Chapter 9

Sequence and Series

Exercise 9.2

Question 1: Find the sum of odd integers from 1 to 2001.

Answer 1:

The odd integers from 1 to 2001 are 1, 3, 5 … 1999, 2001.

This, sequence forms an A.P.

Here, first tern, a = 1

Common difference, d = 2

Here, a + (n – 1) d = 2001

= 1 + (n – 1) (2) = 2001

= 2n – 2 = 2000

= n = 1001

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{1001}{2}[2 \times 1 + (1001 - 1) \times 2]$

$= \frac{1001}{2}[2 + 1000 \times 2]$

$= \frac{1001}{2} \times 2002$

= 1001 × 1001

= 1002001

Thus, the sum of odd numbers from 1 to 2001 is 1002001.

Question 2: Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Answer 2:

The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, …995.

Here, a = 105 and d = 5

$a + (n - 1) d = 995$

$= 105 + (n - 1) 5 = 995$

$= (n - 1) 5 = 995 - 105 = 890$

$= n - 1 = 178$

$= n = 179$

$S_n = \frac{179}{2} [2 (105) + (179 - 1) (5)]$

$= \frac{179}{2} [2 (105) + (178) (5)]$

$= (179) (105 + 445)$

$= (179) (550)$

$= 98450$

Thus, the sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 98450.

Question 3: In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is –112.

Answer 3:

First term = 2

Let d be the common difference of the A.P.

Therefore, the A.P. is 2, 2 +d, 2 +2d, 2 + 3d

Sum of the first five terms = 10 + 10d

Sum of the next five terms = 10 + 35d

According to given condition

$10 + 10d = \frac{1}{4}(10 + 5d)$

$= 40 + 40d = 10 + 5d$

$= 30 = -5d$

$= d = -6$

$a_{20} = a + (20 – 1) d = 2 + (19) (- 6) = 2 – 114 = - 112$

thus, the 20^{th} term of the A.P. is - 112

Question 4: How many terms of the A.P. – 6, $-\frac{11}{2}$, - 5 … are needed to give the sum –25?

Answer 4:

Let, the sum of n terms of the given A.P. be – 25

It is known that

$S_n = \frac{n}{2}[2a + (n – 1)d]$

Where n = number of terms, a = first term, and d = common difference

Here, a = - 6

$d = \frac{11}{2} + 6 = \frac{-11+12}{2} = \frac{1}{2}$

therefore, we obtain

- $25 = \frac{n}{2}[2 \times (-6) + (n – 1)\frac{1}{2}]$

$= -50 = n\left[-12 + \frac{n}{2} - \frac{1}{2}\right]$

$= - 50 = n \left[-\frac{25}{2} + \frac{n}{2}\right]$

$= - 100 = n (-25 + n)$

$= n^2 – 5n – 20n + 100 = 0$

$= n (n – 5) – 20 (n – 5) = 0$

$n = 20$ or 5

Question 5: In an A.P., if pth term is 1/q and qth term is 1/p, prove that the sum of first p q terms is 1⁄2 (p q +1), where p ≠ q.

Answer 5:

It is known that the general term of an A.P. is an = a + (n – 1) d

According to the given information

Pth term = $ap = a + (p – 1)d = \frac{1}{q}$ ... (1)

qth term = aq = a + (q – 1)d = $\frac{1}{p}$ … (2)

subtracting (2) from (1), we obtain

$(p – 1)d – (q – 1)d = \frac{1}{q} - \frac{1}{p}$

$= (p – 1 – q + 1)d = \frac{p-q}{pq}$

$= (p – q)d = \frac{p-q}{pq}$

$= d = \frac{1}{pq}$

Putting the value of din (1), we obtain

$a + (p – 1) \frac{1}{pq} = \frac{1}{q}$

$= a = \frac{1}{q} - \frac{1}{q} + \frac{1}{pq} = \frac{1}{pq}$

$S_n = \frac{pq}{2}$ [2a + (p q – 1)d]

$= \frac{pq}{2} [\frac{2}{pq} + (p\,q - 1)\frac{1}{pq}]$

$= 1 + \frac{1}{2}(p\,q - 1)$

$= \frac{1}{2}p\,q + 1 - \frac{1}{2} = \frac{1}{2}p\,q + \frac{1}{2}$

$= \frac{1}{2}(p\,q + 1)$

Question 6: If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116.

Find the last term

Answer 6:

Let the sum of n terms of the given A.P. be 116.

$S_n = \frac{n}{2}$ [2a + (n – 1)d]

Here, a = 25 and d – 22 – 25 = - 3

$S_n = \frac{n}{2}$ [2×25 + (n – 1) (-3)]

$= 116 = \frac{n}{2}$ [50 – 3n + 3]

$= 232 = n(53 - 3n) = 53n - 3n^2$

$= 3n^2 - 53n + 232 = 0$

$= 3n^2 - 24n - 29n + 232 = 0$

= 3n (n – 8) – 29 (n – 8) = 0

= (n – 8) (3n – 29) = 0

$= n = 8$, or $n = \frac{29}{3}$

However,

n cannot be equal to $\frac{29}{3}$

therefore, n = 8

a_8 = last term = a + (n – 1) d = 25 + (8 – 1) (-3)

= 25 + (7) (-3) = 25 – 21

= 4

Thus, the last term of the A.P. is 4.

Question 7: Find the sum to n terms of the A.P., whose k^{th} term is 5k + 1.

Answer 7:

It is given that the kth term of the A.P. is 5k + 1.

K^{th} term = $ak = a + (k – 1) d$

= a + (k – 1) d = 5k + 1a + k d – d = 5k + 1

Comparing the coefficient of k, we obtain d = 5a – d = 1

= a – 5 = 1

= a = 6

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

= $\frac{n}{2}$ [2 (6) + (n -1) (5)]

= $\frac{n}{2}$ [12 +5n – 5]

= $\frac{n}{2}$ [5n + 7]

Question 8: If the sum of n terms of an A.P. is $(pn + qn2)$, where p and q are constants, find the common difference.

Answer 8:

It is known that:

$S_n = \frac{n}{2}[2a + (n - 1)d]$

According to the given condition,

$\frac{n}{2}[2a + (n - 1)\, d] = p\, n + qn^2$

$= \frac{n}{2}[2a + n\, d - d] = p\, n + qn^2$

$= n\, a + n^2 \frac{d}{2} - n \frac{d}{2} = p\, n + qn^2$

Comparing the coefficient if n^2 on both sides, we obtain

$\frac{d}{2} = q$

$= d = 2q$

Thus, the common difference of the A.P. is 2q.

Question 9: The sums of n terms of two arithmetic progressions are in the ratio 5n + 4: 9n + 6. Find the ratio of their 18^{th} terms.

Answer 9:

Let, a_1, a_2, and d_1 d_2 be the first - terms and the common difference of the first and second arithmetic progression respectively.

According to the given condition,

$\frac{sum\ of\ n\ terms\ of\ first\ A.P.}{sum\ of\ n\ terms\ of\ second\ A.P.} = \frac{5n+4}{9n+6}$

$= \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{5n+4}{9n+6}$

$= \frac{2a_1(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+6}$ … (1)

Substituting n = 35 in (1), we obtain

$\frac{2a_1+34d_1}{2a_2+34d_2} = \frac{5(35)+4}{9\,(35)+5}$

$= \frac{a_1+17d_1}{a_2+17d_2} = \frac{179}{321}$ … (2)

$\frac{18^{th} term\ of\ first\ A.P.}{18^{th}\ term\ of\ second\ A.P.} = \frac{a_1+17d_1}{a_2+17d_2}$ … (3)

$\frac{18_{th} term\ of\ first\ A.P.}{18_{th} term\ of\ second\ A.P.} = \frac{179}{321}$

Thus, the ration of 18^{th} term of both the A.P. is 179: 321.

Question 10: If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first (p + q) terms.

Answer 10:

Let, a and d be the first term and the common difference of the A.P. respectively.

Here,

$Sp\ = \frac{p}{2}$ [2a + (p – 1) d]

$Sq = \frac{q}{2}$ [2a + (q – 1)d]

According to the given condition,

$\frac{p}{2}$ [2a + (p – 1)d] = $\frac{q}{2}$ [2a + (q – 1)d]

= p [2a + (p – 1) d] = q [2a + (q – 1) d]

= 2ap + pd (p – 1) = 2aq + qd (q – 1)

= 2a (p – q) + d [p (p – 1) – q (q – 1)] = 0

= 2a (p – q) + d [p_2 – p – q^2 + q] = 0

= 2a (p – q) + d [(p – q) (p + q) – (p – q)] = 0

= 2a (p – q) + d [(p –q) (p + q – 1)] = 0

$= 2a + d (p + q – 1) = 0$

$= d = \frac{-2a}{p+q-1}$ … (1)

$S_{p+q} = \frac{p+q}{2}$ [2a + (p + q – 1).d]

$= S_{p+q} = \frac{p+q}{2}\left(2a + (p + q - 1)\left(\frac{-2a}{p+q-1}\right)\right)$ [from (1)]

$= \frac{p+q}{2}$ [2a – 2a]

= 0

Thus, the sum of first (p + q) terms pf the A.P. is 0.

Question 11: Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}$ (q – r) + $\frac{b}{q}$ (r – p) + $\frac{c}{r}$ (p – q) = 0

Answer 11:

Let, a1 and d be the first term and the common difference of the A.P. respectively.

According to the given information,

$S_p = \frac{p}{2}$ [$2a_1$ + (p – 1) d] = a

$= 2a_1 + (p – 1) d = \frac{2a}{p}$ … (1)

$S_q = \frac{q}{2}$ [$2a_1$ + (q – 1) d = b

$= 2a_1 + (q – 1) d = \frac{2b}{q}$ … (2)

$S_r = \frac{r}{2}$ [$2a_1$ + (r – 1) d] = c

$= 2a_1 + (r – 1) d = \frac{2c}{r}$ … (3)

Subtracting (2) from (1), we obtain

$(p - 1)\ d - (q - 1)\ d = \frac{2a}{p} - \frac{2b}{q}$

$= d\ (p - 1 - q + 1) = \frac{2aq - 2bp}{pq}$

$= d\ (p - q) = \frac{2aq - 2bp}{pq}$

$= d = \frac{2(aq - bp)}{pq(p - q)}$ … (4)

Subtracting (3) from (2), we obtain

$(q - 1)\ d - (r - 1)\ d = \frac{2b}{q} - \frac{2c}{r}$

$= d\ (q - 1 - r + 1) = \frac{2b}{q} - \frac{2c}{r}$

$= d\ (q - r) = \frac{2br - 2cq}{qr}$

$= d = \frac{2(br - qc)}{qr(q - r)}$ … (5)

Equating both the value of d obtained in (4) and (5), we obtain

$\frac{aq - bp}{pq(p - q)} = \frac{br - qc}{qr(q - r)}$

$= qr\ (q - r)\ (aq - bp) = pq\ (p - a)\ (br - qc)$

$= r\ (aq - bp)\ (q - r) = p\ (br - qc)\ (p - a$

$= (aqr - bpr)\ (q - r) = (bpr - pqc)\ (p - q)$

Dividing both sides by pqr, we obtain

$\left(\frac{a}{p} - \frac{b}{q}\right)(q - r) = \left(\frac{b}{q} - \frac{c}{r}\right)(p - q)$

$\left(\frac{a}{p} - \frac{b}{q}\right)(q - r) = \left(\frac{b}{q} - \frac{c}{r}\right)(p - q)$

$= \frac{a}{p}(q-r) - \frac{b}{q}(q-r+p+q) + \frac{c}{r}(p-q) = 0$

$= \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

Thus, the given result is proved.

Question 12: The ratio of the sums of m and n terms of an A.P. is m^2: n^2. Show that the ratio of m^{th} and n^{th} term is (2m – 1): (2n – 1).

Answer 12:

Let, a and d be the first term and the common difference of the A.P. respectively.

According to the given information,

$\frac{sum\ of\ m\ terms}{sum\ of\ n\ terms} = \frac{m^2}{n^2}$

$= \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

$= \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n} \ldots (1)$

Putting m = 2m – 1 and n = 2n – 1 in (1), we obtain

$\frac{2a+(2m-2)d}{2a+(2n-2)d} = \frac{2m-1}{2n-1}$

$= \frac{a+(m-1)d}{a+(n-1)d} = \frac{2m-1}{2n-1} \ldots (2)$

$\frac{m^{th} term\ of\ A.P.}{n_{th} term\ of\ A.P.} = \frac{a+(m-1)d}{a+(n-1)d} \ldots (2)$

From (2) and (3), we obtain

$\frac{m^{th} term\ of\ A.P.}{n^{th} term\ of\ A.P.} = \frac{2m-1}{2n-1}$

Thus, the given result is proved.

Question 13: If the sum of n terms of an A.P. is $2n^2 + 5n$ and its m^{th} term is 164, find the value of m.

Answer 13:

Let, a and d be the first term and the common difference of the A.P. respectively.

$a_m = a + (m – 1) d = 164 \ldots (1)$

sum of n terms:

$s_n = \frac{n}{2} [2a + (n – 1)d]$

here,

$= \frac{n}{2} [2a + nd – d] = 3n^2 + 5n$

$= na + n^2 .\frac{d}{2} = 3n^2 + 5n$

Comparing the coefficient of n^2 on both sides, we obtain

$\frac{d}{2} = 3$

$= d = 6$

Comparing the coefficient of n on both sides, we obtain

$= a - \frac{d}{2} = 5$

$= a – 3 = 5$

$= a = 8$

Therefore, from (1), we obtain

$8 + (m – 1) 6 = 164$

$= (m – 1) 6 = 164 – 8 = 156$

$= m – 1 = 26$

= m = 27

Thus, the value of m is 27.

Question 14: Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

Answer 14:

Let, A_1, A_2, A_3, A_4, and A_5 be five numbers between 8 and 26 such that 8, A_1, A_2, A_3, A_4, A_5, 26 is an A.P.

Here, a = 8 b = 26, n = 7

Therefore, 26 = 8 + (7 – 1) d

= 6d = 26 – 8 = 18

= d = 3

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4 \times 3 = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 5 \times 3 = 8 + 15 = 23$

Thus, the required five numbers between 8 and 26 are 11, 14, 17, 20, and 23.

Question 15: If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between a and b, then find the value of n.

Answer 15:

A.M. of a and b = $\frac{a+b}{2}$

According to the given condition,

$\frac{a+b}{2} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$

$= (a + b) (a^{n-1} + b^{n-1}) = 2 (a^n + b^n)$

$= a^n + a^{n-1} + ba^{n-1} + b^n = 2a^n + 2b^n$

$= ab^{n-1} + a^{n-1} b = a^n + b^n$

$= ab^{n-1} – b^n = a^n – a^{n-1}b$

$= b^{n-1} (a – b) = a^{n-1} (a – b)$

$= b^{n-1} = a^{n-1}$

$= \left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0$

$= n – 1 = 0$

$= n = 1$

Question 16: Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7^{th} and $(m - 1)^{th}$ numbers is 5:9. Find the value of m.

Answer 16:

Let A_1, A_2 …Am be m numbers such that 1, A_1, A_2, …Am, 31 is an A.P.

Here, a = 1, b = 31, n = m + 2

$31 = 1 + (m + 2 – 1) d$
$= 30 = (m + 1) d$
$= d = \frac{30}{m+1}$ … (1)
$A_1 = a + d$
$A_2 = a + 2d$
$A_3 = a + 3d$
$\therefore A_7 = a + 7d$
$A_{m-1} = a + (m - 1) d$
According to the given condition,

$$\frac{a+7d}{a+(m-1)d} = \frac{5}{9}$$

$$= \frac{1+7\left(\frac{30}{m+1}\right)}{1+(m-1)\left(\frac{30}{m-1}\right)} = \frac{5}{9} \text{ [from (1)]}$$

$$= \frac{m+1+7(30)}{m+1+30(m-1)} = \frac{5}{9}$$

$$= \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$$

$$= \frac{m+211}{31m-29} = \frac{5}{9}$$

$= 9m + 1899 = 155m - 145$

$= 155m - 9m = 1899 + 145$

$= 146m = 2044$

$= m = 14$

Thus, the value of m is 14.

Question 17: A man starts repaying a loan as first installment of ₹100. If he increases the installment by ₹5 every month, what amount he will pay in the 30th installment?

Answer 17:

The first installment of the loan is ₹100.

The second installment of the loan is ₹105 and so on.

The amount that the man repays every month forms an A.P.

The A.P. is 100, 105, 110 …

First term, a = 100

Common difference, d = 5

$A_{30} = a + (30 - 1) d$

= 100 + (29) (5)

= 100 + 145

= 245

Thus, the amount to be paid in the 30^{th} installment is ₹245.

Question 18: The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

Answer 18:

The angles of the polygon will form an A.P. with common difference d as 5° and first term an as 120.

It is known that the sum of all angles of the polygon with n sides 180° (n – 2)

$S_n = 180° (n – 2)$

$= \frac{n}{2} [2a + (n - 1)d] = 180° (n – 2)$

$= \frac{n}{2} [240° + (n – 1) 5°] = 180° (n – 2)$

$= n [240 + (n – 1) 5] = 360 (n – 2)$

$= 240n + 5n^2 – 5n = 360n – 720$

$= 5n^2 + 235n – 360n + 720 = 0$

$= 5n^2 – 125n + 720 = 0$

$= n^2 – 25n + 144 = 0$

$= n^2 – 16n – 9n + 144 = 0$

$= n (n – 16) – 9 (n – 16) = 0$

$= (n – 9) (n – 16) = 0$

$= n = 9$ or 16