DAY SEVENTEEN

Waves

Learning & Revision for the Day

- Wave Motion
- Speed of Waves
- Sound Waves
- Displacement Relation for a Progressive or Harmonic Wave
- Principle of Superposition of Waves
- Intensity
- Power
- Reflection and Transmission of Waves
- Standing or Stationary Waves
- Beats
- Doppler's Effect

Wave Motion

Wave motion involves transfer of disturbance (energy) from one point to the other with particles of medium oscillating about their mean positions i.e. the particles of the medium do not travel themselves along with the wave. Instead, they oscillate back and forth about the same equilibrium position as the wave passes by. Only the disturbance is propagated.

1. **Longitudinal Waves** When particles of the medium vibrate parallel to the direction of propagation of wave, then wave is called **longitudinal wave**. These waves propagate in the form of compressions and rarefactions. They involve changes in pressure and volume. The medium of propagation must possess elasticity of volume. They are set up in solids, liquids and gases.
2. **Transverse Waves** When the particles of the medium vibrate in a direction perpendicular to the direction of propagation of wave, then wave is called transverse waves. These wave propagates in the form of crests and troughs. These waves can be set up in solids, on surface of liquids but never in gases.

Terms Used in Wave Motion

- **Angular Wave Number** Number of wavelength in the distance 2π is called the wave number or propagation constant.

$$K = \frac{2\pi}{\lambda} \text{ rad/m}$$

- **Particle velocity** It is the velocity of the particle executing simple harmonic motion.

  i.e. $$v = \frac{dy}{dt}$$

  where, y denotes displacement at any instant.

- **Wave Velocity** The velocity of transverse wave motion is given by

$$v = \frac{\text{Distance travelled by wave}}{\text{Time taken}}$$

  i.e. $$v = \frac{\lambda}{T} = \frac{\omega}{k} \quad \text{or} \quad v = \nu\lambda$$

- **Differential Equation of Wave Motion**

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2}\frac{d^2 y}{dt^2}$$

Speed of Waves

Speed of waves is divided in two types as per the nature of wave, these are given below

1. Speed of Transverse Wave

The expression for speed of transverse waves in a solid and in case of a stretched string can be obtained theoretically

- In solids, $v = \sqrt{\frac{\eta}{d}}$

  where, η is the modulus of rigidity and d is the density of the medium.

- In a stretched string, $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mg}{\pi r^2 d}}$

  where, T = the tension in the string,
  m = the mass per unit length of the string,
  M = mass suspended from the string,
  r = radius of the string and
  d = density of the material of the string.

2. Speed of Longitudinal Wave (or Sound Wave)

Following are the expressions for the speed of longitudinal waves in the different types of media

- If the medium is solid,

$$v = \sqrt{\frac{B + \frac{4}{3}\eta}{\rho}}$$

  where B, η and ρ are values of bulk modulus, modulus of rigidity and density of the solid respectively.
  If the solid is in the form of a long rod, then

$$v = \sqrt{\frac{Y}{\rho}}$$

  where, Y is the Young's modulus of the solid material.

- In a liquid,

$$v = \sqrt{\frac{B}{\rho}}$$

  where B is the bulk modulus of the liquid.

- According to Newton's formula, speed of sound in a gas is obtained by B replaced by initial pressure p of the gas *i.e.*, $B = p$.

$$v = \sqrt{\frac{p}{\rho}}$$

Factors Affecting Speed of Sound

- **Effect of Temperature on Velocity** With rise in temperature, the velocity of sound increases as

$$v = \sqrt{\frac{\gamma RT}{M}};\ \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{i.e.} \quad v \propto \sqrt{T}$$

  Speed of sound in air increases by 0.61 m/s for every 1°C rise in temperature.

- **Effect of Pressure for Gases Medium** Pressure has no effect on the velocity of sound, provided temperature remains constant.

- **Effect of Humidity** When humidity in air increases, its density decreases and so velocity of sound increases.

  For solids, $v = \sqrt{\frac{Y}{D}}$. For liquids, $v = \sqrt{\frac{K}{D}}$

  where, Y = Young's modulus of elasticity
  K = bulk modulus of elasticity.

Sound Waves

The longitudinal waves which can be heard are called sound waves. They are classified into following categories

- **Infrasonics** The longitudinal waves having frequencies below 20 Hz are called infrasonics. These waves cannot be heard. These waves can be heard by snakes.
- **Audible waves** The longitudinal waves having the frequency between 20 Hz and 20000 Hz are called audible waves. Human can hear these waves.
- **Ultrasonics** The longitudinal waves having the frequencies above 20000 Hz are called ultrasonics. These waves are also called supersonic waves or supersonics.

Displacement Relation for a Progressive or Harmonic Wave

The equation of a plane progressive or simple harmonic wave travelling along positive direction of x-axis is

$$y = a\sin(\omega t - kx) \Rightarrow y = a\sin\frac{2\pi}{T}\left(t - x\frac{T}{\lambda}\right).$$

$$\Rightarrow \quad y = a\sin\frac{2\pi}{\lambda}(vt - x) \Rightarrow y = a\sin\omega\left(t - \frac{x}{v}\right)$$

$$\Rightarrow \quad y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right).$$

- If maximum value of $y = a$, i.e. a is amplitude, then

  dy/dt = velocity of particle

$$v = \frac{dy}{dt} = a\omega\cos\cdot\frac{2\pi}{\lambda}(vt - x)$$

$$\left(\frac{dy}{dt}\right)_{\max} = \frac{2\pi va}{\lambda} = 2\pi na = \omega a \qquad \text{[where, } n = \text{frequency]}$$

- Acceleration of particle

$$\frac{d^2y}{dt^2} = -\omega^2 a \sin \frac{2\pi}{\lambda}(vt - x)$$

- For a wave, velocity of sound wave

v = frequency $(n) \times$ wavelength (λ)

$\Rightarrow \qquad v = n\lambda$

- Angular speed, $\omega = 2\pi n = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi v}{\lambda}$

Relation between Phase Difference, Path Difference and Time Difference

- Phase difference $(\phi) = \frac{2\pi}{\lambda} \times$ path difference (x)

$\Rightarrow \qquad \phi = \frac{2\pi x}{\lambda} \Rightarrow x = \frac{\phi\lambda}{2\pi}$

- Phase difference $(\phi) = \frac{2\pi}{T} \times$ time difference (t)

$\Rightarrow \qquad \phi = \frac{2\pi t}{T} \quad \Rightarrow t = \frac{T\phi}{2\pi}$

- Time difference $(t) = \frac{T}{\lambda} \times$ path difference (x)

$\Rightarrow \qquad t = \frac{Tx}{\lambda} \Rightarrow x = \frac{\lambda t}{T}$

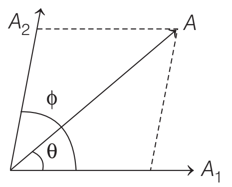
Principle of Superposition of Waves

Two or more waves can traverse the same space independently of one another. The resultant displacement of each particle of the medium at any instant is equal to the vector sum of displacements produced by the two waves separately. This principle is called principle of superposition of waves.

$$\boxed{y = y_1 + y_2 + y_3 + \ldots\ldots}$$

Interference of Waves

When two waves of same frequency (or same wavelength) travelling along same path superimpose each other, there occurs redistribution of energy in the medium. If at a given position (x being constant) displacement due to two waves be



$y_1 = A_1 \sin \omega t$

and $\quad y_2 = A_2 \sin (\omega t + \phi)$

Then, resultant displacement

$$y = y_1 + y_2 = A \sin (\omega t + \phi)$$

where, $\quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$

and $\quad \tan \theta = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$

Constructive and Destructive Interference

- When the wave meet a point with some phase, **constructuve interference** is obtained at that point.
  (i) Phase difference between the waves at the point of observation $\phi = 0°$ or $2\pi n$.
  (ii) Resultant amplitude at the point of observation will be maximum, $A_{max} = A_1 + A_2$.
- When the waves meet a point with opposite phase, **destructive interference** is obtained at that point.
  (i) Phase difference between the waves at the point of observation $\phi = 180°$ or $(2n-1)\pi$.
  (ii) Resultant amplitude at the point of observation will be minimum, $A_{min} = A_1 - A_2$.

Intensity

The intensity of waves is the average amount of energy transported by the wave per unit area per second normally across a surface at the given point.

Intensity $(I_1) \propto$ (Amplitude $A)^2$

$\therefore \qquad \frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2$

If I_1 and I_2 are intensities of the interfering waves and ϕ is the phase difference, then resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2, \text{ for } \phi = 2\pi n$$

and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2, \text{ for } \phi = (2n+1)\pi$$

Power

If P is power of a sound source, then intensity (I) follows inverse square law of distance (d).

$$I = \frac{P}{4\pi d^2}$$

Reflection and Transmission of Waves

When sound waves are incident on a boundary separating two media, a part of it is reflected back into the initial medium while the remaining is partly absorbed and partly transmitted into the second medium.

Standing or Stationary Waves

Standing or stationary wave is formed due to superposition of two progressive waves of same nature, same frequency (or same wavelength), same amplitude travelling with same speed in a bounded medium in mutually opposite directions.

If the incident wave be represented as $y_1 = A \sin(\omega t - kx)$

and the reflected wave as $y_2 = A \sin (\omega t + kx)$,

then $y = y_1 + y_2 = A \sin(\omega t - kx) + A \sin(\omega t + kx)$

$\Rightarrow \quad y = 2A \cos kx \sin \omega t$

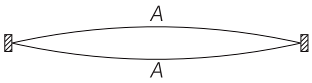
The resultant wave does not represent a progressive wave.

Standing Waves in String

Consider a string of length L stretched under tension T between two fixed points (i.e. clamped at its ends). Transverse wave is set up on the string whose speed is given by $v = \sqrt{T/\mu}$, where μ is the mass per unit length of the string.

Different modes of vibration of stretched string are discuss below

- Let only one anti-node A is formed at the centre and string vibrates in one segment only, it is called **fundamental mode**, then



Fundamental or first harmonic

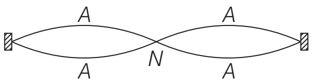
$$L = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2L$$

  Frequency of vibration in fundamental mode

$$\nu_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

  It is known as the **fundamental frequency** or **first harmonic**.

- If string vibrates in two segments, then



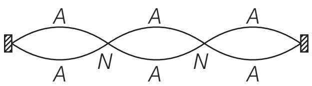
Second harmonic

$$L = \lambda_2$$

  and $\nu_2 = \dfrac{v}{\lambda_2} = \dfrac{1}{L}\sqrt{\dfrac{T}{\mu}} = 2\nu_1$

  It is known as **first overtone** or **second harmonic.**

- If the string vibrates in three segments,



Third harmonic

  then $L = \dfrac{3\lambda_3}{2}$

  and $\nu_3 = \dfrac{v}{\lambda_3} = 3\nu_1$

  It is called **second overtone** or **third harmonic**.

- In general, if a string vibrates in p segments [i.e. have $(p+1)$ nodes and p antinodes], then

  $\nu_{p\text{th}} = \dfrac{p}{2L}\sqrt{\dfrac{T}{\mu}} = p\nu_1$ and it is known as pth harmonic or $(p-1)$th overtone.

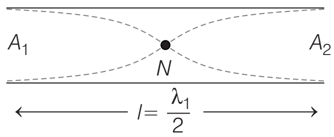
Standing Waves in Organ Pipes (Air Columns)

Organ pipes are those cylindrical pipes which are used for producing musical (longitudinal) sounds. The standing waves in both organ pipes (i.e. open organ pipe and closed organ pipe) are described below.

1. Open Organ Pipe

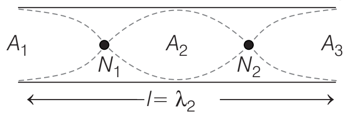
In an open organ pipe, always anti-node is formed at both open ends. Various modes of vibration of air column in an open organ pipe are shown below

- First harmonic $l = \dfrac{\lambda_1}{2} \Rightarrow f_1 = \dfrac{v}{2l}$



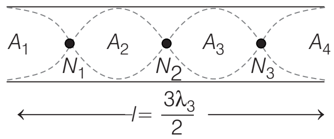
(a)

- Second harmonic or first overlone $l = \lambda_2$; $f = \dfrac{2v}{2l}$



(b)

- Third harmonic or second overtone $l = \dfrac{3\lambda_3}{2}$; $f = \dfrac{3v}{2l}$



(c)

- All harmonics are present in open pipe with their frequencies in the ratio $1:2:3:4\ldots$ and ratio of overtones $= 2 : 3 : 4 : 5\ldots$

  Position of nodes from one end $x = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}\ldots$

  Position of anti-nodes from one end

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}\ldots$$

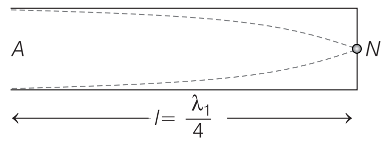
2. Closed Organ Pipe

In a closed organ pipe, always node is formed at the closed end. Various mode of vibration of air column in a closed organ pipe are shown below

- First harmonic

  $l = \dfrac{\lambda_1}{4}$

  $f = \dfrac{v}{4l}$

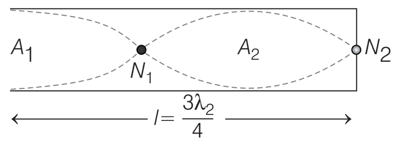


(a)

- Third hormonic (first overtone)

  $l = \dfrac{3\lambda_3}{4}$

  $f_3 = \dfrac{3v}{4l}$

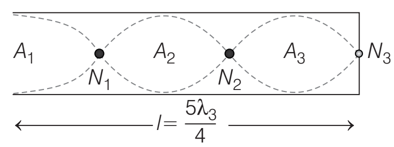


(b)

- Fifth hormonic (second overtone)

  $l = \dfrac{5\lambda_5}{4}$;

  $f_5 = \dfrac{5v}{4l}$



(c)

- In closed organ pipe only odd harmonics are present. Ratio of harmonic is $n_1 : n_3 : n_5 = 1 : 3 : 5$.

- Ratio of overtones = 3 : 5 : 7
- Position of nodes from closed end $x = 0, \dfrac{\lambda}{2}, \lambda, \dfrac{3\lambda}{2}, \ldots$
- Position of antinodes from closed end $x = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}, \ldots$

Beats

When two sound waves of nearly equal (but never equal) or slightly different frequencies and equal or nearly equal amplitudes travelling along the same direction superimpose at a given point, the resultant sound intensity alternately rises and falls. This alternate rise and fall of sound at a given position is called **beats**.

- Number of beats formed per second is called the frequency of beats. If two sound waves of frequencies ν_1 and ν_2 superimpose, then frequency of beats $= (\nu_1 \sim \nu_2)$, i.e. either $(\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$.
- For formation of distinct beats, then difference between the frequencies of two superposing notes should be less than 10 Hz.
- Our perception of loudness is better co-related with the second level measured in decibel (dB) and defined as follows
  $\beta = 10 \log_{10}\left(\dfrac{I}{I_0}\right)$, where $I_0 = 10^{-12}\ \text{Wm}^2$ at 1kHz.

Tuning Fork

The tuning fork is a metallic device that produces sound of a single frequency.

Suppose, a tuning fork of known frequency n_A is sounded together with another tuning fork of unknown frequency (n_B) and x beats heard per second.

There are two possibilities to know frequency of unknown tuning fork

$$n_A - n_B = x \qquad \ldots(\text{i})$$

$$n_B - n_A = x \qquad \ldots(\text{ii})$$

We can find true frequency of tuning fork B from a pair of tuning forks A and B, in which frequency of A is known, where x is the beats per second.

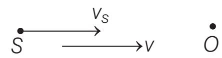
When B is loaded (its frequency decreases)		When B is filled (its frequency increases)	
(i)	If x increases, then $n_B = n_A - x$	(i)	If x increases, then $n_B = n_A + x$
(ii)	If x decreases, then $n_B = n_A + x$	(ii)	If x decreases, then $n_B = n_A - x$
(iii)	If x remains same, then $n_B = n_A + x$	(iii)	If x remains same, then $n_B = n_A - x$
(iv)	If x becomes zero, then $n_B = n_A + x$	(iv)	If x becomes zero, then $n_B = n_A - x$

Doppler's Effect

The phenomena of apparent change in frequency of source due to a relative motion between the source and observer is called Doppler's effect.

- **When Source is Moving and Observer is at Rest** When source is moving with velocity v_s, towards an observer at rest, then apparent frequency

$$n' = n\left(\frac{v}{v - v_s}\right)$$

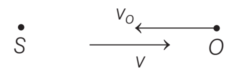


If source is moving away from observer, then

$$n' = n\left(\frac{v}{v + v_s}\right)$$

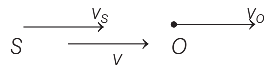
- **When Source is at Rest and Observer is Moving** When observer is moving with velocity v_o, towards a source at rest, then apparent frequency.

$$n' = n\left(\frac{v + v_o}{v}\right)$$



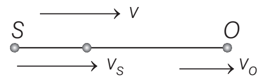
When observer is moving away from source, then

$$n' = n\left(\frac{v - v_o}{v}\right)$$



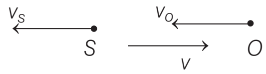
- **When Source and Observer Both are Moving**
  (a) When both are moving in same direction along the direction of propagation of sound, then

$$n' = n\left(\frac{v - v_o}{v - v_s}\right)$$



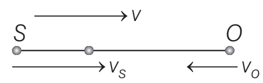
  (b) When both are moving in same direction opposite to the direction of propagation of sound, then

$$n' = n\left(\frac{v + v_o}{v + v_s}\right)$$



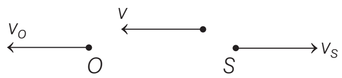
  (c) When both are moving towards each other, then

$$n' = n\left(\frac{v + v_o}{v - v_s}\right)$$



  (d) When both are moving in opposite direction, away from each other, then

$$n' = n\left(\frac{v - v_o}{v + v_s}\right)$$



DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 With propagation of longitudinal waves through a medium, the quantity transmitted is

(a) matter (b) energy
(c) energy and matter
(d) energy, matter and momentum

2 The waves produced by a motorboat sailing in water are

(a) transverse (b) longitudinal
(c) longitudinal and transverse
(d) stationary

3 A transverse wave is represented by $y = A \sin(\omega t - kx)$. For what value of the wavelength is the wave velocity equal to the maximum particle velocity? ➜ CBSE AIPMT 2010

(a) $\pi A/2$ (b) πA (c) $2\pi A$ (d) A

4 A transverse wave propagating along X-axis is represented by

$$y(x, t) = 8.0 \sin\left(0.5\pi x - 4\pi t - \frac{\pi}{4}\right)$$

where, x is in metre and t is in second. The speed of the wave is

(a) 4π ms^{-1} (b) 0.5π ms^{-1} (c) $\frac{\pi}{4}$ ms^{-1} (d) 8 ms^{-1}

5 Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where its speed is $2v$ m/s. Wavelength of sound waves in the second medium is

(a) λ (b) $\frac{\lambda}{2}$ (c) 2λ (d) 4λ

6 Velocity of sound in a gaseous medium is 330 ms^{-1}. If the pressure is increased by 4 times without change in temperature, the velocity of sound in the gas is

(a) 330 ms^{-1} (b) 660 ms^{-1} (c) 156 ms^{-1} (d) 990 ms^{-1}

7 The velocity of sound in air at NTP is 330 m/s. What will be its value when temperature is doubled and pressure is halved?

(a) 330 m/s (b) 165 m/s
(c) $330\sqrt{2}$ m/s (d) $320/\sqrt{2}$ m/s

8 A wave of frequency 500 Hz travels between X and Y, distance of 600 m in 2 s. How many wavelength are there in distance XY?

(a) 1000 (b) 300
(c) 180 (d) 2000

9 Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air ➜ CBSE AIPMT 2011

(a) Increases by a factor 20 (b) Increases by a factor 10
(c) Decreases by a factor 20 (d) Decreases by a factor 10

10 Velocity of sound in the atmosphere of a planet is 500 ms^{-1}. The minimum distance between the source of sound and the obstacle to hear the echo should be

(a) 17 m (b) 20 m (c) 25 m (d) 50 m

11 The displacement Y of a particle in a medium can be expressed as

$$y = 10^{-6} \sin(100t + 20x + \pi/4)\,\text{m}$$

where, t is in second and x in metre.
The speed of the wave is

(a) 2000 ms^{-1} (b) 5 ms^{-1}
(c) 20 ms^{-1} (d) 5π ms^{-1}

12 A wave in a string has an amplitude of 2cm. The wave travels in the positive direction of x-axis with a speed of 128 ms^{-1} and it is noted that 5 complete waves fit in 4 m length of the string. The equation describing the wave is ➜ CBSE AIPMT 2009

(a) $y = (0.02)\,\text{m} \sin(7.85x + 1005t)$
(b) $y = (0.02)\,\text{m} \sin(15.7x - 2010t)$
(c) $y = (0.02)\,\text{m} \sin(15.7x + 2010t)$
(d) $y = (0.02)\,\text{m} \sin(7.85x - 1005t)$

13 A wave is represented by the equation

$$y = 7 \sin\left(7\pi t - 0.04x + \frac{\pi}{3}\right)$$

where, x in metre and t in second. The speed of the wave is

(a) $175\,\pi$ ms^{-1} (b) 49π ms^{-1} (c) $49/\pi$ ms^{-1} (d) $0.28\,\pi$ ms^{-1}

14 In the given progressive wave equation,

$$y = 0.5 \sin(10\pi t - 2x)\,\text{cm}$$

What is the maximum velocity of particle?

(a) 5 ms^{-1} (b) 5π ms^{-1} (c) 10 ms^{-1} (d) 10.5 ms^{-1}

15 The phase difference between two waves, represented by

$$y_1 = 10^{-6} \sin\left[100t + \left(\frac{x}{50}\right) + 0.5\right]\text{m}$$

$$y_2 = 10^{-6} \cos\left[100t + \left(\frac{x}{50}\right)\right]\text{m}$$

where, x is expressed in metre and t is expressed in second, is approximately

(a) 1.07 rad (b) 2.07 rad (c) 0.5 rad (d) 1.5 rad

16 The equation $y = A\cos^2\left[2\pi nt - 2\pi\dfrac{x}{\lambda}\right]$ represents a wave with

(a) amplitude $A/2$, frequency $2n$ and wavelength λ
(b) amplitude $A/2$, frequency $2n$ and wavelength $\lambda/2$
(c) amplitude A, frequency n and wavelength λ
(d) amplitude A, frequency $2n$ and wavelength 2λ

17 A wave travelling in the positive x-direction having displacement along y-direction as 1 m, wavelength 2π m and frequency of $\dfrac{1}{\pi}$ Hz is represented by

(a) $y = \sin(x - 2t)$ (b) $y = \sin(2\pi x - 2\pi t)$
(c) $y = \sin(10\pi x - 20\pi t)$ (d) $y = \sin(2\pi x + 2\pi t)$

18 Ratio of amplitude of two interfering waves is 2:1, then ratio of amplitude of maxima to minima is

(a) 4 : 1 (b) 9 : 1 (c) 3 : 1 (d) 9 : 4

19 If two waves of amplitude a produce a resultant wave of amplitude a, then the phase difference between them will be

(a) 60° (b) 90° (c) 120° (d) 180°

20 A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distance of 2 m and 3 m respectively, from the source. The ratio of the intensities of the waves at P and Q is

(a) 9 : 4 (b) 2 : 3
(c) 3 : 2 (d) 4 : 9

21 The equation of a stationary wave is

$$y = 0.8\cos\left(\frac{\pi x}{20}\right)\sin 200\,\pi t$$

where x is in cm and t is in second. The separation between consecutive nodes will be

(a) 20 cm (b) 10 cm (c) 40 cm (d) 30 cm

22 Standing waves are produced in 10 m long stretched string. If the string vibrates in five segments and wave velocity is 20 ms^{-1}, then its frequency will be

(a) 5 Hz (b) 2 Hz (c) 10 Hz (d) 15 Hz

23 The length of a sonometer wire AB is 110 cm. Where should the two bridges be placed from A to divide the wire in three segments whose fundamental frequencies are in the ratio of 1 : 2 : 3.

(a) 30 cm and 90 cm (b) 40 cm and 80 cm
(c) 60 cm, 30 cm and 20 cm (d) 30 cm and 60 cm

24 If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by → AIPMT 2014

(a) $\dfrac{1}{n} = \dfrac{1}{n_1} + \dfrac{1}{n_2} + \dfrac{1}{n_3}$ (b) $\dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{n_1}} + \dfrac{1}{\sqrt{n_2}} + \dfrac{1}{\sqrt{n_3}}$
(c) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$ (d) $n = n_1 + n_2 + n_3$

25 When a string is divided into three segments of lengths l_1, l_2 and l_3, the fundamental frequencies of these three segments are ν_1, ν_2 and ν_3, respectively. The original fundamental frequency (ν) of the string is

(a) $\sqrt{\nu} = \sqrt{\nu_1} + \sqrt{\nu_2} + \sqrt{\nu_3}$ (b) $\nu = \nu_1 + \nu_2 + \nu_3$
(c) $\dfrac{1}{\nu} = \dfrac{1}{\nu_1} + \dfrac{1}{\nu_2} + \dfrac{1}{\nu_3}$ (d) $\dfrac{1}{\sqrt{\nu}} = \dfrac{1}{\sqrt{\nu_1}} + \dfrac{1}{\sqrt{\nu_2}} + \dfrac{1}{\sqrt{\nu_3}}$

26 An organ pipe, open from both ends produced 5 beats/s when vibrated with a source of frequency 200 Hz in its fundamental mode. The second harmonic of the same pipe produces 10 beats/s with a source of frequency 420 Hz. The fundamental frequency of pipe is

(a) 195 Hz (b) 205 Hz (c) 190 Hz (d) 210 Hz

27 The fundamental frequency of a closed pipe is 220 Hz. If $\dfrac{1}{4}$ of the pipe is filled with water, the frequency of the Ist overtone of the pipe now is

(a) 220 Hz (b) 440 Hz (c) 880 Hz (d) 1760 Hz

28 An organ pipe closed at one end has fundamental frequency of 1500 Hz. The maximum number of overtones generated by this pipe which a normal person can hear is?

(a) 4 (b) 13 (c) 6 (d) 9

29 The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is → AIPMT 2015

(a) 80 cm (b) 100 cm (c) 120 cm (d) 140 cm

30 The number of possible natural oscillations of air column in pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are (velocity of sound = 340 ms^{-1}) → AIPMT 2014

(a) 4 (b) 5 (c) 7 (d) 6

31 The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20 cm, the length of the open organ pipe is → NEET 2018

(a) 12.5 cm (b) 8 cm (c) 13.3 cm (d) 16 cm

32 The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system? → NEET 2017

(a) 10 Hz (b) 20 Hz
(c) 30 Hz (d) 40 Hz

33 A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2. The ratio λ_2/λ_1 is → NEET 2016

(a) $\sqrt{\dfrac{m_1 + m_2}{m_2}}$ (b) $\sqrt{\dfrac{m_2}{m_1}}$ (c) $\sqrt{\dfrac{m_1 + m_2}{m_1}}$ (d) $\sqrt{\dfrac{m_1}{m_2}}$

34 The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe L metre long. The length of the open pipe will be → NEET 2016

(a) L (b) $2L$ (c) $L/2$ (d) $4L$

35 If we study the vibration of a pipe open at both ends, which of the following statements is not true? → NEET 2013

(a) Open end will be antinode
(b) Odd harmonics of the fundamental frequency will be generated
(c) All harmonics of the fundamental frequency will be generated
(d) Pressure change will be maximum at both ends

36 Two sound waves with wavelengths 5.0 m and 5.5 m respectively, each propagate in a gas with velocity 330 ms^{-1}. We expect the following number of beats per second

(a) 12 (b) 0 (c) 1 (d) 6

37 If two waves of wavelengths 50 cm and 51 cm produced 12 beats/s, the velocity of sound is

(a) 360 ms^{-1} (b) 306 ms^{-1} (c) 331 ms^{-1} (d) 340 ms^{-1}

38 Two wave of wavelengths 99 cm and 100 cm both travelling with velocity 396 ms^{-1} are made of interfere. The number of beats produced by them per second are

(a) 1 (b) 2 (c) 4 (d) 8

39 Three sound waves of equal amplitudes have frequencies $(n-1), n, (n+1)$. They superimpose to give beats. The number of beats produced per second will be → NEET 2016

(a) 1 (b) 4
(c) 3 (d) 2

40 Two sources of sound placed closed to each other, are emitting progressive waves given by $y_1 = 4\sin 600\pi t$ and $y_2 = 5\sin 608\pi t$

An observer located near these two sources of sound will hear → CBSE AIPMT 2012

(a) 4 beat/s with intensity ratio 25 : 16 between waxing and waning
(b) 8 beat/s with intensity ratio 25 : 16 between waxing and waning
(c) 8 beat/s with intensity ratio 81 : 1 between waxing and waning
(d) 4 beat/s with intensity ratio 81 : 1 between waxing and waning

41 A source of unknown frequency gives 4 beat/s when sounded with a source of known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 beat/s when sounded with a source of frequency 513 Hz. The unknown frequency is → NEET 2013

(a) 254 Hz (b) 246 Hz
(c) 240 Hz (d) 260 Hz

42 A tuning fork of frequency 512 Hz makes 4 beat/s with the vibrating string of a piano. The beat frequency decreases to 2 beat/s when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was → CBSE AIPMT 2010

(a) 510 Hz (b) 514 Hz (c) 516 Hz (d) 508 Hz

43 A sound source is moving towards stationary listener with 1/10th of the speed of sound. The ratio of apparent to real frequency is

(a) $\frac{11}{10}$ (b) $\left(\frac{11}{10}\right)^2$ (c) $\left(\frac{9}{10}\right)^2$ (d) $\frac{10}{9}$

44 A tuning fork is used to produce resonance in a glass tube. The length of the air column in this tube can be adjusted by a variable piston. At room temperature of 27° C, two successive resonances are produced at 20 cm and 73 cm of column length. If the frequency of the tuning fork is 320 Hz, the velocity of sound in air at 27° C is → NEET 2018

(a) 350 m/s (b) 339 m/s (c) 330 m/s (d) 300 m/s

45 A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of a 9 kg is suspended from a wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is

(a) 25 kg (b) 5 kg (c) 12.5 kg (d) 1/25 kg

46 A train is approaching the platform with a speed of 4 ms^{-1}. Another train is leaving the platform with the same speed. The velocity of sound is 320 ms^{-1}. If both the trains sound their whistles at frequency 280 Hz, the number of beats heard per second will be

(a) 6 (b) 7 (c) 8 (d) 10

47 A speeding motorcyclist sees traffic jam ahead of him. He slows down to 36 km/hour. He finds that traffic has eased and a car moving ahead of him at 18 km/h is honking at a frequency of 1392 Hz. If the speed of sound is 343 m/s, the frequency of the honk as heard by him will be → AIPMT 2014

(a) 1332 Hz (b) 1372 Hz (c) 1412 Hz (d) 1454 Hz

48 Doppler's effect in sound in addition of relative velocity between source and observer, also depends while source and observer or both are moving. Doppler effect in light depend only on the relative velocity of source and observer. The reason of this is

(a) Einstein mass-energy relation
(b) Einstein theory of relativity
(c) Photoelectric effect (d) None of the above

49 Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s respectively. The driver of the first car blows a horn having a frequency 400 Hz. The frequency heard by the driver of the second car is [velocity of sound 340 m/s] ➔ NEET 2017

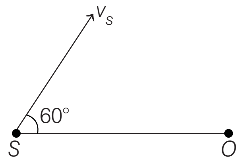
(a) 350 Hz (b) 361 Hz
(c) 411 Hz (d) 448 Hz

50 A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of $15 ms^{-1}$. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is

(Take, velocity of sound in air = $330 ms^{-1}$) ➔ NEET 2016

(a) 800 Hz (b) 838 Hz (c) 885 Hz (d) 765 Hz

51 A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of $19.4 ms^{-1}$ at an angle of 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air is $330 ms^{-1}$), is ➔ CBSE AIPMT 2015



(a) 100 Hz (b) 103 Hz (c) 106 Hz (d) 97 Hz

52 The driver of a car travelling with speed $30 ms^{-1}$ towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is $330 ms^{-1}$, the frequency of reflected sound as heard by driver is ➔ CBSE AIPMT 2009

(a) 550 Hz (b) 555.5 Hz (c) 720 Hz (d) 500 Hz

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 An observer standing on the sea coast finds that 54 ripples reach the surface per minute. If the wavelength of the ripples be 10 m, what is the wave velocity?

(a) 54 m/s (b) 9 m/s
(c) 10 m/s (d) 6 m/s

2 A wave travelling in positive x-direction with $A = 0.2$ m, velocity = $360 ms^{-1}$ and $\lambda = 60$ m, then correct expression for the wave is

(a) $y = 0.2 \sin\left[2\pi\left(6t + \frac{x}{60}\right)\right]$

(b) $y = 0.2 \sin\left[\pi\left(6t + \frac{x}{60}\right)\right]$

(c) $y = 0.2 \sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$

(d) $y = 0.2 \sin\left[\pi\left(6t - \frac{x}{60}\right)\right]$

3 In a resonance pipe the 1st and 2nd resonances are obtained at depths 22.7 cm and 70.2 cm, respectively. What will be the end correction?

(a) 1.05 cm (b) 1115.5 cm (c) 92.5 cm (d) 113.5 cm

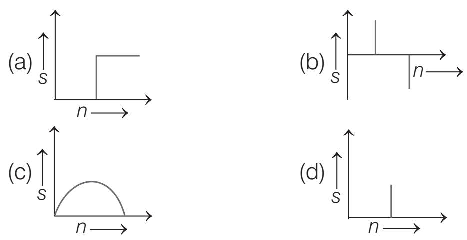
4 A car is moving towards a high cliff. The car driver sounds a horn of frequency f. The reflected sound heard by the driver has a frequency $2f$. If v be the velocity of sound, then the velocity of the car, in the same velocity units will be

(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{3}$
(c) $\frac{v}{4}$ (d) $\frac{v}{2}$

5 The frequency changes by 10% as the source approaches a stationary observer with constant speed v_s. What would be the percentage change in frequency as the source recedes the observer with the same speed? Given, that $v_s << v$ (v = speed of sound in air)

(a) 14.3% (b) 20%
(c) 16.7% (d) 10%

6 The graph between distance of source and observer and apparent frequency in the case of Doppler's effect will be



7 An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz, then the fundamental frequency of open pipe is

(a) 480 Hz (b) 300 Hz
(c) 240 Hz (d) 200 Hz

8 An open pipe resonates with a tunning fork of frequency 500 Hz. It is observed that two successive node are formed at distance 16 cm and 46 cm from the open end. The speed of sound in air in the pipe is

(a) 230 m/s (b) 300 m/s
(c) 320 m/s (d) 350 m/s

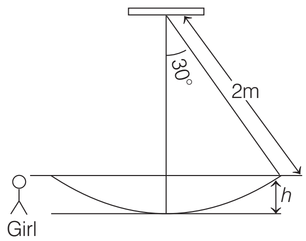
9 Two tuning forks P and Q when set vibrating, give 4 beats/s. If a prong of the fork P is filed, the beats are reduced to 2/s. What is frequency of P, if that of Q is 250 Hz?

(a) 246 Hz (b) 250 Hz (c) 254 Hz (d) 252 Hz

10 An observer standing at station observes frequency 219 Hz when a train approaches and 184 Hz when a train goes away from him. If velocity of sound in air is 340 m/s, then velocity of train and actual frequency of whistle will be

(a) 15.5 m/s, 200 Hz (b) 19.5 Hz, 205 Hz
(c) 29.5 m/s, 200 Hz (d) 32.5 Hz, 205 Hz

11 A boy is sitting on a swing and blowing a whistle at a frequency of 1000 Hz. The swing is moving to an angle of 30° from the vertical. The boy is at 2m from the point of support of swing and a girl stands in front of swing. Then, the maximum frequency she will hear is (given velocity of sound = 330 m/s).



(a) 1000 Hz (b) 1001 Hz
(c) 1007 Hz (d) 1011 Hz

12 A transverse wave propagating on a stretched string of linear density 3×10^{-4} kgm^{-1} is represented by the equation $y = 0.2 \sin(1.5x + 60t)$

where, x is in metres and t is in seconds. The tension in the string (in newton) is

(a) 0.24 (b) 0.48 (c) 1.20 (d) 1.80

13 Two points on a travelling wave having frequency 500 Hz and velocity 300 m/s are 60° out of phase, then the minimum distance between the two points is

(a) 0.2 (b) 0.1
(c) 0.5 (d) 0.4

14 A longitudinal wave is represented by

$$x = x_0 \sin 2\pi \left(nt - \frac{x}{\lambda}\right)$$

The maximum particle velocity will be four times the wave velocity, if

(a) $\lambda = \frac{\pi x_0}{4}$ (b) $\lambda = 2\pi x_0$
(c) $\lambda = \frac{\pi x_0}{2}$ (d) $\lambda = 4\pi x_0$

15 A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz.

What will be the apparent frequency of the source when it is moving away from the observer after crossing him? The velocity of the sound in the medium is 350 m/s.

(a) 750 Hz (b) 857 Hz
(c) 1143 Hz (d) 1333 Hz

16 While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then,

(a) $18 > x$ (b) $x > 54$ (c) $54 > x > 36$ (d) $36 > x > 18$

ANSWERS

SESSION 1	**1** (b)	**2** (a)	**3** (c)	**4** (d)	**5** (c)	**6** (a)	**7** (c)	**8** (a)	**9** (b)	**10** (d)
	11 (b)	**12** (d)	**13** (a)	**14** (b)	**15** (a)	**16** (b)	**17** (a)	**18** (c)	**19** (c)	**20** (a)
	21 (a)	**22** (a)	**23** (c)	**24** (a)	**25** (c)	**26** (b)	**27** (c)	**28** (c)	**29** (c)	**30** (d)
	31 (c)	**32** (b)	**33** (a)	**34** (b)	**35** (d)	**36** (d)	**37** (b)	**38** (c)	**39** (a)	**40** (d)
	41 (a)	**42** (d)	**43** (d)	**44** (b)	**45** (a)	**46** (b)	**47** (c)	**48** (b)	**49** (d)	**50** (b)
	51 (b)	**52** (c)								
SESSION 2	**1** (b)	**2** (c)	**3** (a)	**4** (b)	**5** (d)	**6** (d)	**7** (d)	**8** (b)	**9** (b)	**10** (c)
	11 (c)	**12** (b)	**13** (b)	**14** (c)	**15** (a)	**16** (b)				

Hints and Explanations

1 Propagation of longitudinal waves through a medium leads to transmission of energy through the medium.

2 Transverse waves are produced by a motorboat sailing in water.

3 Given, $y = A\sin(\omega t - kx)$

As we know that wave velocity is given by

$$v_w = \frac{\lambda}{T} = \frac{\omega\lambda}{2\pi} \quad \ldots\text{(i)} \left[T = \frac{2\pi}{\omega}\right]$$

and maximum particle velocity is given by

$$v_\rho = A\omega \quad \ldots\text{(ii)}$$

$$\begin{bmatrix} A = \text{amplitude} \\ \omega = \text{angular frequency} \end{bmatrix}$$

So, as Eq. (i) is equal to Eq. (ii),

$$A\omega = \frac{\omega\lambda}{2\pi},\ \lambda = 2\pi A$$

4 The given equation is

$$y(x, t) = 8.0\sin\left(0.5\,\pi x - 4\,\pi t - \frac{\pi}{4}\right) \quad \ldots\text{(i)}$$

The standard wave equation is

$$y = a\sin(kx - \omega t + \phi) \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get $k = 0.5\pi, \omega = 4\pi$

$\therefore$ Speed of transverse wave

$$v = \frac{\omega}{k} = \frac{4\pi}{0.5\,\pi} = 8\text{ ms}^{-1}$$

5 In the first medium, frequency

$$\nu = \frac{v}{\lambda}$$

as $\quad \nu' = v \quad$ [remain same]

$$\Rightarrow \quad \frac{v'}{\lambda'} = \frac{2v}{\lambda'} = \frac{v}{\lambda} \Rightarrow \lambda' = 2\lambda$$

6 We know the velocity of sound is given by $\quad v = \sqrt{\frac{\gamma\,\pi}{\rho}}$

At constant temperature,

pV = constant

Volume of gas,

$$V = \frac{M}{\rho} \Rightarrow \frac{pM}{\rho} = \text{constant}$$

As M is constant, therefore

$$\frac{p}{\rho} = \text{constant}$$

Hence, change in pressure has no effect on velocity of sound.
So, option (a) is correct.

7 As there is no effect of change in pressure on velocity of sound in air, and velocity $\propto \sqrt{T}$, therefore, when temperature is doubled velocity becomes $330\sqrt{2}$ m/s.

8 Velocity (v) is given by

$$v = \frac{\text{Distance}}{\text{Time}} = \frac{600}{2} = 300\text{ ms}^{-1}$$

Further, wavelength is given by

$$\lambda = \frac{v}{\nu}$$

Substituting $v = 300\text{ ms}^{-1}, \nu = 500\text{ Hz}$,

we get $\quad \lambda = \frac{3}{5}$ m

Therefore, number of wavelengths in 600 m is given by

$$n = \frac{\text{Distance}}{\text{Wavelength}} = \frac{x}{\lambda} = \frac{600}{3/5} = 1000$$

9 Velocity of sound $v = n\lambda$

$$\frac{v_1}{v_2} = \frac{n_1\lambda_1}{n_2\lambda_2} \quad [\text{but } n_1 = n_2]$$

$$\lambda_2 = \lambda_1\frac{v_2}{v_1} = \lambda_1 \times 10$$

$$\lambda_2 = 10\lambda_1$$

10 The time taken by the reflected wave to return to the source should not be more than 0.1s. Hence, total distance travelled by the sound in going and coming back should not be more than 50 m.

11 The general equation of the wave in negative X-axis direction is given by

$$y = A\sin(\omega t + kx + \phi) \quad \ldots\text{(i)}$$

Given equation is

$$y = 10^{-6}\sin(100t + 20x + \pi/4)\text{ m} \quad \ldots\text{(ii)}$$

On comparing Eq. (ii) with Eq. (i), we get

$$\omega = 100,\ k = 20$$

Hence, wave velocity is given by

$$c = \frac{\omega}{k} = \frac{100}{20} = 5\text{ms}^{-1}$$

12 Given, amplitude of wave, $A = 2$ cm

direction = +ve x direction

Velocity of wave $v = 128\text{ ms}^{-1}$

and length of string, $5\lambda = 4$

We know that,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \times 5}{4} = 7.85$$

and $\quad v = \frac{\omega}{k} = 128\text{ ms}^{-1}$

[ω = Angular frequency]

$\Rightarrow \quad \omega = v \times k = 128 \times 7.85 = 1005$

As, the wave travelling towards + x-axis is given by

$$y = A\sin(kx - \omega t)$$

So, $\quad y = 2\text{ cm}\sin(7.85\,x - 1005\,t)$

$$y = (0.02)\text{ m}\sin(7.85\,x - 1005\,t)$$

13 Compare with $y = A\sin(\omega t - kx + \phi)$

Here, $\omega = 7\pi$ and $k = 0.04$

Therefore,

$$c = \frac{\omega}{k} = 7\pi/0.04 = 175\pi\text{ ms}^{-1}$$

14 $\frac{dy}{dt} = 0.5 \times 10\pi\cos(10\pi t - 2x)$

$$= 5\pi\cos(10\pi t - 2x)$$

$$\left(\frac{dy}{dx}\right)_{\max} = 5\pi\text{ ms}^{-1}$$

15 The given waves are

$$y_1 = 10^{-6}\sin\left[100\,t + \left(\frac{x}{50}\right) + 0.5\right]\text{m}$$

and $\quad y_2 = 10^{-6}\cos\left[100\,t + \left(\frac{x}{50}\right)\right]\text{m}$

$$\Rightarrow y_2 = 10^{-6}\sin\left[100\,t + \left(\frac{x}{50}\right) + \frac{\pi}{2}\right]\text{m}$$

$$\left[\because \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta\right]$$

Hence, the phase difference between the waves is

$$\Delta\phi = \left(\frac{\pi}{2} - 0.5\right)\text{rad} = \left(\frac{3.14}{2} - 0.5\right)\text{rad}$$

$$= 1.07\text{ rad}$$

16 $y = A\cos^2\left[2\pi\,nt - 2\pi\frac{x}{\lambda}\right]$

$$= A\left[\frac{1 - \cos 2\,(2\pi nt - 2\pi\ x/\lambda)}{2}\right]$$

$$y = A/2\left[1 - \cos\,(2\pi\,(2n))\,t - \frac{2\pi\ x}{\lambda/2}\right] \quad \ldots\text{(i)}$$

Now, general wave equation is

$$y = A\cos(\omega t - kx)$$

$$= A\cos\left(2\pi\nu t - \frac{2\pi}{\lambda}\,x\right) \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii),
we get Amplitude = $A/2$,
frequency $\nu = 2n$ and wavelength = $\lambda/2$

17 Given, $a = 1$ m, $k = \frac{2\pi}{\lambda}$, $\lambda = 2\pi$

As $\quad y = a\sin(kx - \omega t)$

$$= \sin\left(\frac{2\pi x}{2\pi} - 2\pi \times \frac{1}{\pi}t\right) = \sin(x - 2t)$$

18 We have, $\frac{a_2}{a_1} = \frac{2}{1}$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{a_2 + a_1}{a_2 - a_1}\right)^2 = \left(\frac{\frac{a_2}{a_1} + 1}{\frac{a_2}{a_1} - 1}\right)^2$$

$$= \left(\frac{3}{1}\right)^2 = 9:1$$

$$\therefore \quad \frac{a_{\max}}{a_{\min}} = \sqrt{\frac{9}{1}} = \frac{3}{1} = 3:1$$

19 Here, $R^2 = a^2 + b^2 + 2ab\cos\theta$

Hence, $a^2 = a^2 + a^2 + 2\,aa\cos\phi$

$\Rightarrow\quad \cos\phi = -\dfrac{1}{2}$ or $\phi = 120°$

20 Intensity of sound

$$I = \frac{P}{4\pi r^2} \quad \text{or } I \propto \frac{1}{r^2} \text{ or } \frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

Here, $r_1 = 2$ m, $r_2 = 3$ m

Substituting the values, we have

$$\frac{I_1}{I_2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

21 $\dfrac{2\pi}{\lambda} = \dfrac{\pi}{20} \Rightarrow \lambda = 40$ cm

Separation between two consecutive nodes $= \dfrac{\lambda}{2} = \dfrac{40}{2} = 20$ cm

22 Five segments means five loops. One loop length is $\dfrac{\lambda}{2}$.

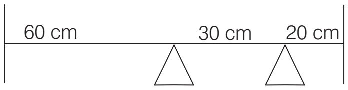
Hence, $\quad \dfrac{5\lambda}{2} = 10$ m

$\therefore \quad \lambda = 4$ m

Now, $\quad f = \dfrac{v}{\lambda} = \dfrac{20}{4} = 5$ Hz

23 Fundamental frequency

$$f \propto \frac{1}{l}$$



Given, $\quad f_1 : f_2 : f_3 = 1:2:3$

or $\quad \dfrac{1}{l_1} : \dfrac{1}{l_2} : \dfrac{1}{l_3} = 1:2:3$

or $\quad l_1 : l_2 : l_3 = \dfrac{1}{1} : \dfrac{1}{2} : \dfrac{1}{3}$

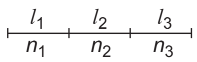
or $\quad l_1 : l_2 : l_3 = 6:3:2$

$$l_1 = \frac{6}{11} \times 110 = 60 \text{ cm}$$

$$l_2 = \frac{3}{11} \times 110 = 30 \text{ cm}$$

and $\quad l_3 = \dfrac{2}{11} \times 110 = 20$ cm

24



$$n_1 = \frac{1}{2l_1}\sqrt{\frac{T}{\mu}},\ n_2 = \frac{1}{2l_2}\sqrt{\frac{T}{\mu}},\ n_3 = \frac{1}{2l_3}\sqrt{\frac{T}{\mu}}$$

or $\quad n = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}} \qquad [l = l_1 + l_2 + l_3]$

$$\therefore \quad \frac{1}{n} = \frac{2l}{\sqrt{\frac{T}{\mu}}} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$$

25 The fundamental frequency of string

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$\therefore \quad \nu_1 l_1 = \nu_2 l_2 = \nu_2 l_3 = k$

From Eq. (i),

$$l_1 = \frac{k}{v_1},\ l_2 = \frac{k}{v_2},\ l_3 = \frac{k}{v_3}$$

Original length $\ l = \dfrac{k}{\nu}$

Here, $\quad l = l_1 + l_2 + l_3$

$$\frac{k}{\nu} = \frac{k}{\nu_1} + \frac{k}{\nu_2} + \frac{k}{\nu_3}$$

$$\frac{1}{\nu} = \frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}$$

26 Initially number of beats per second = 5

$\therefore$ Frequency of pipe

$= 200 \pm 5 = 195$ Hz or 205 Hz $\quad$ ...(i)

Frequency of second harmonic of the pipe = $2n$ and number of beats in this case = 10

$\therefore \quad 2n = 420 \pm 10$

$\Rightarrow \quad n = 205$ Hz or 215 Hz $\quad$ ...(ii)

From Eqs. (i) and (ii), it is clear that

$$n = 205 \text{ Hz}$$

27 Fundamental frequency of closed pipe

$= \dfrac{v}{4l} = 220$ Hz

$\Rightarrow\ v = 220 \times 4l$

If $\dfrac{1}{4}$ of the pipe is filled with water, then remaining length of air column is $\dfrac{3l}{4}$.

Now, fundamental frequency

$$= \frac{v}{4\left(\frac{3l}{4}\right)} = \frac{v}{3l}$$

First overtone

$= 3 \times$ fundamental frequency

$$= \frac{3v}{3l} = \frac{v}{l} = \frac{220 \times 4l}{l} = 880 \text{ Hz}$$

28 The frequency of note emitted from the pipe for v being velocity of sound in air, is

$$f' = n\left(\frac{v}{4l}\right)$$

and l is length of pipe.

$f' = n \times$ fundamental frequency

We know that, human ear can hear frequencies upto 20000 Hz.

$$20000 = n \times 1500$$

$\Rightarrow\quad n = \dfrac{20000}{1500} \approx 13$

Maximum possible harmonics obtained are 1, 3, 5, 7, 9, 11, 13

Hence, man can hear upto 13th harmonic = 7 − 1 = 6

So, number of overtones heard = 6

29 Fundamental frequency of closed organ pipe $n_c = \dfrac{v}{4l}$

Fundamental frequency of open organ pipe $n_0 = \dfrac{v}{2l'}$

Since, fundamental frequency of closed organ pipe is equal to the second overtone of an organ pipe open at both end.

$\therefore \quad n' = 3n_0 \Rightarrow n' = \dfrac{3v}{2l'}$

$\because \quad n_c = n' \Rightarrow \dfrac{v}{4l} = \dfrac{3v}{2l'}$

$l' = 6l = 6 \times 20 = 120$ cm

30 $\quad l_c = 0.85$ m

$$f_0 = \frac{v}{4l_c} = \frac{340 \text{ ms}^{-1}}{4 \times 0.85 \text{ m}} = 100 \text{ Hz}$$

$f_n = (2n+1)f_0 = f_0, 3f_0, 5f_0,$
$7f_0, 9f_0, 11f_0, 13f_0$
$= 100$ Hz, 300 Hz, 500 Hz,
700 Hz, 900 Hz, 1100 Hz

31 Fundamental frequency for an open organ pipe is given as $f = \dfrac{v}{2L}$

where, L is the length of the open organ pipe.

Third harmonic for a closed organ pipe is given as $f' = \dfrac{3v}{4L'}$

where, L' is the length of closed organ pipe.

According to the question,

$f = f'$

$\dfrac{v}{2L} = \dfrac{3v}{4L'} \Rightarrow L = \dfrac{2}{3}L'$

Given, $\ L' = 20$ cm

$\Rightarrow L = \dfrac{2}{3} \times 20$ cm $= \dfrac{40}{3}$ cm $= 13.3$ cm

32 *Thinking Process* Frequency of nth harmonic in a closed end tube

$\Rightarrow\quad f = \dfrac{(2n-1)\,v}{4l};\ n = 1, 2, 3, \ldots$

Also, only odd harmonics exists in a closed end tube.

Now, given two nearest harmonics are of frequency 220 Hz and 260 Hz.

$\therefore \quad \dfrac{(2n-1)\,v}{4l} = 220$ Hz $\quad$ ...(i)

Next harmonic occurs at

$\dfrac{(2n+1)\,v}{4l} = 260$ Hz $\quad$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$\frac{\{(2n+1)-(2n-1)\}\,v}{4l} = 260 - 220$$

$$2\left(\frac{v}{4l}\right) = 40 \quad \Rightarrow \quad \frac{v}{4l} = 20 \text{ Hz}$$

$\therefore$ Fundamental frequency of the system

$= \dfrac{v}{4l} = 20$ Hz

33 According to question, we have

Wavelength of transverse pulse

$$\lambda = \frac{v}{f} \qquad \ldots(\text{i})$$

(v = velocity of the wave; f = frequency of the wave)

As we know $\quad v = \sqrt{\frac{T}{\mu}} \qquad \ldots(\text{ii})$

(T = tension in the spring; μ = mass per unit length of the rope)

From Eqs. (i) and (ii), we get

$$\lambda = \frac{1}{f}\sqrt{\frac{T}{\mu}} \Rightarrow \lambda \propto \sqrt{T}$$

So, for two different case, we get

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{m_1 + m_2}{m_2}}$$

34 For an open organ pipe,

$$\nu_n = \frac{n}{2L}v, \text{ where } n = 1, 2, 3, \ldots$$

For second overtone $n = 3$,

$$\nu_{2_O} = \frac{3}{2L_1}v$$

L_1 = length of open organ pipe $\qquad \ldots(\text{i})$

For closed organ pipe

$$\nu_n = \left(\frac{2n+1}{4L}\right)v$$

where, $n = 0, 1, 2, 3, \ldots$

Ist overtone for closed organ pipe, $n = 1$

$$\nu_{1_C} = \frac{3}{4L}v \qquad \ldots(\text{ii})$$

$\because \quad \nu_{2_O} = \nu_{1_C} \Rightarrow \frac{3v}{2L_1} = \frac{3}{4L}v \Rightarrow L_1 = 2L$

35 Statement (d) is not true, because at the open ends pressure change will be zero.

36 Let $\lambda_1 = 5.0$ m, $v = 330\ \text{ms}^{-1}$

and $\lambda_2 = 5.5$ m

We have, $v = n\lambda \Rightarrow n = \frac{v}{\lambda} \qquad \ldots(\text{i})$

The frequency corresponding to wavelength λ_1,

$$n_1 = \frac{v}{\lambda_1} = \frac{330}{5.0} = 66 \text{ Hz}$$

The frequency corresponding to wavelength λ_2,

$$n_2 = \frac{v}{\lambda_2} = \frac{330}{5.5} = 60 \text{ Hz}$$

Hence, number of beats per second

$$= n_1 - n_2 = 66 - 60 = 6$$

37 Given, $\quad n_1 - n_2 = 12$

$$\frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 12$$

$$\frac{v}{\frac{50}{100}} - \frac{v}{\frac{51}{100}} = 2v - \frac{100\,v}{51}$$

$$\Rightarrow \quad \frac{2\,v}{51} = 12 \Rightarrow v = 306\ \text{ms}^{-1}$$

38 Number of beats is given by

$$n = \nu_1 - \nu_2$$

or $\quad n = \frac{c}{\lambda_1} - \frac{c}{\lambda_2}$ or $n = c\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$

Substituting $c = 396\ \text{ms}^{-1}$, $\lambda_1 = 99$ cm $= 0.99$ m; $\lambda_2 = 100$ cm $= 1$ m, we get $n = 4$

39 As we know that

Beat frequency $= f_1 \sim f_2 = n - (n-1) = 1$

and similarly for n and $n + 1$

Beat frequency $= n + 1 - n = 1$

40 Given, $y_1 = 4\sin 600\pi t$

and $y_2 = 5\sin 608\pi t$

Comparing with general equation

$$y = a\sin 2\pi ft$$

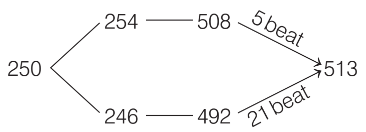
We get, $\quad f_1 = 300$ Hz and $f_2 = 304$ Hz

So, number of beats $= f_2 - f_1 = 4\ \text{s}^{-1}$

We know that,

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{4+5}{4-5}\right)^2 = 81$$

41 Given,



Hence, unknown frequency is 254 Hz.

42 Let n_p be the frequency of piano

As $(n_p \propto \sqrt{T})$

n_f = frequency of tuning fork = 512 Hz

x = Beat frequency = 4 beats/s, which is decreasing $(4 \to 2)$ after changing the tension of piano wire.

Also, tension of piano wire is increasing so $n_p \uparrow$

Hence, $n_p \uparrow - n_f = x \downarrow \longrightarrow$ wrong

$n_f - n_p \uparrow = x \downarrow \longrightarrow$ correct

$$n_p = n_f - x = 512 - 4 = 508 \text{ Hz}$$

43 $\nu' = \frac{c}{c - v_s}\nu$

Substituting $v_s = \frac{1}{10}c$, we get

$$\frac{\nu'}{\nu} = \frac{10}{9}$$

44 For first resonance, $l_1 = \frac{\lambda}{4}$

For second resonance, $l_2 = \frac{3\lambda}{4}$

$\therefore \quad (l_2 - l_1) = \frac{3\lambda}{4} - \frac{\lambda}{4}$

or $\quad \lambda = 2(l_2 - l_1) \qquad \ldots(\text{i})$

As, velocity of sound wave is given as, $v = \nu\lambda$

where, ν is the frequency.

$\Rightarrow \quad v = \nu[2(l_2 - l_1)] \qquad$ [from Eq. (i)]

Here, $\nu = 320$ Hz, $l_2 = 0.73$ m, $l_1 = 0.20$ m

$\Rightarrow \ v = 2[320(0.73 - 0.20)]$

$= 2 \times 320 \times 0.53$

$= 339.2\ \text{ms}^{-1} \simeq 339\ \text{ms}^{-1}$

45 The frequency of vibration of a string

$$n = \frac{P}{2l}\sqrt{\frac{T}{m}}$$

Also number of loops

= Number of antinodes

Hence, with 5 antinodes and hanging mass of 9 kg

we have, $P = 5$ and $T = 9$ g

$$\Rightarrow \quad n_1 = \frac{3}{2l}\sqrt{\frac{9g}{m}}$$

with 3 antinodes and hanging mass M.

we have, $P = 3$ and $T = Mg$

$$n_2 = \frac{B}{2l}\sqrt{\frac{Mg}{m}}$$

$$n_1 = n_2 \Rightarrow \frac{5}{2l}\sqrt{\frac{9g}{m}} = \frac{3}{2l}\sqrt{\frac{Mg}{m}}$$

$$M = 25 \text{ kg}$$

46 $\nu'_a = \frac{c}{c-u}\nu = \frac{320}{320-4} \times 280$

$= \frac{320}{316} \times 280$ Hz

$\nu'_r = \frac{c}{c+u}\nu = \frac{320}{320+4} \times 280$

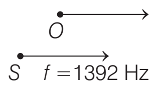
$= \frac{320}{324} \times 280$ Hz

$\nu'_a - \nu'_r = 320 \times 280\left[\frac{1}{316} - \frac{1}{324}\right]$

$= \frac{320 \times 280}{316 \times 324} \times 8 = 7$

47 $v_0 = 36$ km/h $= 10$ m/s

$v_s = 18$ km/h $= 5$ m/s



$$f' = f\left[\frac{v + v_0}{v + v_s}\right]$$

$$= 1392 \times \left(\frac{343 + 10}{343 + 5}\right) \text{Hz}$$

$$= 1392 \times \frac{353}{348} \text{ Hz} = 1412 \text{ Hz}$$

48 According to Einstein theory of relativity, the velocity of the observer is neglected w.r.t. the light velocity.

49 *Thinking Process* When both source and observer are moving towards each other, apparent frequency is given by

$$f_a = f_0\left(\frac{v + v_0}{v - v_s}\right)$$

where, f_0 = original frequency of source

v_s = speed of source

v_0 = speed of observer

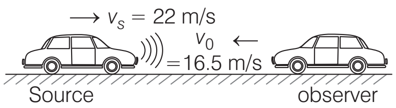
v = speed of sound

Frequency of the horn,

$f_0 = 400$ Hz

Speed of observer in the second car,

$v_0 = 16.5$ m/s



Speed of source,

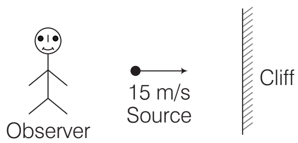
v_s = speed of first car = 22 m/s

Frequency heard by the driver in the second car

$$f_a = f_0\left(\frac{v + v_0}{v - v_s}\right) = 400\left(\frac{340 + 16.5}{340 - 22}\right)$$

$= 448$ Hz

50 According to question, situation can be drawn as follows.



Frequency of sound that the observer hear in the echo reflected from the cliff is given by

$$f' = \left(\frac{v}{v - v_s}\right)$$

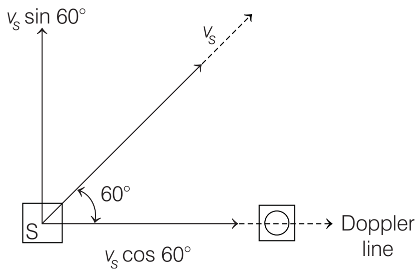
where, f = original frequency of source;

v = velocity of sound

v_s = velocity of source

So, $\quad f' = \left(\frac{330}{330 - 15}\right) 800 = 838$ Hz

51 Given, as a source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance. Such that, source is moving with a speed of 19.4 m/s at angle 60° with source-observer line as shown in figure.



The apparent frequency heared by observer

$$f_0 = f_s\left[\frac{v}{v - v_s \cos 60°}\right]$$

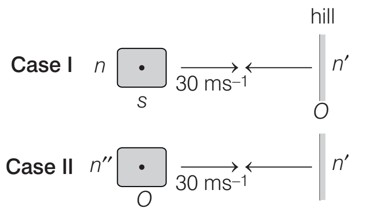
$$= 100\left[\frac{330}{330 - 19.4 \times \frac{1}{2}}\right]$$

$$= 100\left[\frac{330}{300 - 9.7}\right] = 100\left[\frac{330}{320.3}\right]$$

$= 103.02$ Hz ≈ 103 Hz

52 *Concept* Use Doppler's effect.

According to Doppler's effect, whenever there is a relative motion between a source of sound and the observer (listener), the frequency of sound heard by the observer is different from the actual frequency of sound emitted by source.



[for case **I**] $\quad n' = \frac{v}{v - 30} n \qquad \ldots$(i)

[n = frequency emitted by car; v = velocity of sound]

[for case **II**] $\quad n'' = \frac{v + 30}{v} n' \qquad \ldots$(ii)

[n'' = frequency heard by the driver after reflection]

From Eqs. (i) and (ii), we get

$$n'' = \frac{v + 30}{v - 30} n = \frac{360}{300} \times 600 = 720 \text{ Hz}$$

SESSION 2

1 Frequency $= \frac{54}{60}$

and $\quad c = f\lambda = \frac{54}{60} \times 10 = 9$ m/s

2 General equation for a plane progressive wave is given by

$$y = A\sin(\omega t - kx)$$

$$= A \sin 2\pi\left(\nu t - \frac{x}{\lambda}\right) \qquad \ldots\text{(i)}$$

Now, $v = 360$ ms^{-1}, $\lambda = 60$ m $\quad$ [given]

$\Rightarrow \quad \nu = \frac{v}{\lambda} = 6$ Hz

Substituting $A = 0.2$ ms^{-1}, $\nu = 6$ Hz, $\lambda = 60$ m in

Eq. (i), we get

$$y = 0.2\sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$$

3 For end correction x,

$$\frac{l_2 + x}{l_1 + x} = \frac{3\lambda/4}{\lambda/4} = 3$$

$$x = \frac{l_2 - 3l_1}{2}$$

$$= \frac{70.2 - 3 \times 22.7}{2} = 1.05 \text{ cm}$$

4 When the sound is reflected from the cliff, it approaches the driver of the car. Therefore, the driver acts as an observer and both the source (car) and observer are moving.

Hence, apparent frequency heard by the observer (driver) is given by

$$f' = f\left(\frac{v + v_o}{v - v_s}\right) \qquad \ldots\text{(i)}$$

where, v = velocity of sound,

v_o = velocity of car = v_s

Thus, Eq. (i) becomes

$\therefore \quad 2f = f\left(\frac{v + v_o}{v - v_o}\right)$

or $\quad 2v - 2v_o = v + v_o$ or $3v_o = v$

or $\quad v_o = \frac{v}{3}$

5 When the source approaches the observer

$$f_1 = f\left(\frac{v}{v - v_s}\right) = f\left(1 - \frac{v_s}{v}\right)^{-1}$$

$$\approx f\left(1 + \frac{v_s}{v}\right)$$

or $\quad \left(\frac{f_1 - f}{f}\right) \times 100 = \frac{v_s}{v} \times 100 = 10$

In second case when the source recedes the observer

$$f_2 = f\left(\frac{v}{v + v_s}\right) = f\left(1 + \frac{v_s}{v}\right)^{-1}$$

$$\approx f\left(1 - \frac{v_s}{v}\right)$$

$\therefore \quad \left(\frac{f_2 - f}{f}\right) \times 100 = -\frac{v_s}{v} \times 100 = -10\%$

In the first case observed frequency increases by 10% while in the second case observed frequency decreases by 10%.

6 Since, the apparent frequency does not depend on the distance between source and observer, hence option (d) is correct.

7 Fundamental frequency of open organ pipe $= \frac{v}{2l}$

Fundamental of third harmonic of closed pipe $= \frac{3v}{4l}$

$\therefore \quad \frac{3v}{4l} = 100 + \frac{v}{2l} \Rightarrow \frac{3v}{4l} - \frac{2v}{4l} = 100$

$\Rightarrow \quad \frac{v}{4l} = 100 \Rightarrow \frac{v}{2l} = 200$ Hz

8 Distance between two consecutive nodes, $\quad \frac{\lambda}{2} = 46 - 16$

$\lambda = 60$ cm $= 0.6$ m

$v = n\lambda = 500 \times 0.6 = 300$ m/s

9 There are four beats between P and Q, therefore the possible frequencies of P are 246 or 254 (i.e. 250 ± 4) Hz.

When the prong of P is filed, its frequency becomes greater than the original frequency. If we assume that the original frequency of P is 254, then on filing its frequency will be greater than 254. The beats between P and Q will be more than 4. But it is given that the beats are reduced to 2, therefore 254 is not possible.

Therefore, the required frequency must be 246 Hz. (This is true, because on filing the frequency may increase to 248 Hz, giving 2 beats with Q of frequency 250 Hz).

10 When train is approaching frequency heared by observer is

$$n_a = n\left(\frac{v}{v - v_s}\right)$$

$$219 = n\left(\frac{340}{340 - v_s}\right) \quad \ldots\text{(i)}$$

When train is goes away frequency heard by the observer is

$$n_r = n\left(\frac{v}{v + v_s}\right)$$

$$184 = n\left(\frac{340}{340 + v_s}\right) \quad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get $n = 200$ Hz and $v_s = 29.5$ m/s

11 The maximum velocity of swing will be when it crosses the lowest point and minimum velocity when it is at a height of h,

i.e. at its maximum displacement the potential energy $= mgh = mgL\,(1 - \cos\theta)$ $= 2mg\,(1 - \cos 30^\circ)$

Potential energy = kinetic energy

$$\frac{1}{2}mv^2 = 2mg\,(1 - \cos 30^\circ)$$

$$v = 2.3 \text{ m/s}$$

$$n_{\max} = \frac{v + v_0}{v - v_s} \times n$$

$$= \frac{330 + 0}{330 - 2.3} \times 1000$$

$$= 1007 \text{ Hz}$$

12 Equation of wave $y = 0.2\sin(1.5x + 60t)$

Comparing with standard equation, we get $y = A\sin(kx + \omega t)$

$$k = 1.5 \text{ and } \omega = 60$$

$\therefore$ Velocity of wave $v = \dfrac{\omega}{k} = \dfrac{60}{1.5}$

$$= 40 \text{ m/s}$$

Velocity of wave in a stretched string,

$$v = \sqrt{\frac{T}{m}}$$

where, m is linear density and T is tension in the string.

So, $T = v^2 m = (40)^2 \times 3 \times 10^{-4} = 0.48$

13 As $v = n\lambda$

$$\therefore \quad \lambda = \frac{V}{n} = \frac{300}{500} = \frac{3}{5} \text{ m}$$

Now, phase difference

$$= 2\frac{\pi}{\lambda} \times \text{path difference}$$

$$60^\circ = \frac{2\pi}{\lambda} \times \text{ path difference}$$

or $\dfrac{60^\circ \times \pi}{180^\circ} = \dfrac{2\pi \times 5}{3} \times$ path difference

$\therefore$ Path difference $= \dfrac{3 \times 60^\circ \times \pi}{2\pi \times 5 \times 180^\circ} = 0.1$

14 Particle velocity $= \dfrac{d}{dt}(x)$

$$= \frac{d}{dt}\left[x_0 \sin 2\pi\left(nt - \frac{x}{\lambda}\right)\right]$$

$$= 2\pi n x_0 \cos 2\pi\left(nt - \frac{x}{\lambda}\right)$$

Particle velocity will be maximum when $\cos 2\pi\left(nt - \dfrac{x}{\lambda}\right)$ is maximum i.e. 1.

Wave velocity $= \dfrac{\lambda}{T} = n\lambda$

Now, $\quad 2\pi n x_0 = 4n\lambda$

$$\lambda = \frac{2\pi n x_0}{4n} = \frac{\pi x_0}{2}.$$

15 When the source is coming to the stationary observer,

$$n' = \left(\frac{v}{v - v_s}\right)n$$

or $\quad 1000 = \left(\dfrac{350}{350 - 50}\right)n$

or $\quad n = \left(1000 \times \dfrac{300}{350}\right)$ Hz

When the source is moving away from the stationary observer.

$$n'' = \left(\frac{v}{v + v_s}\right)n$$

$$= \left(\frac{350}{350 + 50}\right)\left(\frac{1000 \times 300}{350}\right)$$

$$= 750 \text{ Hz}$$

16 Here, $l_1 = 18$ cm, $f = \dfrac{v_1}{4l_1}$

$$\Rightarrow \quad f = \frac{3v_2}{4l_2}$$

where, $l_2 = x$ according to given situation and also $v_1 < v_2$ as during summer temperature would be higher.

$$\frac{3v_2}{4l_2} = \frac{v_1}{4l_1} \quad \Rightarrow \quad l_2 = 3l_1 \times \frac{v_2}{v_1}$$

$\Rightarrow x = 54 \times$ (a quantity greater than 1)

$\therefore \quad x > 54$