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Chapter : 4
Quadratic Equations

Definations :-

1. Equation :- An algebraic relation between variables and constants and equal to sign is called equation.

2. Quadratic equation : An equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$ is called quadratic equation.

Exercise : 4.1

1. Check whether the following are quadratic equation :-

(i) $(x+1)^2 = 2(x-3)$ $[\because a^2+b^2+2ab]$

$x^2 + (1)^2 + 2(x)(1) = 2x - 6$

$x^2 + 1 + 2x = 2x - 6$

$x^2 + 1 + \cancel{2x} - \cancel{2x} + 6$

$x^2 + 7 = 0$

$x^2 + 7 = 0$

$1 \cdot x^2 + 0 \cdot x + 7 = 0$

It is of the form of $ax^2 + bx + c = 0$

$\therefore$ yes, it is quadratic equation.

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(ii) $x^2 - 2x = (-2)(3-x)$

$x^2 - 2x = -6 + 2x$

$x^2 - 2x + 6 - 2x = 0$

$x^2 - 4x + 6 = 0$

$1 \cdot x^2 - 4x + 6 = 0$

It is of the form of $ax^2 + bx + c = 0$

$\therefore$ yes, it is is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3)$

$\cancel{x^2} + x - 2x - 2 = \cancel{x^2} + 3x - x - 3$

$x - 2x - 2 - 3x + x + 3 = 0$

$\cancel{2x} - \cancel{2x} - 2 - 3x + 3 = 0$

$-3x + 1 = 0$

It is not of the form $ax^2 + bx + c = 0$

$\therefore$ No, it is not quadratic form.

(iv) $(x-3)(2x+1) = x(x+5)$

$2x^2 + x - 6x - 3 = x^2 + 5x$

$2x^2 + x - 6x - 3 - x^2 - 5x = 0$

$x^2 - 5x - 5x - 3 = 0$

$x^2 + 10x - 3 = 0$

$\therefore$ It is in the form $ax^2 + bx + c = 0$ so, it is quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1)$

$2x^2 - 6x - \cancel{x} + 3 = x^2 - \cancel{x} + 5x - 5$

$2x^2 - 6x + 3 - x^2 - 5x + 5$

$0 \cdot x^2 - 11x + 8$

yes it is in the form $ax^2 + bx + c = 0$.

It is quadratic equation.

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(vi) $x^2 + 3x + 1 = (x-2)$

$x^2 + 3x + 1 = x^2 + (2)^2 - 2(x)(2)$

$\cancel{x^2} + 3x + 1 - \cancel{x^2} - 4 + 4x = 0$

$7x - 3 = 0$

It is not in the form of $ax^2 + bx + c = 0$
So, It is not quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1)$

$a^3 + b^3 + 3ab(x+2)$

Now :- $x^3 + (2)^3 + 3 x \times 2 (x+2) = 2x^3 - 2x$

$x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$

$x^3 + 8 + 6x^2 + 12x - 2x^3 - 2x = 0$

$-x^3 + 6x^2 + 14x + 8 = 0$

it is not in the form of $ax^2 + bx + c = 0$
So, It is not a quadratic polynomial

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$ $[a^3 - b^3 - 3ab(a-b)]$

$x^3 - 4x^2 - x + 1 = x^3 - (2)^3 - 3 \times x \times 2 (x-2)$

$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x(x-2)$

$\cancel{x^3} - 4x^2 - x + 1 = -8 + \cancel{x^3} - 6x^2 + 12x$

$-4x^2 - x + 1 + 8 + 6x^2 - 12x$

$2x^2 - 13x + 9 = 0$

yes it is in the form $ax^2 + bx + c = 0$
So, It is quadratic equation.

Statement Question

Ex: 4.1

I (ii) The area of a rectangular plot is 528 m^2. The length of the plot is one more than twice its breadth. We need to find the length and breadth of plot.

Solⁿ: Let the breadth of rectangular plot = x
$\therefore$ length = $2x+1$

Now, area of rectangular plot = 528 m^2

$$L \times B = 528$$
$$(2x+1) \times x = 528$$
$$2x^2 + x = 528$$
$$2x^2 + x - 528 = 0$$

Here, $a = 2$, $b = 1$, $c = -528$

$$D = b^2 - 4ac$$
$$= (1)^2 - 4 \times 2 \times (-528)$$
$$= 1 + 4224$$
$$= 4225$$

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$$x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$$

$$= \frac{-1+\sqrt{4225}}{2(2)}, \frac{-1-\sqrt{4225}}{2(2)}$$

$$= \frac{-1+\sqrt{4225}}{4}, \frac{-1-\sqrt{4225}}{4}$$

$$= \frac{-1+65}{4}, \frac{-1-65}{4}$$

$$= \frac{64}{4}\ 16, \frac{-66}{4\ 2}\ 33$$

$$= 16, \frac{-33}{2}$$

Now :- Reject $x = \frac{-33}{2}$ [∵ breadth of rectangle can't be negative.

$\therefore x = 16$

Now :-

breadth $= 16$ m

length $= 2x+1$

$= 2 \times 16 + 1$

$= 33$ m

(ii) The product of two consective positive integers is 306. we need to find the integers.

Solⁿ :- Let the two consective integers are $= x, x+1$

A.T.Q

Product of Integers $= 306$

$x \times (x+1) = 306$

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$$x^2 + x = 306$$
$$x^2 + x - 306 = 0$$

Here: $a = 1$, $b = 1$, $c = -306$

$$D = b^2 - 4ac$$
$$= (1)^2 - 4 \times 1 \times (-306)$$
$$= 1 + 1224$$
$$= 1225$$

$$x = \frac{-b + \sqrt{D}}{2a}, \quad \frac{-b - \sqrt{D}}{2a}$$

$$= \frac{-1 + \sqrt{1225}}{2(1)}, \quad \frac{-1 - \sqrt{1225}}{2(1)}$$

$$= \frac{-1 + 35}{2}, \quad \frac{-1 - 35}{2}$$

$$= \frac{\cancel{34}}{\cancel{2}}\ 17, \quad \frac{-\cancel{36}}{\cancel{2}}\ -18$$

Now: $x = 17, -18$

$x = 17$

1st integer $= 17$

2nd " $= 17 + 1$

$= 18$

$x = -18$

1st integer $= -18$

2nd " $= -18 + 1$

$= -17$

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Rohan's mother is 26 years older than him. The Product of their ages 3 year from now will be 360. we would like to find Rohan's present age.

Let Rohan's Present age = x year
$\therefore$ Mother's " " = $x + 26$

Now :- After three years,

Rohan's age = $x + 3$
= $x + 26 + 3$
= $x + 29$

A.T.Q

Product of ages = 360

$(x+3)(x+29) = 360$

$x^2 + 29x + 3x + 87 = 360$

$x^2 + 32x + 87 - 360 = 0$

$x^2 + 32x - 273 = 0$

Here, $a = 1$, $b = 32$, $c = -273$

$D = b^2 - 4ac$

$= (32)^2 - 4 \times 1 \times (-273)$

$= 1024 + 1092$

$= 2116$

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Here, $x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$

$= \frac{-32 + \sqrt{2116}}{2}, \frac{-32 - \sqrt{2116}}{2}$

$= \frac{-32 + 46}{2}, \frac{-32 - 46}{2}$

$= \frac{14}{2}\ 7, \frac{-78}{2}\ -39$

$x = 7, x = -39$

Rejected, $x = -39$ [$\because$ age cannot be negative]

$x = 7$

Ist Rohan age $= 7$

Mother's age $= x + 26$

$= 7 + 26$

$= 33$

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to over the same distance. We need to find the speed of the train.

Solⁿ: Let the speed of the train $= x$ km/h

Distance covered $= 480$

Let, Time $= t = \frac{\text{distance}}{\text{speed}}$

$= \frac{480}{x}$

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A.T.Q

Speed = $(x-8)$ km/h
Distance = 480 Km
Now,
time = $\frac{480}{x-8}$

but, Now time is 3 hours more than the earlier time,

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left(\frac{1}{x-8} - \frac{1}{x}\right) = 83$$

$$\frac{1}{x-8} - \frac{1}{x} = \frac{\cancel{8}}{480} = \frac{1}{160}$$

$$\frac{x-(x-8)}{x(x-8)} = \frac{1}{160}$$

$$\frac{\cancel{x}-\cancel{x}+8}{x(x-8)} = \frac{1}{160}$$

$$8 \times 160 = x(x-8)$$
$$1280 = x^2 - 8x$$

$$x^2 - 8x = 1280$$
$$x^2 - 8x - 1280 = 0$$

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Exercise : 4·2

Method to Solve Quadratic equation

1. Factorisation Method
2. Discriminent Method
3. Perfect square method

Nature of Roots

1. $D>0$, Roots are real and different.
2. $D=0$, Roots are real and equal.
3. $D<0$, Roots are not real.

1. Find the roots of the following quadratic equations :-

(i) $x^2 - 3x - 10 = 0$ [By factorisation Method]

$x^2 - (5x - 2x) - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x-5) + 2(x-5) = 0$

$(x-5)(x+2)$

$x - 5 = 0$ | $x + 2 = 0$

$x = 5$ | $x = -2$

$\therefore$ Roots are 5, -2.

OR

$x^2 - 3x - 10 = 0$ [By Discriminent Method

$a = 1, \ b = -3, \ c = -10$

$D = b^2 - 4ac$

$= (-3)^2 - 4 \times 1 \times (-10)$

$= 9 + 40$

$= 49$

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for Roots : $x = \frac{-b + \sqrt{D}}{2a}$, $\frac{-b - \sqrt{D}}{2a}$

$= \frac{-(-1) + \sqrt{49}}{2(1)}$, $\frac{-(-1) - \sqrt{49}}{2(1)}$

$\frac{1+7}{2}$, $\frac{1-7}{2}$

$\frac{10}{2} = 5$, $\frac{-4}{2} = -2$

(ii) $2x^2 + x - 6 = 0$ [By factorisation Method]

$2x^2 + (4x - 3x) - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$

$2x(x + 2) - 3(x + 2) = 0$

$(2x - 3)(x + 2)$

$2x - 3 = 0$ | $x + 2 = 0$

$2x = 3$ | $x = -2$

$x = \frac{3}{2}$

Roots are $\frac{3}{2}$, -2

OR

$2x^2 + x - 6 = 0$ [By Discriminent Method]

$a = 2$, $b = 1$, $c = -6$

$D = b^2 - 4ac$

$= (1)^2 - 4 \times 2 \times (-6)$

$= 1 - (-48)$

$= 1 + 48$

$= 49$

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for Roots,

$$x = \frac{-b+\sqrt{D}}{2a}, \quad \frac{-b-\sqrt{D}}{2a}$$

$$= \frac{-1+\sqrt{49}}{2(2)}, \quad \frac{-1-\sqrt{49}}{2(2)}$$

$$= \frac{-1+7}{4}, \quad \frac{-1-7}{4}$$

$$= \frac{6}{4} = \frac{3}{2}, \quad \frac{-8}{4} = \frac{-2}{1}$$

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ [By factorisation Method]

$\sqrt{2}x^2 + (2x + 5x) + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\sqrt{2}x(x+\sqrt{2}) + 5(x+\sqrt{2}) = 0$

$(x+\sqrt{2})(\sqrt{2}x+5)$

$x + \sqrt{2} = 0$

$x = -\sqrt{2}$

$\sqrt{2}x + 5 = 0$

$\sqrt{2}x = -5$

$x = \frac{-5}{\sqrt{2}}$

OR

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ [By Discriminent Meth

Here $a = \sqrt{2}$, $b = 7$, $c = 5\sqrt{2}$

$$D = b^2 - 4ac$$

$$= (7)^2 - 4(\sqrt{2})(5\sqrt{2})$$

$$= 49 - 20(\sqrt{2})^2$$

$$= 49 - 40$$

$$D = 9$$

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Roots are :-

$x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$

$= \frac{-7+\sqrt{9}}{2(\sqrt{2})}, \frac{-7-\sqrt{9}}{2(\sqrt{2})}$

$= \frac{-7+3}{2\sqrt{2}}, \frac{-7-3}{2\sqrt{2}}$

$= \frac{-\cancel{2}\,4}{\cancel{2}\sqrt{2}} = -\frac{2}{\sqrt{2}}, \frac{-\cancel{10}}{\cancel{2}\sqrt{2}} = \frac{-5}{\sqrt{2}}$

Dinominator

$-\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{(\sqrt{2})^2} = \frac{-2\sqrt{2}}{2}$

$\frac{-5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-5\sqrt{2}}{(\sqrt{2})^2} = \frac{-5\sqrt{2}}{2}$

$2x^2 - x + \frac{1}{8} = 0$ [By Discriminant Method]

Here $a = 2$, $b = -1$, $c = \frac{1}{8}$

$D = b^2 - 4ac$

$= (-1)^2 - 4 \times \cancel{2} \times \frac{1}{\cancel{8}\,4}$

$= 1 - 1$

$D = 0$

Roots are :-

$x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$

$$= \frac{-(-1)+\sqrt{0}}{2(2)}, \quad \frac{(-1)-\sqrt{0}}{2(2)}$$

$$= \frac{1+0}{4}, \quad \frac{1+0}{4}$$

$$= \frac{1}{4}, \quad \frac{1}{4}$$

Roots are $\frac{1}{4}, \frac{1}{4}$

$100x^2 - 20x + 1 = 0$ [factorisation method]

$100x^2 - (10x + 10x) + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x - 1) - 1(10x - 1) = 0$

$(10x - 1)(10x - 1)$

$10x - 1 = 0$	$10x - 1 = 0$
$10x = +1$	$x = \frac{1}{10}$
$x = \frac{+1}{10}$	

OR

$100x^2 - 20x + 1 = 0$ [Discriminent Method

$a = 100, \quad b = -20, \quad c = 1$

$D = b^2 - 4ac$

$= (-20)^2 - 4 \times 100 \times 1$

$= 400 - 400$

$= 0$, Roots are real and equal.

for roots :- $x = \frac{-b+\sqrt{D}}{2a}, \quad \frac{-b-\sqrt{D}}{2a}$

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$\frac{20+0}{200}$, $\frac{20-0}{200}$

$\frac{20}{200} = 10$, $\frac{20}{200} = 10$

Roots are 10.

Ex: 4.2

Find two numbers whose sum is 27 and product is 182.

Let the first number $= x$

$\therefore$ 2nd number $= 27 - x$

A.T.Q

Product $= 182$

$x \times (27 - x) = 182$

$27x - x^2 = 182$

$-x^2 + 27x - 182 = 0$

Here

$a = -1,\ b = 27,\ c = -182$

$D = b^2 - 4ac$

$= (27)^2 - 4(-1)(-182)$

$= 729 - 728$

$= 1$

$x = \dfrac{-b + \sqrt{d}}{2a},\ \dfrac{-b - \sqrt{d}}{2a}$

$= \dfrac{-27 + \sqrt{1}}{2 \times (-1)},\ \dfrac{-27 - \sqrt{1}}{2 \times (-1)}$

$= \dfrac{-27 + 1}{-2},\ \dfrac{-27 - 1}{-2}$

$= \dfrac{\cancel{+26}}{\cancel{+2}}\ 13,\ \dfrac{\cancel{-28}}{\cancel{-2}}\ 14$

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Ist number = 13
2nd number = 14

4. Find two consective positive integers, sum of whose squares is 365.

Solⁿ :- Let the two consective positive integer are = x, $x+1$

A.T.Q

Sum of square = ~~365~~
$(x)^2 + (x+1)^2 = 365$

$x^2 + x^2 + (1)^2 + 2x \times 1 = 365$
$x^2 + x^2 + 1 + 2x = 365$
$2x^2 + 2x + 1 - 365 = 0$
$2x^2 + 2x - 364 = 0$

Here,
$a = 2,\ b = 2,\ c = -364$

$D = b^2 - 4ac$
$= (2)^2 - 4 \times 2 \times (-364)$
46 $= 4 - 8 \times (-364)$
$= 4 + 2912$
$= 2916$

$x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$

$= \frac{-2 + \sqrt{2916}}{2(2)}, \frac{-2 - \sqrt{2916}}{2(2)}$

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$$= \frac{-2+\sqrt{2916}}{4}, \quad \frac{-2-\sqrt{2916}}{4}$$

$$= \frac{-2+54}{4}, \quad \frac{-2-54}{4}$$

$$= \frac{\cancel{52}}{\cancel{4}}, \quad -\frac{\cancel{56}}{\cancel{4}}$$

$$= 13, \quad -14$$

Reject, $x = -14$ [$\because$ two positive consecutive can't be negative]

$x = 13$

Ist no $= 13$

2nd no $= x + 1$

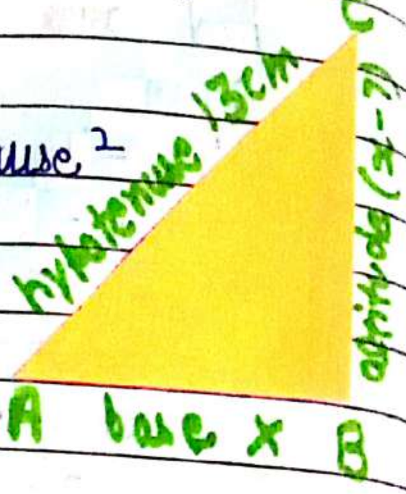
$= 13 + 1$

$= 14$ Ans

The altitude of a right triangle is 7cm less than its base. If the hypotenuse is 13m, find the other two sides.

let us say, the base of the right triangle be x cm.

Given, the altitude of right triangle = $(x-7)$ cm



From Pythagoras Theorem.

Base^2 + Altitude^2 = Hypotenuse^2

$\therefore x^2 + (x-7)^2 = 13^2$

$x^2 + x^2 + 49 - 14x = 169$

$2x^2 - 14x - 120 = 0$

$x^2 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x-12) + 5(x-12) = 0$

$(x-12)(x+5) = 0$

Thus, either = $x - 12 = 0$ or $x + 5 = 0$

$\Rightarrow x = 12$ or $x = -5$

Since, sides cannot be negative x, can be 12.

$\therefore$, the bas of the given triangle is 12cm and the altitude of this triangle will be $(12-7)$cm = 5 cm.

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6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

Ans. let us say, the number of articles produced be x

$\therefore$, cost of production of each article = Rs$(2x+3)$

Given, total cost of production is Rs. 90

$\therefore x(2x+3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x+15) - 6(2x+15) = 0$

$\Rightarrow (2x+15)(x-6) = 0$

Thus, either $2x+15 = 0$ or $x - 6 = 0$

$x = -\frac{15}{2}$ or $x = 6$

As, the number of articles produced can only be a positive integer.

$\therefore$, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs 15

Exercise : 4.3

1. Find the root of the following quadratic equations, if they exist, by the method of (completing the square) :Optional

(i) $2x^2 - 7x + 3 = 0$ [By Descriminent Method]

$a = 2, \quad b = -7, \quad c = 3$

$D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 3$

$= 49 - 24$

$= 25 > 0$, Roots are real and different

for roots : $x = \frac{-b + \sqrt{D}}{2a}, \quad \frac{-b - \sqrt{D}}{2a}$

$= \frac{-(-7) + \sqrt{25}}{2(2)}, \quad \frac{-(-7) - \sqrt{25}}{2(2)}$

$= \frac{7 + 5}{4}, \quad \frac{7 - 5}{4}$

$= \frac{\cancel{12}}{\cancel{4}} \frac{3}{1}, \quad \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2}$

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(ii) $2x^2 + x - 4 = 0$

$a = 2, \quad b = 1, \quad c = -4$

$D = b^2 - 4ac$

$= (1)^2 - 4 \times 2 \times (-4)$

$= 1 + 32$

$= 33$

for roots :- $x = \frac{-b + \sqrt{D}}{2a}, \quad \frac{-b - \sqrt{D}}{2a}$

$= \frac{-1 + \sqrt{33}}{2(2)}, \quad \frac{-1 - \sqrt{33}}{2a}$

$= \frac{-1 + \sqrt{33}}{4}, \quad \frac{-1 - \sqrt{33}}{4}$

Roots are $\frac{-1 + \sqrt{33}}{4}, \quad \frac{-1 - \sqrt{33}}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$a = 4, \quad b = 4\sqrt{3}, \quad c = 3$

$D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4 \times 4 \times 3$

$= (4)^2 (\sqrt{3})^2 - 48$

$= (16)(3) - 48$

$= 48 - 48$

$= 0$

for roots :- $x = \frac{-b + \sqrt{D}}{2a}, \quad \frac{-b - \sqrt{D}}{2a}$

$= \frac{-4\sqrt{3} + \sqrt{0}}{2(4)}, \quad \frac{-4\sqrt{3} - \sqrt{0}}{2(4)}$

$= \frac{-\cancel{4}\sqrt{3} + 0}{\cancel{8}\,2}, \quad \frac{-\cancel{4}\sqrt{3} - 0}{\cancel{8}\,2}$

$= \frac{-\sqrt{3}}{2}, \quad \frac{-\sqrt{3}}{2}$

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(iv) $2x^2 + x + 4 = 0$

$a = 2$, $b = 1$, $c = 4$

$D = b^2 - 4ac$

$= (1)^2 - 4 \times 2 \times 4$

$= 1 - 32$

$= -31 < 0$

roots do not exist.

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2. Find the roots of the quadratic equation given in Q 1 above by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

Sol^n On comparing the given equation with $ax^2 + bx + c = 0$

$a = 2, b = -7, c = 3$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= x = \frac{(7 \pm \sqrt{(49 - 24)})}{4}$$

$$= x = \frac{(7 \pm \sqrt{25})}{4}$$

$$= x = \frac{(7 \pm 5)}{4}$$

$$= x = \frac{(7+5)}{4} \text{ or } x = \frac{(7-5)}{4}$$

$$= x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$

$a = 2, b = 1, c = -4$

By using formula

$$-b \pm \sqrt{b^2 - 4ac}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore x = \frac{-1 + \sqrt{33}}{4} \quad \text{or} \quad x = \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Sol^n On comparing the given equation with $ax^2 + bx + c = 0$

$a = 4$, $b = 4\sqrt{3}$; $c = 3$

By using quadratic formula ;-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2} \quad \text{or} \quad x = \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

Sol^n On comparing the given equation with $ax^2 + bx + c = 0$,

$a = 2$, $b = 1$ and $c = 4$

By using quadratic formula ;-

$$x = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

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$\Rightarrow x = \frac{-1 \pm \sqrt{1-32}}{4}$

$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$

As we know, the square of a number can never be negative. Therefore, there is no real solution for the given equation.

3. Find the roots of the following equations:

(i) $\frac{x-1}{x} = 3, x \neq 0$

Sol^n $\frac{x-1}{x} = 3$

$\Rightarrow x^2 - 3x - 1 = 0$

On comparing the given equation with $ax^2+bx+c=0$

$a = 1, b = -3, c = -1$

By using quadratic formula, we get,

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow x = \frac{3 \pm \sqrt{9+4}}{2}$

$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$

$\therefore x = \frac{3+\sqrt{13}}{2}, x = \frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

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$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$

$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$

$\Rightarrow (x+4)(x-7) = -30$

$\Rightarrow x^2 - 3x - 28 = 30$

$\Rightarrow x^2 - 3x + 2 = 0$

we can solve this equation by factorization method.

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x-2) - 1(x-2) = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\Rightarrow \boxed{x = 1 \text{ or } 2}$

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol^n let us say, present age of Rahman is x years

Three years ago, Rehmans age was $(x-3)$ years

Five years after, his age will be $(x+5)$ years.

Given, the sum of the reciprocals of Rehman's ages 3 years ago and after 5 years is equal to $\frac{1}{3}$.

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$$\therefore \frac{1}{x-3} + \frac{1}{x-5} = \frac{1}{3}$$

$$\Rightarrow \frac{(x+5+x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow \frac{(2x+2)}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow 3(2x+2) = (x-3)(x+5)$$

$$\Rightarrow 6x+6 = x^2+2x-15$$

$$\Rightarrow x^2-4x-21=0$$

$$\Rightarrow x^2-7x+3x-21=0$$

$$\Rightarrow x(x-7)+3(x-7)=0$$

$$\Rightarrow (x-7)(x+3)=0$$

$$\Rightarrow x=7, -3$$

As we kno, age cannot be negative
∴ Rahman's present age is 7 years.

5. In a class test, the sum of shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English. the product of their marks would have been 210. Find her marks in the two subjects.

Sol^n let us say, the marks of shefali in Maths be

Then, the marks in English will be $30-x$.

As, per the given question.

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$(x+2)(30-x-3) = 210$

$(x+2)(27-x) = 210$

$\Rightarrow -x^2 + 25x + 54 = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

$\Rightarrow x^2 - 12x - 13x + 156 = 0$

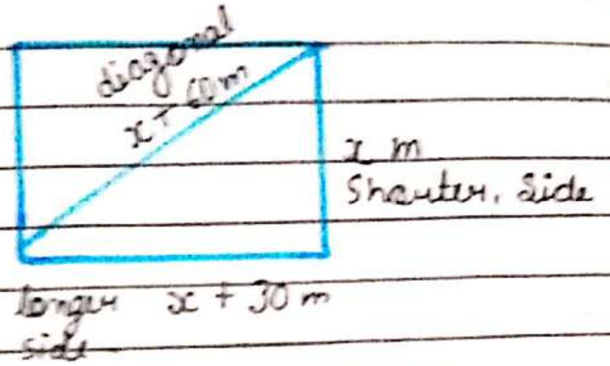
$\Rightarrow (x-12)(x-13) = 0$

$\Rightarrow x = 12, 13$

$\therefore$, if the marks in Maths are 12, then marks in English will be $30-12 = 18$ and the marks in Maths are 13, the marks in English will be $30-13 = 17$.

5. The diagonals of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol:



let, Shorter side = x m

longer side = $x + 60$ m

digonal = $x + 30$ m

A. T. Q

By Pythagoras Th^m

$$H^2 = B^2 + P^2$$

$$(x+60)^2 = (x+30)^2 + x^2$$

$$x^2 + (60)^2 + 2(x)(60) = (x^2) + (30)^3 + 2(x)(30) + x^2$$

$$x^2 + 3600 + 120x = x^2 + 900 + 60x + x^2$$

$$3600 - 900 = x^2 + 60x - 120x$$

$$2700 = x^2 - 60x$$

$$x^2 - 60x - 2700 = 0$$

$a = 1$, $b = -60$, $c = -2700$

$$D = b^2 - 4ac$$

$$= (-60)^2 - 4(1)(-2700)$$

$$= 3600 + 10800$$

$$= 14400$$

$x = \frac{-b+\sqrt{D}}{2a}$, $\frac{-b-\sqrt{D}}{2a}$

$\frac{-(-60)+\sqrt{14400}}{2(1)}$, $\frac{-(-60)-\sqrt{14400}}{2(1)}$

$= \frac{60+120}{2}$, $\frac{60-120}{2}$

$= \frac{180}{2}$ 90 , $\frac{-60}{2}$, -30 [reject side cannot be negative

shorter side

$\therefore x = 90$ m

longer side

$x + 30 \Longrightarrow 90 + 30 = 120$ m

diagonal

$x + 60 \Longrightarrow 90 + 60 = 150$ m

7. The difference of square of two numbers is 180. The square of the smaller number is 8 ~~two~~ times the larger number. Find the two numbers.

Sol^n let the smaller number $= y$

larger number $= x$

A.T.Q

$x^2 - y^2 = 180$ —— ①

$y^2 = 8x$ —— ②

Put $y^2 = 8x$ in —— ①, we get

$x^2 - 8x = 180$

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Here: $a = 1$, $b = 8$, $c = -180$

$D = b^2 - 4ac$
$= (-8)^2 - 4(1)(-180)$
$= 64 + 720$
$D = 784$

$x = \frac{-b + \sqrt{D}}{2a}$, $\frac{-b - \sqrt{D}}{2a}$
$= \frac{-(-8) + \sqrt{784}}{2(1)}$, $\frac{-(-8) - \sqrt{784}}{2(1)}$
$= \frac{8 + 28}{2}$, $\frac{8 - 28}{2}$
$= \frac{36}{2}$, $\frac{-20}{2} = -10$ [∵ reject cannot negative

$\boxed{x = 18}$ Put in — (2)

$y^2 = 8(18)$
$y^2 = 144$
$y = \sqrt{144}$
$\boxed{y = 12}$

Q. A train travels 360 Km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol^n Let the speed of train $= x$ km/h
Total distance covered $= 360$ km/h
If the speed increase $= (x + 5)$ km/h

$t_1 = \frac{D}{S} = \frac{360}{(x+5)}$ hr

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$$\frac{360}{x} - \frac{360}{(x+5)} = 1$$

$$\frac{360(x+5) - 360x}{x(x+5)} = 1$$

$$\frac{360x + 1800 - 360x}{x^2 + 5x} = 1$$

$$x^2 + 5x = 1800$$

$$x^2 + 5x - 1800 = 0$$

there $a = 1$, $b = 5$, $c = -1800$

$$D = b^2 - 4ac$$

$$= (5)^2 - 4(1)(-1800)$$

$$= 25 + 7200$$

$$= 7225$$

$$x = \frac{-b + \sqrt{d}}{2a}, \quad \frac{-b - \sqrt{d}}{a}$$

$$= \frac{-5 + \sqrt{7225}}{2(1)}, \quad \frac{-5 - \sqrt{7225}}{2(1)}$$

$$= \frac{-5 + 85}{2}, \quad \frac{-5 - 85}{2}$$

$$= \cancel{\frac{80}{2}}\ 40, \quad -\frac{90}{2}\ -45$$ [reject cannot be negative]

$x = 40$

$\therefore$ Speed of train is $\boxed{40 \text{ km/h}}$

Q. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time which each tap can separately fill the tank.

big x small y

[Hint]

$y = x + 10$ — (2)

$\frac{1}{x}$ $\frac{1}{y}$

$\frac{1}{x} + \frac{1}{y} = 9\frac{3}{8}$ hours — (1)

Solution

let the larger pipe $= x$

smaller pipe $= y$

Tank fill by larger pipe in one hour $= \frac{1}{x}$

Tank fill by smaller pipe in one hour $= \frac{1}{y}$

A. T. Q

$\frac{1}{x} + \frac{1}{y} = \frac{75}{8}$ — (1)

$y = x + 10$ — (2)

put $y = x + 10$ in (1) we get

$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{75/8}$

$\frac{(x+10) + x}{x(x+10)} = \frac{8}{75}$

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$$\frac{x+10+x}{x^2+10x} = \frac{8}{75}$$

$$\frac{2x+10}{x^2+10x} = \frac{8}{75}$$

$$75(2x+10) = 8(x^2+10x)$$
$$150x+750 = 8x^2+80x$$
$$150x-80x = 8x^2-750$$
$$70x = 8x^2-750$$
$$8x^2-70x-750=0$$

Here, $a=8$, $b=-70$, $c=-750$

$$D = b^2-4ac$$
$$= (-70)^2-4(8)(-750)$$
$$= 4900+24000$$
$$= 28900$$

$$x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$$

$$= \frac{-(-70)+\sqrt{28900}}{2(8)}; \frac{(-70)-\sqrt{28900}}{2(8)}$$

$$= \frac{70+170}{16}, \frac{70-170}{16}$$

$$= \cancel{\frac{240}{16}}, \cancel{\frac{-100}{16}} = \frac{-25}{4}$$

$= 15$, $\frac{-25}{4}$ [reject can't be negative]

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$x = 15$ Put in – (2)

$y = x + 10$

$y = 15 + 10$

$y = 25$

larger tap tape = 15 hours
smaller tap tape = 25 hours

11. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Banglore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train. find the average speed of the two trains.

Sol^n let the speed of passenger train = x km/h
let the speed of express train = $(x+11)$ km/h

Distance covered = 132 km

t_1 = time taken by passenger train = $\frac{132}{x}$ hr

t_2 = time taken by express train = $\frac{132}{(x+11)}$ hr

A.T.Q

$$\frac{132}{x+11} = \frac{132}{x} - 1$$

$$\frac{132}{x+11} = \frac{132 - x}{x}$$

$$132x = (132 - x)(x + 11)$$

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$132x = 132x + 1452 - x^2 - 11x$

$x^2 + 11x - 1452 = 0$

Here : $a = 1$, $b = 11$, $c = -1452$

$D = b^2 - 4ac$

$= (11)^2 - 4(1) \cdot (-1452)$

$= 121 + 5808$

$= 5929$

$$x = \frac{-b + \sqrt{D}}{2a} , \frac{-b - \sqrt{D}}{2a}$$

$$= \frac{-11 + \sqrt{5929}}{2(1)} , \frac{-11 - \sqrt{5929}}{2(1)}$$

$$= \frac{-11 + 77}{2} , \frac{-11 - 77}{2}$$

$$= \frac{66}{2}\ 33 , \frac{-88}{2}\ -44$$

$= 33$, -44 [∵ reject can't be negative]

∴ Speed of Passenger train $= x = 33$ km/h

Speed of express train $= x + 11$

$= 33 + 11$

$= 44$ km/h

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11. Sum of the areas of two squares is $468\ m^2$. If the difference of their perimeters is 24 m find the sides of the two squares.

Ans. let the sides of the two squares by x m and y m.

∴ their perimeter will be $4x$ and $4y$ respectively

And area of the squares will be x^2 and y^2 respectively

Given

$4x - 4y = 24$

$x - y = 6$

also, $x^2 + y^2 = 468$

$\Rightarrow (6 + y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y + 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

$y = -18, 12$

We know, the side of a square cannot be negative.

Hence, the sides of the square are 12 m and $(12 + 6)$ m $= 18$ m.

$D = b^2 - 4ac$
$= (1)^2 - 4 \times 2 \times 4$
$= 1 - 32$
$= -31 < 0$

roots do not exist.

Exercise : 4.4

The nature of the roots of the following quadratic equations. If the real roots exist find them.

$2x^2 - 3x + 5 = 0$ [By Discriminent Method]

$a = 2, \quad b = -3, \quad c = 5$

$D = b^2 - 4ac$
$= (-3)^2 - 4 \times 2 \times 5$
$= 9 - 40$
$= -31$ (Roots are not real)

Roots do not exist.

$3x^2 - 4\sqrt{3}x + 4 = 0$
$a = 3, \quad b = -4\sqrt{3}, \quad c = 4$
$D = b^2 - 4ac$
$= (-4\sqrt{3})^2 - 4 \times 3 \times 4$
$= (-4)^2 (\sqrt{3})^2 - 4 \times 3 \times 4$

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$= (16)(3) - 48$

$= 48 - 48$

$= 0$ (Roots are real and equal)

for roots :- $x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$

$= \frac{-(-4\sqrt{3}) + \sqrt{0}}{2(3)}, \frac{-(-4\sqrt{3}) - \sqrt{0}}{2(3)}$

$= \frac{4\sqrt{3} + 0}{6}, \frac{4\sqrt{3} - 0}{6}$

$= \frac{\cancel{4}^{2}\sqrt{3}}{\cancel{6}_{3}}, \frac{\cancel{4}^{2}\sqrt{3}}{\cancel{6}_{3}}$

$= \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

$2x^2 - 6x + 3 = 0$

$a = 2, b = -6, c = 3$

$D = b^2 - 4ac$

$= (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24$

$= 12$

$D = 12 > 0$, Roots are real and different.

for Roots :- $x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$

$= \frac{-(-6) + \sqrt{12}}{2(2)}, \frac{-(-6) - \sqrt{12}}{2(2)}$

$= \frac{6 + 2\sqrt{3}}{4}, \frac{6 - 2\sqrt{3}}{4}$

$= \frac{\cancel{2}(3 + \sqrt{3})}{\cancel{4}_{2}}, \frac{\cancel{2}(3 - \sqrt{3})}{\cancel{4}_{2}}$

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$= \frac{3+\sqrt{3}}{2} , \frac{3-\sqrt{3}}{2}$

Nature of Roots

$D > 0$, Roots are real and different.

$D = 0$, Roots are real and equal.

$D < 0$, Roots are not real.

Find the values of k for each of the following quadratic equations, so that they have two equal roots

(i) $2x^2 + kx + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$

$a = 2$, $b = k$ and $c = 3$

As we know, Discriminant $= b^2 - 4ac$

$= (k)^2 - 4(2)(3)$

$k^2 - 24$

For equal roots, we know

$D = 0$

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) $kx(x-2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$

$a = k$, $b = -2k$, $c = 6$

$D = b^2 - 4ac$

$= (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

For equal roots, we know

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k-6) = 0$

Either $4k = 0$ or $k - 6 = 0$

$k = 0$ or $k = 6$

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However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'

$\therefore$ if this equation has two equal roots, k should be 6 only

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Sol^n Let the breadth of mango grove be l.
length of mango grove will be $2l$.
Area of mango grove $= (2l)(l) = 2l^2$

$2l^2 = 800$

$l^2 = \frac{800}{2} = 400$

$l^2 - 400 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$
$a = 1$, $b = 0$, $c = 400$

$D = b^2 - 4ac$
$\Rightarrow (0)^2 - 4 \times (1) \times (-400) = 1600$
Here $b^2 - 4ac > 0$

Thus, the equation will have real roots. And hence, the desired rectangular mango grove can be designed
$l = \pm 20$

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As we know the value of length cannot be negative

$\therefore$ breadth of mango grove = 20 m
length of mango grove = 2 × 20 = 40 m

Q. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48

A. let's say, the age of one friend be x years.

Then, the age of other friend will be (20 - x) years.

Four years ago,
Age of first friend = (x - 4) years

Age of second friend = (20 - x - 4) = (16 - x) years

As per the given question, we can write.

$(x-4)(16-x) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing the equation with $ax^2 + bx + c = 0$

$a = 1, b = -20, c = 112$

$D = b^2 - 4ac$

$\Rightarrow (-20)^2 - 4 \times 112$

$\Rightarrow 400 - 448 = -48$

$b^2 - 4ac < 0$

$\therefore$ there will be no real solⁿ possible for the equations. Hence, condition doesn't exist.

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5. Is it possible to ~~desi~~ design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Ans let the length and breadth of the park be l and b.

Perimeter of the rectangular park $= 2(l+b) = 80$

so, $l + b = 40$

or $b = 40 - l$

Area of the rectangular park $= l \times b$

$l(40 - l) = 40l - l^2 = 400$

$l^2 - 40l + 400 = 0$, which is a quadratic equation.

Comparing the equation with $ax^2 + bx + c = 0$

$a = 1, b = -40, c = 400$

$D = b^2 - 4ac$

$\Rightarrow (-40)^2 - 4 \times 400$

$\Rightarrow 1600 - 1600 = 0$

$b^2 - 4ac = 0$

$\therefore$ this equation has equal real roots. Hence, the equation is possible

Root of the equation,

$l = \frac{-b}{2a}$

$l = \frac{40}{2(1)} = \frac{40}{2} = 20$

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∴ length of rectangular park, $l = 20 m$

breadth of the park, $b = 40 - l = 40 - 20 = 20 m$

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