda$ (let)

Now, putting any point $(\lambda+1, 2\lambda+3, \lambda+2)$ in plane L_1,

$\lambda+1-2\lambda-3+3(\lambda+2)=6$

$\Rightarrow \lambda=1$

So, point Q (2, 5, 3)

Equation of line through Q (2, 5, 3) perpendicular to L_1 is $\frac{x-2}{1}=\frac{y-5}{-1}=\frac{z-3}{3}=\mu$ (Let)

Putting any point $(\mu+2, -\mu+5, 3\mu+3)$ in plane L_2

$\Rightarrow \mu=-1$

So, point R (1, 6, 0)

(a) PQ = $\sqrt{1+4+1}=\sqrt{6}$

(b) R (1, 6, 0)

(c) Centroid $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$

(d) PQ + QR + PR = $\sqrt{6}+\sqrt{11}+\sqrt{13}$

19. **(a, b)** We have planes P_1 and P_2 given as

$P_1: 10x+15y+12z-60=0$ and

$P_2: -2x+5y+4z-20=0$

Thus, equation of pair of planes is

$S: (10x+15y+12z-60)(-2x+5y+4z-20)=0$

Now we will obtain a general point of each line and we will solve it with *S*. If we get more than one value of variable λ, then the line can be the edge of given tetrahedron.

(a) From option we have $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}$

Let $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}=\lambda$

So, point is $(1, 1, 5\lambda+1)$

So, $(60\lambda-23)(20\lambda-17)=0$

$\lambda=\frac{23}{60}$ and $\frac{17}{20}$

So, it can be the edge of tetrahedron.

(b) Similarly for option (b)

point is $(-5\lambda+6, 2\lambda, 3\lambda)$

So, $(16\lambda)(32\lambda-32)=0$

$\Rightarrow \lambda=0$ and 1

So, it can be the edge of tetrahedron.

(c) Similarly for option (c)

Point is $(-2\lambda, 5\lambda+4, 4\lambda)$

So, $(103\lambda)(45\lambda)=0$

$\lambda=0$ only

So, it cannot be the edge of tetrahedron.

(d) Similarly for option (d)
Point is $(\lambda, -2\lambda+4, 3\lambda)$

$\Rightarrow \quad (16\lambda)(-2\lambda)=0$

$\Rightarrow \quad \lambda = 0$ only

Hence, it cannot be the edge of tetrahedron.

20. **(a, b, c)** We are given that equation of plane is

$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$

This can be written as

$\vec{r} = \hat{k} + t\left(-\hat{i}+\hat{j}\right) + p\left(-\hat{i}+\hat{k}\right)$

Now, equation of plane in standard form is

$\left[\vec{r} - \hat{k} - \hat{i} + \hat{j} - \hat{i} + \hat{k}\right] = 0$

$\therefore \quad x + y + z = 1 \quad$ ...(i)

Coordinate of $Q = (10, 15, 20)$

Coordinate of $S = (\alpha, \beta, \gamma)$

$$\therefore \quad \frac{\alpha-10}{1} = \frac{\beta-15}{1} = \frac{\gamma-20}{1} = \frac{-2(10+15+20-1)}{3}$$

$$\left[\because \text{ point of reflection is given as } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}\right]$$

$$\therefore \quad \alpha - 10 = \beta - 15 = \gamma - 20 = -\frac{88}{3}$$

$$\therefore \quad \alpha = -\frac{58}{3}, \beta = -\frac{43}{3}, \gamma = -\frac{28}{3}$$

$\therefore \quad 3(\alpha+\beta) = -101,\ 3(\beta+\gamma) = -71$

$3(\gamma+\alpha) = -86$ and $3(\alpha+\beta+\gamma) = -129$

21. **(a, b)** The point of intersection of L_1 and L_2 is (1, 0, 1)

$\because$ Line L passes through the point of intersection (1, 0, 1) of L_1 and L_2

$$\therefore \frac{1-\alpha}{\ell} = -\frac{1}{m} = \frac{1-\gamma}{-2} \quad \text{....(i)}$$

$\because$ Line L_1 bisects the acute angle between the lines L_1 and L_2, then

$$\vec{r} = \hat{i} + \hat{k} + \lambda\left(\frac{\hat{i}-\hat{j}+3\hat{k}-3\hat{i}-\hat{j}+\hat{k}}{\sqrt{11}}\right)$$

$$\Rightarrow \vec{r} = \hat{i} + \hat{k} + t\left(\hat{i}+\hat{j}-2\hat{k}\right)$$

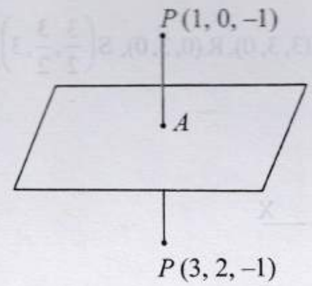
$$\Rightarrow \frac{\ell}{1} = \frac{m}{1} = \frac{-2}{-2} \Rightarrow \ell = m = 1$$

From (i), $\frac{1-\alpha}{1} = -1 \Rightarrow \alpha = 2$

and $\frac{1-\gamma}{-2} = -1 \Rightarrow \gamma = -1$

$\therefore \alpha - \gamma = 2-(-1) = 3$ and $l + m = 1 + 1 = 2$

22. **(a, b, c)**



Mid-point of PQ = A (2, 1, –1)

D.r's of PQ = 2, 2, 0

Since PQ perpendicular to plane and mid-point lies on plane

$\therefore$ Equation of plane :

$2(x-2) + 2(y-1) + 0(z+1) = 0$

$\Rightarrow x - 2 + y - 1 = 0$

$\Rightarrow x + y = 3$ comparing with $\alpha x + \beta y + \gamma z = \delta$,

we get $\alpha = 1, \beta = 1, \gamma = 0$ and $\delta = 3$.

$\therefore$ option (a), (b), (c) are true.

23. **(c, d)**

(a) Direction vector of line of intersection of two planes will be given by $\vec{n}_1 \times \vec{n}_2$.

$$\therefore \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} + 3\hat{k}$$

$\therefore$ dr's of line of intersection of P_1 and P_2 are 1, – 1, 1

$\therefore$ (a) is not correct.

(b) The standard form of given line as

$$\frac{x-\frac{4}{3}}{3} = \frac{y-\frac{1}{3}}{-3} = \frac{z}{3}$$

$\because 1 \times 3 + (-1)(-3) + 1(3) = 9 \neq 0$

$\therefore$ This line is not perpendicular to line of intersection

$\therefore$ (b) is not correct.

(c) Let θ be the angle between P_1 and P_2 then

$$\cos\theta = \left|\frac{2\times1+1\times2+(-1)\times1}{\sqrt{6}\sqrt{6}}\right| = \frac{3}{6} = \frac{1}{2}$$

$\therefore \theta = 60°$

Hence (c) is correct.

(d) Equation of plane P_3 :

$1(x-4) - 1(y-2) + 1(z+2) = 0$

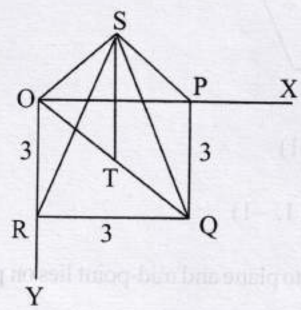
$\Rightarrow x - y + z = 0$

Distance of (2, 1, 1) from $P_3 = \frac{2-1+1}{\sqrt{1+1+1}} = \frac{2}{\sqrt{3}}$

$\therefore$ (d) is correct.

24. **(b, c, d)** According to question the coordinates of vertices of pyramid OPQRS will be

O(0, 0, 0), P (3, 0, 0), Q (3, 3, 0), R (0, 3, 0), $S\left(\frac{3}{2},\frac{3}{2},3\right)$



dr's of OQ = 1, 1, 0
dr's of OS = 1, 1, 2
$\therefore$ acute angle between OQ and OS

$$=\cos^{-1}\left(\frac{2}{\sqrt{2}\times\sqrt{6}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\neq\frac{\pi}{3}$$

$\therefore$ (a) is not correct

$$\text{Eqn of plane OQS}=\begin{vmatrix}x & y & z\\ 1 & 1 & 0\\ 1 & 1 & 2\end{vmatrix}=0$$

$\Rightarrow 2x-2y=0$ or $x-y=0$
$\therefore$ (b) is correct.
length of perpendicular from P (3, 0, 0) to plane $x-y=0$ is =

$$\left|\frac{3-0}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$$

$\therefore$ (c) is correct.

$$\text{Eqn of RS}: \frac{x}{\frac{3}{2}}=\frac{y-3}{\frac{-3}{2}}=\frac{z}{3} \text{ or } \frac{x}{1}=\frac{y-3}{-1}=\frac{z}{2}=\lambda$$

$\therefore$ Any point ON RS is N $(\lambda, -\lambda+3, 2\lambda)$
Since ON is perpendicular to RS,

$$\because \ ON\perp RS \Rightarrow 1\times\lambda-1(-\lambda+3)+2\times 2\lambda=0$$

$$\Rightarrow \lambda=\frac{1}{2}\Rightarrow N\left(\frac{1}{2},\frac{5}{2},1\right)$$

$$\therefore ON=\sqrt{\frac{1}{4}+\frac{25}{4}+1}=\sqrt{\frac{15}{2}}$$

$\therefore$ (d) is correct

25. **(a, b)** $\because$ All the points on L are at a constant distance from P_1 and P_2 that means L is parallel to both P_1 and P_2

$$\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ 2 & -1 & 1\end{vmatrix}=\hat{i}-3\hat{j}-5\hat{k}$$

$$\therefore \quad L:\frac{x}{1}=\frac{y}{-3}=\frac{z}{-5}=\lambda\,(\text{say})$$

$\therefore$ Any point on line L is $(\lambda, -3\lambda, -5\lambda)$
Equation of line perpendicular to P_1 drawn from any point on L is

$$\frac{x-\lambda}{1}=\frac{y+3\lambda}{2}=\frac{z+5\lambda}{-1}=\mu$$

$\therefore \quad M(\mu+\lambda, 2\mu-3\lambda, -\mu-5\lambda)$
But M lies on P_1 so, it satisfy the eqn. of P_1.

$$\therefore \quad \mu+\lambda+4\mu-6\lambda+\mu+5\lambda+1=0\Rightarrow \ \mu=\frac{-1}{6}$$

$$\therefore \quad M\left(\lambda-\frac{1}{6},-3\lambda\frac{-1}{3},-5\lambda+\frac{1}{6}\right)$$

For locus of M,

$$x=\lambda-\frac{1}{6}, y=-3\lambda-\frac{1}{3}, z=5\lambda+\frac{1}{6}$$

$$\Rightarrow \quad \frac{x+1/6}{1}=\frac{y+1/3}{-3}=\frac{z-1/6}{-5}=\lambda$$

On checking the given point, we find $\left(0,\frac{-5}{6},\frac{-2}{3}\right)$ and $\left(\frac{-1}{6},\frac{-1}{3},\frac{1}{6}\right)$ satisfy the above eqn.

26. **(b, d)** $P_3:(x+z-1)+\lambda y=0\Rightarrow x+\lambda y+z-1=0$
Distance of point (0, 1, 0) from P_3 :

$$\left|\frac{\lambda-1}{\sqrt{2+\lambda^2}}\right|=1\Rightarrow\lambda^2-2\lambda+1=\lambda^2+2\Rightarrow\lambda=\frac{-1}{2}$$

Distance of point (α, β, γ) from P_3 :

$$\left|\frac{\alpha+\lambda\beta+\gamma-1}{\sqrt{2+\lambda^2}}\right|=2\Rightarrow\frac{\alpha-\frac{1}{2}\beta+\gamma-1}{\frac{3}{2}}=\pm 2$$

$$\Rightarrow \quad \alpha-\frac{1}{2}\beta+\gamma-1=\pm 3 \ \Rightarrow 2\alpha-\beta+2\gamma-2=\pm 6$$

$$\Rightarrow \quad 2\alpha-\beta+2\gamma-8=0 \text{ or } 2\alpha-\beta+2\gamma+4=0$$

27. **(a, d)** Given that L_1 and L_2 are coplanar, therefore

$$\begin{vmatrix}5-\alpha & 0 & 0\\ 0 & 3-\alpha & -2\\ 0 & -1 & 2-\alpha\end{vmatrix}=0$$

$\Rightarrow (5-\alpha)[6-5\alpha+\alpha^2-2]=0$

$\Rightarrow (5-\alpha)(\alpha-1)(\alpha-4)=0 \Rightarrow \alpha=1,4,5.$

28. **(b, c)** Given that lines are coplanar.

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \Rightarrow k=\pm 2$$

For $k=2$, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0 \Rightarrow y-z+1=0$$

For $k=-2$, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0 \Rightarrow y+z+1=0$$

29. **(b, d)** Normal vector of plane P_1 is

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 3 \\ 0 & 4 & -3 \end{vmatrix} = -18\hat{i}$$

Normal vector of plane P_2 is

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 3 & 3 & 0 \end{vmatrix} = 3\hat{i}-3\hat{j}-3\hat{k}$$

$\therefore$ $\vec{A}$ is parallel to $\pm(\vec{n}_1 \times \vec{n}_2) = \pm(-54\hat{j}+54\hat{k})$

Now, angle between $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is given by

$$\cos\theta = \pm\frac{(-54\hat{j}+54\hat{k}).(2\hat{i}+\hat{j}-2\hat{k})}{54\sqrt{2}.3} = \pm\frac{1}{\sqrt{2}}$$

$\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

30. **(b)** For largest possible distance between plane H_o and l_2, the line l_2 must be parallel to plane H_o.

$\therefore$ H_o will be the plane containing the line l_1 and parallel to l_2

$$\text{Normal vector } \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}-\hat{k}$$

$\therefore H_o : x-z = c/(0, 0, 0) \Rightarrow c=0$

$\therefore H_o : x-z=0$

(P) Distance of point (0, 1, −1) from H_o.

$$d(H_o) = \left|\frac{0-(1)}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

(Q) The distance of the point (0, 1, 2) from $H_o = \left|\frac{0-2}{\sqrt{2}}\right| = \sqrt{2}$

(R) The distance of origin from $H_o = \left|\frac{0}{\sqrt{2}}\right| = 0$

(S) Point of intersection of planes $y=z$, $x=1$ and H_o is (1, 1, 1).

Distance $=\sqrt{1+1+1}=\sqrt{3}.$

31. **(a)** Let any point on L_1 is $(2\lambda+1, -\lambda, \lambda-3)$

and that on L_2 is $(\mu+4, \mu-3, 2\mu-3)$

For point of intersection of L_1 and L_2

$2\lambda+1=\mu+4, -\lambda=\mu-3, \lambda-3=2\mu-3$

$\Rightarrow \lambda=2, \mu=1$

$\therefore$ Intersection point of L_1 and L_2 is (5, −2, − 1)

Equation of plane passing through, (5, −2, −1) and perpendicular to P_1 & P_2 is given by

$$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$$

$\Rightarrow x-3y-2z=13$

$\therefore$ a = 1, b = −3, c = −2, d = 13

or (P) → (3) (Q) → (2) (R) → (4) (S) → (1)

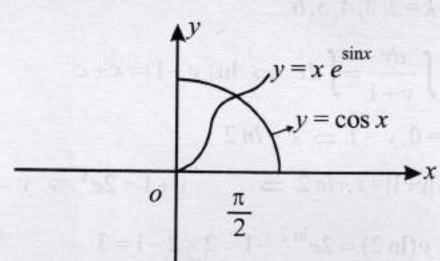
32. **A → p; B → q,s; C → q,r,s,t; D → r**

(A) Let us consider two functions

$y = xe^{\sin x}$ and $y=\cos x$

The range of $y=xe^{\sin x}$ is $\left(0, \frac{\pi e}{2}\right)$ and

$\frac{dy}{dx} = e^{\sin x} + xe^{\sin x}\cos x \geq 0$, for $x \leftarrow \left(0, \frac{\pi}{2}\right)$, so, it is an increasing function on $\left(0, \frac{\pi}{2}\right)$. Their graph are as shown in the figure below :



Clearly the two curves meet only at one point, therefore the given equation has only one solution in $\left(0, \frac{\pi}{2}\right)$.

(B) Since given planes intersect in a straight-line

$$\therefore \begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad k(k-4)-4(4-4)+1(8-2k)=0$

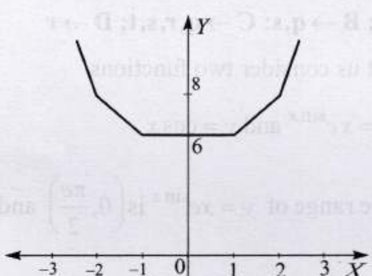
$\Rightarrow \quad k^2-6k+8=0 \Rightarrow (k-2)(k-4)=0$

$\Rightarrow \quad k=2 \text{ or } 4$

(C) We have $f(x)=|x-1|+|x-2|+|x+1|+|x+2|$

$$=\begin{cases} -4x & , \ x \le -2 \\ -2x+4\,, & -2 < x \le -1 \\ 6\,, & -1 < x \le 1 \\ 2x+4\,, & 1 < x \le 2 \\ 4x\,, & x \ge 2 \end{cases} \quad \left[\because |x-1| \begin{cases} x-1, \text{is } x \ge 1 \\ -(x-1) \text{ is } x < 1 \end{cases}\right]$$

The graph of the above function is as given below



Clearly, from graph, $f(x) \ge 6$

$\Rightarrow \quad 4k \ge 6 \ \Rightarrow \ k \ge \dfrac{3}{2}$

$\therefore \quad k=2,3,4,5,6,.....$

(D) $\displaystyle\int \frac{dy}{y+1} = \int dx \ \Rightarrow \ \ln|y+1| = x+c$

At $x=0, y=1 \Rightarrow c=ln\,2$

$\therefore \ln|y+1| = x + ln\,2 \ \Rightarrow \qquad y+1=2e^x \Rightarrow y=2e^x-1$

$\therefore \quad y(\ln 2)=2e^{\ln 2}-1=2\times 2-1=3$

33. **(A) → r ; (B) → q; (C) → p ; (D) → s**

The determinant of the coefficient matrix of given equation, as

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

(A) When $a+b+c \ne 0$ and

$a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \quad (a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow \quad a=b=c \quad (\text{but} \ne 0 \text{ as } a+b+c \ne 0)$

This equation represent identical planes.

(B) When $a+b+c=0$ and

$a^2+b^2+c^2-ab-bc-ca \ne 0$

$\Rightarrow \quad \Delta=0$ and *a, b, c* are not all equal.

$\therefore$ All equations are not identical but have infinite many solutions.

$\therefore \quad ax+by=(a+b)z$ ... (i) (using $a+b+c=0$)

and $bx+cy=(b+c)z$ ...(ii)

On Solving eqn. (i) and (ii) we, get

$\Rightarrow \quad (b^2-ac)y=(b^2-ac)z \ \Rightarrow \ y=z$

$\Rightarrow \quad ax+by+cy=0 \ \Rightarrow \ ax=ay \ \Rightarrow \ x=y$

$\Rightarrow \quad x=y=z$

$\therefore$ The equations represent the line $x=y=z$

(C) When $a+b+c \ne 0$ and

$a^2+b^2+c^2-ab-bc-ca \ne 0$

$\Rightarrow \quad \Delta \ne 0 \ \Rightarrow$ Equations have only trivial solution

i.e., $x=y=z=0$

$\therefore$ the equations represents the three planes meeting at a single point namely origin.

(D) When $a+b+c=0$ and

$a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \quad a=b=c$ and $\Delta=0 \Rightarrow a=b=c=0$

$\Rightarrow$ All equations are satisfied by all *x, y,* and *z.*

$\Rightarrow$ The equations represent the whole of the three dimensional space (all points in 3–*D*)

34. **(A) → (s); (B) → (p); (C) → (q), (r); (D) → (s)**

(A) $\quad x+y=|a|$

$\quad ax-y=1$

$\quad \overline{(1+a)x=1+|a|}$

$\Rightarrow x=\dfrac{1+|a|}{a+1} \Rightarrow y=\dfrac{a|a|-1}{a+1}$

$\because$ Rays intersect each other in I quad i.e. $x>0.\ y\ge 0$

$\Rightarrow a+1>0$ and $a|a|-1>0 \Rightarrow a>1$

$\therefore a_0=1$ (A) → (s)

(B) Given that (α, β, γ) lies on the plane $x+y+z=2$

$\Rightarrow \alpha+\beta+\gamma=2$

Also $\hat{k} \times (\hat{k} \times \vec{a}) = (\hat{k}.\hat{a})\hat{k} - (\hat{k}.\hat{k})\vec{a}$

$\Rightarrow \gamma\hat{k} - \alpha\hat{i} - \beta\hat{j} - \gamma\hat{k} = 0 \Rightarrow \alpha\hat{i} + \beta\hat{j} = 0$

$\Rightarrow \alpha = 0 = \beta \Rightarrow \gamma = 2 \qquad (\because \alpha + \beta + \gamma = 2)$

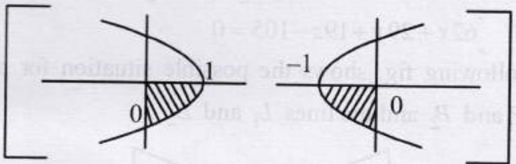
(B) $\rightarrow$ (p)

(C) $\left|\int_0^1 (1-y^2)dy\right| + \left|\int_0^1 (y^2-1)dy\right|$

$= 2\left|\int_0^1 (1-y^2)dy\right| = \frac{4}{3}$

$\because y = \sqrt{1-x}, \Rightarrow y^2 = -(x-1)$ and $y = \sqrt{1+x}$

$\Rightarrow y^2 = (x+1)$ It is clear from above figure that



$\left|\int_0^1 \sqrt{1-x}\,dx\right| + \left|\int_{-1}^0 \sqrt{1+x}\,dx\right| = 2\int_0^1 \sqrt{1-x}\,dx$

$= 2\int_0^1 \sqrt{x}\,dx \quad \left[\text{Using} \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \left[2.\frac{2}{3}x^{3/2}\right]_0^1 = \frac{4}{3}$, (C) $\rightarrow$ (r) and (q)

(D) Given that $\sin A \sin B \sin C + \cos A \cos B = 1$

We know that $\sin A \sin B \sin C + \cos A \cos B \le \sin A \sin B + \cos A \cos B = \cos(A-B)$

$\Rightarrow \cos(A-B) \ge 1 \Rightarrow \cos(A-B) = 1$

$\Rightarrow A - B = 0 \Rightarrow A = B$

$\therefore$ Given relation becomes $\sin^2 A \sin C + \cos^2 A = 1$

$\Rightarrow \sin C = 1$,

(D) $\rightarrow$ (s)

35. **(b)** Vector in the direction of $L_1 = \vec{b_1} = 3\hat{i} + \hat{j} + 2\hat{k}$

Vector in the direction of $L_2 = \vec{b_2} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Vector perpendicular to both L_1 and L_2

$$= \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$$

$\therefore$ Required unit vector

$$= \hat{b} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{1+49+25}} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$$

36. **(d)** The shortest distance between L_1 and L_2 is

$$= \frac{(\vec{a}_2 - \vec{a}_1).\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} = (\vec{a}_2 - \vec{a}_1).\hat{b}$$

Since, $a_1 = -\hat{i} - 2\hat{j} - \hat{k} \qquad a_2 = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 4\hat{k} \qquad \therefore (\vec{a}_2 - \vec{a}_1).\hat{b}$

$$\therefore (3\hat{i} + 4\hat{k}).\left(\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}\right) = \frac{-3+20}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$$

37. **(c)** The plane passing through $(-1, -2, -1)$ and having normal along $\vec{b}$ is

$-1(x+1) - 7(y+2) + 5(z+1) = 0$

$\Rightarrow x + 7y - 5z + 10 = 0$

$\therefore$ Distance of point $(1, 1, 1)$ from the above plane is

$$= \frac{1 + 7 \times 1 - 5 \times 1 + 10}{\sqrt{1+49+25}} = \frac{13}{\sqrt{75}}$$

38. **(d)** The given planes are

$P_1 : x - y + z = 1$ ...(1)

$P_2 : x + y - z = -1$ ...(2)

$P_3 : x - 3y + 3z = 2$ ...(3)

Since, line L_1 is intersection of planes P_2 and P_3.

$\therefore L_1$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = -4\hat{j} - 4\hat{k}$$

Line L_2 is intersection of P_3 and P_1

$\therefore L_2$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -3 & 3 \end{vmatrix} = -2\hat{j} - 2\hat{k}$$

And line L_3 is intersection of P_1 and P_2

$\therefore L_3$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\hat{j} + 2\hat{k}$$

Clearly lines L_1, L_2 and L_3 are parallel to each other.

$\therefore$ Statement–1 is False

Also family of planes passing through the intersection of P_1 and P_2 is $P_1 + \lambda P_2 = 0$.

$\Rightarrow x(1+\lambda) + y(\lambda - 1) + z(1 - \lambda) + (\lambda - 1) = 0$

The three planes have a common point

$$\frac{1+\lambda}{1}=\frac{\lambda-1}{-3}=\frac{1-\lambda}{3}=\frac{1-\lambda}{2} \quad ...(i)$$

Taking $\frac{1+\lambda}{1}=\frac{1-\lambda}{2}$, we get $\lambda=-\frac{1}{3}$ and taking

$\frac{1+\lambda}{1}=\frac{1-\lambda}{3}$, we get $\lambda=-\frac{2}{3}$.

$\therefore$ There is no value of λ which satisfies eq (i).

$\therefore$ The three planes do not have a common point.

$\Rightarrow$ Statement 2 is true.

$\therefore$ (d) is the correct option.

39. **(d)** The line of intersection of given plane is

$3x-6y-2z-15=0=2x+y-2z-5$

For $z=0$, we get $x=3$ and $y=-1$

$\therefore$ Line passes through $(3,-1,0)$.

Direction vector of line is

$$\vec{b}=\vec{x}_1\times\vec{x}_2$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -6 & -2\\ 2 & 1 & -2\end{vmatrix}$$

$$=14\hat{i}+2\hat{j}+15\hat{k}$$

$\therefore$ Eqn. of line is $\frac{x-3}{14}=\frac{y+1}{2}=\frac{z}{15}=t$

whose parametric form is

$x=3+14t, y=2t-1, z=15t$

$\therefore$ Statement-I is false

$\therefore$ Statement 2 is true.

40. Equation of plane containing line of intersection of two given planes is given by

$$(2x-y+z-3)+\lambda(3x+y+z-5)=0$$

$$\Rightarrow (3\lambda+2)x+(\lambda-1)y+(\lambda+1)z+(-5\lambda-3)=0$$

since distance of this plane from the pt. $(2, 1, -1)$ is $\frac{1}{\sqrt{6}}$

$$\therefore \left|\frac{(3\lambda+2)2+(\lambda-1)1+(\lambda+1)(-1)+(-5\lambda-3)}{\sqrt{(3\lambda+2)^2+(\lambda-1)^2+(\lambda+1)^2}}\right|=\frac{1}{\sqrt{6}}$$

$$\Rightarrow \left|\frac{\lambda-1}{\sqrt{11\lambda^2+12\lambda+6}}\right|=\frac{1}{\sqrt{6}}$$

Squaring both sides, we get

$$\frac{(\lambda-1)^2}{11\lambda^2+12\lambda+6}=\frac{1}{6}$$

$$\Rightarrow 6\lambda^2-12\lambda+6-11\lambda^2-12\lambda-6=0$$

$$\Rightarrow 5\lambda^2+24\lambda=0 \Rightarrow \lambda(5\lambda+24)=0$$

$$\Rightarrow \lambda=0 \text{ or } -24/5$$

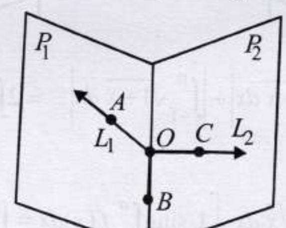
$\therefore$ The required equations of planes are

$$2x-y+z-3=0$$

$$\text{or } \left[3\left(\frac{-24}{5}\right)+2\right]x+\left[-\frac{24}{5}-1\right]y+\left[-\frac{24}{5}+1\right]z-5\left(\frac{-24}{5}\right)-3=0$$

$$\text{or } 62x+29y+19z-105=0$$

41. Following fig. shows the possible situation for planes P_1 and P_2 and the lines L_1 and L_2



A corresponds to one of A', B', C' and B corresponds to one of the remaining of A', B', C' and C corresponds to third of A', B', C'.

Hence six such permutations are possible

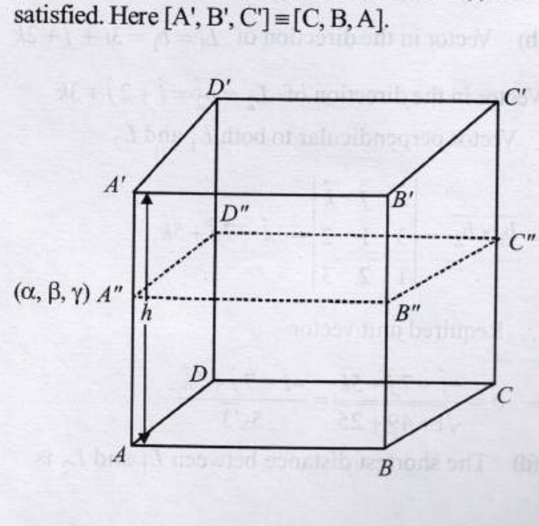
e.g., One of the permutations may A = A', B = B', C = C'

From the given conditions : A lies on L_1, B lies on the line of intersection of P_1 and P_2 and 'C' lies on the line L_2 on the plane P_2.

Now, A' lies on L_2 = C, B' lies on the line of intersection of P_1 and P_2 = B and C' lie on L_1 on plane P_1 = A.

Hence there exist a particular set [A', B', C'] which is the permutation of [A, B, C] such that both (i) and (ii) is satisfied. Here [A', B', C'] ≡ [C, B, A].

42.



Let

Let equation of plane $ABCD$ be

$ax+by+cz+d=0$, h be the height of original parallelepiped S. and $A''(\alpha,\beta,\gamma)$

Then height of new parallelepipe dT is the length of perpendicular from A'' to $ABCD$

i.e. $\dfrac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}$

$$V_T=\frac{90}{100}V_s$$

$$\therefore\quad (ar\ ABCD)\times\frac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}$$

$$=(ar\,ABCD)\times h\times 0.9$$

But given that,

$$\frac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}=0.9h$$

$$\Rightarrow\quad a\alpha+b\beta+c\gamma+(d-0.9h\sqrt{a^2+b^2+c^2})=0$$

$\therefore$ Locus of $A''(\alpha,\beta,\gamma)$ is

$$ax+by+cz+(d-0.9h\sqrt{a^2+b^2+c^2})=0$$

which is a plane parallel to $ABCD$. Hence proved.

43. Equation of plane through (1, 1, 1) is

$$\begin{vmatrix}x-1 & y-1 & z-1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{vmatrix}=0$$

$$\Rightarrow\quad (x-1)(0-1)-(y-1)(0+1)+(z-1)(-1-0)=0$$

$$\Rightarrow\quad -1(x-1)-1(y-1)-1(z-1)=0\ \Rightarrow\ x+y+z=3$$

$$\Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{3}=1 \qquad ...(1)$$

$\therefore$ plane intersect the axes at

$A(3,0,0), B(0,3,0)$, and $C(0,0,3)$

$\therefore$ Vol.of tetrahedron $OABC$

$$=\frac{1}{6}\times\text{Area of base}\times\text{altitude}$$

$$=\frac{1}{6}\times\text{Ar}(\Delta ABC)\times\text{length of }\perp^{\text{lar}}\ (0,0,0)\text{ to plane}(1)$$

$$=\frac{1}{6}\times\frac{1}{2}\left[\frac{\sqrt{3}}{4}\times|\overrightarrow{AB}|^2\right]\times\left[\left|\frac{-3}{\sqrt{1+1+1}}\right|\right]$$

($\therefore\ \Delta ABC$ is an equilateral triangle)

$$=\frac{1}{12}\times\frac{\sqrt{3}}{4}\times(3\sqrt{2})^2\times\sqrt{3}\ =\frac{3\times 18}{48}=\frac{9}{2}\text{ cubic units.}$$

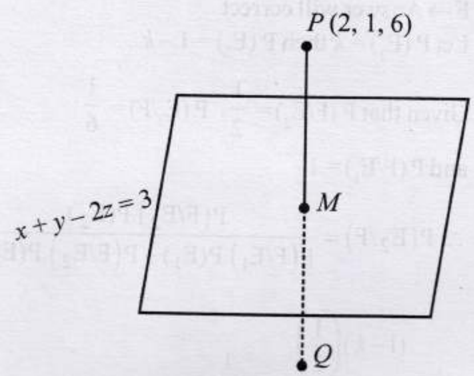
44. (i) Equation of plane passing through (2, 1, 0), (5, 0, 1) and (4, 1, 1) is

$$\begin{vmatrix}x-2 & y-1 & z-0\\ 5-2 & 0-1 & 1-0\\ 4-2 & 1-1 & 1-0\end{vmatrix}=0\Rightarrow\begin{vmatrix}x-2 & y-1 & z\\ 3 & -1 & 1\\ 2 & 0 & 1\end{vmatrix}=0$$

$$\Rightarrow\quad (x-2)(-1-0)-(y-1)(3-2)+z(0-(-2))=0$$

$$\Rightarrow\quad -x+2-y+1+2z=0\quad\Rightarrow\quad x+y-2z=3$$

(ii)



Eq^n of PQ passing through $P(2, 1, 6)$ and $\perp$ to plane $x+y-2z=3$, is given by

$$\frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\lambda$$

$\therefore\quad Q(\lambda+2,\lambda+1,-2\lambda+6)$

$\therefore\quad$ Mid. pt. of PQ

i.e. $M\left(\dfrac{2+\lambda+2}{2},\dfrac{1+\lambda+1}{2},\dfrac{6-2\lambda+6}{2}\right)$

$$=\left(\frac{\lambda+4}{2},\frac{\lambda+2}{2},\frac{12-2\lambda}{2}\right)$$

But M lies on plane $x+y-2z=3$

$$\therefore\quad \frac{\lambda+4}{2}+\frac{\lambda+2}{2}-(12-2\lambda)=3$$

$$\Rightarrow\lambda+4+\lambda+2-24+4\lambda=6\Rightarrow 6\lambda=24\ \Rightarrow\lambda=4$$

$$\therefore\quad Q(4+2,4+1,-8+6)=(6,5,-2)$$