Series EF1GH/5

SET~1

प्रश्न-पत्र कोड
Q.P. Code **65/5/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*निर्धारित समय : **3** घण्टे* — *अधिकतम अंक : **80***

*Time allowed : **3** hours* — *Maximum Marks : **80***

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका पालन कीजिए :

(i) *इस प्रश्न-पत्र में कुल **38** प्रश्न हैं । **सभी** प्रश्न अनिवार्य हैं ।*

(ii) *प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – खण्ड-**क, ख, ग, घ** तथा **ङ** ।*

(iii) *खण्ड – **क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **एक-एक** अंक के प्रश्न हैं ।*

(iv) *खण्ड – **ख** में प्रश्न संख्या **21** से **25** तक अति लघु उत्तरीय प्रकार के **दो-दो** अंकों के प्रश्न हैं ।*

(v) *खण्ड – **ग** में प्रश्न संख्या **26** से **31** तक लघु उत्तरीय प्रकार के **तीन-तीन** अंकों के प्रश्न हैं ।*

(vi) *खण्ड – **घ** में प्रश्न संख्या **32** से **35** तक दीर्घ उत्तरीय प्रकार के **पाँच-पाँच** अंकों के प्रश्न हैं ।*

(vii) *खण्ड – **ङ** में प्रश्न संख्या **36** से **38** प्रकरण अध्ययन आधारित **चार-चार** अंकों के प्रश्न हैं । प्रत्येक में **एक-एक** अंक के **दो** प्रश्न तथा **दो** अंक का **एक** प्रश्न है । आंतरिक विकल्प **2** अंकों के प्रश्न में दिया गया है ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड – **ख** के **2** प्रश्नों में, खण्ड – **ग** के **3** प्रश्नों में, खण्ड – **घ** के **2** प्रश्नों में तथा खण्ड-**ङ** के **2** प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग वर्जित है ।*

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This question paper contains **38** questions. **All** questions are compulsory.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQ) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.*

(v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E** – Question Number **36** to **38** are case study based questions carrying **4** marks each where **2 VSA** type questions are of **1** mark each and **1** SA type question is of **2** marks. Internal choice is provided in **2** marks question in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, 2 questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculators is **NOT** allowed.*

खण्ड – क

(बहुविकल्पीय प्रश्न)

प्रत्येक प्रश्न का 1 अंक है ।

दिए गए चार विकल्पों में से सही विकल्प का चयन कीजिए :

1. माना A = {3, 5} है, तो A में स्वतुल्य संबंधों की संख्या है :

   (a) 2 (b) 4

   (c) 0 (d) 8

2. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ का मान है :

   (a) 1 (b) $\frac{1}{2}$

   (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

3. एक वर्ग आव्यूह A के लिए यदि $A^2 - A + I = O$ है, तो A^{-1} बराबर है :

   (a) A (b) A + I

   (c) I – A (d) A – I

4. यदि $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ तथा $A = B^2$ है, तो x बराबर है :

   (a) ± 1 (b) –1

   (c) 1 (d) 2

5. यदि $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$ है, तो α का मान है :

   (a) 1 (b) 2

   (c) 3 (d) 4

SECTION – A

(Multiple Choice Questions)

Each question carries **1** mark.

Select the correct option out of the four given options :

1. Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4

(c) 0 (d) 8

2. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

3. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals

(a) A (b) A + I

(c) I – A (d) A – I

4. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals

(a) ± 1 (b) –1

(c) 1 (d) 2

5. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

(a) 1 (b) 2

(c) 3 (d) 4

6. x^{2x} का x के सापेक्ष अवकलज है :

(a) x^{2x-1}

(b) $2x^{2x} \log x$

(c) $2x^{2x}(1 + \log x)$

(d) $2x^{2x}(1 - \log x)$

7. फलन $f(x) = [x]$, जहाँ $[x]$ महत्तम पूर्णांक जो कि x से छोटा या x के समान है, संतत है :

(a) $x = 1$ पर

(b) $x = 1.5$ पर

(c) $x = -2$ पर

(d) $x = 4$ पर

8. यदि $x = A \cos 4t + B \sin 4t$ है, तो $\frac{d^2x}{dt^2}$ बराबर है :

(a) x

(b) $-x$

(c) $16x$

(d) $-16x$

9. फलन $f(x) = 2x^3 + 9x^2 + 12x - 1$ जिस अंतराल में ह्रासमान है, वह है :

(a) $(-1, \infty)$

(b) $(-2, -1)$

(c) $(-\infty, -2)$

(d) $[-1, 1]$

10. $\int \frac{\sec x}{\sec x - \tan x} dx$ बराबर है :

(a) $\sec x - \tan x + c$

(b) $\sec x + \tan x + c$

(c) $\tan x - \sec x + c$

(d) $-(\sec x + \tan x) + c$

11. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$ का मान है :

(a) 1

(b) -1

(c) 2

(d) -2

6. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x} \log x$

(c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

7. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$ (b) $x = 1.5$

(c) $x = -2$ (d) $x = 4$

8. If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

(a) x (b) $-x$

(c) $16x$ (d) $-16x$

9. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $(-1, \infty)$ (b) $(-2, -1)$

(c) $(-\infty, -2)$ (d) $[-1, 1]$

10. $\int \frac{\sec x}{\sec x - \tan x}\, dx$ equals

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$

(c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

11. $\int_{-1}^{1} \frac{|x-2|}{x-2}\, dx,\ x \neq 2$ is equal to

(a) 1 (b) –1

(c) 2 (d) –2

12. अवकल समीकरण $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ की कोटि और घात का योगफल है :

(a) 2 (b) 3

(c) 5 (d) 0

13. दो सदिश $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ तथा $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ सरेख हैं, यदि

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

14. सदिश $6\hat{i} - 2\hat{j} + 3\hat{k}$ का परिमाण है :

(a) 1 (b) 5

(c) 7 (d) 12

15. यदि कोई रेखा x, y तथा z-अक्ष से क्रमशः 90°, 135° तथा 45° के कोण बनाती है, तो इसके दिक् कोसाइन हैं :

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

16. रेखाओं $2x = 3y = -z$ तथा $6x = -y = -4z$ के बीच का कोण है :

(a) 0° (b) 30°

(c) 45° (d) 90°

17. किन्हीं दो घटनाओं A तथा B के लिए यदि $P(A) = \frac{4}{5}$ तथा $P(A \cap B) = \frac{7}{10}$ है, तो P(B/A) बराबर है :

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

12. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is

(a) 2
(b) 3
(c) 5
(d) 0

13. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$
(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$
(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$
(d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

14. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is

(a) 1
(b) 5
(c) 7
(d) 12

15. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
(b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$
(d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

16. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 0°
(b) 30°
(c) 45°
(d) 90°

17. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to

(a) $\frac{1}{10}$
(b) $\frac{1}{8}$
(c) $\frac{7}{8}$
(d) $\frac{17}{20}$

18. पाँच अनभिनत सिक्कों को एक साथ उछाला जाता है । कम से कम एक चित आने की प्रायिकता है :

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{32}$

अभिकथन – तर्क आधारित प्रश्न

निम्नलिखित प्रश्न **19** व **20** में एक अभिकथन (A) के बाद एक तर्क कथन (R) दिया गया है । निम्न विकल्पों में से सही उत्तर चुनिए :

(a) (A) तथा (R) दोनों सत्य हैं और (R), कथन (A) की पूरी व्याख्या करता है ।

(b) (A) तथा (R) दोनों सत्य हैं, परंतु (R), कथन (A) की सही व्याख्या नहीं करता है ।

(c) (A) सत्य है और (R) सत्य नहीं है ।

(d) (A) असत्य है, जबकि (R) सत्य है ।

19. **अभिकथन (A) :** दो सिक्के एक साथ उछाले गए । यदि यह ज्ञात है कि कम से कम एक चित आया है, तो दोनों चितों के आने की प्रायिकता $\frac{1}{3}$ है ।

तर्क (R) : माना E तथा F, एक यादृच्छिक प्रयोग की दो घटनाएँ हैं, तो $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

20. **अभिकथन (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$ है ।

तर्क (R) : $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

18. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

(a) $\frac{27}{32}$

(b) $\frac{5}{32}$

(c) $\frac{31}{32}$

(d) $\frac{1}{32}$

Assertion – Reason Based Questions

In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

20. **Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$

Reason (R) : $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

खण्ड – ख

इस खण्ड में अति लघु उत्तरीय (VSA) प्रकार के प्रश्न हैं जिनमें प्रत्येक के 2 अंक हैं ।

21. निम्न फलन की मुख्य मान शाखा का प्रान्त व परिसर ज्ञात कीजिए :

$$f(x) = \tan^{-1} x$$

22. (a) यदि $f(x) = \begin{cases} x^2, & \text{यदि } x \geq 1 \\ x, & \text{यदि } x < 1 \end{cases}$ है, तो दर्शाइए कि $x = 1$ पर f अवकलनीय नहीं है ।

अथवा

(b) यदि फलन $f(x) = \begin{cases} \dfrac{\sin^2 \lambda x}{x^2}, & \text{यदि } x \neq 0 \\ 1, & \text{यदि } x = 0 \end{cases}$ $x = 0$ पर संतत है, तो 'λ' के मान ज्ञात कीजिए ।

23. रेखाओं $2x + y = 8$, $y = 2$, $y = 4$ तथा y-अक्ष द्वारा घिरे क्षेत्र को आलेखित कीजिए । अतः समाकलन के प्रयोग से इस क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

24. (a) यदि सदिश $\vec{a}$ तथा $\vec{b}$ ऐसे हैं कि $|\vec{a}| = 3$, $|\vec{b}| = \dfrac{2}{3}$ तथा $\vec{a} \times \vec{b}$ एक मात्रक सदिश है, तो $\vec{a}$ और $\vec{b}$ के बीच का कोण ज्ञात कीजिए ।

अथवा

(b) उस समांतरचतुर्भुज का क्षेत्रफल ज्ञात कीजिए जिसकी संलग्न भुजाएँ सदिशों $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ तथा $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$ द्वारा निर्धारित हैं ।

25. बिंदु A (1, 2, –1) से होकर जाने वाली तथा रेखा $5x - 25 = 14 - 7y = 35z$ के समांतर एक रेखा के सदिश व कार्तीय समीकरण ज्ञात कीजिए ।

SECTION – B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. Write the domain and range (principle value branch) of the following functions :

$f(x) = \tan^{-1} x$

22. (a) If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

OR

(b) Find the value(s) of 'λ', if the function

$$f(x) = \begin{cases} \dfrac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$ is continuous at $x = 0$.

23. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

24. (a) If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

25. Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

खण्ड – ग

इस खण्ड में लघु उत्तरीय (SA) प्रकार के प्रश्न हैं जिनमें प्रत्येक के 3 अंक हैं ।

26. यदि $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ है, तो दर्शाइए कि $A^3 - 23A - 40I = O$.

27. (a) $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ का $\sin^{-1}(2x\sqrt{1-x^2})$ के सापेक्ष अवकलन कीजिए ।

अथवा

(b) यदि $y = \tan x + \sec x$ है, तो सिद्ध कीजिए कि $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$

28. (a) मान ज्ञात कीजिए : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$

अथवा

(b) ज्ञात कीजिए : $\int \frac{x^4}{(x-1)(x^2+1)}\, dx$

29. निम्न द्वारा दर्शाए गए क्षेत्र का समाकलन से क्षेत्रफल ज्ञात कीजिए :

$\{(x, y) : y^2 \le 2x$ तथा $y \ge x - 4\}$

SECTION – C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.

27. (a) Differentiate $\sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If $y = \tan x + \sec x$, then prove that $\dfrac{d^2y}{dx^2} = \dfrac{\cos x}{(1-\sin x)^2}$.

28. (a) Evaluate : $\displaystyle\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$

OR

(b) Find : $\displaystyle\int \frac{x^4}{(x-1)(x^2+1)}\, dx$

29. Find the area of the following region using integration :

$\{(x, y) : y^2 \le 2x \text{ and } y \ge x - 4\}$

30. (a) बिंदु P(0, 2, 3) से रेखा $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ पर खींचे गए लंब के पाद के निर्देशांक ज्ञात कीजिए ।

अथवा

(b) तीन सदिश $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ इस प्रकार हैं कि $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ है । राशि $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ का मान ज्ञात कीजिए, यदि $|\vec{a}| = 3$, $|\vec{b}| = 4$ तथा $|\vec{c}| = 2$ है ।

31. निम्न रेखाओं के बीच की दूरी ज्ञात कीजिए :

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k});$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

खण्ड – घ

इस खण्ड में दीर्घ उत्तरीय (LA) प्रकार के प्रश्न हैं जिनमें प्रत्येक के 5 अंक हैं ।

32. (a) एक समबाहु त्रिभुज की माध्यिका $2\sqrt{3}$ cm/s की दर से बढ़ रही है । इसकी भुजा के बढ़ने की दर ज्ञात कीजिए ।

अथवा

(b) दो संख्याओं का योग 5 है । यदि इन संख्याओं के घनों का योगफल न्यूनतम हो, तो इनके वर्गों का योगफल ज्ञात कीजिए ।

30. (a) Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

OR

(b) Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

31. Find the distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) ;$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

SECTION – D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

33. मान ज्ञात कीजिए : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x)\, dx$

34. निम्न रैखिक प्रोग्रामन समस्या को आलेख द्वारा हल कीजिए :

व्यवरोधों : $3x + 2y \le 9$,

$3x + y \le 9$,

$x \ge 0$, $y \ge 0$ के अंतर्गत

$P = 70x + 40y$ का अधिकतम मान ज्ञात कीजिए ।

35. (a) एक बहुविकल्पी प्रश्न का उत्तर देने में एक विद्यार्थी या तो प्रश्न का उत्तर जानता है या वह अनुमान लगाता है । मान लें कि उसके उत्तर जानने की प्रायिकता $\frac{3}{5}$ है और अनुमान लगाने की प्रायिकता $\frac{2}{5}$ है । मान लें कि छात्र के प्रश्न के उत्तर का अनुमान लगाने पर सही उत्तर देने की प्रायिकता $\frac{1}{3}$ है, तो क्या प्रायिकता है कि कोई छात्र प्रश्न का उत्तर जानता है, दिया है कि उसने सही उत्तर दिया है ?

अथवा

(b) एक बक्से में 10 टिकटें हैं, जिनमें 2 पर ₹ 8 प्रति टिकट का इनाम है, 5 पर ₹ 4 प्रति टिकट का इनाम है तथा शेष 3 पर ₹ 2 प्रति टिकट का इनाम है । यदि एक टिकट यादृच्छया निकाला गया तो इनाम की राशि का माध्य ज्ञात कीजिए ।

33. Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x)\, dx$

34. Solve the following Linear Programming Problem graphically :

Maximize : $P = 70x + 40y$

subject to : $3x + 2y \leq 9,$

$3x + y \leq 9,$

$x \geq 0, y \geq 0$

35. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

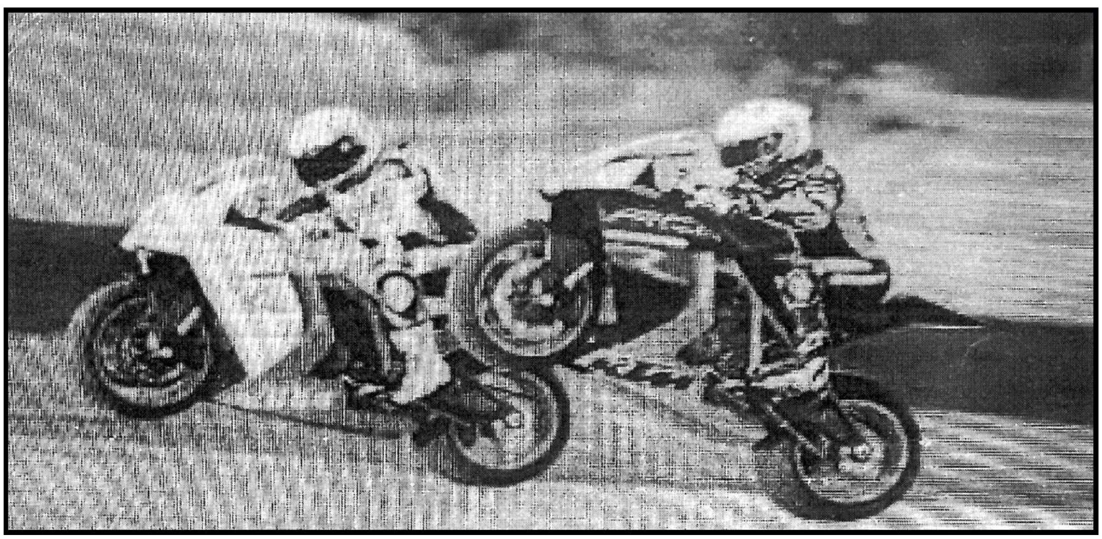
खण्ड – ङ

इस खण्ड में 3 प्रकरण/परिच्छेद आधारित प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं । प्रथम दो प्रकरण अध्ययन प्रश्नों में क्रमशः 1, 1, 2 अंकों के तीन उप-भाग (I), (II), (III) हैं । तीसरे प्रकरण अध्ययन प्रश्न में प्रत्येक 2 अंकों के दो उप-भाग हैं ।

प्रकरण अध्ययन-I

36. एक संस्था ने दो वर्गों–छात्र व छात्राओं के लिए बाईक दौड़ का आयोजन किया । कुल 28 भाग लेने वाले थे । अन्त में वर्ग 1 में से तीन तथा वर्ग 2 में दो को अंतिम दौड़ के लिए चुना गया । रवि ने अपने कॉलेज प्रोजेक्ट के लिए इन प्रतिभागियों से दो समुच्चय B और G बनाए ।

मान B = $\{b_1, b_2, b_3\}$ तथा G = $\{g_1, g_2\}$, जहाँ B अन्तिम दौड़ के लिए चुने गए छात्रों तथा G चुनी गई छात्राओं को निरूपित करते हैं



उपरोक्त के आधार पर निम्न के उत्तर दीजिए :

(I) B से G में कितने संबंध सम्भव हैं ?

(II) B से G के सभी संभव संबंधों में कितने B से G के फलन हैं ?

(III) माना $R : B \to B$, $R = \{(x, y) : x$ तथा y एक ही लिंग के हैं$\}$ द्वारा परिभाषित है । जाँच कीजिए कि क्या R एक तुल्यता संबंध है ।

अथवा

(III) यदि फलन $f : B \to G$, $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$ द्वारा परिभाषित है तो जाँच कीजिए कि क्या f एकैकी तथा आच्छादक है । अपने उत्तर का औचित्य दीजिए ।

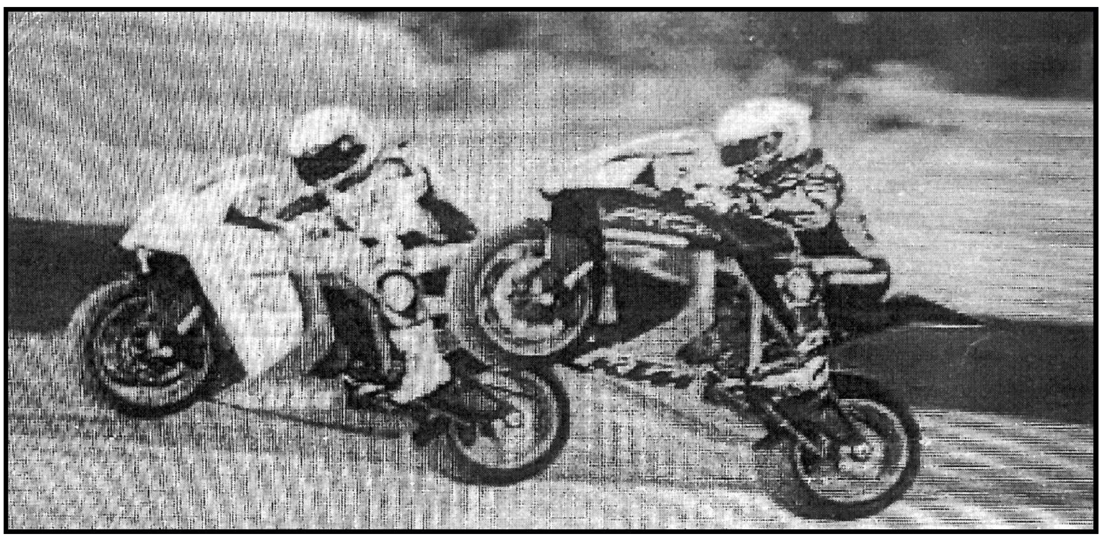
SECTION – E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub – parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub – parts (I) and (II) of marks 2 each.

Case Study-I

36. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective. Justify your answer.

प्रकरण अध्ययन-II

37. गौतम 5 पैन, 3 बैग तथा 1 उपकरण बॉक्स ₹ 160 में खरीदता है । उसी दुकान से विक्रम 2 पैन, 1 बैग तथा 3 उपकरण बॉक्स ₹ 190 में खरीदता है । अंकुर भी वहीं से 1 पैन, 2 बैग तथा 4 उपकरण बॉक्स ₹ 250 में खरीदता है ।

उपरोक्त सूचनाओं के आधार पर निम्न के उत्तर दीजिए :

(I) उपरोक्त सूचनाओं से $AX = B$ के रूप की एक आव्यूह समीकरण लिखो ।

(II) $|A|$ ज्ञात कीजिए ।

(III) A^{-1} ज्ञात कीजिए ।

अथवा

(III) $P = A^2 - 5A$ ज्ञात कीजिए ।

प्रकरण अध्ययन-III

38. एक ऐसा समीकरण जिसमें स्वतंत्र चरों के सापेक्ष आश्रित चर के अवकलज सम्मिलित हों, अवकलज समीकरण कहलाता है । $\frac{dy}{dx} = F(x, y)$ के रूप वाला अवकल समीकरण समघातीय कहलाता है यदि $F(x, y)$ शून्य घात वाला समघातीय फलन है, जहाँ फलन $F(x, y)$, n घात वाला समघातीय फलन कहलाता है यदि $F(\lambda x, \lambda y) = \lambda^n F(x, y)$ । एक समघातीय अवकल समीकरण $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$ को हल करने के लिए हम $y = vx$ प्रतिस्थापित करते हैं तथा चरों को अलग – अलग करते हैं ।

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(I) दर्शाइए कि $(x^2 - y^2)\, dx + 2xy\, dy = 0$ एक $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$ प्रकार की समघातीय अवकल समीकरण है ।

(II) उपरोक्त अवकल समीकरण का व्यापक हल ज्ञात कीजिए ।

Case Study-II

37. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions :

(I) Convert the given above situation into a matrix equation of the form AX = B.

(II) Find $|A|$.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)\, dx + 2xy\, dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
MATHEMATICS PAPER CODE 65/5/1

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/5/1)

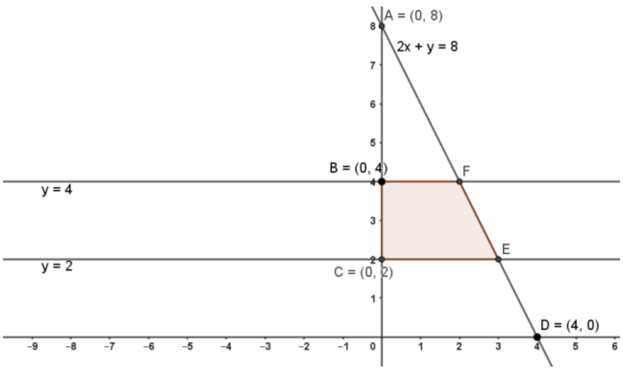
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	Let A = {3, 5}. Then number of reflexive relations on A is (a) 2 (b) 4 (c) 0 (d) 8	
Sol.	(b) 4	**1**
2.	$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$	
Sol.	(a) 1	**1**
3.	If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals (a) A (b) A + I (c) I – A (d) A – I	
Sol.	(c) I – A	**1**
4.	If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals (a) ± 1 (b) –1 (c) 1 (d) 2	
Sol.	(c) 1	**1**

5.	If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is (a) 1 (b) 2 (c) 3 (d) 4	
Sol.	(d) 4	**1**
6.	The derivative of x^{2x} w.r.t. x is (a) x^{2x-1} (b) $2x^{2x} \log x$ (c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$	
Sol.	(c) $2x^{2x}(1 + \log x)$	**1**
7.	The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at (a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$	
Sol.	(b) $x = 1 \cdot 5$	**1**
8.	If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to (a) x (b) $-x$ (c) $16x$ (d) $-16x$	
Sol.	(d) $\boldsymbol{-16\,x}$	**1**
9.	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is (a) $(-1, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -2)$ (d) $[-1, 1]$	
Sol.	(b) $\boldsymbol{(-2, -1)}$	**1**
10.	$\int \frac{\sec x}{\sec x - \tan x}\, dx$ equals (a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$ (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$	
Sol.	(b) $\boldsymbol{\sec x + \tan x + c}$	**1**

11.	$\int_{-1}^{1} \frac{\lvert x-2 \rvert}{x-2}\, dx,\ x \neq 2$ का मान है : (a) 1 (b) –1 (c) 2 (d) –2	
Sol.	(d) $-\mathbf{2}$	**1**
12.	The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is (a) 2 (b) 3 (c) 5 (d) 0	
Sol.	Due to error in the question, 1 mark should be awarded to each student who attempted the question	**1**
13.	Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if (a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ (c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$	
Sol.	(b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$	**1**
14.	The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is (a) 1 (b) 5 (c) 7 (d) 12	
Sol.	(c) 7	**1**
15.	If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	
Sol.	(a) $\mathbf{0}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	**1**

16.	The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is (a) 0° (b) 30° (c) 45° (d) 90°	
Sol.	(d) $\mathbf{90^\circ}$	**1**
17.	If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to (a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$	
Sol.	(c) $\mathbf{\frac{7}{8}}$	**1**
18.	Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is (a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$	
Sol.	(c) $\mathbf{\frac{31}{32}}$	**1**
	Assertion – Reason Based Questions In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices : (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true, but (R) is not the correct explanation of (A). (c) (A) is true and (R) is false. (d) (A) is false, but (R) is true.	
19.	**Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$. **Reason (R) :** Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.	

Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
20.	**Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$ **Reason (R) :** $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$	
Sol.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A)	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	Write the domain and range (principle value branch) of the following functions : $f(x) = \tan^{-1} x$	
Sol.	Domain = R ; Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	**1+1**
22(a).	If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.	
Sol.	Here $\left.\begin{array}{l} \text{RHD} = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h} = 2 \\ \\ \text{LHD} = \lim_{h \to 0} \left[\frac{f(1-h)-f(1)}{-h}\right] = 1 \end{array}\right\}$ Since RHD $\neq$ LHD $\therefore$ f is not differentiable at x = 1.	$1\frac{1}{2}$ $\frac{1}{2}$
22(b).	Find the value(s) of 'λ', if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.	

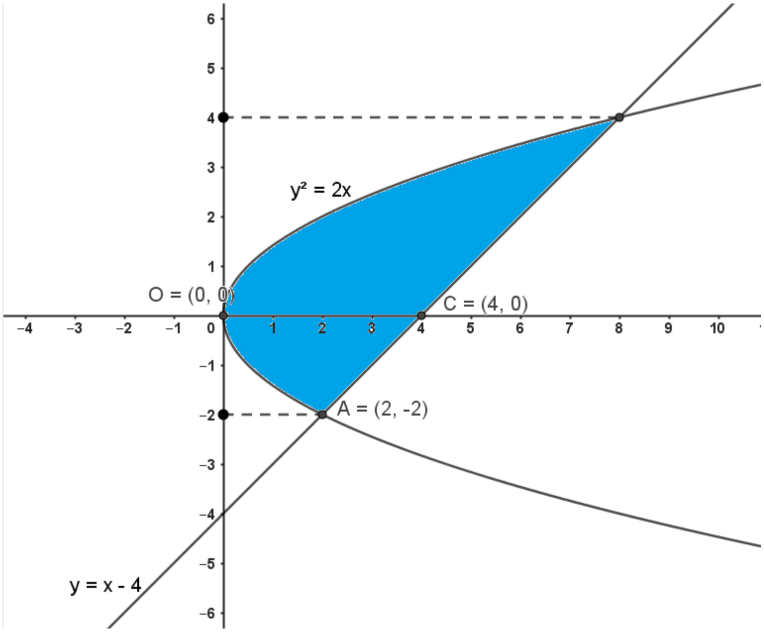
Sol.	$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{sin^2 \lambda x}{x^2}\right) = \lim_{x \to 0} \left[\frac{sin^2 \lambda x}{(\lambda x)^2} . \lambda^2\right] = \lambda^2$ Since f(x) is continuous at x = 0 $\lim_{x \to 0} f(x) = f(0)$ $\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$	**1** $\frac{1}{2}$ $\frac{1}{2}$
23.	Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.	
Sol.	 Required area = $\int_2^4 \frac{1}{2}\ (8 - y)dy$ $= \frac{1}{2} \left\|8y - \frac{y^2}{2}\right\|_2^4$ = 5	**$\frac{1}{2}$ for correct figure** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
24(a).	If the vectors $\vec{a}$ and $\vec{b}$ are such that $\|\vec{a}\| = 3$, $\|\vec{b}\| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.	

Sol.	Let θ be the angle between $\vec{a}$ and $\vec{b}$ Since $\vec{a} \times \vec{b}$ is a unit vector, we have $\lvert\vec{a} \times \vec{b}\rvert = 1$ $\Rightarrow \lvert\vec{a}\rvert\ \lvert\vec{b}\rvert \sin\theta = 1$ $\Rightarrow \sin\theta = \frac{1}{2}$, or $\theta = 30°$ (or $\frac{\pi}{6}$)	 **1** **1**
24(b).	Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.	
Sol.	Here $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$ $\Rightarrow \lvert\vec{a} \times \vec{b}\rvert = \sqrt{400 + 25 + 25} = \sqrt{450}$ **Area of parallelogram** = $\lvert\vec{a} \times \vec{b}\rvert = \sqrt{450} = 15\sqrt{2}$	 $\mathbf{1\frac{1}{2}}$ $\mathbf{\frac{1}{2}}$
25.	Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.	
Sol.	The given line is $\frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}$, or $\frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$ So, the required vector equation of the line passing through (1,2,-1) is $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$	 **1** $\mathbf{\frac{1}{2}}$

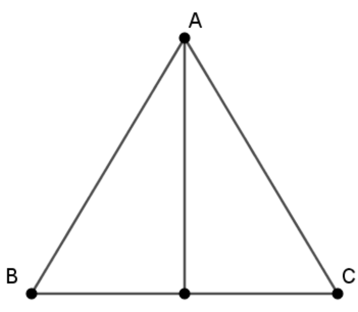
	Cartesian equation of the line is $$\frac{x-1}{7}=\frac{y-2}{-5}=\frac{z+1}{1}$$	$\frac{1}{2}$
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	If $A=\begin{bmatrix}1&2&3\\3&-2&1\\4&2&1\end{bmatrix}$, then show that $A^3-23A-40I=O$.	
Sol.	Getting $A^2=\begin{bmatrix}19&4&8\\1&12&8\\14&6&15\end{bmatrix}$ Getting $A^3=\begin{bmatrix}63&46&69\\69&-6&23\\92&46&63\end{bmatrix}$ $A^3-23A-40I=$ $\begin{bmatrix}63&46&69\\69&-6&23\\92&46&63\end{bmatrix}-\begin{bmatrix}23&46&69\\69&-46&23\\92&46&23\end{bmatrix}-\begin{bmatrix}40&0&0\\0&40&0\\0&0&40\end{bmatrix}$ $=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{0}\end{bmatrix}=\mathbf{O}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$
27(a).	Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.	
Sol.	Let $x=\sin\theta$. Then $U=\sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)=\sec^{-1}\left(\frac{1}{\cos\theta}\right)$	

	$= \sec^{-1}(\sec\theta) = \theta = \sin^{-1}x$ $\Rightarrow \frac{dU}{dx} = \frac{1}{\sqrt{1-x^2}}$ and $V = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$ $= \sin^{-1}[2\sin\theta\cos\theta] = 2\theta = 2\sin^{-1}x$ $\Rightarrow \frac{dV}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\Rightarrow \frac{dU}{dV} = \frac{dU/dx}{dV/dx} = \frac{1}{2}$ **Note: If the substitution is made as $x = \cos\theta$, answer will be $-\frac{1}{2}$**	1 1 1
27(b).	If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.	
Sol.	$y = \tan x + \sec x = \frac{\sin x + 1}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{\cos x(\cos x) + (\sin x + 1)\sin x}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1+\sin x}{1-\sin^2 x} = \frac{1}{1-\sin x}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-\sin x).0 - 1(0-\cos x)}{(1-\sin x)^2} = \frac{\cos x}{(1-\sin x)^2}$	 $1\frac{1}{2}$ $1\frac{1}{2}$
28(a).	Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$	

Sol.	Let I = $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi - x)}}\,dx$ $= \int_0^{2\pi} \frac{1}{1+e^{-\sin x}}dx = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1}dx$ $\Rightarrow 2I = \int_0^{2\pi} \frac{e^{\sin x}+1}{e^{\sin x}+1}dx = \int_0^{2\pi} 1\,.\,dx = 2\pi$ $\Rightarrow$ **I = π**	**1** **1** $\frac{1}{2}$ $\frac{1}{2}$
28(b).	Find : $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$	
Sol.	$I = \int \frac{x^4}{(x-1)(x^2+1)}dx = \int \left[x+1+\frac{1}{(x-1)(x^2+1)}\right]dx$ $= \frac{x^2}{2} + x + \int \left[\frac{1}{2(x-1)} - \frac{1}{2}\frac{(x+1)}{(x^2+1)}\right]dx$ (Using partial fractions) $= \frac{x^2}{2} + x + \frac{1}{2}\log\|x-1\| - \frac{1}{4}\log\|x^2+1\| - \frac{1}{2}\tan^{-1}x + C$	**1** $\frac{1}{2}$+1 $\frac{1}{2}$
29.	Find the area of the following region using integration : $\{(x, y) : y^2 \le 2x \text{ and } y \ge x - 4\}$	

Sol.	 Solving $y^2 = 2x$ and $y = x - 4$, we get $y = 4$ or -2 Required area = $\int_{-2}^{4} \left[(y + 4) - \frac{y^2}{2}\right] dy$ $= \left\| \frac{y^2}{2} + 4y - \frac{1}{6}y^3 \right\|_{-2}^{4}$ **= 18**	**1 mark for correct figure** $\mathbf{\frac{1}{2}}$ **1** $\mathbf{\frac{1}{2}}$
30(a).	Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.	
Sol.	General point on the given line is M $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$ Direction ratios of PM are $5\lambda - 3, 2\lambda - 1, 3\lambda - 7$	**1**

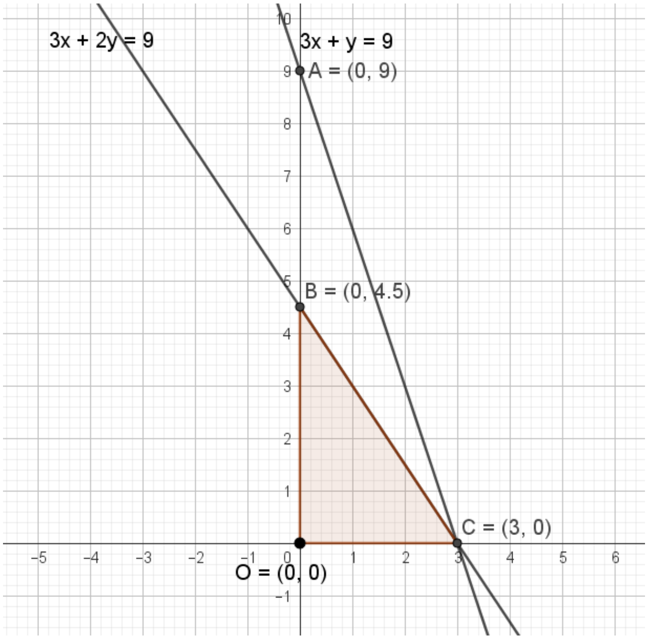
	If this point is the foot of the perpendicular from the point P (0, 2, 3), then PM is perpendicular to the line. Thus, $$(5\lambda - 3).5 + (2\lambda - 1).2 + (3\lambda - 7).3 = 0$$ $\Rightarrow \lambda = 1$ **Hence co-ordinates of M are (2, 3, – 1)**	**1** **1**
30(b).	Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = 4$ and $\lvert\vec{c}\rvert = 2$.	
Sol.	$(\vec{a} + \vec{b} + \vec{c})^2 = 0$ $\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\mu) = 0$ $\Rightarrow \mu = -\frac{29}{2}$	**1** **1** **1**
31.	Find the distance between the lines : $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$; $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$	
Sol.	Here $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$ Here, $\vec{b_1}$ and $\vec{b_2}$ are parallel vectors.	 $\frac{1}{2}$

	$\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}, \qquad \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$	$\frac{1}{2}$
	Thus, $(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$	**1**
	Distance between the lines = $\left\lvert \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}}{\lvert \vec{b} \rvert} \right\rvert$ $= \frac{\sqrt{81+196+16}}{\sqrt{4+9+36}}$ $= \frac{\sqrt{293}}{7}$ units.	**1**
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32(a).	The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.	
Sol.	In an equilateral triangle, median is same as altitude. Let 'h' denote the length of the median (or altitude) and 'x' be the side of Δ ABC.  Then, $h = \frac{\sqrt{3}}{2}x$ or $x = \frac{2h}{\sqrt{3}}$ ________ (i)	**1** **2**

	It is given that $\frac{dh}{dt} = 2\sqrt{3}$ So, by (i) we have $\frac{dx}{dt} = \frac{2}{\sqrt{3}}\frac{dh}{dt} \Rightarrow \frac{dx}{dt} = 4$ **Thus, the side of Δ ABC is increasing at the rate of 4 cm/sec.**	**1** **1**
32(b).	Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.	
Sol.	Let the two numbers be x and y. Then, x + y = 5 or y = 5 – x Let S denote the sum of the cubes of these numbers. Then $S = x^3 + y^3 = x^3 + (5 - x)^3$ $\frac{dS}{dx} = 3x^2 - 3(5-x)^2 = 15(2x - 5)$ Now $\frac{dS}{dx} = 0$, gives $x = \frac{5}{2}$ Showing S is minimum at $x = \frac{5}{2}$ So, the two numbers are $\frac{5}{2}$ and $\frac{5}{2}$ $\Rightarrow \boldsymbol{x^2 + y^2 = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}}$	$\mathbf{\frac{1}{2}}$ **1** **1** $\mathbf{\frac{1}{2}}$ **1** **1**
33.	Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x)\, dx$	
Sol.	Let I = $\int_0^{\pi/2} \sin 2x \tan^{-1} (\sin x)\, dx$.	

	$= \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1} (\sin x)\, dx$	$\frac{1}{2}$
	Put sin x = t so that cos x dx = dt	$\frac{1}{2}$
	Thus, $I = 2\int_0^1 t \tan^{-1} t\, dt$	$\frac{1}{2}$
	$= 2\left[\left\|\frac{t^2}{2} \tan^{-1} t\right\|_0^1 - \int_0^1 \frac{1}{1+t^2} \cdot \frac{t^2}{2}\, dt\right]$	1
	$= 2 \cdot \frac{1}{2} \cdot \frac{\pi}{4} - \int_0^1 \frac{t^2}{1+t^2}\, dt$	
	$= \frac{\pi}{4} - \int_0^1 \left[1 - \frac{1}{1+t^2}\right] dt$	1
	$= \frac{\pi}{4} - \| t \|_0^1 + \left\|\tan^{-1} t\right\|_0^1$	1
	$= \frac{\pi}{4} - 1 + \frac{\pi}{4}$	
	$= \frac{\pi}{2} - 1$	$\frac{1}{2}$
34.	Solve the following Linear Programming Problem graphically : Maximize : $P = 70x + 40y$ subject to : $3x + 2y \leq 9$, $3x + y \leq 9$, $x \geq 0, y \geq 0$	

Sol.



3 for correct figure and shading

Corner Points	Value of P
O (0,0)	0
B (0,4.5)	180
C (3,0)	210 → Max Value

$\mathbf{1\frac{1}{2}}$

Maximum value of P =210 at $x = 3$ and y=0

$\mathbf{\frac{1}{2}}$

35(a).	In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?	
Sol.	Let events A, B and E be defined as: A : Student knows the answer B : Student guesses the answer E : student answered correctly $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$ Here, $P\left(\frac{E}{A}\right) = 1$ and $P\left(\frac{E}{B}\right) = \frac{1}{3}$ By Bayes' Theorem $P\left(\frac{A}{E}\right) = \frac{P(A) \cdot P\left(\frac{E}{A}\right)}{P(A) \cdot P\left(\frac{E}{A}\right) + P(B) \cdot P\left(\frac{E}{B}\right)}$	**1** $\mathbf{\frac{1}{2}}$ **1**

	$= \dfrac{\frac{3}{5} \times 1}{\left(\frac{3}{5} \times 1\right) + \left(\frac{2}{5} \times \frac{1}{3}\right)}$ $= \dfrac{9}{11}$	**$1\frac{1}{2}$** **1**
35(b).	A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.	
Sol.	Let X denote the prize value. Here X can take values of 8, 4 and 2. $P(X = 8) = \frac{2}{10}$, or $\frac{1}{5}$ $P(X = 4) = \frac{5}{10}$, or $\frac{1}{2}$ $P(X = 2) = \frac{3}{10}$	**1**
	<table><tr><td>X</td><td>8</td><td>4</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{1}{5}$</td><td>$\frac{1}{2}$</td><td>$\frac{3}{10}$</td></tr><tr><td>XP(X)</td><td>$\frac{8}{5}$</td><td>$\frac{4}{2}$</td><td>$\frac{6}{10}$</td></tr></table> Hence, Mean value of X = $\sum X\, P(X) = \frac{8}{5} + 2 + \frac{6}{10}$	**3**

<table>
<tr><td></td><td>$= \frac{42}{10}$ or ₹ 4.20</td><td>1</td></tr>
<tr><td></td><td>SECTION E
This section comprises of 3 case-study based questions of 4 marks each.</td><td></td></tr>
<tr><td>36.</td><td>An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let B = {b_1, b_2, b_3} and G = {g_1, g_2}, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions :
(I) How many relations are possible from B to G ?
(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?
(III) Let R : B → B be defined by R = {(x, y) : x and y are students of the same sex}. Check if R is an equivalence relation.
OR
(III) A function f : B → G be defined by f = {(b_1, g_1), (b_2, g_2), (b_3, g_1)}.
Check if f is bijective. Justify your answer.</td><td></td></tr>
<tr><td>Sol.</td><td>(I) Number of relations = $2^6 = 64$

(II) Number of possible functions = $2^3 = 8$

(III) R is an equivalence relation as it is reflexive, symmetric and transitive</td><td>1

1

2</td></tr>
</table>

	OR Since f is not one-one function $\therefore f$ is not bijective	 **1** **1**
37.	Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250. Based on the above information, answer the following questions : (I) Convert the given above situation into a matrix equation of the form AX = B. (II) Find $\|A\|$. (III) Find A^{-1}. **OR** (III) Determine $P = A^2 - 5A$.	
Sol.	(I) Matrix equation is AX = B, where $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$ where x is the number of pens bought, y the number of bags and z the number of instrument boxes. (II) $\|A\| = 5(4-6) - 3(8-3) + 1(4-1) = -22$ (III) $\text{adj}(A) = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	 **1** **1** **1**

	$\Rightarrow A^{-1} = \frac{1}{(-22)}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$	**1**
	OR	
	$P = A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$	**1+$\frac{1}{2}$**
	$= \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$	**$\frac{1}{2}$**
38.	An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables. Based on the above, answer the following questions : (I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$. (II) Solve the above equation to find its general solution.	
Sol.	(I) $(x^2 - y^2)dx + 2xydy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$	**1**

	$= \dfrac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$	
	$= g\left(\frac{y}{x}\right)$	**1**
	(II) $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dy}{dx}$	$\frac{1}{2}$
	$v + x\dfrac{dv}{dx} = \dfrac{v^2 - 1}{2v} - v = \dfrac{-1 - v^2}{2v}$	
	$\Rightarrow \displaystyle\int \frac{2v}{1 + v^2}\,dv = -\int \frac{dx}{x}$	**1**
	$\Rightarrow \log\left\|1 + v^2\right\| + \log\left\|x\right\| = \log C$	$\frac{1}{2}$
	or $x\left(1 + \dfrac{y^2}{x^2}\right) = C$	
	or $x^2 + y^2 = Cx$	