Logarithms

TALENT & OLYMPIAD

Introduction

In the previous classes we have studied that whenever a number is raised to certain power, than we write it in exponential from which tells us that how many times the number appearing in the base is being multiplied by itself and the number of times is being indicated by exponent.

Logarithms are mathematical statement which is used to answer a slightly different question for exponents whose base is a positive real number. This if a is a positive real number other than 1 and $a^x = n$, then x is called the logarithm of n to the base a, and the equation may be written as $x = \log_a^n$.

Note: If no base is mentioned in a logarithm then it is taken as Logarithms to the base 10 are known as common logarithms.

Properties of Logarithms

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_x x = 1$
- $\log_a x^n = n\log_a x$
- $\log_a x = \dfrac{1}{\log_x a}$
- $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$
- $\log_a x = \dfrac{\log_b x}{\log_b a}$

The last formula given above is also known as base changing formula.

Antilogarithms

The logarithm of a number always contains two parts which are characteristic and mantissa/The integral part is known as characteristic and the decimal part is known as mantissa. Mantissa is always kept positive. The number whose logarithm is x is called the antilogarithm of x and is denoted by antilog x.

Illustrative EXAMPLE

The value of $\log_{343} 7$ is:

(a) 0 (b) 7

(c) $\frac{1}{3}$ (d) $\frac{1}{7}$

(e) None of these

Answer (c)

Explanation let $\log_{343} 7 = x$, then $343^x = 7$

$$\Rightarrow (7^3)^x = 7 \Rightarrow 7^{3x} = 7 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$$

Illustrative EXAMPLE

If $\log 2+\frac{1}{2}\log x+\frac{1}{2}\log y=\log(x+y)$ **, then:**

(a) $x=y$ (b) $x+y=1$

(c) $x=2y$ (d) $x-y=1$

(e) None of these

Answer (a)

Explanation $\log 2+\frac{1}{2}\log x+\frac{1}{2}\log y=\log(x+y)$

$\Rightarrow \log(2\times\sqrt{x}\times\sqrt{y})=\log(x+y)$

$\Rightarrow (x-y)^2=0 \quad \Rightarrow \quad x=y$

Self Evaluation TEST

Duration 10 Minutes

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1. **What is the value of $\log_x x$?**

(a) 0 (b) 1
(c) x (d) -1
(e) None of these

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2. **If $\log 2 = 0.3010$, then what is the value of log 0.0005?**

(a) 3.302 (b) 1
(c) -3.3010 (d) 0.5
(e) None of these

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3. **log 28 is equal to:**

(a) $\log 2 + \log 5$ (b) $\log 7 - \log 2$
(c) $21\log 3$ (d) $\log 7 + 21\log 2$
(e) None of these

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4. **If $\log_3 a = 4$, then find the value of a.**

(a) 89 (b) 81
(c) 92 (d) 85
(e) None of these

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5. **Write the value of $2^{\log_2 5}$.**

(a) 5 (b) 1
(c) 0 (d) 2
(e) None of these

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6. **What is the value of $\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32$?**

(a) 0 (b) -1
(c) 1 (d) -2
(e) None of these

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7. **If $\log_x 4 = \frac{1}{4}$, then x is equal to;**

(a) 16 (b) 64
(c) 128 (d) 256 (e) None of these

8. **If $10^x = x^{50}$ then x is equal to:**

(a) 100 (b) 200

(c) $\sqrt{10}$ (d) $\sqrt{15}$

(e) None of these

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9. **The value of $\frac{\log_a x}{\log_{ab} x} - \log_a b$ is:**

(a) 0 (b) 1

(c) a (d) ab

(e) None of these

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10. **$\log_2 8 + \log_4 8 + \log_{16} 8$ equals:**

(a) $\frac{21}{4}$ (b) 5

(c) 6 (d) 4

(e) None of these

Answers – Self Evaluation Test																			
1.	B	**2.**	C	**3.**	D	**4.**	B	**5.**	A	**6.**	C	**7.**	D	**8.**	A	**9.**	B	**10.**	A

Self Evaluation Test
SOLUTIONS

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2. $\log 0.0005 = \log \frac{5}{10000} = \log \frac{1}{2000}$

$= \log(2\times 10^3)^{-1} = -\log^2 - \log 10^3 = -\log 2 - 3\log 10 = -3 - \log 2 = -3 - 0.3010 = -3.3010$

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3. $\log 28 = \log 2\times 2\times 7 = \log 2^2 \times 7 = 2\log 2 + \log 7$

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5. Let $2^{\log_2 5} = x$, then $\log_2 5 \log_2 2 = \log_2 x \Rightarrow x = 5$

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6. $\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32$

$= \frac{1}{3}\log_{10} 5^3 - \log_{10} 4^2 + \log_{10} 32$

$= \log_{10} 5 - \log_{10} 16 + \log_{10} 32$

$= \log_{10} \frac{5\times 32}{16} = \log_{10} 10 = 1$

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7. $10^x = x^{50}$, . Taking logarithms on both sides we get, $\log 10^x = \log x^{50}$

$\Rightarrow x\log 10 = 50\log x \Rightarrow x = 50\log x \Rightarrow x = 100$
