Sample Question Paper - 20
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

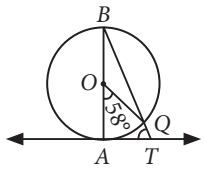
Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. In the given figure, AB is a diameter of the circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, then find $\angle ATQ$.



2. If the numbers $2n - 1$, $3n + 2$ and $6n - 1$ are in A.P., find n and hence find the numbers.

3. Find the nature of the roots of the quadratic equation : $4x^2 - 2\sqrt{3}x + 9 = 0$.

OR

Find the values of k for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has real and equal roots.

4. Find the mean of the following distribution :

x_i	10	30	50	70	89
f_i	7	8	10	15	10

5. 20 years ago, when Meera's parents got married, their average age was 23 years, now the average age of her family consisting of Meera and her parents is 34 years. What is Meera's present age?

6. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone and that of hemisphere is 18 cm and the height of cone is 12 cm. Calculate the surface area of the toy.[Take $\pi = 3.14$]

OR

A wooden toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the volume of cone.

SECTION - B

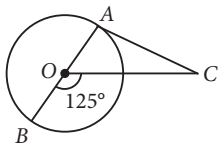
7. If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}(mn + 1)$.

8. From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower.

OR

The angle of elevation of the top of a tower from certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top increased by 15°. Find the height of the tower.

9. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 125°$, then find $\angle ACO$.



10. The first and the last terms of an A.P. are 7 and 49 respectively. If the sum of all its terms is 420, then find its common difference.

SECTION - C

11. Mary's mother organised a new year party at her home. She used the data on number of persons who attended the party at her home based on their age group, which is given in the following frequency distribution :

Age	0-6	6-12	12-18	18-24	24-30
Number of persons	1	4	8	3	4

Find the median of the data.

OR

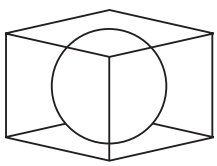
The following table shows the daily pocket allowance given to the children of a multistory building. The mean of the pocket allowance is ₹ 18. Find out the missing frequency.

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	?	5	4

12. Draw a circle of radius 7 cm and draw a tangent to this circle making an angle of 30° with a line passing through the centre.

Case Study - 1

13. A carpenter was carving a sphere of maximum size out of a cuboidal wooden block of dimensions 21 cm by 21 cm by 24 cm.

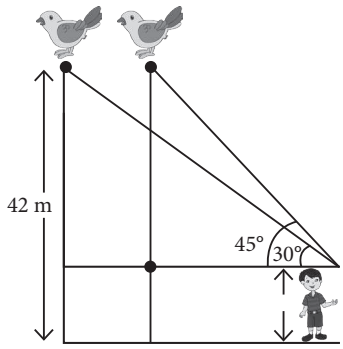


(i) What is the volume of the complete block of wood?

(ii) What is the volume of the wood wasted?

Case Study - 2

14. A boy 2 m tall spots a bird sitting on the top of a pole of height 42 m from the ground. The angle of elevation of the bird from the eye of boy at any instant is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 4 seconds, the angle of elevation of the bird from the same point is 30°. (Take $\sqrt{3} = 1.732$)



(i) Find the distance of first position of the bird from the eyes of the boy.

(ii) How much distance the bird covers in 4 seconds?

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. Since AT is a tangent.

$\therefore\ AT \perp OA \Rightarrow \angle BAT = 90°$ ...(i)

Now, angle subtended by an arc at centre is twice the angle subtended by it at remaining part of the circle.

$\therefore\ \angle AOQ = 2\angle ABQ$

$\Rightarrow\ \angle ABQ = \frac{1}{2} \times 58° = 29°$ ...(ii)

In ΔABT, $\angle ABT + \angle BAT + \angle ATB = 180°$

[By angle sum property]

$\Rightarrow\ \angle ATQ = 180° - (90° + 29°)$ [Using (i) and (ii)]

$\Rightarrow\ \angle ATQ = 61°$

2. Let $2n - 1$, $3n + 2$ and $6n - 1$ are in A.P.

$\therefore\ (3n + 2) - (2n - 1) = (6n - 1) - (3n + 2)$

$\Rightarrow\ n + 3 = 3n - 3 \Rightarrow 6 = 2n \Rightarrow n = 3$

$\therefore$ Numbers are : $2(3) - 1 = 5$,

$3(3) + 2 = 11$ and $6(3) - 1 = 17$.

3. We have, $4x^2 - 2\sqrt{3}\,x + 9 = 0$

Here, $a = 4$, $b = -2\sqrt{3}$ and $c = 9$

$\therefore\ D = b^2 - 4ac = \left(-2\sqrt{3}\right)^2 - 4(4)(9) = 12 - 144 = -132 < 0$

Thus, the given equation has no real roots.

OR

We have, $(k + 4)x^2 + (k + 1)x + 1 = 0$

Here, $a = k + 4$, $b = k + 1$ and $c = 1$.

$\therefore\ D = b^2 - 4ac = (k + 1)^2 - 4(k + 4)(1)$

$= k^2 + 1 + 2k - 4k - 16 = k^2 - 2k - 15$

$= k^2 - 5k + 3k - 15 = k(k - 5) + 3(k - 5) = (k - 5)(k + 3)$

Now, the roots of the given equation are real and equal.

$\therefore\ D = 0 \Rightarrow (k - 5)(k + 3) = 0 \Rightarrow k = 5$ or $k = -3$.

4. Let us construct the following table for the given data.

x_i	f_i	$f_i x_i$
10	7	70
30	8	240
50	10	500
70	15	1050
89	10	890
Total	$\sum f_i = 50$	$\sum f_i x_i = 2750$

$\therefore$ Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{2750}{50} = 55$

5. Average age of parents 20 years ago = 23 years

$\therefore$ Their total ages 20 years ago = $23 \times 2 = 46$ years

Let Meera's present age = x years

Sum of present ages of parents = $46 + 20 \times 2 = 86$ years

Now, average age of family $= \frac{86 + x}{3}$

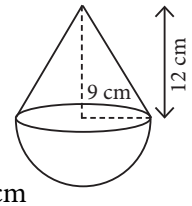
According to question, we have

$34 = \frac{86 + x}{3} \Rightarrow 102 = 86 + x \Rightarrow x = 16$

Hence, present age of Meera is 16 years.

6. Radius of the base of the cone and hemisphere (r)

$= \frac{18}{2} = 9$ cm



Height of cone (h) = 12 cm

Slant height of cone (l)

$= \sqrt{r^2 + h^2} = \sqrt{9^2 + 12^2} = \sqrt{225} = 15$ cm

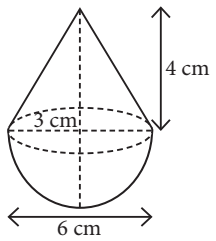
Total surface area of toy = Curved surface area of hemisphere + Curved surface area of cone

$= 2\pi r^2 + \pi rl = \pi r(2r + l) = 3.14 \times 9(2 \times 9 + 15)$

$= 3.14 \times 9 \times 33 = 932.58$ cm^2

OR

Given, diameter of conical part = Diameter of hemispherical part = 6 cm



$\therefore$ Radius of conical part (r) = Radius of hemispherical part (r)

$= \frac{6}{2} = 3$ cm

Height of conical part (h) = 4 cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 4$

$= 37.71$ cm^3

7. Let a be the first term and d be the common difference of given A.P.

Given, $a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ ...(i)

and $a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ ...(ii)

On subtracting (ii) from (i), we get $(m-n)d = \frac{1}{n} - \frac{1}{m}$

$\Rightarrow \quad (m-n)d = \frac{m-n}{mn} \quad \Rightarrow \quad d = \frac{1}{mn}$

On putting $d = \frac{1}{mn}$ in (i), we get $a + (m-1)\frac{1}{mn} = \frac{1}{n}$

$\Rightarrow \quad a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \quad \Rightarrow \quad a = \frac{1}{mn}$

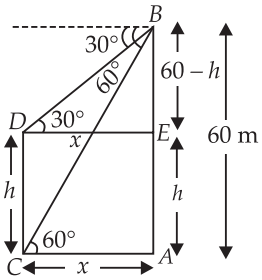
$\therefore$ Sum of mn terms, $S_{mn} = \frac{mn}{2}[2a + (mn-1)d]$

$= \frac{mn}{2}\left[\frac{2}{mn} + (mn-1)\frac{1}{mn}\right] = \frac{1}{2}(mn+1)$

8. Let AB be the building and CD be the tower. Let $CD = h$. It is given that from the top of the building B, the angles of depression of the top D and the bottom C of the tower CD are 30° and 60° respectively.

$\therefore \quad \angle EDB = 30°$ and $\angle ACB = 60°$

Let $AC = DE = x$



In ΔDEB, we have

$\tan 30° = \frac{BE}{DE}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60-h}{x}$

$\Rightarrow \quad x = \sqrt{3}\,(60-h) \qquad$ ...(i)

In ΔCAB, we have

$\tan 60° = \frac{AB}{CA} \Rightarrow \sqrt{3} = \frac{60}{x} \Rightarrow x = 20\sqrt{3}$

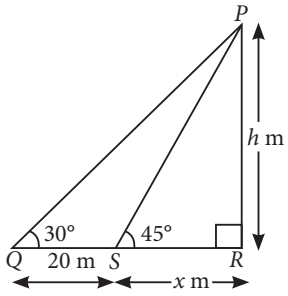
Putting the value of x in (i), we get

$20\sqrt{3} = \sqrt{3}(60-h) \Rightarrow 20 = 60 - h \Rightarrow h = 60 - 20 = 40$

Thus, the height of the tower is 40 metres.

OR

Let the height of the tower PR be h m, the angle of elevation at point Q is 30° *i.e.*, $\angle PQR = 30°$ and S be the position of observer after moving 20 m towards the tower. Let $SR = x$ m



According to the question,

$\angle PSR = \angle PQR + 15° = 30° + 15° \quad \Rightarrow \angle PSR = 45°$

Now in right angle ΔPRS, we have

$\tan 45° = \frac{h}{x} \quad \Rightarrow \quad 1 = \frac{h}{x} \Rightarrow x = h \qquad$ ...(i)

And in right angle ΔPRQ,

$\tan 30° = \frac{PR}{QR} \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20+x} \quad \Rightarrow \quad 20 + x = \sqrt{3}h$

$\Rightarrow \quad 20 + h = \sqrt{3}h \qquad$ [From (i)]

$\Rightarrow \quad \sqrt{3}h - h = 20$

$\Rightarrow \quad h(\sqrt{3}-1) = 20 \quad \Rightarrow \quad h = \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow \quad h = \frac{20(\sqrt{3}+1)}{3-1} = 10(\sqrt{3}+1)$ m

Hence, the required height of the tower is $10(\sqrt{3}+1)$ m.

9. Since the tangent at a point to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad OA \perp AC \quad \Rightarrow \quad \angle CAO = 90°$

Now, in ΔAOC,

$\angle CAO + \angle ACO = \angle BOC$ (By exterior angle property)

$\Rightarrow \quad 90° + \angle ACO = 125°$

$\Rightarrow \quad \angle ACO = 125° - 90° = 35°$

10. Let a and d denotes the first term and common difference respectively of the A.P.

Given, $a = 7$ and $l = 49 = a + (n-1)d$

$\Rightarrow \quad 49 = 7 + (n-1)d \quad \Rightarrow \quad (n-1)d = 42 \qquad$ ...(i)

Now, $S_n = \frac{n}{2}[a+l] \quad \Rightarrow \quad \frac{n}{2}[7+49] = 420$

$\Rightarrow \quad \frac{n}{2}(56) = 420 \Rightarrow n = \frac{420}{28} = 15$

Putting $n = 15$ in (i), we get

$14d = 42 \quad \Rightarrow \quad d = 3$

11. We have the following table :

Age	**Number of persons (f_i)**	**Cumulative frequency (*c.f.*)**
0-6	1	1
6-12	4	5
12-18	8	13
18-24	3	16
24-30	4	20
Total	$\sum f_i = 20$	

Here, $\frac{N}{2} = \frac{20}{2} = 10$. The cumulative frequency just greater than 10 is 13 and its corresponding class is 12-18. So, median class is 12-18.

$\therefore \quad l = 12$, *c.f.* $= 5$, $f = 8$ and $h = 6$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h = 12 + \frac{10-5}{8} \times 6$$

$$= 12 + \frac{15}{4} = 15.75$$

OR

Let the missing frequency be f.

Class Interval	Frequency (f_i)	Class Mark (x_i)	$f_i x_i$
11-13	3	12	36
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	$20f$
21-23	5	22	110
23-25	4	24	96
Total	$\sum f_i = 40 + f$		$\sum f_i x_i = 704 + 20f$

Mean of the data = ₹ 18 [Given]

$$\Rightarrow \frac{\sum f_i x_i}{\sum f_i} = 18 \Rightarrow \frac{704 + 20f}{40 + f} = 18$$

$\Rightarrow 704 + 20f = 18 \times (40 + f)$

$\Rightarrow 20f - 18f = 720 - 704 \Rightarrow 2f = 16 \Rightarrow f = 8$

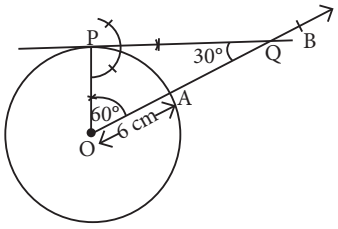
12. Steps of construction :

Step-I : Draw a circle with centre O and radius 7 cm.

Step-II : Draw a radius OA and produce it to B.

Step-III : Construct an $\angle AOP$ equal to the complement of 30° *i.e.*, 60°.

Step-IV : Draw a perpendicular to OP at P which intersects OB at Q.



Hence, PQ is the required tangent such that $\angle OQP = 30°$.

13. (i) Volume of wooden block = $l \times b \times h$

$= 21 \times 21 \times 24 = 10584 \text{ cm}^3$

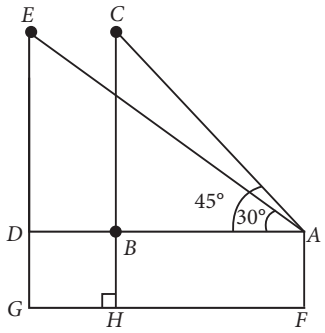
(ii) Volume of sphere carved out $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (10.5)^3 = 4851 \text{ cm}^3$$

$\therefore$ Volume of wood wasted = Volume of wooden block – Volume of sphere

$= 10584 - 4851 = 5733 \text{ cm}^3$

14. (i) Distance of first position of bird and the eyes of boy = AC



In ΔABC, $\sin 45° = \frac{BC}{AC}$

$$\Rightarrow AC = \frac{CH - BH}{\sin 45°} = \frac{42 - 2}{1/\sqrt{2}} = 40\sqrt{2} \text{ m}$$

(ii) Distance between boy and pole = AB

Now, In ΔABC, $\tan 45° = \frac{BC}{AB}$

$\Rightarrow AB = BC \Rightarrow AB = 40$ m

In ΔAED, $\tan 30° = \frac{ED}{AD}$

$\Rightarrow \quad AD = \sqrt{3}\, BC \qquad (\because ED = BC)$

$\Rightarrow \quad AD = 40\sqrt{3}$ m

Now, distance between two positions of bird

$= BD = AD - AB$

$= 40(\sqrt{3} - 1)$ m $= 40\ (1.732 - 1) = 29.28$ m