Probability

TALENT & OLYMPIAD

Introduction

The word 'probability" is one of the most commonly used word in our day to day life. Like probably today will raining, probably India will win the world cup etc. In mathematics, the concept of probability originated in the beginning of eighteenth century in problem of game of chance. Probability is a concept which numerically represents the degree of certainly of the event. Now a days it is widely used as the basic tools of statistics. Science and Engineering.

Terms Related to Probability

The following are the terms related to describe the concept of probability:

🖡 🛛 Event

The outcomes of a random experiment is called its elementary event. When one coin is tossed, H and T is possible outcome. That is why Getting head is an elementary event. When two coins are tossed, the sample space of this experiment is {HH, HT, TH, TT} Events are HH, HT, TH and TT are the events of random experiment of tossing two coins.

Compound Events

When two or more than two elementary events occur with a random experiment, it s said to be Compound Event. When we throw a dice and getting odd number is a compound event.

Equally Likely Events

A given number of events are said to be equally likely, if none of them is expected to occur in preference to the others.

If you roll an unbiased dice then each number is equally likely to occur. If a dice is so formed that a particular face occurs most often then the dice is biased. In this case, the outcomes are not equally likely to happen.

Possible Outcomes

The total number of the events which are possible to occur is called possible outcomes.

When one dice is thrown, the possible outcomes are {1, 2, 3, 4, 5, 6}

When two dices are thrown, the possible outcomes are {(1,1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

Favorable Outcomes

The outcomes which satisfy the given condition of chance is called favorable outcomes. When an unbiased dice is thrown, the number obtained which is greater than 2 3, 4, 5, 6 are the favorable outcomes.

Probability of an Event

Let event is denoted by E the probability of occurrence ratio of event and sample space mathematically.

Let the number of events is denoted by n (e) and number of sample space denoted by n(S) then $P(E) = \frac{n(E)}{n(S)}$

Illustrative EXAMPLE

For examination you have selected 100 question for Preparation. In which 10 question match with exam

paper. The probability = $\frac{10}{100} = \frac{1}{10}$

Sure Event

Sure event is an event whose probability is always one.

Illustrative **EXAMPLE**

In a single toss of dice, what will be the probability of getting a number which is less then 7? Solution:

Sample space = {1, 2, 3, 4, 5, 6} $\therefore n(E) = 6$

 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1.$ So it is sure event.

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EXAMPLE

Find the probability of getting head, if the coin is two headed.

(a) 1	(b) $\frac{1}{2}$
(c) $\frac{1}{3}$	(d) $\frac{1}{4}$
(e) None of these	

Answer: (a)

Explanation

In two headed coin, sample space = {H, H} number of sample space = 2 =n(s).

 \therefore E be the event of getting head = {H, H}

$$\therefore n(E) = 2$$

 $\therefore n(E) = \frac{2}{2} = 1.$

Impossible Event

Impossible event is an event whose probability is 0 (zero).

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EXAMPLE

If a coin is tossed then what is the probability of getting I?

Solution:

We know that when a coin is tossed, only head or tail will come. There is no any event which is one. Due to this region number of event = 0. Therefore, probability is also zero.

Note: From the above calculation we can say that probability of any event lies between 0 and 1. i.e. $0 \le P(E) \le 1$. It means that maximum probability = 1, and minimum probability = 0. Also, probability can never be negative.

Complementary Event

Let E be an event and E' (not E) be an event which occur only when E does not occur. So, E' is called complementary event of E.

$$\therefore P(E) + P(E^1) = 1$$

 $P(E^{1}) = 1 - P(E)$

Illustrative EXAMPLE

The Probability that it will rain today is 0.84. What is the probability that it will not rain today?(a) 0.12(b) 0.1(c) 0.3(d) 0.4(e) None of these

Answer: (e)

Explanation

P(E) = 0.84, we know that $P(E) + P(E^{1}) = 1$ or $0.84 + P(E^{1}) = 1, P(E^{1}) = -0.84 = 0.16$

Illustrative EXAMPLE

What is the probability that an ordinary year has 53 Sunday?



Answer: (d) Explanation

An ordinary year has 365 days, i.e., 52 weeks and 1 day then PC that this days in Sunday) $=\frac{1}{7}$

Drawing a Card

We know that there are 52 cards in pack of card.

It is of four types, such as Spades, Clubs, Hearts and Diamonds. All are equally divided. It means that 13 spades, 13 clubs, 13 hearts and 13 diamonds.

In 52 cards, there are two colour red and black which is also equally divided. It means that 26 red cards and 26 black cards. Hearts and diamonds are red cards and spades and clubs are black cards.



Name all the 13 cards given below:

2 3, 4, 5, 6, 7, 8, 9, 10, Ace, Jack, King, Queen. The total number of cards are 52. Out of 52 some of the cards are shown below:



Face Cards

Kings, queens and jakes are called face cards, therefore, total number of face cards is 12: In which 6 is Red face cards and 6 is black face cards.

Number Card

The card on which numbers are written is called number card. 2, 3, 4, 5, 6, 7, 8, 9, 10 are the number cards.



- Mathematics arises from many different kinds of problems. At first these were found in commerce, land measurement, architecture and later astronomy nowadays, all sciences suggest problems studied by mathematicians, and mar problems arise within mathematics itself.
- The best-known award in mathematics is the Fields Medal established in 1936 and now awarded every 4 years.

SUMMARY



- The outcomes of a random experiment is called its elementary event.
- When two or more than two elementary events occurs with random experiment then it is said to be Compound Event.
- The outcomes which satisfies the given condition of chance is called favorable outcomes.
- $P(E) = \frac{n(E)}{n(S)}$
- $\mathbf{O}(E) + P(\overline{E}) = 1$

Commonly Asked



One card is drawn from a well shuffled deck of 52 cards. What is the probability of getting a black face card?

(a) $\frac{1}{26}$	(b) $\frac{3}{14}$
(c) $\frac{3}{26}$	(d) $\frac{3}{13}$
(e) None of these	

Answer: (c)

Explanation Number of black face card = (2 king + 2 queen + 2 jack) = 6.

$$\therefore P(E) = \frac{6}{52} = \frac{3}{26}$$

There are 20 tickets numbered as 1, 2, 3 - 20 respectively one ticket is drawn at random. What is the probability that the number on the ticket is a multiple of 3 or 5?

(a) $\frac{1}{4}$	(b) $\frac{1}{5}$
(c) $\frac{2}{5}$	(d) $\frac{4}{10}$
(e) None of these	

Answer: (b)

There are 25 cards numbered as 1, 2, 3, ---25 respectively one card is drawn at random. What is the probability that the number on the card is a multiple of 4 or 5?

(a) $\frac{2}{5}$	(b) $\frac{11}{25}$
(c) $\frac{12}{25}$	(d) $\frac{13}{25}$
(e) None of these	

Answer: (b)

A cartoon contains 144 bulbs out of which 20 are defective and others are good. Peter will buy one bulb if it is good, but will not buy if it is defective. The shopkeeper draws one bulb at random and gives it to him. The probability that he will buy it?

$(2) \frac{5}{5}$	(b) $\frac{31}{31}$
$\frac{(a)}{36}$	(0) 36
(c) 13	(d) 15
$(c) \frac{1}{36}$	$(u) = \frac{1}{36}$
(e) None of these	

Answer: (b)

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In a class, there are 20 boys and 18 girls. The class teacher wants to choose one student as class representative. He writes the name of each student on a flash card and puts them into a bag and mixes it. A student picks one card from the bag. Find the probability that the name written on the card is the name of a girl.

(a) $\frac{8}{19}$	(b) $\frac{6}{19}$
(c) $\frac{7}{19}$	(d) $\frac{9}{19}$
(e) None of these	

Answer: (d)

Self Evaluation



1. A bag contains 8 red 2 black and 5 white balls. One ball is drawn at random. What is the probability that the ball drawn is not black?

(a) $\frac{2}{15}$	(b) $\frac{13}{15}$
(c) $\frac{8}{15}$	(d) $\frac{1}{3}$
(e) None of these	

2. A bag contains 3 white, 4 red, 5 black balls. One ball is drawn at random. What is the probability that the ball drawn that neither black nor white.

(a) $\frac{1}{4}$	(b) $\frac{1}{2}$
(c) $\frac{1}{3}$	(d) $\frac{3}{4}$
(e) None of these	

3. What is the probability that an ordinary year has 53 Monday?

(a) $\frac{2}{7}$	(b) $\frac{1}{7}$
(c) $\frac{7}{53}$	(d) $\frac{7}{52}$
(e) None of these	

4. In a lottery there are 8 prizes and 16 blanks. What is the probability of getting a prize?

(a) $\frac{1}{2}$	(b) $\frac{1}{3}$
(c) $\frac{2}{3}$	(d) $\frac{4}{5}$
(e) None of these	C C

5. There are 25 cards numbered from 1 to 25. One card is drown at random. What is the probability that the number on this card is not divisible by 4?

(a) $\frac{4}{25}$	(b) $\frac{21}{25}$
(c) $\frac{6}{25}$	(d) $\frac{19}{25}$

(e) None of these

6. If Q(x) denotes the probability of a random even x then:

(a) $Q(x) > 0$	(b) $Q(x) > -1$
(c) $0 \le Q(x) \le 1$	(d) $-2 \le Q(x) > 1$
(e) None of these	

7. The probability that the three friends have same birthday, if they are born in 1997 is:

(a) $\frac{3}{365}$	(b) $\frac{1}{365}$
(c) $\frac{2}{365}$	(d) $\frac{6}{365}$
(e) None of these	

8. A bag contains 12 balls out of which x are red. If three more red balls are put in the bag the probability of drawing the red balls is twice then the value of *x* is:

(a) 2	(b) 4
(c) 6	(d) 5

(e) None of these

9. A number "a" is selected from numbers 1, 2, 3 and then the second number "b" is randomly selected from the numbers 1, 4 and 9. The probability that the product ab of the two numbers will be less than 8 is.....

(a) $\frac{4}{9}$	(b) $\frac{1}{3}$
(c) $\frac{5}{9}$	(d) $\frac{2}{9}$
(e) None of these	

10. Two dices are thrown simultaneously then the probability of getting an even number as the sum is:

(a) $\frac{1}{3}$	(b) $\frac{1}{4}$
(c) $\frac{1}{2}$	(d) $\frac{4}{5}$
(e) None of these	-

Answers – Self Evaluation Test																		
1.	В	2.	С	3.	В	4.	В	5.	D	6.	С	7.	D	8.	А	9.	Α	10. C

Self Evaluation Test SOLUTIONS

1. Total number of balls = 15. Number of non black balls = 13.

:
$$P(E) = \frac{15}{15}$$

2. Total number of balls = 12. Number of favourable cases = 4. Required probability = $\frac{4}{12} = \frac{1}{3}$

3. An ordinary year has 365 days, i.e. 52 weeks 1 day. This day can be any of 7 days of the week.

$$\therefore P(E) = \frac{1}{7}$$

4. Total number of tickets = 8 + 16 = 24. Number of prizes = 8. $\therefore P(E) = \frac{8}{24} = \frac{1}{3}$.

6. We know that probabilities must lie between 0 and (i).

8. According to given condition $\frac{x+3}{15} = \frac{2x}{12}$ Solve for x.

9. Possible outcomes = {(1, 1), (1, 4), (1, 9), (2, 1), (2, 4), (2, 9), (3, 1), (3, 4), (3, 9) Favourable outcomes = {(1, 1), (1, 4), (2, 1), (3, 1)} Probability = $\frac{4}{9}$ 10. Possible outcomes = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 5), (6, 6)} Favourable outcomes are: {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)} Probability = $\frac{18}{36} = \frac{1}{2}$