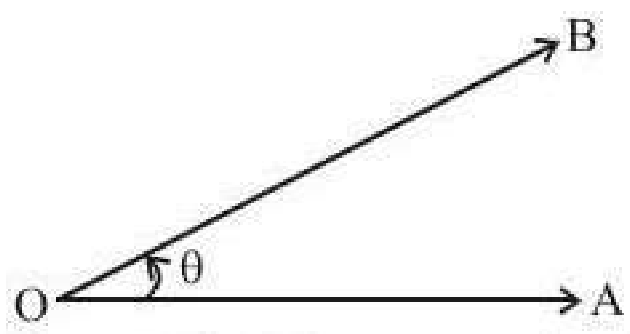
CHAPTER 16 TRIGONOMETRY

DEFINITION

In this chapter we intend to study an important branch of mathematics called '**trigonometry**'. It is the science of measuring angle of triangles, side of triangles.

Angle



Consider a ray OA if this ray rotate about its end point O and takes the position OB then we say that the angle ∠AOB has been generated.

Measure of an angle

The measure of an angle is the amount of rotation from initial side to the terminal side.

NOTE :

Relation between degree and radian measurement

π radians = 180 degree

$$\text{radian measure} = \frac{\pi}{180} \times \text{degree measure}$$

$$\text{degree measure} = \frac{180}{\pi} \times \text{radian measure}$$

1° = 60' (60 minutes)
1' = 60" (60 seconds)

Example ***1* : Find radian measure of 270°.**

Solution :

$$\text{Radian meausre} = \frac{\pi}{180} \times 270 = \frac{3\pi}{2}$$

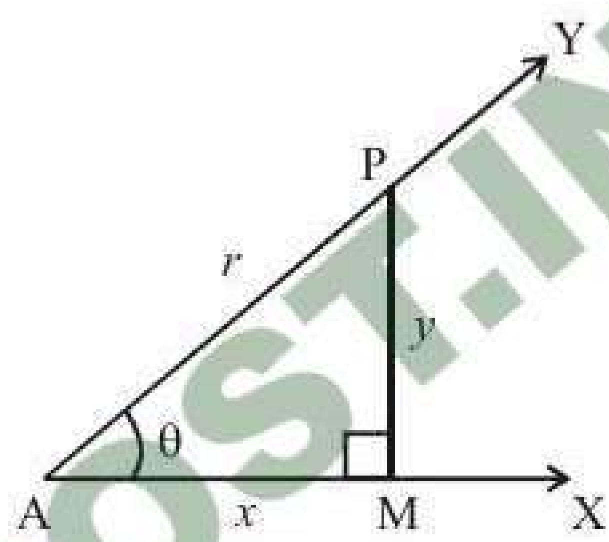
Example ***2* : Find degree measure of $\frac{5\pi}{9}$.**

Solution :

$$\text{degree measure} = \frac{180}{\pi} \times \frac{5\pi}{9} = 100°$$

TRIGONOMETRIC RATIOS

The most important task of trigonometry is to find the remaining side and angle of a triangle when some of its side and angles are given. This problem is solved by using some ratio of sides of a triangle with respect to its acute angle. These ratio of acute angle are called trigonometric ratio of angle. Let us now define various trigonometric ratio.



Consider an acute angle ∠YAX = θ with initial side AX and terminal side AY. Draw PM perpendicular from P on AX to get right angle triangle AMP. In right angle triangle AMP.

Base = AM = x
Perpendicular = PM = y and
Hypotenuse = AP = r.

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

We define the following six Trigonometric Ratios:

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{x}{r}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{r}{y}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{r}{x}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$$

IMPORTANT FORMULA

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\text{cosec}^2\theta - \cot^2\theta = 1$
3. $\sec^2\theta + \tan^2\theta = 1$
4. $\sin(90° - \theta) = \cos\theta$.
5. $\cos(90° - \theta) = \sin\theta$.
6. $\tan(90° - \theta) = \cot\theta \Rightarrow \cot(90° - \theta) = \tan\theta$.
7. $\text{cosec}(90° - \theta) = \sec\theta$.
8. $\sec(90° - \theta) = \text{cosec}\,\theta$.

9.

θ / T-ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

10. Trignometric functions is Quadrants

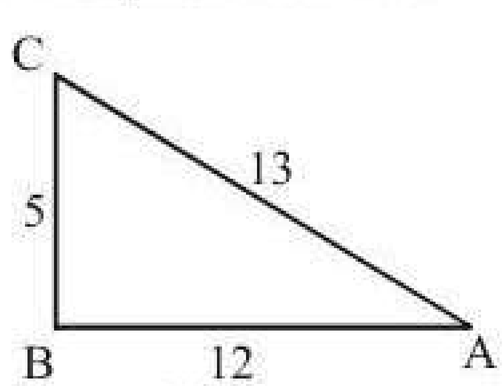
S Sine and cosecant positive II	A All functions positive I
T Tangent and cotangent positive III	C Cosine and Secant positive IV

RELATION AMONG T-RATIONS

	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
sin θ	sin θ	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\text{cosec}\theta}$
cos θ	$\sqrt{1-\sin^2\theta}$	cos θ	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{1}{\sqrt{\frac{\text{cosec}^2\theta-1}{\text{cosec}\theta}}}$
tan θ	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	tan θ	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\text{cosec}^2\theta-1}}$
cot θ	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	cot θ	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\text{cosec}^2\theta-1}$
sec θ	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1-\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	sec θ	$\frac{\text{cosec}\theta}{\sqrt{\text{cosec}^2\theta-1}}$
cosec θ	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	cosec θ

Example 3 : In a ΔABC right angled at B if AB = 12, and BC = 5 find sin A and tan A, cos C and cot C

Solution :



$AC = \sqrt{(AB)^2+(BC)^2} = \sqrt{12^2+5^2}$

$= \sqrt{144+25} = \sqrt{169} = 13$

When we consider t-ratios of $\angle A$ we have

Base AB = 12

Perpendicular = BC = 5

Hypotenuse = AC = 13

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{5}{13}$

$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{5}{12}$

When we consider t-ratios of $\angle C$, we have

Base = BC = 5

Perpendicular = AB = 12

Hypotenuse = AC = 13

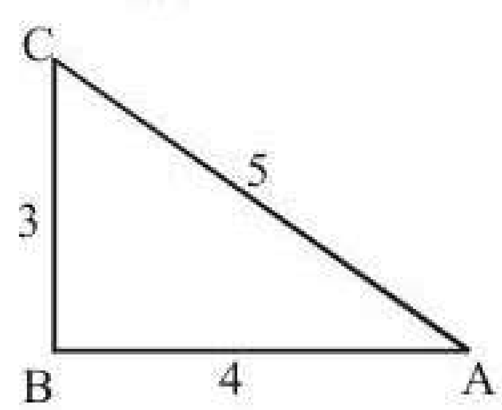
$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13}$

$\cot C = \frac{\text{Base}}{\text{Perpendicular}} = \frac{5}{12}$

Example 4 : In a right triangle ABC right angle at B if sin A $= \frac{3}{5}$ find all the six trigonometric ratios of $\angle C$

Solution :

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$



$\text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicualr})^2}$

$= \sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9} = \sqrt{16} = 4$

Now

$\sin C = \frac{BC}{AC} = \frac{4}{5}$, $\text{cosec } C = \frac{5}{4}$

$\cos C = \frac{3}{5} = \frac{AB}{AC}$, $\sec C = \frac{5}{3}$

$\tan C = \frac{AB}{AC} = \frac{4}{3}$, $\cot C = \frac{3}{4}$.

Example 5 : Find the value of 2 sin² 30° tan 60° – 3cos² 60° sec² 30°

Solution :

$$2\left(\frac{1}{2}\right)^2 \times \sqrt{3} - 3\left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 2 \times \frac{1}{4} \times \sqrt{3} - 3 \times \frac{1}{4} \times \frac{4}{3} = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3} - 2}{2}$$

Example 6 : If θ is an acute angle tan θ + cot θ = 2 find the value of tan⁷ θ + cot⁷ θ.

Solution :

$\tan\theta + \cot\theta = 2$

$\tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$(\tan\theta - 1)^2 = 0$

$\tan\theta = 1$

$\theta = 45°$

Now, $\tan^7\theta + \cot^7\theta$.

$= \tan^7 45° + \cot^7 45° = 1 + 1 = 2$

Example 7 : Find the value of $\frac{\cos 37°}{\sin 53°}$

Solution :

We have

$$\frac{\cos 37°}{\sin 53°} = \frac{\cos(90° - 53°)}{\sin 53°} = \frac{\sin 53°}{\sin 53°} = 1$$

Example 8 : Find the value of

$$\frac{\sin 36°}{\cos 54°} - \frac{\sin 54°}{\cos 36°}$$

Solution :

We have

$$= \frac{\sin 36°}{\cos 54°} - \frac{\sin 54°}{\cos 36°}$$

$$= \frac{\sin(90° - 54°)}{\cos 54°} - \frac{\sin(90° - 36°)}{\cos 36°}$$

$$= \frac{\cos 54°}{\cos 54°} - \frac{\cos 36°}{\cos 36°} = 1 - 1 = 0$$

Example 9 : Evaluate the cot 12° cot 38° cot 52° cot 60° cot 78°

Solution :

We have

$\cot 12° \cot 38° \cot 52° \cot 60° \cot 78°$

$= (\cot 12° \cot 78°)(\cot 38° \cot 52°) \cot 60°$

$= [\cot 12° \cot(90° - 12°)][\cot 38° \cot(90° - 38°)] \cot 60°$

$= [\cot 12° \tan 12°][\cot 38° \tan 38°] \cot 60°$

$= 1 \times 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

Example 10 : If tan 2θ = cot (θ + 6°), where 2θ and θ + 6° are acute angles find the value of θ.

Solution :

We have

$\tan 2\theta = \cot(\theta + 6°)$

$\cot(90° - 2\theta) = \cot(\theta + 6°)$

$90 - 2\theta = \theta + 6°$

$3\theta = 84°$

$\theta = 28°$

Example 11 : Find the value of (1 – sin² θ) sec² θ.

Solution :

We have,

$(1 - \sin^2\theta)(\sec^2\theta)$

$= \cos^2\theta \sec^2\theta$

$= \cos^2\theta \times \frac{1}{\cos^2\theta} = 1$

Example *12 :* $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$ **find its value**

Solution :

We have

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta}=\frac{2}{\cos^2\theta}=2\sec^2\theta.$$

Example *13 :* **Find the value of** $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

Solution :

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$=\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\frac{1-\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\sec\theta-\tan\theta.$$

Example *14 :* **Find the value of [(1 + cot θ) – cosec θ] [1 + tan θ + sec θ]**

Solution :

$$(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)$$

$$=\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$=\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$$

$$=\frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$=\frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$$

Example *15 :* **If** $\sin\theta=\frac{3}{5}$**, find the value of sin θ cos θ.**

Solution :

$$\sin\theta=\frac{3}{5}$$

$$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$$

$$\sin\theta\times\cos\theta=\frac{3}{5}\times\frac{4}{5}=\frac{12}{25}$$

Example *16 :* **If** $\cos\theta=\frac{1}{2}$**, find the value if** $\frac{2\sec\theta}{1+\tan^2\theta}$.

Solution :

$$\cos\theta=\frac{1}{2}$$

$$\sec\theta=2$$

$$\frac{2\sec\theta}{1+\tan^2\theta}=\frac{2\sec\theta}{\sec^2\theta}=\frac{2}{\sec\theta}=\frac{2}{2}=1$$

Example *17 :* **If** $\tan\theta=\frac{12}{5}$**, find the value of** $\frac{1+\sin\theta}{1-\sin\theta}$

Solution :

$$\tan\theta=\frac{12}{5}$$

$$\sec\theta=\sqrt{1+\tan^2\theta}$$

$$=\sqrt{1+\left(\frac{12}{5}\right)^2}=\frac{13}{5}$$

$$\cos\theta=\frac{5}{13}$$

$$\sin\theta=\sqrt{1-\cos^2\theta}=\frac{12}{13}$$

$$\text{thus }\frac{1+\sin\theta}{1-\sin\theta}=\frac{1+\frac{12}{13}}{1-\frac{12}{13}}=\frac{\frac{25}{13}}{\frac{1}{13}}=25$$

Example *18 :* **If** $\sin\theta=\frac{a}{\sqrt{a^2+b^2}}$ $0<\theta<90°$ **find the value of tan θ.**

Solution :

$$\sin\theta=\frac{a}{\sqrt{a^2+b^2}}$$

$$\cos\theta=\sqrt{1-\sin^2\theta}$$

$$\cos\theta=\sqrt{1-\frac{a^2}{a^2+b^2}}=\sqrt{\frac{b^2}{a^2+b^2}}=\frac{b}{\sqrt{a^2+b^2}}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{a}{\sqrt{a^2+b^2}}}{\frac{b}{\sqrt{a^2+b^2}}}=\frac{a}{b}$$

EXERCISE

1. $\cos 1^\circ . \cos 2^\circ . \cos 3^\circ$ .... $\cos 179^\circ$ is equal to
(a) -1 (b) 0
(c) 1 (d) $1/\sqrt{2}$

2. $\sin^2\theta + \text{cosec}^2\theta$ is always
(a) greater than 1
(b) less than 1
(c) greater than or equal to 2
(d) equal to 2

3. If $\sin\theta + \cos\theta = a$ and $\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = b$, then
(a) $b = \frac{2a}{a^2 - 1}$ (b) $a = \frac{2b}{b^2 - 1}$
(c) $ab = b^2 - 1$ (d) $a + b = 1$

4. The value of $(\sin^2 7\frac{1}{2}^\circ + \cos^2 7\frac{1}{2}^\circ) - (\sin^2 30^\circ + \cos^2 30^\circ) + (\sin^2 7^\circ + \sin^2 83^\circ)$ is equal to
(a) 3 (b) $3\frac{1}{2}$
(c) 2 (d) 1

5. If $\tan 15^\circ = 2 - \sqrt{3}$, then the value of $\cot^2 75^\circ$ is
(a) $7 + \sqrt{3}$ (b) $7 - 2\sqrt{3}$
(c) $7 - 4\sqrt{3}$ (d) $7 + 4\sqrt{3}$

6. If $x = p\sec\theta$ and $y = q\tan\theta$ then
(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$
(c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

7. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$
(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$
(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

8. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then the value of $m^2 - n^2$ is equal to
(a) $4mn$ (b) $2\sqrt{mn}$
(c) $4\sqrt{mn}$ (d) $2\sqrt{m/n}$

9. $\tan 9^\circ \times \tan 27^\circ \times \tan 63^\circ \times \tan 81^\circ =$
(a) 4 (b) 3
(c) 2 (d) 1

10. The value of $(\sin^2 1^\circ + \sin^2 3^\circ + \sin^2 5^\circ + \ldots\ldots + \sin^2 85^\circ + \sin^2 87^\circ + \sin^2 89^\circ)$
(a) $21\frac{1}{2}$ (b) 22 (c) $22\frac{1}{2}$ (d) $23\frac{1}{2}$

11. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90^\circ$, then the value of $\sin\theta + \cos\theta$ is.
(a) $\frac{17}{13}$ (b) $\frac{13}{17}$
(c) $\frac{1}{13}$ (d) $\frac{1}{17}$

12. The minimum value of $\cos 2\theta + \cos\theta$ for real values of θ is–
(a) $-9/8$ (b) 0
(c) -2 (d) None of these

13. The value of $\frac{\cos^3 20^\circ - \cos^3 70^\circ}{\sin^3 70^\circ - \sin^3 20^\circ}$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 2

14. If 5θ & 4θ are acute angles satisfying $\sin 5\theta = \cos 4\theta$ then $2\sin 3\theta - \sqrt{3}\tan 3\theta = ?$
(a) 1 (b) 0
(c) -1 (d) $\frac{1}{\sqrt{3}}$

15. If $\sec\theta + \tan\theta = x$, then $\sec\theta = ?$
(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2+1}{2x}$
(c) $\frac{x^2-1}{2x}$ (d) $\frac{x^2-1}{x}$

16. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then $a^2 + b^2 =$
(a) $m^2 - n^2$ (b) m^2n^2
(c) $n^2 - m^2$ (d) $m^2 + n^2$

17. If $\sin\alpha + \cos\beta = 2$ $(0^\circ \le \beta < \alpha \le 90^\circ)$, then $\sin\left(\frac{2\alpha+\beta}{3}\right) =$
(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{3}$
(c) $\sin\frac{\alpha}{3}$ (d) $\cos\frac{2\alpha}{3}$

18. If $A = \sin^2\theta + \cos^4\theta$, for any value of θ, then the value of A is
(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 2$
(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

19. If $x = a \sec\theta \cos\phi$, $y = b \sec\theta \sin\phi$, $z = c \tan\theta$, then, the value of $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}$ is :
(a) 1 (b) 4 (c) 9 (d) 0

20. If $0 \le \theta \le \frac{\pi}{2}$ and $\sec^2\theta + \tan^2\theta = 7$, then θ is
(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian
(c) $\frac{\pi}{5}$ radian (d) $\frac{\pi}{6}$ radian

21. If $x = a \sin\theta$ and $y = b \tan\theta$ then prove that $\frac{a^2}{x^2}-\frac{b^2}{y^2}$ is
(a) 1 (b) 2
(c) 3 (d) 4

22. If $2y \cos\theta = x \sin\theta$ and $2x \sec\theta - y \operatorname{cosec}\theta = 3$, then the relation between x and y is
(a) $2x^2 + y^2 = 2$ (b) $x^2 + 4y^2 = 4$
(c) $x^2 + 4y^2 = 1$ (d) $4x^2 + y^2 = 4$

23. If $\sec\theta + \tan\theta = \sqrt{3}$, then the positive value of sinθ is
(a) 0 (b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) 1

24. The radian measure of $63°14'51''$ is
(a) $\left(\frac{2811\pi}{8000}\right)^c$ (b) $\left(\frac{3811\pi}{8000}\right)^c$
(c) $\left(\frac{4811\pi}{8000}\right)^c$ (d) $\left(\frac{5811\pi}{8000}\right)^c$

25. If $\frac{\cos^4\alpha}{\cos^2\beta}+\frac{\sin^4\alpha}{\sin^2\beta}=1$, then the value of $\frac{\cos^4\beta}{\cos^2\alpha}+\frac{\sin^4\beta}{\sin^2\alpha}$ is
(a) 4 (b) 0 (c) $\frac{1}{8}$ (d) 1

26. If $\sin^2\alpha = \cos^3\alpha$, then the value of $(\cot^6\alpha - \cot^2\alpha)$ is
(a) 1 (b) 0
(c) –1 (d) 2

27. The value of
$$\frac{\cot 5°.\cot 10°.\cot 15°.\cot 60°.\cot 75°.\cot 80°.\cot 85°}{(\cos^2 20° + \cos^2 70°) + 2}$$ is
(a) $\frac{9}{\sqrt{3}}$ (b) $\frac{1}{9}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{9}$

28. In a triangle, the angles are in the ratio 2 : 5 : 3. What is the value of the least angle in the radian ?
(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{10}$ (c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{5}$

29. If $x = a\cos\theta - b\sin\theta$, $y = b\cos\theta + a\sin\theta$, then find the value of $x^2 + y^2$.
(a) a^2 (b) b^2
(c) $\frac{a^2}{b^2}$ (d) $a^2 + b^2$

30. The value of $\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$ is equal to
(a) –1 (b) 1
(c) 2 (d) 0

31. If $\sin 17° = \frac{x}{y}$, then $\sec 17° - \sin 73°$ is equal to
(a) $\frac{y}{\sqrt{y^2-x^2}}$ (b) $\frac{y^2}{\left(x\sqrt{y^2-x^2}\right)}$
(c) $\frac{x}{\left(y\sqrt{y^2-x^2}\right)}$ (d) $\frac{x^2}{\left(y\sqrt{y^2-x^2}\right)}$

32. If θ is a positive acute angle and $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$, then the value of cosec θ is
(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$
(c) $\frac{2}{\sqrt{3}}$ (d) 1

33. If $\cos\alpha + \sec\alpha = \sqrt{3}$, then the value of $\cos^3\alpha + \sec^3\alpha$ is
(a) 2 (b) 1
(c) 0 (d) 4

34. If x lies in the first quadrant and $\cos x = \frac{5}{13}$, what is the value of tan x – cot x?
(a) $\frac{-139}{60}$ (b) $\frac{139}{60}$
(c) $\frac{119}{60}$ (d) None of these

35. Consider the following :
I. $\frac{\cot 30° + 1}{\cot 30° - 1} = 2(\cos 30° + 1)$
II. $2\sin 45° \cos 45° - \tan 45° \cot 45° = 0$
Which of the above identities is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

36. If $0° < \theta < 90°$, then all the trigonometric ratios can be obtained when
(a) only sin θ is given
(b) only cos θ is given
(c) only tan θ is given
(d) any one of the six ratios is given

37. A wheel makes 12 revolutions per min. The angle in radian described by a spoke of the wheel in 1 s is
(a) $5\pi/2$ (b) $2\pi/5$
(c) $3\pi/5$ (d) $4\pi/5$

38. Consider the following statements for $0 \le \theta \le 90°$.
I. The value of $\sin\theta + \cos\theta$ is always greater than 1.
II. The value of $\tan\theta + \cot\theta$ is always greater than 1.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

39. Consider the following statements :
I. $\tan\theta$ increases faster than $\sin\theta$ as θ increases.
II. The value of $\sin\theta + \cos\theta$ is always greater than 1.
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

40. If $\cos x + \sec x = 2$, then what $\cos^n x + \sec^n x$ equal to, where n is a positive integer?
(a) 2 (b) 2^{n-2}
(c) 2^{n-1} (d) 2^n

41. Consider the following statements
1. $\sin 1° > \sin 1$ 2. $\cos 1° < \cos 1$
Which of the above statements is / are correct ?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

42. If $\sin x + \operatorname{cosec} x = 2$, then what is $\sin^9 x + \operatorname{cosec}^9 x$ equal to ?
(a) 2 (b) 18
(c) 512 (d) 1024

43. If $\sin x + \cos x = p$ and $\sin^3 x + \cos^3 x = q$, then what is $p^3 - 3p$ equal to ?
(a) 0 (b) $-2q$
(c) $2q$ (d) $4q$

44. If $\dfrac{3-\tan^2 A}{1-3\tan^2 A} = K$
where K is a real number, then cosec A(3 sinA – 4 sin^{3}A) is equal to **(CDS)**
(a) $\dfrac{2K}{K-1}$
(b) $\dfrac{2K}{K-1}$, where $\dfrac{1}{3} \le K \le 3$
(c) $\dfrac{2K}{K-1}$, where $K < \dfrac{1}{3}$ or $K > 3$
(d) $\dfrac{2K}{K+1}$

45. If tan A + cot A = 4
then $\tan^4 A + \cot^4 A$ is equal to **(CDS)**
(a) 110 (b) 191
(c) 80 (d) 194

46. If $p = \sqrt{\dfrac{1-\sin x}{1+\sin x}}, q = \dfrac{1-\sin x}{\cos x}, r = \dfrac{\cos x}{1+\sin x}$
then which of the following is/are correct ? **(CDS)**
1. $p = q = r$
2. $p^2 = qr$
Select the correct answer using the code given below.
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

47. Consider the following : **(CDS)**
1. $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \cos A + \sin A$
2. $(1-\sin A-\cos A)^2 = 2(1-\sin A)(1+\cos A)$
Which of the above is/are identity/identities?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

48. ABC is a triangle right angled at B and $AB : BC = 3 : 4$. What is $\sin A + \sin B + \sin C$ equal to? **(CDS)**
(a) 2 (b) $\dfrac{11}{5}$
(c) $\dfrac{12}{5}$ (d) 3

49. The value of $\operatorname{cosec}^2 67° + \sec^2 57° - \cot^2 33° - \tan^2 23°$ is **(CDS)**
(a) $2\sqrt{2}$ (b) 2
(c) $\sqrt{2}$ (d) 0

50. Consider the following statements : **(CDS)**
1. There exists at least one value of x between 0 and $\dfrac{\pi}{2}$ which satisfies the equation $\sin^4 x - 2\sin^2 x - 1 = 0$.
2. sin 1.5 is greater than cos 1.5. Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

51. If $\sin x + \cos x = c$ then $\sin^6 x + \cos^6 x$ is equal to **(CDS)**
(a) $\dfrac{1+6c^2-3c^4}{16}$ (b) $\dfrac{1+6c^2-3c^4}{4}$
(c) $\dfrac{1+6c^2+3c^4}{16}$ (d) $\dfrac{1+6c^2+3c^4}{4}$

52. Consider the following statements : **(CDS)**
1. There exists no value of x such that
$$\frac{1}{1-\sin x} = 4 + 2\sqrt{3},\quad 0 < x < \frac{\pi}{2}$$
2. $\sin x = 3^{\sin^2 x}$ does not hold good for any real x.
Which of the above statements is /are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

53. The complement angle of 80° is **(CDS)**
(a) $\dfrac{18}{\pi}$ radian (b) $\dfrac{5\pi}{9}$ radian
(c) $\dfrac{\pi}{18}$ radian (d) $\dfrac{9}{5\pi}$ radian

54. Consider the following statements : **(CDS)**

1. $\frac{1+\tan^2\theta}{1+\cot^2\theta}=\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$ is true for all $0<\theta<\frac{\pi}{2}, \theta\neq\frac{\pi}{4}$.

2. $\cot\theta=\frac{1}{\tan\theta}$ is true for $\theta=45°$ only.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

55. If $x = a\cos\theta$ and $y = b\cot\theta$, then $(ax^{-1}-by^{-1})(ax^{-1}+by^{-1})$ is equal to **(CDS)**

(a) 0 (b) 1
(c) $\tan^2\theta$ (d) $\sin^2\theta$

56. $\frac{\cos\theta}{1-\sin\theta}$ is equal to $\left(\text{where}\ \theta\neq\frac{\pi}{2}\right)$ **(CDS)**

(a) $\frac{\tan\theta-1}{\tan\theta+1}$ (b) $\frac{1+\sin\theta}{\cos\theta}$
(c) $\frac{\tan\theta+1}{\tan\theta-1}$ (d) $\frac{1+\cos\theta}{\sin\theta}$

57. If $\tan(x+40)°\tan(x+20)°\tan(3x)°\tan(70-x)°\tan(50-x)°=1$, then the value of x is equal to **(CDS)**

(a) 30 (b) 20
(c) 15 (d) 10

58. If θ is an acute angle and $\sin\theta\cos\theta = 2\cos^3\theta - 1.5\cos\theta$, then what is sin θ equal to? **(CDS)**

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{1-\sqrt{5}}{4}$
(c) $\frac{\sqrt{5}+1}{4}$ (d) $-\frac{\sqrt{5}+1}{4}$

59. Consider the following statements : **(CDS)**

1. sin66° is less than cos66°
2. sin26° is less than cos26°

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

60. If a and b are positive, then the relation $\sin\theta=\frac{2a+3b}{3b}$ is **(CDS)**

(a) not possible (b) possible only if a = b
(c) possible if a > b (d) possible if a < b

61. The minimum value of $\cos^2 x + \cos^2 y - \cos^2 z$ is **(CDS)**

(a) –1 (b) 0
(c) 1 (d) 2

62. The value of **(CDS)**

$32\cot^2\left(\frac{\pi}{4}\right)-8\sec^2\left(\frac{\pi}{3}\right)+8\cos^3\left(\frac{\pi}{6}\right)$ is equal to

(a) $\sqrt{3}$ (b) $2\sqrt{3}$
(c) 3 (d) $3\sqrt{3}$

63. If $\frac{x}{a}-\frac{y}{b}\tan\theta=1$ and $\frac{x}{a}\tan\theta+\frac{y}{b}=1$, then the value of $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ is **(CDS)**

(a) $2\sec^2\theta$ (b) $\sec^2\theta$
(c) $2\cos^2\theta$ (d) $2\cos^2\theta$

64. Consider the following : **(CDS)**

1. $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\operatorname{cosec}\theta-\cot\theta$

2. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}=\operatorname{cosec}\theta+\cot\theta$

Which of the above is/are identity identities?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

65. If $p = \cot\theta + \tan\theta$ and $q = \sec\theta - \cos\theta$, then $(p^2p)^{\frac{2}{3}} - (q^2p)^{\frac{2}{3}}$ is equal to **(CDS)**

(a) 0 (b) 1
(c) 2 (d) 3

66. Which of the following is correct in respect of the equation $3-\tan^2\theta = \alpha(1-3\tan^2\theta)$? (Given that α is a real number.) **(CDS)**

(a) $\alpha\in\left[\frac{1}{3},3\right]$ (b) $\alpha\in\left[-\infty,\frac{1}{3}\right][3,\infty]$
(c) $\alpha\in\left[-\infty,\frac{1}{3}\right][3,\infty]$ (d) None of the above

67. If $\tan\theta+\cot\theta=\frac{4}{\sqrt{3}}$, where $0<\theta<\frac{\pi}{2}$, then $\sin\theta + \cos\theta$ is equal to **(CDS)**

(a) 1 (b) $\frac{\sqrt{3}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) $\sqrt{2}$

HINTS & SOLUTIONS

1. (b) $\cos 90° = 0$
So while expression becomes 0.

2. (c)

3. (a) $(\sin\theta + \cos\theta)^2 = a^2$
$\Rightarrow 1 + 2\sin\theta\cos\theta = a^2$
$\Rightarrow 1 + 2 \times \frac{a}{b} = a^2, \therefore\ b = \frac{2a}{a^2 - 1}$

4. (d) $\sin 83° = \cos 7°$ $(\sin(90-\theta) = \cos\theta)$
$\therefore$ The given expression is $1 - 1 + 1 = 1$

5. (c) $\cot^2 75° = \left(2-\sqrt{3}\right)^3 = 7 - 4\sqrt{3}$

6. (d) We know $\sec^2\theta - \tan^2\theta = 1$ and $\sec\theta = \frac{x}{p},\ \tan\theta = \frac{y}{q}$
$\therefore\ x^2q^2 - p^2y^2 = p^2q^2$

7. (d) $\tan\theta = \frac{a}{b}$
$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2 - b^2}{a^2 + b^2}$$

8. (c) $\tan\theta + \sin\theta = m,\ n = \tan\theta - \sin\theta$
$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$
$= 4\tan\theta\sin\theta$ ... (1) $(\because (a+b)^2 - (a-b)^2 = 4ab)$
Now,
$mn = (\tan\theta + \sin\theta)(\tan\theta + \sin\theta)$
$= \tan^2 - \sin^2\theta$
$mn = \text{san}^2\theta\left(\frac{1}{\cos^2\theta} - 1\right)$
$= \sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}$
$mn = \sin^2\theta + \tan^2\theta$
$\sqrt{mn} = \sin\theta + \tan\theta$ ... (2)
from (1) and (2)
$\therefore\ m^2 - n^2 = 4\sqrt{mn}$

9. (d) $\tan 9° \times \tan 27° \times \tan(90° - 27°) \times \tan(90° - 9°)$
$= \tan 9° \times \tan 27° \times \frac{1}{\tan 9°} \times \frac{1}{\tan 9°} \times \frac{1}{\tan 27°} = 1$

10. (c) To find total number of terms
First term = 1, last term = 89, common diff = 2.
$a_n = a_1 + (n-1)d$
$89 = 1 + (n-1)^2$
$\Rightarrow\ 88 = (n-1)^2$
$\Rightarrow\ n - 1 = 44$
$\Rightarrow$ 45 terms.
Now, $\sin^2 1° + \sin^2 3° + \sin^2 5° + ... + \sin^2 85° + \sin^2 87° + \sin^2 89°$
$= (\sin^2 1° + \sin^2 89°) + (\sin^2 3° + \sin^2 87°) + ...$ 22 terms $+ \sin^2 45°$
$= (\sin^2 1° + \cos^2 1°) + (\sin^2 3° + \cos^2 3°) + ...$ 22 terms $+ \left(\frac{1}{\sqrt{2}}\right)^2$
$= (1 + 1 + ...\ 22\ \text{terms}) + \frac{1}{2}$
$= 22 + \frac{1}{2} = 22\frac{1}{2}$

11. (a) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$
$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta \cdot \cos\theta + (\sin^2\theta + \cos^2\theta) - 2\sin\theta \cdot \cos\theta.$
$= 1 + 1 = 2$
So, $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$
or, $(\sin\theta + \cos\theta)^2 + \left(\frac{7}{13}\right)^2 = 2$
or, $(\sin\theta + \cos\theta)^2 = 2 - \frac{49}{169} = \frac{289}{169}$
$\sin\theta + \cos\theta = \sqrt{\left(\frac{17}{13}\right)^2} = \frac{17}{13}$

12. (a) Let $S = \cos 2\theta + \cos\theta = 2\cos^2\theta - 1 + \cos\theta$
$= -1 + 2\left(\cos^2\theta + \frac{1}{2}\cos\theta + \frac{1}{16}\right) - \frac{1}{8}$
$= -\frac{9}{8} + 2\left(\cos\theta + \frac{1}{4}\right)^2 \geq -\frac{9}{8}$
So, the minimum value $S = -9/8$

13. (c) $\cos 20° = \cos(90° - 70°) = \sin 70°$
$\cos 70° = \sin 20°$
$$\therefore\ \frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°} = \frac{\sin^3 70° - \sin^3 20°}{\sin^3 70° - \sin^3 20°} = 1$$

14. (b) Given, $\sin 5\theta = \cos 4\theta = \sin(90° - 4\theta)$
$\Rightarrow\ 5\theta = 90° - 4\theta$
$\theta = 10°$
$2\sin 3\theta - \sqrt{3}\tan 3\theta$
$= 2\sin 30° - \sqrt{3}\tan 30°$
$= 2 \times \frac{1}{2} - \sqrt{3} \times \frac{1}{\sqrt{3}} = 1 - 1 = 0.$

15. (b) Given, $\sec\theta + \tan\theta = x$ .....(i)
$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow$ $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

or $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta} = \dfrac{1}{x}$ .....(ii)

Adding (i) & (ii), we get

$2\sec\theta = x + \dfrac{1}{x} = \dfrac{x^2+1}{x}$

$\sec\theta = \dfrac{x^2+1}{2x}$

16. (d) $(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 = m^2 + n^2$.
$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta.\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta.\cos\theta = m^2 + n^2$.

or $a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$.

or $\boxed{a^2 + b^2 = m^2 + n^2}$

17. (b) $\sin\alpha + \cos\beta = 2$
$\sin\alpha \le 1 : \cos\beta \le 2$
$\Rightarrow \alpha = 90°\,;\ \beta = 0°$

$\therefore \sin\left(\dfrac{2\alpha+\beta}{3}\right) = \sin\left(\dfrac{180°}{3}\right)$

$= \sin 60° = \dfrac{\sqrt{3}}{2}$

$\cos\dfrac{\alpha}{3} = \cos 30° = \dfrac{\sqrt{3}}{2}$

18. (b) When $\theta = 0°$
$\sin^2\theta + \cos^4\theta = 1$
When $\theta = 45°$,

$\sin^2\theta + \cos^4\theta = \dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$

When $\theta = 30°$

$\sin^2\theta + \cos^4\theta = \dfrac{1}{4} + \dfrac{9}{16} = \dfrac{13}{16}$

19. (a) $x = a\sec\theta.\cos\phi;\ y = b\sec\theta.\sin\phi;\ z = c\tan\theta$

$\therefore \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2}$

$= \dfrac{a^2\sec^2\theta\cos^2\phi}{a^2} + \dfrac{b^2\sec^2\theta\sin^2\phi}{b^2} - \dfrac{c^2\tan^2\theta}{c^2}$

$= \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi - \tan^2\theta$
$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$
$= \sec^2\theta - \tan^2\theta = 1$

20. (b) $\sec^2\theta + \tan^2\theta = 7$
$1 + \tan^2\theta + \tan^2\theta = 7$
$(\because 1 + \tan^2\theta = \sec^2\theta)$

$\tan^2\theta = \dfrac{6}{2} = 3$

$\tan\theta = \pm\sqrt{3}$

$\tan\theta = \sqrt{3}$ or $\tan\theta = -\sqrt{3}$

As $0 \le \theta \le \pi/2$

$\therefore$ $\theta = \tan^{-1}\sqrt{3}$

$\theta = \dfrac{\pi}{3}$

21. (a) $\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = \dfrac{a^2}{a^2\sin^2\theta} - \dfrac{b^2}{b^2\tan^2\theta}$

$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1$

22. (b) $2y\cos\theta = x\sin\theta$

$\Rightarrow \sin\theta = \dfrac{2y}{x}\cos\theta$

And $2x\sec\theta - y\operatorname{cosec}\theta = 3$

$\Rightarrow 2x\sec\theta - \dfrac{y}{\sin\theta} = 3$

$\Rightarrow \dfrac{2x}{\cos\theta} - \dfrac{yx}{2y\cos\theta} = 3$

$\Rightarrow 3\cos\theta = \dfrac{3}{2}x \Rightarrow \cos\theta = \dfrac{x}{2}$

Now $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow y^2 + \dfrac{x^2}{4} = 1$

$\Rightarrow 4y^2 + x^2 = 4$

23. (b) $\sec^2\theta - \tan^2\theta = 1$
$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\sqrt{3}(\sec\theta - \tan\theta) = 1 \Rightarrow \sec\theta - \tan\theta = \dfrac{1}{\sqrt{3}}$

...(1)

$\sec\theta + \tan\theta = \sqrt{3}$ (Given)
...(2)

Adding eqns. (1) and (2)

$2\sec\theta = \sqrt{3} + \dfrac{1}{\sqrt{3}} \Rightarrow 2\sec\theta = \dfrac{4}{\sqrt{3}} \Rightarrow \sec\theta = \dfrac{2}{\sqrt{3}}$

$\therefore \cos\theta = \dfrac{\sqrt{3}}{2}$ $\left[\because \sec\theta = \dfrac{1}{\cos\theta}\right]$

Therefore, $\sin\theta = \sqrt{1-\cos^2\theta}$

$\Rightarrow \sqrt{1-\dfrac{3}{4}} = \dfrac{1}{2}$

24. (a) $63°\ 14'\left(\dfrac{51}{60}\right)'$

[1 minute = 60 seconds]

$\Rightarrow 63°\left[14 + \dfrac{17}{20}\right]' \Rightarrow 63°\left[\dfrac{297}{20}\right]' \Rightarrow 63° + \dfrac{297}{20\times 60}$

[1 degree = 60 minutes]

$\Rightarrow \left(\frac{75897}{1200}\right)^\circ \Rightarrow \frac{75897}{1200} \times \frac{\pi}{180}$ radian $\Rightarrow \left(\frac{2811}{8000}\pi\right)^c$

25. (d) $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$

$\Rightarrow \cos^4\alpha \sin^2\beta + \sin^4\alpha \cos^2\beta = \cos^2\beta \sin^2\beta$

$\Rightarrow \cos^4\alpha (1 - \cos^2\beta) + \cos^2\beta (1 - \cos^2\alpha)^2 = \cos^2\beta (1 - \cos^2\beta)$

$\Rightarrow \cos^4\alpha - \cos^4\alpha \cos^2\beta + \cos^2\beta - 2\cos^2\alpha \cos^2\beta + \cos^4\alpha \cos^2\beta = \cos^2\beta - \cos^4\beta$

$\Rightarrow \cos^4\alpha - 2\cos^2\alpha \cos^2\beta + \cos^4\beta = 0$

$\Rightarrow (\cos^2\alpha - \cos^2\beta)^2 = 0$

$\Rightarrow \cos^2\alpha = \cos^2\beta$

$\Rightarrow \sin^2\alpha = \sin^2\beta$

Then, $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha} = \frac{\cos^4\beta}{\cos^2\beta} + \frac{\sin^4\beta}{\sin^2\beta} = 1$

26. (a) If $\sin^2\alpha = \cos^3\alpha$

$\tan^2\alpha = \cos\alpha$ ...(1)

Now consider, $\cot^6\alpha - \cot^2\alpha$

$= \frac{1}{\tan^6\alpha} - \frac{1}{\tan^2\alpha}$ Since $\cot\alpha = \frac{1}{\tan\alpha}$

Substituting for $\tan^2\alpha$ with $\cos\alpha$ from (1) above equation will be

$= \frac{1}{\cos^3\alpha} - \frac{1}{\cos\alpha} = \frac{1-\cos^2\alpha}{\cos^3\alpha} = \frac{\sin^2\alpha}{\cos^3\alpha} = \frac{\tan^2\alpha}{\cos\alpha} = 1$

27. (d) $\frac{\cot 5^\circ . \cot 10^\circ . \cot 15^\circ . \cot 60^\circ . \cot 75^\circ . \cot 80^\circ . \cot 85^\circ}{(\cos^2 20 + \cos^2 70^\circ) + 2}$

$\Rightarrow \frac{\cot(90^\circ - 85^\circ).\cot(90^\circ - 80^\circ).\cot(90^\circ - 75^\circ).\cot 60^\circ.\cot 75^\circ.\cot 80^\circ \cot 85^\circ}{(\cos^2(90^\circ - 70^\circ) + \cos^2 70^\circ) + 2}$

$\Rightarrow \frac{\cot 60^\circ}{(1+2)} = \frac{\frac{1}{\sqrt{3}}}{3} = \frac{1}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{9}$

28. (d) Let angles are $2x$, $5x$ and $3x$.

$2x + 5x + 3x = 180^\circ$

(sum of interior angle of triangles is 180°)

$10x = 18^\circ$

$x = 18^\circ$

$\therefore$ Least angle in degree $= 2x = 2 \times 18 = 36^\circ$

In radian $= \frac{\pi}{180^\circ} \times 36^\circ = \frac{\pi}{5}$

29. (d) $x = a\cos\theta - b\sin\theta$

$y = b\cos\theta + a\sin\theta$

$x^2 + y^2 = (a\cos\theta - b\sin\theta)^2 + (b\cos\theta + a\sin\theta)^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta + b^2\cos^2\theta + a^2\sin^2\theta + 2ab\cos\theta\sin\theta$

$\Rightarrow (a^2 + b^2)\cos^2\theta + (a^2 + b^2)\sin^2\theta$

$\Rightarrow a^2 + b^2 (\cos^2\theta + \sin^2\theta)$

$\Rightarrow a^2 + b^2. (1) \Rightarrow a^2 + b^2$

30. (c) $\frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$

$+ \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$

$= 2\cos^2\theta + 2\sin^2\theta - \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 2$

31. (d) $\sin 17^\circ = \frac{x}{y}$

$\cos 17^\circ = \sqrt{1 - \frac{x^2}{y^2}} = \frac{\sqrt{y^2 - x^2}}{y}$

$\sec 17^\circ - \sin 73^\circ$

$= \sec 17^\circ - \cos 17^\circ$

$= \frac{y}{\sqrt{y^2 - x^2}} - \frac{\sqrt{y^2 - x^2}}{y}$

$= \frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$

32. (c) $\text{cosec}\theta + \cot\theta = \sqrt{3}$

$\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \sqrt{3}$

$\frac{1 + \cos\theta}{\sin\theta} = \sqrt{3}$

$\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \sqrt{3}$

$\cot\frac{\theta}{2} = \sqrt{3}$

$\tan\frac{\theta}{2} = \frac{1}{\sqrt{3}}; \frac{\theta}{2} = 30^\circ; \theta = 60^\circ$

$\text{cosec}\,\theta = \text{cosec}\,60^\circ = \frac{2}{\sqrt{3}}$

33. (c) $\cos\alpha + \sec\alpha = \sqrt{3}$

taking cube both sides

$\cos^3\alpha + \sec^3\alpha + 3\cos\alpha\sec\alpha(\cos\alpha + \sec\alpha) = 3\sqrt{3}$

$\cos^3\alpha + \sec^3\alpha + 3\sqrt{3} = 3\sqrt{3}$

$\cos^3\alpha + \sec^3\alpha = 0$

34. (c) Given that, $\cos x = \frac{5}{13} = \frac{\text{Base}}{\text{Hypotenuse}}$

$P = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2}$

$= \sqrt{169-25} = \sqrt{144} = 12$

$\therefore \tan x - \cot x = \frac{p}{b} - \frac{b}{p}$

$= \frac{12}{5} - \frac{5}{12} = \frac{144-25}{60} = \frac{119}{60}$

35. (c) I. $\frac{\cot 30^\circ + 1}{\cot 30^\circ - 1} = 2(\cos 30^\circ + 1)$

$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2\left(\frac{\sqrt{3}}{2}+1\right)$

$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2\left(\frac{\sqrt{3}+2}{2}\right)$

$\Rightarrow \frac{3+1+2\sqrt{3}}{3-1} = \sqrt{3}+2$

$\Rightarrow \frac{4+2\sqrt{3}}{2} = \sqrt{3}+2$

$\Rightarrow \frac{2(2+\sqrt{3})}{2} = \sqrt{3}+2$

$\Rightarrow \sqrt{3}+2 = \sqrt{3}+2$

Hence, it is true.

II. $2 \sin 45^\circ \cos 45^\circ - \tan 45^\circ \cot 45^\circ = 0$

$\Rightarrow 2 \times \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) - 1 \times 1 = 0$ or $2 \times \frac{1}{2} - 1 \times 1 = 0$

$\Rightarrow 1 - 1 = 0$

Hence, both Statements I and II are true.

36. (d) If $0^\circ < \theta < 90^\circ$, then all the trigonometric ratios can be obtained when any one of the six ratios is given. Since, we use any of the following identity to get any trigonometric ratios.

$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$

and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

37. (b) In 1 min = 60 s, distance travelled by the wheel

$= 12 \times$ Its circumference

$= 12 \times 2\pi r$

$\therefore$ In 1 s distance travelled by the wheel $= \frac{12 \times 2\pi r}{60} = \frac{2}{5}\pi r$

$\because$ Angle $= \frac{\text{Distance}}{\text{Radius}} = \frac{\frac{2}{5}\pi r}{r} = \frac{2}{5}\pi$

Which is the required angle.

38. (b) Let $f(\theta) = \sin\theta + \cos\theta$

Maximum and minimum value of $a\cos\theta + b\sin\theta$ is

$-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

$\therefore -\sqrt{1+1} \le \cos\theta + \sin\theta \le \sqrt{1+1}$

$\Rightarrow -\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2}$

$\Rightarrow -1.414 \le \cos\theta + \sin\theta \le 1.414$

$\therefore f(\theta) = (\sin\theta + \cos\theta) \in [-1.414, 1.414]$

and let $g(\theta) = \tan\theta + \cot\theta = \tan\theta + \frac{1}{\tan\theta}$

($\because$ AM $\ge$ GM)

$\Rightarrow \frac{\tan\theta + \frac{1}{\tan\theta}}{2} \ge \left(\tan\theta \cdot \frac{1}{\tan\theta}\right)^{\frac{1}{2}}$

$\Rightarrow (\tan\theta + \cot\theta) \ge 2$

So, $(\tan\theta + \cot\theta)$ is always greater than 1.

Hence, Statement 1 is false and Statement II is true.

39. (a) Only Statement I is correct as $\tan\theta$ increases faster than $\sin\theta$ as θ increases while Statement II is wrong as the value of $\sin\theta + \cos\theta$ is not always greater than 1. It may also be equal to 1.

40. (a) $\cos x + \sec x = 2$ ... (i)

On squaring both sides, we get

$\cos^2 x + \sec^2 x + 2 = 4$

$\Rightarrow \cos^2 x + \sec^2 x = 2$ ... (ii)

On cubing equation (i), we get

$\cos^3 x + \sec^3 x + 3(\cos x + \sec x) = 8$

$\Rightarrow \cos^3 x + \sec^3 x + (3 \times 2) = 8$

$\Rightarrow \cos^3 x + \sec^3 x = 2$ ... (iii)

Similarly, when we multiply n in power both sides,

$\cos^n x + \sec^n x = 2$

Alternate Method

of we put $x = 0^\circ$, :

then $\cos 0^\circ + \sec 0^\circ = 1 + 1 = 2$

Similary $\operatorname{cosec}^h x + \sec^h x$

$= \operatorname{cosec}^h 0^\circ + \sec^h 0^\circ$

$= 1 + 1 = 2$

41. (d) We know that, $\sin 1^\circ > \sin 1$ and $\cos 1 < \cos 1^\circ$

Hence, neither Statement 1 nor 2 is correct.

42. (a) Here put $\theta = 90^\circ$

$\sin 90^\circ + \operatorname{cosec} 90^\circ$

$= 1 + 1 = 2$

Now, $\sin^9 x + \operatorname{cosec}^9 x = (\sin 90)^9 + (\operatorname{cosec} 90)^9$

$= 1 + 1 = 2$

43. (b) Let $\sin x + \cos x = p$ ....(i)

$\sin^3 x + \cos^3 x = q$ ....(ii)

On cubing Eq. (i) both sides

$\sin^3 x + \cos^3 x + 3\sin x\cos x(\sin x + \cos x) = p^3$

Put $\sin^3 x + \cos^3 x = q$ from equation (ii)

$\Rightarrow q + 3\sin x\cos x(p) = p^3$....(iii)

On squaring Eq. (i) both sides, we get

$\sin^2 x + \cos^2 x + 2\sin x\cos x = p^2$

$\Rightarrow \quad \sin x \cos x = \dfrac{p^2 - 1}{2} \quad [\because \sin^2 x + \cos^2 x = 1]$

From Eq. (iii),

$$q + \frac{3(p^2 - 1)p}{2} = p^3$$

$\Rightarrow \quad 2q + 3p^3 - 3p = 2p^3 \quad \Rightarrow p^3 - 3p = -2q$

44. (c) $\dfrac{3 - \tan^2 A}{1 - 3\tan^2 A} = K$

$3 - \tan^2 A = K - 3K \tan^2 A$
$3K \tan^2 A - \tan^2 A = K - 3$
$\tan^2 A\,(3K - 1) = K - 3$

$\tan^2 A = \dfrac{K - 3}{3K - 1} \quad$ ...(i)

Subject to the condition $K > 3$ or $K < \dfrac{1}{3}$.

$\text{cosec}A\,(3\sin A - 4\sin^3 A) = 3 - 4\sin^2 A$

$\cot^2 A = \dfrac{3K - 1}{K - 3}$

$\text{cosec}^2 A = \dfrac{K - 3 + 3K - 1}{K - 3} = \dfrac{4K - 4}{K - 3}$

$\sin^2 A = \dfrac{K - 3}{4(K - 1)}$

$3 - 4\sin^2 A = 3 - \dfrac{4(K - 3)}{4(K - 1)}$

$= \dfrac{3K - 3 - K + 3}{K - 1} = \dfrac{2K}{K - 1}$

where $K > 3$ or $K < \dfrac{1}{3}$.

45. (d) $\tan A + \cot A = 4$

$\Rightarrow$ Squaring both sides
$\tan^2 A + \cot^2 A + 2 = 16$
$\tan^2 A + \cot^2 A = 14$
Again, squaring both sides
$\tan^4 A + \cot^4 A + 2 = 196$
$\tan^4 A + \cot^4 A = 194$

46. (c) Statement 1

$1 = \sqrt{\dfrac{1 - \sin x}{1 + \sin x}} = \sqrt{\dfrac{(1 - \sin x)(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}}$

$= \dfrac{1 - \sin x}{\sqrt{1 - \sin^2 x}} = \dfrac{1 - \sin x}{\sqrt{\cos^2 x}} = \dfrac{1 - \sin x}{\cos x}$

$P = q$

$r = \dfrac{\cos x}{1 + \sin x} = \dfrac{\cos x(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} = \dfrac{\cos x(1 - \sin x)}{1 - \sin^2 x}$

$= \dfrac{\cos x(1 - \sin x)}{\cos^2 x} = \dfrac{1 - \sin x}{\cos x}$

$P = q = r$

Now, Statement 2, $P^2 = qr$

$= \dfrac{1 - \sin x}{\cos x} \cdot \dfrac{\cos x}{1 + \sin x} = \dfrac{1 - \sin x}{1 + \sin x} = P^2$

So, Both are correct.

47. (a) Statement 1

$\dfrac{\cos A}{1 - \tan A} + \dfrac{\sin A}{1 - \cot A}$

$\Rightarrow \dfrac{\cos A.\cos A}{\cos A - \sin A} + \dfrac{\sin A.\sin A}{\sin A - \cos A}$

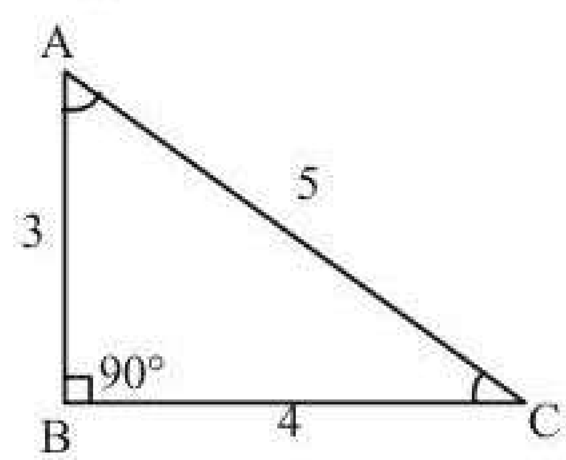
$= \dfrac{\cos^2 A - \sin^2 A}{(\cos A - \sin A)} = \cos A + \sin A.$

Statement 2

$(1 - \sin A - \cos A)^2 = 2\,(1 - \sin A)\,(1 + \cos A)$

LHS $= (1 - \sin A - \cos A)^2$
$= 1 + \sin^2 A + \cos^2 A - 2\sin A + 2\sin A \cos A - 2\cos A$
$= 2 - 2\sin A - 2\cos A + 2\sin A \cos A$
$\Rightarrow 2\,\{(1 - \sin A) - \cos A\,(1 - \sin A)\}$
$= 2(1 - \sin A)\,(1 - \cos A)$
Only 1 is correct.

48. (c)



Given $\dfrac{AB}{BC} = \dfrac{3}{4}$

$\sin A + \sin B + \sin C$

$= \dfrac{4}{5} + 1 + \dfrac{3}{5} = \dfrac{4 + 3 + 5}{5} = \dfrac{12}{5}$

49. (b) $\text{cosec}^2 67° + \sec^2 57° - \cot^2 33° - \tan^2 23°$

$= (\text{cosec}^2 67° - \tan^2 23°) + (\sec^2 57° - \cot^2 33°)$
$= (\sec^2 23° - \tan^2 23°) + (\text{cosec}^2 33° - \cot^2 33°)$
$(\because \text{cosec}\,(90 - \theta) = \sec\theta)$
$= 1 + 1$
$= 2$

50. (b) Statement (1)

$\sin^4 x - 2\sin^2 x - 1 = 0$
Let $\sin^2 x = t$
$\Rightarrow t^2 - 2t - 1 = 0$

$\Rightarrow \quad t = \dfrac{2 \pm 2\sqrt{2}}{2} \Rightarrow t = 1 \pm \sqrt{2}$

$\Rightarrow \quad \sin^2 x = 1 \pm \sqrt{2}$ i.e., $1-\sqrt{2}$ or $1+\sqrt{2}$

$\sin^2 x$ cannot be –ve.

and $\sin x$ lies between –1 and 1

So, for $0 \le x \le \pi/2$ there is no value that satisfies the equation.

$\therefore$ (1) is not true

Statement (2) sin (1.5) > cos (1.5)

1.5 radian is $1.5 \times \dfrac{180^\circ}{\pi} > 90^\circ$ in 2^{nd} quadrant.

sin (1.5) is +ve but cos(1.5) falls in second where it is –ve.

So, it is always cos (1.5) < sin (1.5).

51. (b) $\sin x + \cos x = c$ ....(i)

Squaring both sides.

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = c^2$

$\Rightarrow \sin x \cos x = \dfrac{c^2 - 1}{2}$ ...(ii)

Now, cubing eqn (i) both sides

$\Rightarrow \sin^3 x + \cos^3 x + 3\sin x \cos x (\sin x + \cos x) = c^3$

$\Rightarrow \sin^3 x + \cos^3 x + 3.\dfrac{(c^2 - 1)}{2} \times c = c^3$

$\Rightarrow \sin^3 x + \cos^3 x = c^3 - \dfrac{3}{2}(c^2 - 1)c$

$\Rightarrow \sin^3 x + \cos^3 x = c^3 - \dfrac{3c^3 + 3c}{2}$

$\sin^3 x + \cos^3 x = \dfrac{3c - c^3}{2}$ ...(iii)

On squaring both sides.

$\Rightarrow \sin^6 x + \cos^6 x + 2\sin^3 x \cos^3 x = \dfrac{(3c - c^3)^2}{4}$

$\Rightarrow \sin^6 x + \cos^6 x + 2\left\{\dfrac{(c^2 - 1)}{2}\right\}^3 = \dfrac{9c^2 + c^6 - 6c^4}{4}$

$\Rightarrow \sin^6 x + \cos^6 x$

$= \dfrac{9c^2 + c^6 - 6c^4 - c^6 + 1 + 3c^2(c^2 - 1)}{4}$

$\sin^6 x + \cos^6 x . = \dfrac{1 + 6c^2 - 3c^4}{4}$

52. (b) Statement 1

$\dfrac{1}{1 - \sin x} = 4 + 2\sqrt{3}$

$\Rightarrow 1 = 4 + 2\sqrt{3} - 4\sin x - 2\sqrt{3}\sin x$

$\Rightarrow \sin x = \dfrac{3 + 2\sqrt{3}}{4 + 2\sqrt{3}}$

$\Rightarrow \sin x = .866 < 1$

$0 < \sin x < 1$, Therefore, value of x exists between

0 to $\dfrac{\pi}{2}$

Statement 2

$\sin x = 3^{\sin^2 x}$

For example $x = 45$,

$\sin 45^\circ = 3 \sin^2 45$

$\Rightarrow \dfrac{1}{\sqrt{2}} = 3\left(\dfrac{1}{\sqrt{2}}\right)^2$; $\dfrac{1}{\sqrt{2}} = 3\left(\dfrac{1}{2}\right)$; $\dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{3}}$

So this does not hold good for any values.

So only statement 2 is true.

53. (c) Complementary angle of 80° = 90° – 80° = 10°

10° can be written as $= 10 \times \dfrac{\pi}{180}$ Rad $= \dfrac{\pi}{18}$ rad.

54. (a) (LHS) $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} = \dfrac{\sec^2\theta}{\operatorname{cosec}^2\theta}$

$= \left(\dfrac{\sec\theta}{\operatorname{cosec}\theta}\right)^2 = \left(\dfrac{\sin\theta}{\cos\theta}\right)^2$

$= \tan^2\theta$

R.H.S. $\left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2 = \left(\dfrac{1 - \tan\theta}{1 - \dfrac{1}{\tan\theta}}\right)^2$

$\left[\tan\theta\left(\dfrac{1 - \tan\theta}{\tan\theta - 1}\right)\right]^2 = (-\tan\theta)^2$

$\Rightarrow \tan^2\theta$

LHS = RHS.

$\therefore$ Statement 1 is true when $\theta \ne \dfrac{\pi}{4}$.

Statement 2

$(\cot\theta)(\tan\theta) = 1$ for all values of θ. Except when $\theta = 0, 90^\circ, 180^\circ$..... .

$\therefore$ Statement 2 is not true.

55. (b) $x = a\cos\theta,\ y = b\cot\theta$

$\Rightarrow (ax^{-1} - by^{-1})(ax^{-1} + by^{-1})$

$\Rightarrow \left(\dfrac{a}{x} - \dfrac{b}{y}\right)\left(\dfrac{a}{x} + \dfrac{b}{y}\right)$

$\Rightarrow \left(\dfrac{a}{a\cos\theta} - \dfrac{b}{b\cot\theta}\right)\left(\dfrac{a}{a\cos\theta} + \dfrac{b}{b\cot\theta}\right)$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

$\Rightarrow \sec^2\theta - \tan^2\theta$

$\Rightarrow 1$

56. (b) $\dfrac{\cos\theta}{(1-\sin\theta)}\times\dfrac{(1+\sin\theta)}{(1+\sin\theta)}$

$\Rightarrow \dfrac{\cos\theta(1+\sin\theta)}{\left(1-\sin^2\theta\right)}$

$\Rightarrow \dfrac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$\Rightarrow \dfrac{(1+\sin\theta)}{\cos\theta}$

So, option (b) is correct.

57. (c) $\tan(x+40)^\circ \tan(x+20)^\circ \tan 3x^\circ \tan(70-x)^\circ \tan(50-x)^\circ = 1$

$\Rightarrow \tan(x+40)\tan(x+20)\tan 3x \cot[90-(70-x)]$ $\cot[90-(50-x]=1$

$\Rightarrow \tan(x+40)\tan(x+20)\tan 3x \cot(x+20)\cot(x+40) = 1$

$\Rightarrow \tan 3x = \tan 45^\circ$

$\Rightarrow 3x = 45^\circ$

$\Rightarrow x = 15^\circ$

So, option (c) is correct.

58. (a) $\sin\theta\cos\theta = 2\cos^3\theta - 1.5\cos\theta$

$\sin\theta\cos\theta = [2\cos^2\theta - 1.5]\cos\theta$

$\sin\theta = 2(1-\sin^2\theta) - 1.5$

$2\sin^2\theta + \sin\theta - 0.5 = 0$

$\sin\theta = \dfrac{-1\pm\sqrt{(1)^2+4\times2\times0.5}}{4}$

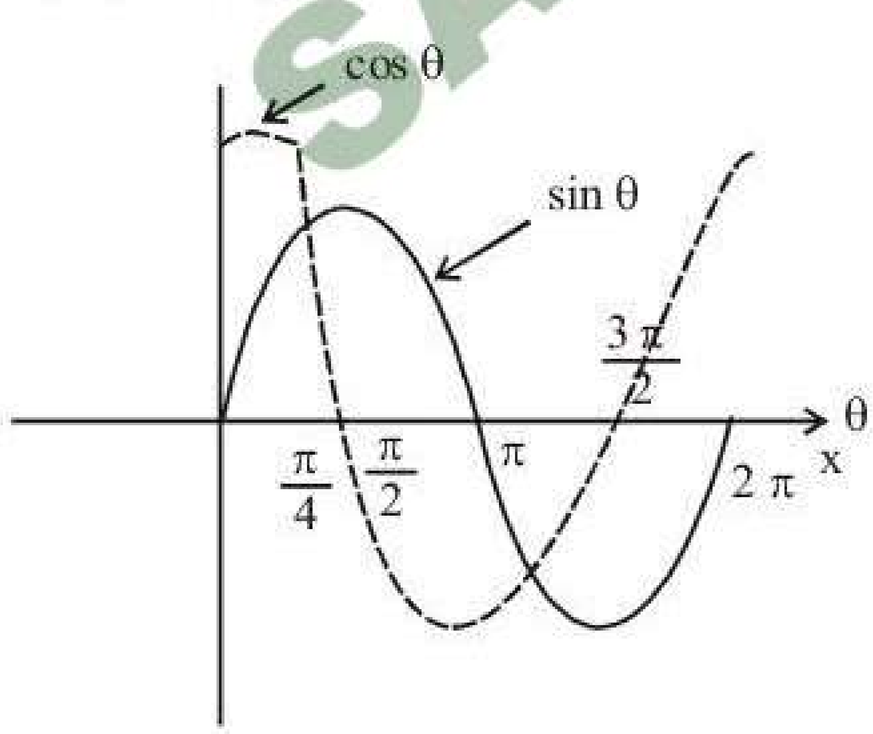
$\dfrac{-1\pm\sqrt{5}}{4}$

as $-1\le\sin\theta\le1$

$\sin\theta = \dfrac{-1\pm\sqrt{5}}{4}$

So, option (a) is correct.

59. (b)



From the graph it is clear that

$\sin\theta > \cos\theta$, when $\dfrac{\pi}{4} < \theta \le \dfrac{\pi}{2}$

and $\sin\theta < \cos\theta$, when $0 \le \theta < \dfrac{\pi}{4}$

So, option (b) is correct

60. (a) $\sin\theta = \dfrac{2a+3b}{3b}$

$\Rightarrow \sin\theta = \dfrac{2a}{3b}+1$

as a and b is positive so $\left(1+\dfrac{2a}{3b}\right)$ will be always greater than 1 that is not possible for $\sin\theta$.

61. (a) $0\le\cos^2 x\le1$ ------------(i)

$0\le\cos^2 y\le1$ ------------(ii)

$0\le\cos^2 z\le1$

$0\ge-\cos^2 z\ge-1$

$-1\le-\cos^2 z\le0$ ------------(iii)

Adding eq (i), (ii) and (iii)-

$0+0-1\le\cos^2 x+\cos^2 y-\cos^2 z\le1+1+0$

$-1\le\cos^2 x+\cos^2 y-\cos^2 z\le2$

so minimum value is -1.

62. (d) $32\cot^2\left(\dfrac{\pi}{4}\right)-8\sec^2\left(\dfrac{\pi}{3}\right)+8\cos^3\left(\dfrac{\pi}{6}\right)$

$\Rightarrow 32^2(1)^2-8(2)^2+8\left(\dfrac{\sqrt3}{2}\right)^2$

$\Rightarrow 32-32+8\times\dfrac{3\sqrt3}{8}$

$\Rightarrow 3\sqrt3$

63. (d) $\dfrac{x}{a}-\dfrac{y}{b}\tan\theta=1$. ..(1)

$\dfrac{x}{a}\tan\theta+\dfrac{y}{b}=1$ ...(2)

Multiplying (2) by $\tan\theta$ and add in (1) we get

$\dfrac{x}{a}-\dfrac{y}{b}\tan\theta=1$

$\dfrac{x}{a}\tan^2\theta+\dfrac{y}{b}\tan\theta=\tan\theta$

$\dfrac{x}{a}\left(1+\tan^2\theta\right)=1+\tan\theta$

$\Rightarrow \dfrac{x}{a}=\dfrac{1+\tan\theta}{1+\tan^2\theta}$ ...(3)

$\Rightarrow \dfrac{y}{b}=1-\dfrac{x}{a}\tan\theta$ ...(from (2))

$=1-\dfrac{(1+\tan\theta)}{1+\tan^2\theta}\tan\theta$

$\dfrac{y}{b}=\dfrac{1-\tan\theta}{1+\tan^2\theta}$ ...(4)

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{(1+\tan\theta)^2}{(1+\tan^2\theta)^2}+\frac{(1-\tan\theta)^2}{(1+\tan^2\theta)^2}$$

$$=\frac{1+\tan^2\theta+2\tan\theta+1+\tan^2\theta-2\tan\theta}{(1+\tan^2\theta)^2}$$

$$=\frac{2(1+\tan^2\theta)}{(1+\tan^2\theta)^2}=\frac{2}{1+\tan^2\theta}=\frac{2}{\sec^2\theta}=2\cos^2\theta$$

64. (c) Statement (1) $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\operatorname{cosec}\theta-\cot\theta$

$$\Rightarrow \sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}}=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \sqrt{\tan^2\theta/2}=\frac{1-\cos\theta}{\sin\theta}=\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\Rightarrow \tan\frac{\theta}{2}=\tan\frac{\theta}{2}$$

Hence, (1) is identity.

(2) Statement $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}=\operatorname{cosec}\theta+\cot\theta$

$$\sqrt{\frac{2\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}}=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}=\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\sqrt{\cot^2\frac{\theta}{2}}=\cot^2\frac{\theta}{2}$$

$$\Rightarrow \cot\frac{\theta}{2}=\cot\frac{\theta}{2}$$

Hence, (2) is also an identity

65. (b) $p=\cot\theta+\tan\theta$ $\quad q=\sec\theta-\cos\theta$

$$=\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta} \qquad =\frac{1}{\cos\theta}-\cos\theta$$

$$\Rightarrow p=\frac{1}{\sin\theta\cos\theta} \qquad q=\frac{\sin^2\theta}{\cos\theta}$$

$$\therefore (p^2q)^{\frac{2}{3}}-(q^2p)^{\frac{2}{3}}$$

$$=\left(\frac{1}{\sin^2\theta.\cos^2\theta}\times\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}}-\left(\frac{\sin^4\theta}{\cos^2\theta}\times\frac{1}{\sin\theta\cos\theta}\right)^{\frac{2}{3}}$$

$$=\left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}}-\left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}}=\frac{1}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}$$

$$=\frac{1-\sin^2\theta}{\cos^2\theta}=\frac{\cos^2\theta}{\cos^2\theta}=1$$

66. (d) $3-\tan^2\theta=\alpha(1-3\tan^2\theta)$

$$\Rightarrow \alpha=\frac{3-\tan^2\theta}{1-3\tan^2\theta}$$

$$\Rightarrow \alpha=\frac{4-4\tan^2\theta}{2-2\tan^2\theta}\ \text{[By componendo and dividendo]}$$

$$=2\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$\Rightarrow \alpha=2\cos 2\theta$$

Now $-1\le\cos 2\theta\le 1\Rightarrow -2\le 2\cos 2\theta\le 2$

67. (c) $\tan\theta+\cot\theta=\frac{4}{\sqrt{3}}$

$$\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{4}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{\sin\theta\cos\theta}=\frac{4}{\sqrt{3}}$$

$$\Rightarrow \sin\theta\cos\theta=\frac{\sqrt{3}}{4}$$

$$\Rightarrow 2\sin\theta\cos\theta=\frac{\sqrt{3}}{2}$$

$\Rightarrow \sin 2\theta=\sin 60°\Rightarrow 2\theta=60°\Rightarrow\theta=30°$

$\Rightarrow \sin\theta+\cos\theta$

$=\sin 30°+\cos 30°$

$$=\frac{1}{2}+\frac{\sqrt{3}}{2}$$

$$=\frac{\sqrt{3}+1}{2}$$