15 Chapter
Quadratic and other Equations

We have already seen in the *Back to School* part of this block the key interrelationship between functions, equations and inequalities. In this chapter we are specifically looking at questions based on equations—with an emphasis on quadratic equations. Questions based on equations are an important component of the CAT exam and hence your ability to formulate and solve equations is a key skill in the development of your thought process for QA.

As you go through with this chapter, focus on understanding the core concepts and also look to create a framework in your mind which would account for the typical processes that are used for solving questions based on equations.

THEORY OF EQUATIONS

Equations in one variable

An equation is any expression in the form $f(x) = 0$; the type of equation we are talking about depends on the expression that is represented by $f(x)$. The expression $f(x)$ can be linear, quadratic, cubic or might have a higher power and accordingly the equation can be referred to as linear equations, quadratic equations, cubic equations, etc. Let us look at each of these cases one by one.

Linear Equations

$2x + 4 = 0$; we have the expression for $f(x)$ as a linear expression in x. Consequently, the equation $2x + 4 = 0$ would be charaterised as a linear equation. This equation has exactly 1 root (solution) and can be seen by solving $2x + 4 = 0$ Æ $x = –2$ which is the root of the equation. Note that the root or solution of the equation is the value of 'x' which would make the LHS of the equation equal the RHS of the equation. In other words, the equation is satisfied when the value of x becomes equal to the root of the equation.

The linear function $f(x)$ when drawn would give us a straight line and this line would be intersecting with the x-axis at the point where the value of x equals the root (solution) of the equation.

Quadratic Equations

An equation of the form: $2x^2 – 5x + 4 = 0$; we have the expression for $f(x)$ as a quadratic expression

in x**.** Consequently, the equation $2x^2 - 5x + 4 = 0$ would be charaterised as a quadratic equation. This equation has exactly 2 roots (solutions) and leads to the following cases with respect to whether these roots are real/imaginary or equal/unequal.

Case 1: Both the roots are real and equal;

Case 2: Both the roots are real and unequal;

Case 3: Both the roots are imaginary.

A detailed discussion of quadratic equations and the analytical formula based approach to identify which of the above three cases prevails follows later in this chapter.

The graph of a quadratic function is always U shaped and would just touch the X-Axis in the first case above, would cut the X-Axis twice in the second case above and would not touch the X-Axis at all in the third case above.

Note that the roots or solutions of the equation are the values of 'x' which would make the LHS of the equation equal the RHS of the equation. In other words, the equation is satisfied when the value of x becomes equal to the root of the equation.

Cubic Equations

An equation of the form: $x^3 + 2x^2 - 5x + 4 = 0$ **where the expression** $f(x)$ **is a cubic expression in** x**.** Consequently, the expression would have three roots or solutions.

Depending on whether the roots are real or imaginary we can have the following two cases in this situation:

Case 1: All three roots are real; (Graph might touch/cut the x-axis once, twice or thrice.)

In this case depending on the equality or inequality of the roots we might have the following cases:

Case (i) All three roots are equal; (The graph of the function would intersect the X-axis only once as all the three roots of the equation coincide.)

Case (ii) Two roots are equal and one root is distinct; (In this case the graph cuts the X-axis at one point and touches the X-Axis at another point where the other two roots coincide.)

Case (iii) All three roots are distinct from each other. (In this case the graph of the function cuts the x-axis at three distinct points.)

Case 2: One root is real and two roots are imaginary. (Graph would cut the X-axis only once.)

The shapes of the graph that a cubic function can take has already been discussed as a part of the discussion on the *Back to School* write up of this block.

Note: For a cubic equation $ax^3 + bx^2 + cx + d = 0$ with roots as l, m and n:

The product of its three roots viz: $l \times m \times n = -d/a$;

The sum of its three roots viz: $l + m + n = -b/a$

The pairwise sum of its roots taken two at a time viz: $lm + ln + mn = c/a$.

THEORY OF QUADRATIC EQUATIONS

An equation of the form

$$ax^2 + bx + c = 0 \qquad (1)$$

where a, b and c are all real and a is not equal to 0, is a quadratic equation. Then,

$D = (b^2 - 4ac)$ is the discriminant of the quadratic equation.

If $D < 0$ (i.e. the discriminant is negative) then the equation has no real roots.

If $D > 0$, (i.e. the discriminant is positive) then the equation has two distinct roots, namely,

$x_1 = (-b + \sqrt{D})/2a$, and $x_2 = (-b - \sqrt{D})/2a$

and then $\quad ax^2 + bx + c = a(x - x_1)(x - x_2) \qquad (2)$

If $D = 0$, then the quadratic equation has equal roots given by

$x_1 = x_2 = -b/2a$

and then $\quad ax^2 + bx + c = a(x - x_1)^2 \qquad (3)$

To represent the quadratic $ax^2 + bx + c$ in form (2) or (3) is to expand it into linear factors.

Properties of Quadratic Equations and Their Roots

(i) If D is a *perfect square* then the roots are *rational* and in case it is not a perfect square then the roots are *irrational*.

(ii) In the case of imaginary roots ($D < 0$) and if $p + iq$ is one root of the quadratic equation, then the other must be the conjugate $p - iq$ and vice versa (where p and q are real and $i = \sqrt{-1}$)

(iii) If $p + \sqrt{q}$ is one root of a quadratic equation, then the other must be the conjugate $p - \sqrt{q}$ and vice versa. (where p is rational and $\sqrt{q}$ is a surd).

(iv) If $a = 1$, b, c Œ I and the roots are rational numbers, then the root must be an integer.

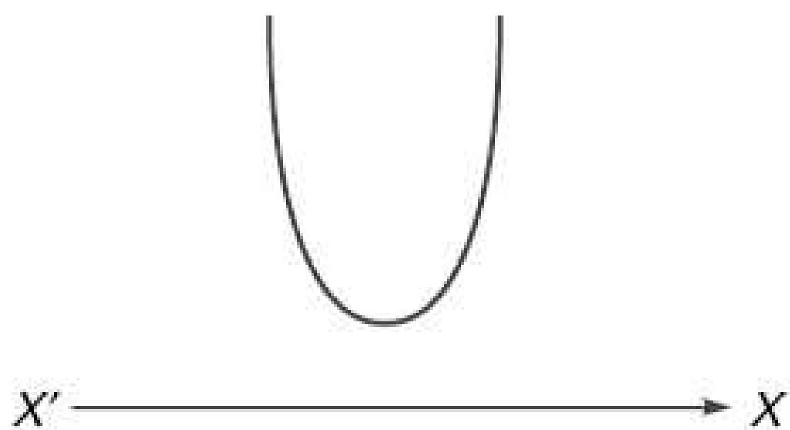
(v) If a quadratic equation in x has more than two roots, then it is an identity in x.

The Sign of a Quadratic Expression

Let $f(x) = y = ax^2 + bx + c$ where a, b, c are real and a π 0, then $y = f(x)$ represents a parabola whose axis is parallel to y-axis. For some values of x, $f(x)$ may be positive, negative or zero. Also, if $a > 0$, the parabola opens upwards and for $a < 0$, the parabola opens downwards. This gives the following cases:

(i) $a > 0$ and $D < 0$ (The roots are imaginary)

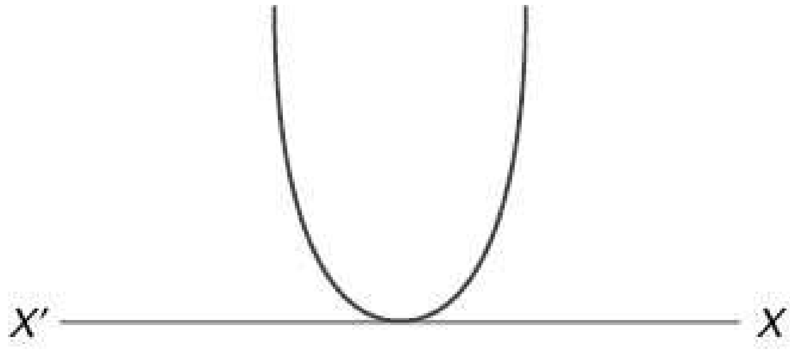
The function $f(x)$ will always be positive for all real values of x. So $f(x) > 0$ " x Œ R. Naturally the graph as shown in the figure does not cut the X-Axis.



(ii) When $a > 0$ and $D = 0$ (The roots are real and identical.)

$f(x)$ will be positive for all values of x except at the vertex where $f(x) = 0$.

So $f(x) \geq 0$ " x Œ R. Naturally, the graph touches the X-Axis once.



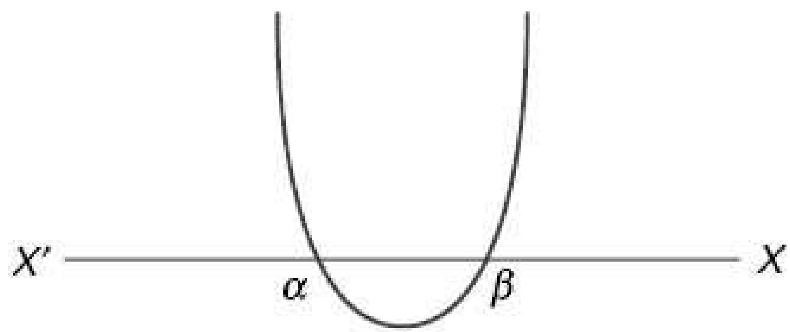
(iii) When $a > 0$ and $D > 0$ (The roots are real and distinct)

Let $f(x) = 0$ have two real roots a and b ($a < b$) then $f(x)$ will be positive for all real values of x which are lower than a or higher than b; $f(x)$ will be equal to zero when x is equal to either of a or b.

When x lies between a and b then $f(x)$ will be negative. Mathematically, this can be represented as

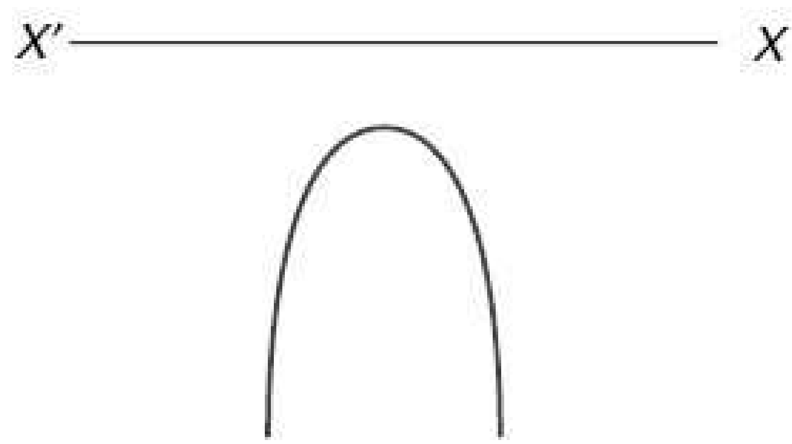
$f(x) > 0$ " x Œ (–•, a) » (b, •)

and $f(x) < 0$ " x Œ (a, b) (Naturally the graph cuts the X axis twice)



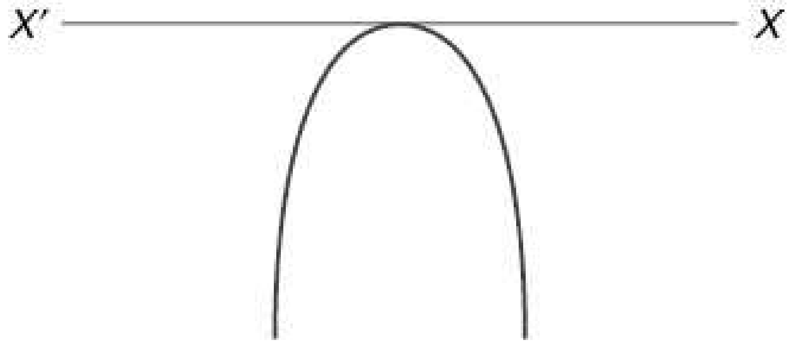
(iv) When $a < 0$ and $D < 0$ (Roots are imaginary)

$f(x)$ is negative for all values of x. Mathematically, we can write $f(x) < 0$ " x ŒR. (The graph will not cut or touch the X axis.)



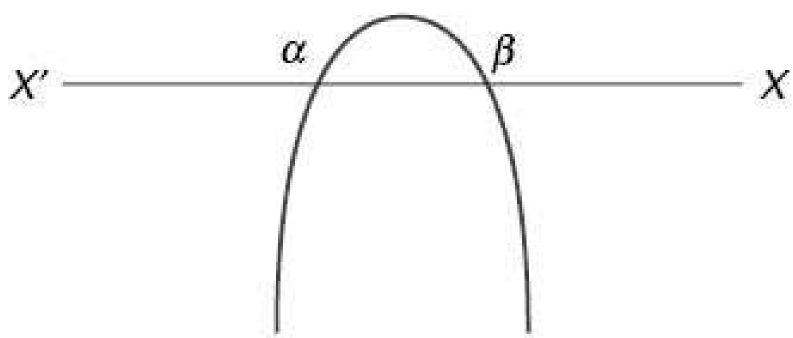
(v) When $a < 0$ and $D = 0$ (Roots are real and equal)

$f(x)$ is negative for all values of x except at the vertex where $f(x) = 0$. i.e. $f(x)$ £ 0 " x Œ R (The graph touches the X axis once.)



(vi) When $a < 0$ and $D > 0$

Let $f(x) = 0$ have two roots a and b ($a < b$) then $f(x)$ will be negative for all real values of x that are lower than a or higher than b. $f(x)$ will be equal to zero when x is equal to either of a or b. The graph cuts the X axis twice.



When x lies between a and b then $f(x)$ will be positive.

Mathematically, this can be written as

f(*x*) < 0 " *x* Œ (–•, a) » (*b*, •) and *f*(*x*) > 0 " *x* Œ(*a*, *b*).

Sum of the roots of a quadratic

Equation = $-b/a$.

Product of the roots of a quadratic equation = c/a

Equations in more than one Variable

Sometimes, an equation might contain not just one variable but more than one variable. In the context of an aptitude examination like the CAT, multiple variable equations may be limited to two or three variables. Consider this question from an old CAT examination which required the student to understand the interrelationship between the values of x and y.

The question went as follows:

$4x - 17y = 1$ where x and y are integers with $x,y > 0$ and $x, y < 1000$. How many pairs of values of (x, y) exist such that the equation is satisfied?

In order to solve this equation, you need to consider the fact that $4x$ in this equation should be looked upon as a multiple of 4 while $17y$ should be looked upon as a multiple of 17. A scan of values which exist such that a multiple of 4 is 1 more than a multiple of 17 starts from $52 - 51 = 1$, in which case the value of $x = 13$ and $y = 3$. This represents the first set of values for (x, y) that satisfies the equation.

The next pair of values in this case would happen if you increase x from 13 to 30 (increase by 17 which is the coefficient of y); at the same time increase y from 3 to 7 (increase by 4 which is the coefficient of x). The effect this has on $4x$ is to increase it by 68 while $17y$ would also increase by 68 keeping the value of

$4x$ exactly 1 more than $17y$. In other words, at $x = 30$ and $y = 7$ the equation would give us 120–119 = 1. Going further, you should realise that the same increases need to be repeated to again identify the pair of x, y values. ($x = 47$ and $y = 11$ gives us 188–187 = 1)

Once, you realise this, the next part of the visualisation in solving this question has to be on creating the series of values which would give us our desired outcomes everytime.

This series can be viewed as

(13,3); (30,7); (47,11); (64,15)….and the series would basically be two arithmetic progressions running parallel to each other (viz: 13, 30, 47, 64, 81,….) and (3, 7, 11, 15, 19,…) and obviously the number of such pairs would depend on the number of terms in the first of these arithmetic progressions (since that AP would cross the upper limit of 1000 first).

You would need to identify the last term of the series below 1000. The series can be visualised as 13, 30, 47, 64,… 999 and the number of terms in this series is 986/17 + 1 = 59 terms. [Refer to the chapter on arithmetic Progressions for developing the thinking that helps us do these last two steps.]

WORKED-OUT PROBLEMS

Problem 15.1 Find the value of $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$

(a) 4 (b) 3

(c) 3.5 (d) 2.5

Solution Let $y = \sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$

Then, $y = \sqrt{6+y}$ fi or $y^2 - y - 6 = 0$

or, $(y + 2)(y - 3) = 0$ fi $y = -2, 3$

$y = -2$ is not admissible

Hence$y$ = **3**

Alternative: Going through options:

Option (a): For 4 to be the solution, value of the whole expression should be equal to 16. Looking into the expression, it cannot be equal to 16. So, option (a) cannot be the answer. Option (b): For 3 to be the solution, the value of the expression should be 9.

So, the expression is = $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$

But $\sqrt{6}$ ª 2.9, hence = $\sqrt{6+2.9+\sqrt{\cdots}}$ @ $\sqrt{8.9+\cdots}$ @ **3**

(Since, the remaining part is negligible in value)

Problem 15.2 One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as –6 and 1 respectively. The correct roots are

(a) 3, –2 (b) –3, 2

(c) – 6, –1 (d) 6, –1

Solution Let a, b be the roots of the equation. Then $a + b = 5$ and $ab = -6$. So, the equation is $x^2 - 5x - 6 = 0$. The roots of the equation are 6 and –1.

Problem 15.3 If p and q are the roots of the equation $x^2 + px + q = 0$

(a) $p = 1$ (b) $p = 1$ or 0 or $-\frac{1}{2}$

(c) $p = -2$ (d) $p = -2$ or 0

Solution Since p and q are roots of the equation $x^2 + px + q = 0$,

$p^2 + p^2 + q = 0$ and $q^2 + pq + q = 0$

fi $2p^2 + q = 0$ and $q(q + p + 1) = 0$

fi $2p^2 + q = 0$ and $q = 0$ or $q = -p - 1$

When we use, $q = 0$ and $2p^2 + q = 0$

we get $p = 0$.

or when we use $q = -p - 1$ and $2p^2 + q = 0$

we get $2p^2 - p - 1 = 0$ Æ which gives us

$p = 1$ or $p = -1/2$

Hence, there can be three values for p

i.e. $p = 1$, $p = 0$, or $p = -\frac{1}{2}$

Problem 15.4 If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then a, b, c are in

(a) AP (b) GP

(c) HP (d) Cannot be determined

Solution Since roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal

fi $b^2 (c - a)^2 - 4ac(b - c)(a - b) = 0$

fi $b^2(c + a)^2 - 4abc(a + c) + 4a^2c^2 = 0$

fi $[b(c + a) - 2ac]^2 = 0$ fi $b(c + a) - 2ac = 0$

fi $b = (2ac)/(a + c)$ fi a, b, c, are in HP

Problem 15.5 The number of roots of the equation

$x - \frac{2}{(x-1)} = 1 - \frac{2}{(x-1)}$ is

(a) 0 (b) 1

(c) 2 (d) infinite

Solution The equation gives $x = 1$.

But $x = 1$ is not admissible because it gives $x - 1 = 0$ which, in turn, makes the whole expression like this: $x-2/0 = 1-2/0$. But $2/0$ gives the value as infinity so, no solution is possible.

Problem 15.6 If the roots of the equation $x^2 - bx + c = 0$ differ by 2, then which of the following is true?

(a) $c^2 = 4(c + 1)$ (b) $b^2 = 4c + 4$

(c) $c^2 = b + 4$ (d) $b^2 = 4(c + 2)$

Solution Let the roots be a and $a + 2$.

Then $a + a + 2 = b$ fi $a = (b - 2)/2$ (1)

and $a(a + 2) = c$ fi $a^2 + 2a = c$ (2)

Putting the value of a from (1) in (2).

$((b - 2)/2)^2 + 2(b - 2)/2) = c$

fi $(b^2 + 4 - 4b)/4 + b - 2 = c$

fi $b^2 + 4 - 8 = 4c$

fi $b^2 = 4c + 4$

\ Option (b) is the correct answer.

Problem 15.7 What is the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be twice the other?

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$

(c) $c^2 = 4a + b^2$ (d) $c^2 = 9a – b^2$

Solution

\ $a + 2a = –(b/a)$ and $a \times 2a = c/a$

fi $3a = –(b/a)$ fi $a = –b/3a$

and $2a^2 = c/a$ fi $2(–b/3a)^2 = c/a$

fi $2b^2/9a^2 = c/a$ fi $2b^2 = 9ac$

Hence the required condition is $2b^2 = 9ac$.

Alternative: Assume any equation having two roots as 2 and 4 or any equation having two roots one of which is twice the other.

When roots are 2 and 4, then equation will be $(x – 2)(x – 4) = x^2 – 6x + 8 = 0$.

Now, check the options one by one and you will find only **(b)** as a suitable option.

Problem 15.8 Solve the system of equations

$$\begin{cases} 1/x + 1/y = 3/2, \\ 1/x^2 + 1/y^2 = 5/4 \end{cases}$$

Solution Let $1/x = u$ and $1/y = v$. We then obtain

$$\begin{cases} u + v = 3/2, \\ u^2 + v^2 = 5/4 \end{cases}$$

From the first equation, we find $v = (3/2) – u$ and substitute this expression into the second equation

$u^2 + ((3/2) – u)^2 = 5/4$ or $2u^2 – 3u + 1 = 0$

when $u_1 = 1$ and $u_2 = 1/2$; consequently, $v_1 = 1/2$ and $v_2 = 1$. Therefore, $x_1 = 1$,

$y_1 = 2$ and $x_2 = 2$, $y_2 = 1$

Answer: (1, 2) and (2, 1).

Problem 15.9 The product of the roots of the equation $mx^2 + 6x + (2m – 1) = 0$ is – 1. Then m is

(a) 1 (b) 1/3

(c) –1 (d) –1/3

Solution We have $(2m – 1)/m = –1$ fi $m = 1/3$

Problem 15.10 If $13x + 17y = 643$, where x and y are natural numbers, what is the value of two times the product of x and y?

(a) 744 (b) 844

(c) 924 (d) 884

Solution The solution of this question depends on your visualisation of the multiples of 13 and 17 which would satisfy this equation. (Note: your reaction to 13*x* should be to look at it as a multiple of 13 while 17*y* should be looked at as a multiple of 17). A scan of multiples of 13 and 17 gives us the solution at 286 + 357 which would mean 13 × 22 and 17 × 21 giving us *x* as 22 and *y* as 21. The value of 2*xy* would be 2 × 22 × 21 = 2 × 462 = 924.

Problem 15.11 If $[x]$ denotes the greatest integer $\leq x$, then

$$\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{99}\right]+\left[\frac{1}{3}+\frac{2}{99}\right]+\cdots\left[\frac{1}{3}+\frac{198}{99}\right]=$$

(a) 99 (b) 66

(c) 132 (d) 165

Solution When the value of the sum of the terms inside the function is less than 1, the value of its greatest integer function would be 0. In the above sequence of values, the value inside the bracket would become equal to 1 or more from the value $\left[\frac{1}{3}+\frac{66}{99}\right]$.

Further, this value would remain between 1 to 2 till the term $\left[\frac{1}{3}+\frac{164}{99}\right]$.

There would be 99 terms each with a value of 1 between 66/99 to 164/99. Hence, this part of the expression would each yield a value of 1 giving us:

$$\left[\frac{1}{3}+\frac{66}{99}\right]+\left[\frac{1}{3}+\frac{67}{99}\right]+\cdots\left[\frac{1}{3}+\frac{164}{99}\right] = 99$$

Further, from $\left[\frac{1}{3}+\frac{166}{99}\right]+\left[\frac{1}{3}+\frac{167}{99}\right]+\cdots\left[\frac{1}{3}+\frac{198}{99}\right]$

= 2 × 33 = 66

This gives us a total value of 99 + 66 = 165 which means that Option (d) is the correct answer.

LEVEL OF DIFFICULTY (I)

1. Find the maximum value of the expression

$$\frac{1}{x^2+5x+10}$$

(a) $\frac{15}{2}$ (b) 1

(c) $\frac{4}{15}$ (d) $\frac{1}{3}$

2. Find the maximum value of the expression ($x^2 + 8x + 20$).

(a) 4 (b) 2

(c) 29 (d) None of these

3. Find the minimum value of the expression ($p + 1/p$); $p > 0$.

(a) 1 (b) 0

(c) 2 (d) Depends upon the value of p

4. If the product of roots of the equation $x^2 - 3(2a + 4)x + a^2 + 18a + 81 = 0$ is unity, then a can take the values as

(a) 3, – 6 (b) 10, – 8

(c) –10, –8 (d) –10, – 6

5. For the equation $2^{a+3} = 4^{a+2} - 48$, the value of a will be

(a) $\frac{-3}{2}$ (b) –3

(c) –2 (d) 1

6. The expression $a^2 + ab + b^2$ is __________for $a < 0$, $b < 0$

(a) π 0 (b) < 0

(c) > 0 (d) = 0

7. If the roots of equation $x^2 + bx + c = 0$ differ by 2, then which of the following is true?

(a) $a^2c^2 = 4(1 + c)$ (b) $4b + c = 1$

(c) $c^2 = 4 + b$ (d) $b^2 = 4(c + 1)$

8. If $f(x) = (x + 2)$ and $g(x) = (4x + 5)$, and $h(x)$ is defined as $h(x) = f(x) \lozenge g(x)$, then sum of roots of $h(x)$ will be

(a) $\frac{3}{4}$ (b) $\frac{13}{4}$

(c) $\frac{-13}{4}$

(d) $\frac{-3}{4}$

9. If equation $x^2 + bx + 12 = 0$ gives 2 as its one of the roots and $x^2 + bx + q = 0$ gives equal roots then the value of b is

(a) $\frac{49}{4}$

(b) – 8

(c) 4

(d) $\frac{25}{2}$

10. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal then which of the following is true?

(a) $ab = cd$

(b) $ad = bc$

(c) $ad = \sqrt{bc}$

(d) $ab = \sqrt{cd}$

11. For what value of c the quadratic equation $x^2 - (c + 6)\,x + 2(2c - 1) = 0$ has sum of the roots as half of their product?

(a) 5

(b) – 4

(c) 7

(d) 3

12. Two numbers a and b are such that the quadratic equation $ax^2 + 3x + 2b = 0$ has – 6 as the sum and the product of the roots. Find $a + b$.

(a) 2

(b) –1

(c) 1

(d) –2

13. If a and b are the roots of the Quadratic equation $5y^2 - 7y + 1 = 0$ then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(a) $\frac{7}{25}$

(b) –7

(c) $\frac{-7}{25}$

(d) 7

14. Find the value of the expression

$$(\sqrt{x + (\sqrt{x + (\sqrt{x \cdots +))}})$$

(a) $\frac{1}{2}\left[2\sqrt{(2x-1)}+1\right]$

(b) $\frac{1}{2}\left[\sqrt{(4x+1)}+1\right]$

(c) $\frac{1}{2}\left[2\sqrt{(2x-1)}-1\right]$

(d) $\frac{1}{2}\left[\sqrt{(4x-1)}-1\right]$

15. If $a = \sqrt{(7+4\sqrt{3})}$, what will be the value of $\left(a+\frac{1}{a}\right)$?

(a) 7 (b) 4

(c) 3 (d) 2

16. If the roots of the equation $(a^2 + b^2)\, x^2 - 2b(a + c)\, x + (b^2 + c^2) = 0$ are equal then a, b, c, are in

(a) AP (b) GP

(c) HP (d) Cannot be said

17. If a and b are the roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $a + \frac{1}{\beta}$ and $b + \frac{1}{\alpha}$ is

(a) $abx^2 + b(c + a)\, x + (c + a)^2 = 0$

(b) $(c + a)\, x^2 + b(c + a)\, x + ac = 0$

(c) $cax^2 + b(c + a)\, x + (c + a)^2 = 0$

(d) $cax^2 + b(c + a)\, x + c(c + a)^2 = 0$

18. If $x^2 + ax + b$ leaves the same remainder 5 when divided by $x - 1$ or $x + 1$ then the values of a and b are respectively

(a) 0 and 4 (b) 3 and 0

(c) 0 and 3 (d) 4 and 0

19. Find all the values of b for which the equation $x^2 - bx + 1 = 0$ does not possess real roots.

(a) $-1 < b < 1$ (b) $0 < b < 2$

(c) $-2 < b < 2$ (d) $-1.9 < b < 1.9$

Directions for Questions 20 to 24: Read the data given below and it solve the questions based on.

If a and b are roots of the equation $x^2 + x - 7 = 0$ then.

20. Find $a^2 + b^2$

(a) 10 (b) 15

(c) 5 (d) 18

21. Find $a^3 + b^3$

(a) 22 (b) –22

(c) 44 (d) 36

22. For what values of c in the equation $2x^2 - (c^3 + 8c - 1)x + c^2 - 4c = 0$ the roots of the equation would be opposite in signs?

(a) c Œ (0, 4) (b) c Œ (– 4, 0)

(c) c Œ (0, 3) (d) c Œ (– 4, 4)

23. The set of real values of x for which the expression $x^2 - 9x + 20$ is negative is represented by

(a) $4 < x < 5$ (b) $4 < x < 5$

(c) $x < 4$ or $x > 5$ (d) $-4 < x < 5$

24. The expression $x^2 + kx + 9$ becomes positive for what values of k (given that x is real)?

(a) $k < 6$ (b) $k > 6$

(c) $|k| < 6$ (d) $|k|$ £ 6

25. If $9^{a-2} \div 3^{a+4} = 81^{a-11}$, then find the value of $3^{a-8} + 3^{a-6}$.

(a)972 (b) 2916

(c)810 (d) 2268

26. Find the number of solutions of $a^3 + 2^{a+1} = a^4$, given that n is a natural number less than 100.

(a) 0 (b) 1

(c) 2 (d) 3

27. The number of positive integral values of x that satisfy $x^3 - 32x - 5x^2 + 64$ £ 0 is/are

(a) 4 (b) 5

(c) 6 (d) More than 6

28. Find the positive integral value of x that satisfies the equation $x^3 - 32x - 5x^2 + 64 = 0$.

(a) 5 (b) 6

(c) 7 (d) 8

29. If a, b, c are positive integers, such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{29}{72}$ and $c < b < a < 60$, how many sets of (a, b, c) exist?

(a) 3 (b) 4

(c) 5 (d) 6

30. The variables p, q, r and s are correlated with each other with the following relationship: $s^{0.5}/p = q/r^2$. The ranges of values for p, q and r are respectively:
–0.04 £ p £ – 0.03, – 0.25 £ q £ – 0.09, 1 £ r £ 7
Determine the difference between the maximum and minimum value of s?

(a) – 0.2 (b) 0.02

(c) 0.1 (d) None of these

LEVEL OF DIFFICULTY (II)

1. In the Maths Olympiad of 2020 at Animal Planet, two representatives from the donkey's side, while solving a quadratic equation, committed the following mistakes:

   (i) One of them made a mistake in the constant term and got the roots as 5 and 9.

   (ii) Another one committed an error in the coefficient of x and he got the roots as 12 and 4.

   But in the meantime, they realised that they are wrong and they managed to get it right jointly. Find the quadratic equation.

   (a) $x^2 + 4x + 14 = 0$ (b) $2x^2 + 7x - 24 = 0$

   (c) $x^2 - 14x + 48 = 0$ (d) $3x^2 - 17x + 52 = 0$

2. If the roots of the equation $a_1x^2 + a_2x + a_3 = 0$ are in the ratio $r_1 : r_2$ then

   (a) $r_1 \Diamond r_2 \Diamond a_1^2 = (r_1 + r_2)\, a_3^2$

   (b) $r_1 \Diamond r_2 \Diamond a_2^2 = (r_1 + r_2)^2\, a_1 \Diamond a_3$

   (c) $r_1 \Diamond r_2 \Diamond a_2 = (r_1 + a_2)^2\, a_1 \Diamond a_2$

   (d) $r_1 \Diamond r_2 \Diamond a_1^2 = (r_1 + r_2)^2\, a_3 \Diamond a_2$

3. For what value of a do the roots of the equation $2x^2 + 6x + a = 0$, satisfy the conditions $\left(\frac{\alpha}{\beta}\right) + \left(\frac{\beta}{\alpha}\right) < 2$.

   (a) $a < 0$ or $a > \frac{9}{2}$ (b) $a > 0$

   (c) $-1 < a < 0$ (d) $-1 < a < 1$

4. For what value of b and c would the equation $x^2 + bx + c = 0$ have roots equal to b and c.

   (a) (0, 0) (b) (1, –2)

   (c) (1, 2) (d) Both (a) and (b)

5. The sum of a fraction and its reciprocal equals 85/18. Find the fraction.

   (a) $\frac{2}{6}$ (b) $\frac{2}{3}$

   (c) $\frac{2}{9}$ (d) $\frac{4}{9}$

6. A journey between Mumbai and Pune (192 km apart) takes two hours less by a car than by a truck. Determine the average speed of the car if the average speed of the truck is 16 km/h less than the car.

   (a) 48 km/h (b) 64 km/h

(c) 16 km/h

(d) 24 km/h

7. If both the roots of the quadratic equation $ax^2 + bx + c = 0$ lie in the interval (0, 3) then a lies in

(a) (1, 3)

(b) (–1, –3)

(c) $(-\sqrt{121}/91, -\sqrt{8})$

(d) None of these

8. If the common factor of $(ax^2 + bx + c)$ and $(bx^2 + ax + c)$ is $(x + 2)$ then

(a) $a = b$, or $a + b + c = 0$

(b) $a = c$, or $a + b + c = 0$

(c) $a = b = c$

(d) $b = c, a + b + c = 0$

9. If $P = 2^{2/3} + 2^{1/3}$ then which of the following is true?

(a) $p^3 - 6p - 6 = 0$

(b) $p^3 - 6p + 6 = 0$

(c) $p^3 + 6p - 6 = 0$

(d) $p^3 + 6p + 6 = 0$

10. If $f(x) = x^2 + 2x - 5$ and $g(x) = 5x + 30$, then the roots of the quadratic equation $g[f(x)]$ will be

(a) – 1, –1

(b) 2, –1

(c) $-1 + \sqrt{2}, -1 - \sqrt{2}$

(d) 1, 2

11. If one root of the quadratic equation $ax^2 + bx + c = 0$ is three times the other, find the relationship between a, b and c.

(a) $3b^2 = 16\,ac$

(b) $b^2 = 4ac$

(c) $(a + c)^2 = 4b$

(d) $(a^2 + c^2)/ac = \frac{b}{2}$

12. If $x^2 - 3x + 2$ is a factor of $x^4 - ax^2 + b = 0$ then the values of a and b are

(a) –5, –4

(b) 5, 4

(c) –5, 4

(d) 5, –4

13. Value of the expression $(x^2 - x + 1)/(x - 1)$ cannot lie between

(a) 1, 3

(b) –1, –3

(c) 1, –3

(d) –1, 2

14. The value of p satisfying $\log_3 (p^2 + 4p + 12) = 2$ are

(a) 1, –3

(b) –1, –3

(c) – 4, 2

(d) – 4, –2

15. If $q, r > 0$ then roots of the equation $x^2 + qx - r = 0$ are

(a) Both negative

(b) Both positive

(c) Of opposite sign but equal magnitude

(d) Of opposite sign

16. If two quadratic equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ have a common root $x = 1$ then which of the following statements hold true?

(a) $a + b = -3.5$ (b) $ab = 3$

(c) $\frac{a}{b} = \frac{3}{4}$ (d) $a - b = -0.5$

(a) *A, B, C* (b) *B, C, D*

(c) *A, C, D* (d) *A, B, D*

17. If the expression $ax^2 + bx + c$ is equal to 4 when $x = 0$ leaves a remainder 4 when divided by $x + 1$ and a remainder 6 when divided by $x + 2$, then the values of a, b and c are respectively

(a) 1, 1, 4 (b) 2, 2, 4

(c) 3, 3, 4 (d) 4, 4, 4

18. If p and q are the roots of the equation $x^2 - px + q = 0$, then

(a) $p = 1, q = -2$ (b) $p = 0, q = 1$

(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

19. Sum of the real roots of the equation $x^2 + 5|x| + 6 = 0$

(a) Equals to 5 (b) Equals to 10

(c) Equals to –5 (d) None of these

20. The value of p for which the sum of the square of the roots of $2x^2 - 2(p - 2)x - p - 1 = 0$ is least is

(a) 1 (b) $\frac{3}{2}$

(c) 2 (d) –1

21. For what values of p would the equation $x^2 + 2(p - 1)x + p + 5 = 0$ possess at least one positive root?

(a) *P* Œ (–•, –5) (b) *P* Œ (–•, –1]

(c) *P* Œ (1, •) (d) *P* Œ (–5, 1)

22. If *a*, *b* Œ {1, 2, 3, 4}, then the number of the equations of the form $ax^2 + bx + 1 = 0$ having real roots is

(a) 10 (b) 7

(c) 6 (d) 12

23. If $a^2 + b^2 + c^2 = 1$, then which of the following cannot be a value of $(ab + bc + ca)$?

(a) 0 (b) 1/2

(c) $\frac{-1}{4}$ (d) –1

24. If one root of the equation $(I - m)\,x^2 + Ix + 1 = 0$ is double of the other and is real, find the greatest value of m.

(a) $\frac{9}{8}$ (b) $\frac{8}{7}$

(c) $\frac{8}{6}$ (d) $\frac{7}{5}$

25. The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite sign is

(a) (–•, 0) (b) (0, 1)

(c) (1, •) (d) (0, •)

26. One day each of Neha's friends consumed some cold drink and some orange squash. Though the quantities of cold drink and orange squash varied for the friends, the total consumption of the two liquids was exactly 9 litres for each friend. If Neha had one-ninth of the total cold drink consumed and one-eleventh of the total orange squash consumed. Find the ratio of the quantity of cold drink to that of orange squash consumed by Neha on that day?

(a) 3 : 2 (b) 5:4

(c) 2 : 1 (d) 1:1

27. $Q_1(x)$ and $Q_2(x)$ are quadratic functions such that $Q_1(10) = Q_2(8) = 0$. If the corresponding equations $Q_1(x) = 0$ and $Q_2(x) = 0$ have a common root and $Q_1(4) \times Q_2(5) = 36$, what is the value of the common root?

(a) 10 (b) 6

(c) Either 9 or 6 (d) Cannot be determined

28. For the above question if it is known that the coefficient of x^2 for $Q_1(x) = 0$ is 1/15 and that of $Q_2(x) = 0$ is 1, then which of the following options is a possible value of the sum of the roots of $Q_2(x) = 0$?

(a) 18 (b) 36

(c) 25 (d) 11

29. Which of the following could be a possible value of 'x' for which, each of the fractions is in its simplest form, where $[x]$ stands for the greatest integer less than or equal to 'x'?

$$\frac{[x]+7}{10}, \frac{[x]+18}{11}, \frac{[x]+31}{12}, \frac{[x]+46}{13}, \ldots, \frac{[x]+1489}{39} \text{ and } \frac{[x]+1567}{40}$$

(a) 95.71 (b) 93.71

(c) 94.71 (d) 92.71

30. $x - y = 8$ and $P = 7x^2 - 12y^2$, where $x, y > 0$. What is the maximum possible value of P?

(a) Infinite (b) 352.8

(c) 957.6

(d) 604.8

31. If the equations $5x + 9y + 17z = a$, $4x + 8y + 12z = b$ and $2x + 3y + 8z = c$ have at least one solution for x, y and z and a, b, and c π 0, then which of the following is true?

(a) $4a - 3b - 3c = 0$

(b) $3a - 4b - 3c = 0$

(c) $4a - 3b - 4c = 0$

(d) Nothing can be said

32. If the roots of the equation $x^3 - ax^2 + bx - 1080 = 0$ are in the ratio 2 : 4 : 5, find the value of the coefficient of x^2.

(a) 33

(b) 66

(c) –33

(d) 99

33. If the roots of the equation $px^3 - 20x^2 + 4x - 5 = 0$, where m π 0, are l, m and n, then what is the value of $\frac{1}{lm}+\frac{1}{mn}+\frac{1}{ln}$?

(a) 5

(b) –4

(c) 4

(d) 8

34. The cost of 10 pears, 8 grapes and 6 mangoes is ` 44. The cost of 5 pears, 4 grapes and 3 mangoes is ` 22. Find the cost of 4 mangoes and 3 grapes, if the cost of each of the items, in rupees, is a natural number and the cost of no two items is the same.

(a) ` 17

(b) `18

(c) ` 14

(d) Cannot be determined

35. The number of positive integral solutions to the system of equations $a_1 + a_2 + a_3 + a_4 + a_5 = 47$ and $a_1 + a_2 = 37$ is

(a) 2044

(b) 2246

(c) 2024

(d) 2376

36. If $f(x)$ is a quadratic polynomial, such that $f(5) = 75$ and $f(-5) = 55$, and $f(p) = f(q) = 0$, then find $p \times q$, given that the value of the constant term in the polynomial is 10.

(a) 2

(b) –3

(c) 5

(d) Cannot be determined

37. If $|a| + a + b = 75$ and $a + |b| - b = 150$, then what is the value of $|a| + |b|$?

(a) 105

(b) 60

(c) 90

(d) Cannot be determined

38. If $[x]$ represents the greatest integer less than or equal to x, then the value of $\left[315^{\frac{1}{3}}\right]+\left[316^{\frac{1}{3}}\right]+\left[317^{\frac{1}{3}}\right]+\dots+\left[515^{\frac{1}{3}}\right]$ is

(a) 1383

(b) 1379

(c) 1183

(d) 1351

39.

a, b, c, d and e are five consecutive integers $a < b < c < d < e$ and $a^2 + b^2 + c^2 = d^2 + e^2$. What is/are the possible value(s) of d?

(a) –2 and 10 (b) –1 and 11

(c) 0 and 12 (d) None of these

40. The 9 to 9 supermarket purchases x liters of fruit juice from the Fruit Garden Inc for a total price of \$ $4x^2$ and sells the entire x liters at a total price of \$ $10 \times (15 + 16x)$. Find the minimum amount of profit that the 9 to 9 supermarket makes in the process.

(a) 1700 (b) 1,000

(c) 1,750 (d) 1,300

ANSWER KEY

Level of Difficulty (I)

1. (c)	2. (d)	3. (c)	4. (c)
5. (d)	6. (c)	7. (d)	8. (c)
9. (b)	10. (b)	11. (c)	12. (b)
13. (d)	14. (b)	15. (b)	16. (b)
17. (c)	18. (a)	19. (c)	20. (b)
21. (b)	22. (a)	23. (b)	24. (c)
25. (c)	26. (b)	27. (d)	28. (d)
29. (d)	30. (b)		

Level of Difficulty (II)

1. (c)	2. (b)	3. (d)	4. (d)
5. (c)	6. (a)	7. (d)	8. (a)
9. (a)	10. (a)	11. (a)	12. (b)
13. (d)	14. (b)	15. (d)	16. (a)
17. (a)	18. (c)	19. (d)	20. (b)
21. (b)	22. (b)	23. (d)	24. (a)
25. (b)	26. (a)	27. (d)	28. (a)
29. (c)	30. (d)	31. (c)	32. (c)
33. (c)	34. (b)	35. (d)	36. (c)
37. (a)	38. (a)	39. (d)	40. (c)

Solutions and Shortcuts

Level of Difficulty (I)

1. For the given expression to be a maximum, the denominator should be minimized. (Since, the function in the denominator has imaginary roots and is always positive). $x^2 + 5x + 10$ will be minimized at $x = –2.5$ and its minimum values at $x = –2.5$ is 3.75.

   Hence, required answer = 1/3.75 = 4/15.

2. Has no maximum.
3. The minimum value of $(p + 1/p)$ is at $p = 1$. The value is 2.
4. The product of the roots is given by: $(a^2 + 18a + 81)/1$.

   Since product is unity we get: $a^2 + 18a + 81 = 1$

   Thus, $a^2 + 18a + 80 = 0$

   Solving, we get: $a = -10$ and $a = -8$.
5. Solve through options. LHS = RHS for $a = 1$.
6. For a, b negative the given expression will always be positive since, a^2, b^2 and ab are all positive.
7. To solve this take any expression whose roots differ by 2.

   Thus, $(x - 3)(x - 5) = 0$

   fi$x^2 - 8x + 15 = 0$

   In this case, $a = 1$, $b = -8$ and $c = 15$.

   We can see that $b^2 = 4(c + 1)$.
8. $h(x) = 4x^2 + 13x + 10$.

   Sum of roots – 13/4.
9. $x^2 + bx + 12 = 0$ has 2 as a root.

   Thus, $b = -8$.
10. Solve this by assuming each option to be true and then check whether the given expression has equal roots for the option under check.

    Thus, if we check for option (b).

    $ad = bc$.

    We assume $a = 6$, $d = 4$ $b = 12$ $c = 2$ (any set of values that satisfies $ad = bc$).

    Then $(a^2 + b^2)x^2 - 2(ac + bc)x + (c^2 + d^2) = 0$

    $180x^2 - 120x + 20 = 0$.

    We can see that this has equal roots. Thus, option (b) is a possible answer. The same way if we check for a, c and d we see that none of them gives us equal roots and can be rejected.
11. $(c + 6) = 1/2 \times 2(2c - 1)$ fi $c + 6 = 2c - 1$ fi $c = 7$
12. $-3/a = -6$ fi $a = ½$,

    $2b/a = -6$ and $a = ½$

    Gives us $b = -1.5$.

    $a + b = -1$.
13. $1/a + 1/b = (a + b)/ab$

    $= (7/5)/(1/5) = 7$.
14. $y = \sqrt{x} + \sqrt{x} + \sqrt{x} + \ldots$

    fi$y = \sqrt{x} + y$

    fi$y^2 = x + y$

    $y^2 - y - x = 0$

Solving quadratically, we have option (b) as the root of this equation.

15. The approximate value of $a = \sqrt{13.92} = 3.6$ (approx).

$a + 1/a = 3.6 + 1/3.6$ is closest to 4.

16. Solve by assuming values of a, b, and c in AP, GP and HP to check which satisfies the condition.

17. Assume any equation:

Say $x^2 – 5x + 6 = 0$

The roots are 2, 3.

We are now looking for the equation, whose roots are:

$(2 + 1/3) = 2.33$ and $(3 + 1/2) = 3.5$.

Also $a = 1$, $b = –5$ and $c = 6$.

Put these values in each option to see which gives 2.33 and 3.5 as its roots.

18. Remainder when $x^2 + ax + b$ is divided by $x – 1$ is got by putting $x = 1$ in the expression. Thus, we get.

$a + b + 1 = 5$ and

$b – a + 1 = 5$

fi$b = 4$ and $a = 0$

19. $b^2 – 4 < 0$ fi $– 2 < b < 2$

20. $(a^2 + b^2) = (a + b)^2 – 2ab$

$= (–1)^2 – 2 \times (–7) = 15.$

21. $(a^3 + b^3) = (+ b)^2 – 3 \times b(a + b)$

$= (–1)^3 – 3 \times (–7)\ (–1)$

$= –1 – 21 = –22$

22. For the roots to be opposite in sign, the product should be negative.

$(c2 – 4x)/2 < 0$ fi $0 < c < 4$.

23. The roots of the equation $x^2 – 9x + 20 = 0$ are 4 and 5. The expression would be negative for $4 < x < 5$.

24. The roots should be imaginary for the expression to be positive

i.e. $k^2 – 36 < 0$

thus $– 6 < k < 6$ or $|k| < 6$.

25. Simplifying the equation $9^{a – 2}\ 3^{a + 4} = 81^{a – 11}$ we will get: $3^{2a – 4} \div 3^{a + 4} = 3^{4a – 44}$. This gives us:

$2a–4 – a – 4 = 4a–44$ Æ $a–8 = 4a–44$ Æ $3a = 36$ Æ $a = 12$.

Hence, we have to evaluate the value of $3^4 + 3^6 = 81 + 729 = 810$. Option (c) is correct.

26. In order to think of this situation, you need to think of the fact that “the cube of a number + a power of two” (LHS of the equation) should add up to the fourth power of the same number.

The only in which situation this happens is for $8 + 8 = 16$ where $a = 2$ giving us $2^3 + 2^3 = 2^4$.

Hence, Option (b) is the correct answer.

27. The values of x, where the above expression turns out to be negative or 0 are $x = 2, 3, 4, 5, 6, 7$ or 8. Hence, Option (d) is correct.

28. The value of the LHS would become 512 – 256 – 320 + 64 = 0 when $x = 8$.

29. The above equation gets satisfied at $a = 9$, $b = 8$ and $c = 6$. (In order to visualise this, look for sets of 3 numbers with an LCM of 72). All integral multiples of (9, 8, 6) till each of the three values is lower than 60 would satisfy this equation. Thus, for instance (18, 16, 12); (27, 24, 18)… all the way up to (54, 48, 36). Thus, there are a total of 6 such sets possible.

30. The equation can be rewritten as $s^{0.5} = pq/r^2$

The maximum value of s^2 will happen when we take extreme values of p, q and r in order to support the maximisation of s^2. Taking $p = -0.4$, $q = -0.25$ and $r = 1$ we get $s^2 = ((-0.04) \times (-0.25))/1$ Æ $s^{0.5} = 0.1$

Maximum value of $s = 0.01$

Minimum value of $s = -0.01$

So, required difference is = 0.02

Level of Difficulty (II)

1. From (i) we have sum of roots = 14

and from (ii) we have product of roots = 48.

Option (c) is correct.

2. Assume the equation to be $(x - 1)(x - 2) = 0$

which gives $a_1 = 1$, $a_2 = -3$ and $a_3 = 2$ and $r_1 = 1$, $r_2 = 2$. With this information check the options.

3. $\dfrac{\alpha}{b} + \dfrac{\beta}{\alpha} < 2$ Æ $\dfrac{\alpha^2 + \beta^2}{\alpha\beta} < 2$

Æ $\dfrac{(\alpha + \beta)^2 - 2\,\alpha^2\ \beta^2}{\alpha\beta} < 2$

Use the formulae for sum of the roots and product of the roots.

4. Solve using options. It can be seen that at $b = 0$ and $c = 0$ the condition is satisfied. It is also satisfied at $b = 1$ and $c = -2$.

5. 2/9 + 9/2 = 85/18.

6. Solve using options, If the car's speed is 48 kmph, the truck's speed would be 32 kmph. The car would take 4 hours and the bus 6 hours.

7. For each of the given options it can be seen that the roots do not lie in the given interval. Thus, option (d) is correct.

8. Using $x = -2$, we get $4a - 2\,b + c = 4\,b - 2a + c = 0$.

Thus, $a = b$ and $a + b + c = 0$.

9. Use an approximation of the value of p to get the correct option. Such questions are generally not worth solving through mathematical approaches under the constraint of time in the examination.

10. $g(f((x) = 5x^2 + 10\,x + 5$

Roots are – 1 and –1.

11. Solve using options.

For option a, $3b^2 = 16\,ac$

We can assume $b = 4$, $a = 1$ and $c = 3$.

Then the equation $ax^2 + bx + c = 0$ becomes:

$x^2 + 4x + 3 = 0$ fi $x = -3$ or $x = -1$

which satisfies the given conditions.

12. $x^2 - 3x + 2 = 0$ gives its roots as $x = 1, 2$.

    Put these values in the equation and then use the options.

13. Trial and error gives us value as –1 at $x = 0$. If you try more values, you will see that you cannot get a value between –1 and 2 for this expression.

14. $p^2 + 4p + 12 = 9$

    fi$p^2 + 4p + 3 = 0$

    $p = -3$ and -1.

15. The roots would be of opposite sign as the product of roots is negative.

16. Use the value of $x = 1$ in each of the two quadratic equations to get the value of a and b respectively. With these values check the options for their validity.

17. We get $c = 4$ (by putting $x = 0$)

    Then, at $x = -1$, $a - b + 4 = 4$. So $a - b = 0$.

    At $x = -2$, $4a - 2b + 4 = 6$ fi $4a - 2b = 2$ fi $2a = 2$ fi $a = 1$, Thus, option (a) is correct.

18. Solve by checking each option for the condition given. Option c gives: $x^2 + 2x = 0$ whose roots are –2 and 0.

19. The equation is:

    $X^2 + 5x + 6 = 0$ and $X^2 - 5x + 6 = 0$

    – Sum of roots = $-5 + 5 = 0$.

20. We have to minimize : $R^2{}_1 + R^2{}_2$ or $(R_1 + R_2)^2 - 2R_1R_2$

    fi $(p - 2)^2 - 2 \times (-(p + 1)/2) = p^2 - 4p + 4 + p + 1$

    $= p^2 - 3p + 5$.

    This is minimized at $p = 1.5$. or 3/2.

21. Go through trial and error. At $p = 2$, both roots are negative. So, option (c) is rejected.

    At $p = 0$, roots are imaginary. So option (d) is rejected.

    At $p = -1$, we have both roots positive and equal.

    At $p > -1$, roots are imaginary Thus, option (b) is correct.

22. $b^2 - 4a \geq 0$ for real roots.

    If $b = 1$, no value possible for a.

    If $b = 2$, $a = 1$ is possible.

    If $b = 3$, a can be either 1 or 2 and if $b = 4$, a can be 1, 2, 3 or 4.

    Thus, we have 7 possibilities overall.

23. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

    fi $1 + 2(ab + bc + ca)$.

    Since, $(a + b + c)^2 = \geq 0$ fi $ab + bc + ca \geq -1/2$

Option (d) is not in this range and is hence not possible.

25. $p(p - 1)/3 < 0$ (Product of roots should be negative).

fi$p\ (p - 1) < 0$

$p^2 - p < 0.$

This happens for $0 < p < 1$.

Option (b) is correct.

26. Let there be 'n' friends of Neha which would mean that the amount of total liquids consumed by the group would be $9n$. Further let the total amount (in litres) of cold drink consumed by Neha and her friends be 'x' liters. Then, the amount of orange squash consumed by the friends will be $9n–x$. As per the given information, we know that Neha has consumed one-ninth of the total cold drink (i.e. $x/9$) and also that she has consumed one-eleventh of the total orange squash $(9n–x)/11$. Also, since Neha has consumed a total of 9 liters of the liquids we will get:

$$\frac{x}{9} + \frac{(9n - x)}{11} = 9$$

fi $11x + 81n - 9x = 891$

Æ $2x + 81n = 891$ Æ $x = (891 - 81n) \div 2$.

In this equation, n is an integer and x should be less than $9n$. This gives rise to the inequality:

$0 < x < 9n$ Æ $0 < (891–81n)/2 < 9n$ Æ $0 < 891 - 81n < 18n$.

The only value of n that satisfies this inequality is at $n = 10$.

This means that there were 10 friends in the group and the amount of liquids consumed in total would have been 90 liters (9 liters each). Putting this value in the equation, we get: $2x + 810 = 891$ Æ $x = 40.5$. This would mean that Neha consumes $40.5/9 = 4.5$ liters of cold drink and hence she would consume 4.5 liters of orange squash. The required ratio would be 1:1.

27. Let the common root be 'm'. Since we know that $Q_1(10) = 0$, it means that 10 would be one of the roots of Q_1. By the same logic it is given to us that 8 is one of the roots of Q_2. Using this information we have:

$Q_1 = c_1 \times (x - 10) \times (x - m)$

and $Q_2 = c_2 \times (x - 8) \times (x - m)$. Note: c_1 and c_2 are constants (each not equal to 0).

The only information we have beyond this is that the product $Q_1\ (4) \times Q_2\ (5) = 36$. However, it is evident that by replacing $x = 4$ and $x = 5$ in the expressions for Q_1 and Q_2 respectively, we would not get any conclusive value for m since the value of m would depend on the values of c_1 and c_2. You can see this happening here:

$c_1 \times -6 \times (4 - m) \times c_2 \times -3 \times (5 - m) = 36$

$c_1 \times c_2 \times (20 - 9m + m^2) = 2$

In this equation it can be clearly seen that the value of the common root 'm' would be dependent on the values of c_1 and c_2 and hence we cannot determine the answer to the question. Option (d) becomes the correct answer.

28. Since we got: $c_1 \times c_2 \times (20 - 9m + m^2) = 2$ as the equation in the previous solution, we can see that if we insert $c_1 = 1/15$ and $c_2 = 1$ in this equation we will get:

$m^2 - 9m + 20 = 30$ Æ $m^2 - 9m - 10 = 0$ Æ $(m-10)(m+1) = 0$ Æ $m = 10$ or $m = -1$. i.e., the common roots for the two equations could either be 10 or –1 giving rise to two cases for the quadratic equation $Q_2 = 0$:

Case 1: When the common root is 10; Q_2 would become Æ $(x - 8)(x - 10) = x_2 - 18x + 80$. The sum of roots for $Q_2(x) = 0$ in this case would be 18.

Case 2: When the common root is –1; Q_2 would become Æ $(x - 8)(x + 1) = x_2 - 7x - 8$. The sum of roots for $Q_1(x) = 0$ in this case would be 7.

Option (a) gives us a possible sum of roots as 18 and hence is the correct answer.

29. In order to solve this question, the first thing we need to do is to identify the pattern of the numbers in the expression. The series 7, 18, 31, 46 etc can be identified as 7, 7 + 11 × 1; 7 + 12 × 2; 7 + 13 × 3 and so on. Thus, the logic of the term when 39 is in the denominator is 7 + 39 × 38 = 1489 and the last term is 7 + 40 × 39 = 1567.

The series can be rewritten as:

$$\frac{[x]+7}{10}, 1+\frac{[x]+8}{11}, 2+\frac{[x]+7}{12}, 3+\frac{[x]+7}{13} \cdots 29+\frac{[x]+7}{39}$$

and $30 + \dfrac{[x]+7}{40}$

For each of these to be in their simplest forms, the value of $[x]$ should be such that $[x] + 7$ is co-prime to each of the 31 denominators (from 10 to 40). From amongst the options, Option (c) gives us a value such that $[x] + 7 = 101$ which is a prime number and would automatically be co-prime with the other values.

30. $x - y = 6$ $x = 6 + y$. Substituting this value of x in the expression for the value of P we get:

$P = 7(6 + y)^2 - 12y^2$

$P = 252 + 7y^2 + 84y - 12y^2 = 84y - 5y^2 + 252$

Differentiating P with respect to y and equating to zero we get:

$84 - 10y = 0$ Æ $y = 8.4$

The maximum value of P would be got by inserting $y = 8.4$ in the expression. It gives us:

$84 \times 8.4 - 5 \times 8.4^2 + 252 = 705.6 - 5 \times 70.56 + 252 = 957.6 - 352.8 = 604.8$.

31. Only in the case of Option (c) do we get the LHS of the equation $4a - 3b - 4c = 0$ such that all the x, y and z cancel each other out. Hence, Option (c) is the sole correct answer.

32. The equation can be thought of as $(x - 2m)(x - 4m)(x - 5m) = 0$. The value of the constant term would be given by $(-2m) \times (-4m) \times (-5m)$ which would give us an outcome of $-40m^3$ which is equal to – 320. Solving $-40m^3 = -1080$ Æ $m = 3$. Hence, the roots of the equation being $2m$, $4m$ and $5m$ would be 6,12 and 15 respectively. Hence, the equation would become $(x - 6)(x - 12)(x - 15) = 0$. The coefficient of x^2 would be $(-15x^2 - 6x^2 - 12x^2) = -33x^2$. Hence, the value of '$a$' would be –33. Option (c) would be the correct answer.

33. The value of the expression $\dfrac{1}{lm}+\dfrac{1}{mn}+\dfrac{1}{ln} = l\,m\,n\,(l + m + n)/l^2\,m^2\,n^2 = (l + m + n)/l\,m\,n$. For any cubic equation of the form $ax^3 + bx^2 + cx + d = 0$, the sum of the roots is given by $-b/a$; while the

product of the roots is given by $-d/a$. The ratio $(l + m + n)/l\,m\,n = b/d = -20/-5 = 4$.

Option (c) is correct.

34. When you look at this question it seems that there are two equations with three unknowns. However a closer look of the second equation shows us that the second equation is the same as the first equation-

i.e. $10p + 8g + 6m = 44$ and $5p + 4g + 3m = 22$ are nothing but one and the same equation. Hence you have only one equation with three unknowns. However, before you jump to the 'cannot be determined' answer consider this thought process.

The cost of 10 pears would be a multiple of 10 (since all costs are natural numbers). Similarly, the cost of 8 grapes would be a multiple of 8 while the cost of 6 mangoes would be a multiple of 6.

Thus, the first equation can be numerically thought of as follows :

By fixing the cost of 10 pears as a multiple of 10 and the cost of 8 grapes as a multiple of 8, we can see whether the cost of mangoes turns out to be a multiple of 6.

Total cost	*Total cost of 10 pears*	*Total cost of 8 grapes*	*Total cost of 6 mangoes*	
44	10	16	18	Possible
44	10	24	10	Not possible
44	10	32	2	Not possible
44	20	8	16	Not possible
44	20	24	0	Not possible
44	30	8	6	Not possible since both the cost of mangoes and grapes turns out to be Re 1 per unit. (They have to be distinct.)

Thus, there is only one possibility that fits into the situation. The cost per pear = Re 1. The cost per grape = ` 2 per unit and the cost per mango = ` 3 per unit. Hence, the total cost of 4 mangoes + 3 grapes = 12 + 6 = ` 18.

Option (b) is correct.

35. There are 36 ways of distributing the sum of 37 between a_1 and a_2 such that both a_1 and a_2 are positive and integral. (From 1,36; 2,35; 3,34; 4,33…; 36,1)

Similarly, there are ${}^{12}C_2$ (= 66) ways of distributing the residual value of 10 amongst a_3, a_4 and a_5. Thus, there are a total of $36 \times 66 = 2376$ ways of distributing the values amongst the five variables such that each of them is positive and integral. Option (d) is the correct answer.

36. Let the polynomial be $f(x) = ax^2 + bx + 10$.

The value of $f(5)$ in this case would be:

$f(5) = 25a + 5b + 10 = 75$

$f(-5) = 25a - 5b + 10 = 45$

$f(5) - f(-5) = 10b = 30$ Æ $b = 3$. The polynomial expression is: $ax^2 + 3x + 10$.

Further, if we put the value of $b = 3$ in the equation for $f(5)$ we would get: $25a + 15 + 10 = 75$ Æ

$25a = 50$ Æ $a = 2$.

Since $f(p)$ and $f(q)$ are both equal to zero it means that p and q are the roots of the equation $2x^2 + 3x + 10 = 0$. Finding $p \times q$ would mean that we have to find the sum of the roots of the equation. The sum of the roots would be equal to $10/2 = 5$. Option (c) is the correct answer.

37. The various possibilities for the values of a and b (in terms of their being positive or negative) would be as follows:

Possibility 1: a positive and b positive;

Possibility 2: a positive and b negative;

Possibility 3: a negative and b positive;

Possibility 4: a negative and b negative.

Let us look at each of these possibilities one by one and check out which one of them is possible.

Possibility 1: If a and b are both positive the second equation becomes $a = 150$ (since the value of $|b| - b$ would be equal to zero if b is positive). However, this value of 'a' does not fit the first equation since the value of the LHS would easily exceed 75 if we use $a = 150$ in the first equation. Hence, the possibility of both a and b being positive is not feasible and can be rejected.

Through similar thinking the possibilities 3 and 4 are also rejected. Consider this:

For possibility 3: a negative and b positive the second equation would give us $a = 150$ which contradicts the presupposition that a is negative. Hence, this possibility can be eliminated.

For possibility 4: a negative and b negative- the first equation would give us $b = 75$ (since the value of $|a| - a$ would be equal to zero if a is negative) which contradicts the presupposition that b is negative. Hence, this possibility can be eliminated.

The only possibility that remains is Possibility 2: a positive and b negative. In this case, the equations would transform as follows:

$|a| + a + b = 75$ would become $2a + b = 75$;

$a + |b| - b = 150$ would become $a - 2b = 150$.

Solving the two equations simultaneously we will get the value of $a = 60$ and $b = -45$.

The sum of $|a| + |b| = 60 + 45 = 105$

Option (a) is the correct answer.

38. The value of the expression would be dependent on the individual values of each of the terms in the expression. $[315^{1/3}]$ would give us a value of 6 and so would all the terms upto $[342^{1/3}]$. (as $6^3 = 216$ and $7^3 = 343$) Hence, the value of the expression from $[315^{1/3}] + [316^{1/3}] + \ldots + [342^{1/3}] = 28 \times 6 = 168$

Similarly, the value of the expression from $[343^{1/3}] + [344^{1/3}] + \ldots + [511^{1/3}] = 169 \times 7 = 1183$.

Also, the value of the expression from $[512^{1/3}] + [513^{1/3}] + \ldots + [515^{1/3}] = 4 \times 8 = 32$.

Thus, the answer $= 168 + 1183 + 32 = 1383$. Option (a) is correct.

39. The five consecutive integers can be represented by: $(c - 2)$; $(c - 1)$; c; $(c + 1)$ and $(c + 2)$.

Then we have $a^2 + b^2 + c^2 = d^2 + e^2$ giving us:

$(c - 2)^2 + (c - 1)^2 + c^2 = (c + 1)^2 + (c + 2)^2$ Æ

$3c^2 - 6c + 5 = 2c^2 + 6c + 5$ Æ

$c^2 - 12c = 0$ Æ

$c = 0$ or $c = 12$.

Hence, the possible values of $d = c + 1$ would be 1 or 13.

Option (d) is correct.

40. The profit of the 9 to 9 supermarket would be:

Total Sales Price – Total cost price

$= 10 \times (15 + 16x) - 4x^2$

$= -4x^2 + 160x + 150$

The minimum value of this function can be traced by differentiating it with respect to x and equating to 0. We get:

$-8x + 160 = 0$ Æ $x = 20$. The minimum value of Profit would occur at $x = 20$.

The minimum profit would be $= -4 \times 20^2 + 160 \times 20 + 150 = -1600 + 3200 + 150 = 1750$.

Option (c) is the correct answer.