

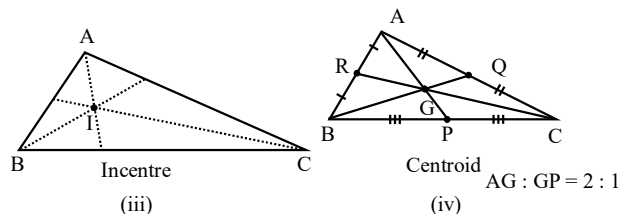
TRIANGLES

7

CHAPTER

CONTENTS

- Introduction
- Angle sum property for triangle
- Congruent triangles
- Criteria for congruent triangles
- Isosceles triangle



Note :

- (1) All four points are coincide for an equilateral triangle.
- (2) Orthocentre (H), Centroid (G), Circumcentre (O) are always collinear points and G divides OH in ratio 1 : 2.



ANGLE SUM PROPERTY FOR TRIANGLE

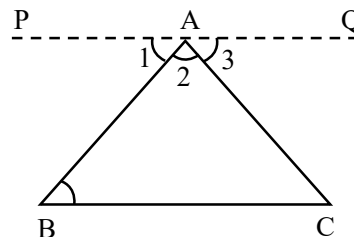
Theorem :

Prove that sum of all three angles is 180° or 2 right angles.

Given : $\triangle ABC$

To prove : $\angle A + \angle B + \angle C = 180^\circ$

Construction : Draw $PQ \parallel BC$, passes through point A.



Proof : $\left. \begin{array}{l} \angle 1 = \angle B \\ \text{and } \angle 3 = \angle C \end{array} \right\} \begin{array}{l} \text{alternate angles } \because PQ \parallel BC \\ \dots\dots(i) \end{array}$

\therefore PAQ is a line

$\therefore \angle 1 + \angle 2 + \angle 3 = 180^\circ$ (linear pair application)

$$\angle B + \angle 2 + \angle C = 180^\circ$$

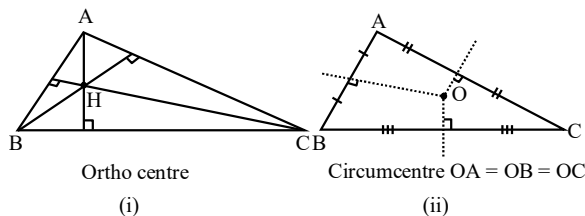
$$\angle B + \angle CAB + \angle C = 180^\circ$$

= 2 right angles.

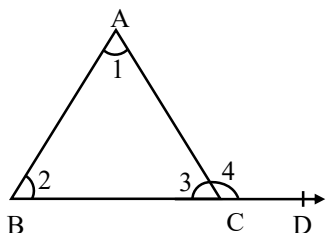
Proved.

IN A TRIANGLE (INTRODUCTION)

- Orthocenter is the point of intersection of the altitudes.
- Circumcentre is the point of intersection of the perpendicular bisectors of the sides.
- In centre is the point of intersection of the angular bisectors.
- Centroid is the point of intersection of the medians.
- The circumcentre of a triangle is equidistant from its vertices.
- The in centre of a triangle is equidistant from its sides.
- The centroid divides a median in the ratio 2 : 1.
- The orthocentre of a right angled triangle lies at the vertex containing the right angle.



Theorem : If one side of a triangle is produced then the exterior angle so formed is equal to the sum of two interior opposite angles.



Means $\angle 4 = \angle 1 + \angle 2$

Proof : $\angle 3 = 180^\circ - (\angle 1 + \angle 2)$ (1)
(by angle sum property)

and BCD is a line

$\therefore \angle 3 + \angle 4 = 180^\circ$ (linear pair)

or $\angle 3 = 180^\circ - \angle 4$ (2)

by (1) & (2)

$180^\circ - (\angle 1 + \angle 2) = 180^\circ - \angle 4$

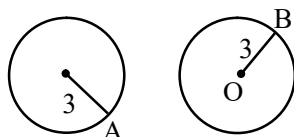
$\Rightarrow \angle 1 + \angle 2 = \angle 4$ Proved.

CONGRUENT FIGURES

Two geometrical figures having exactly the same shape & size are known as congruent figures. Lines, polygons, circles etc. can congruent.

Note :

(1) If radius of a circle is same as other circle then only both circles are congruent.



(2) Two line segment are congruent only when their length are equal.

$\overline{AB} \quad 4.0 \text{ cm} \quad \overline{CD} \quad 4.0 \text{ cm}$

CONGRUENT TRIANGLES

Two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.

Thus, congruent triangles are exactly identical.

Example 1 : If $\triangle ABC \cong \triangle DEF$ then we have :

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$; and
 $AB = DE$, $BC = EF$ and $AC = DF$.

Example 2 If $\triangle ABC \cong \triangle EDF$ then we have:

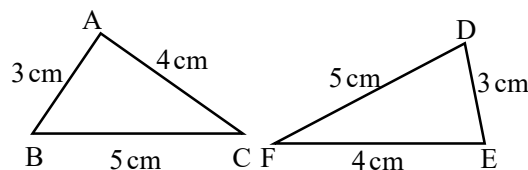
$\angle A = \angle E$, $\angle B = \angle D$, $\angle C = \angle F$; and
 $AB = ED$, $BC = DF$ and $AC = EF$.

Note :

- (1) Every triangle is congruent to itself, i.e.,
 $\triangle ABC \cong \triangle ABC$.
- (2) If $\triangle ABC \cong \triangle DEF$ then $\triangle DEF \cong \triangle ABC$.
- (3) If $\triangle ABC \cong \triangle DEF$, and $\triangle DEF \cong \triangle PQR$, then
 $\triangle ABC \cong \triangle PQR$.
- (4) 'c.p.c.t.' for 'corresponding parts of congruent triangles'.

CRITERIA FOR CONGRUENT TRIANGLES

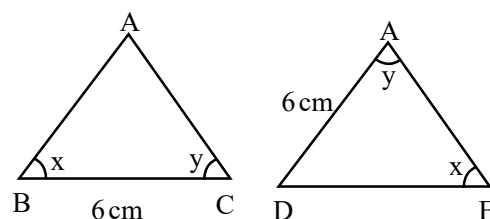
(1) SSS (Side Side Side)



\therefore By SSS criteria $\triangle ABC \cong \triangle EDF$

$\therefore \angle A = \angle E$, $\angle B = \angle D$, $\angle C = \angle F$ (c.p.c.t.)

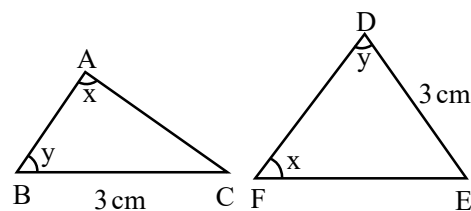
(2) ASA (Angle Side Angle)



\therefore By ASA criteria $\triangle ABC \cong \triangle DEF$

$\therefore \angle A = \angle D$, $AB = DE$, $AC = DF$ (c.p.c.t.)

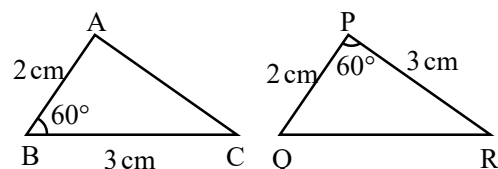
(3) AAS (Angle Angle Side)



\therefore By AAS, $\triangle ABC \cong \triangle FDE$

$\therefore \angle C = \angle E$, $AB = FD$, $AC = FE$ (c.p.c.t.)

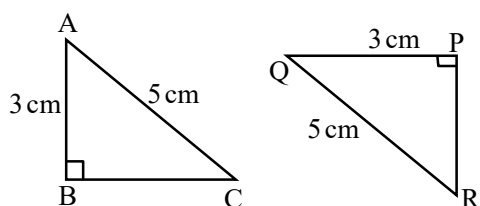
(4) SAS (Side Angle Side)



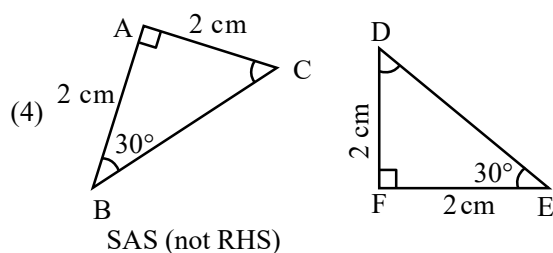
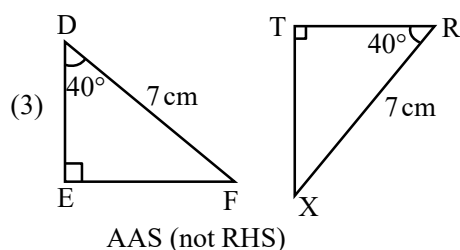
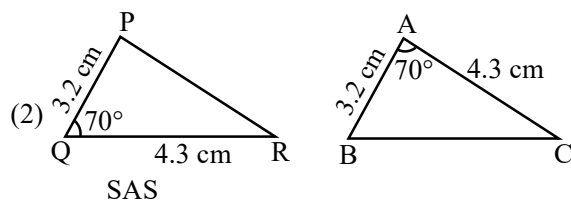
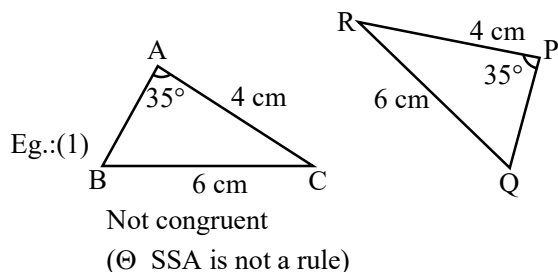
By SAS, $\triangle ABC \cong \triangle PQR$

$\therefore \angle A = \angle P$, $\angle C = \angle R$, $AC = PR$ (c.p.c.t.)

(5) RHS (Right Hypotenuse Side)



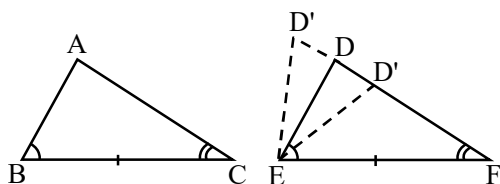
\therefore By RHS, $\triangle ABC \cong \triangle PQR$
 $\therefore \angle A = \angle Q, \angle C = \angle R, BC = PR$ (c.p.c.t.)



Theorem 1 : If two angles and the included side of one triangle are equal to two angles and the included side of other triangle, then both triangles are congruent.

Proof :

Given : $\triangle ABC$ and $\triangle DEF$ in which
 $\angle ABC = \angle DEF, \angle ACB = \angle DFE$ and $BC = EF$.



To prove : $\triangle ABC \cong \triangle DEF$.

Proof :

Case I

Let $AC = DF$.

In this case, $AC = DF, BC = EF$ and $\angle C = \angle F$.

$\therefore \triangle ABC \cong \triangle DEF$ (SAS-criteria)

Case II

If possible, let $AC \neq DF$.

Then, construct $D'F = AC$. Join $D'E$.

Now, in $\triangle ABC$ and $\triangle D'EF$, we have $AC = D'F$,
 $BC = EF$ and $\angle C = \angle F$.

$\therefore \triangle ABC \cong \triangle D'EF$ (SAS-criteria)

$\therefore \angle ABC = \angle D'EF$ (c.p.c.t.)

But, $\angle ABC = \angle DEF$ (given)

$\therefore \angle D'EF = \angle DEF$.

This is possible only when D and D' coincide.

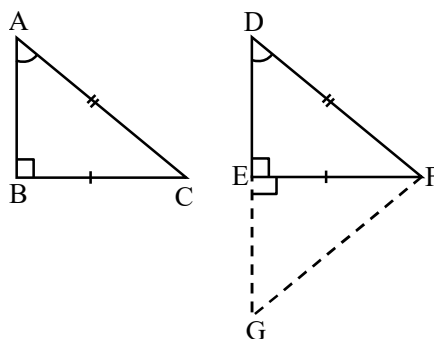
$\therefore \triangle ABC \cong \triangle DEF$.

Theorem 2 : Two right-angled triangles are congruent if one side and the hypotenuse of the one are respectively equal to the corresponding side and the hypotenuse of the other. (i.e. RHS)

Given : Two right-angled triangles $\triangle ABC$ & $\triangle DEF$ in which $\angle B = \angle E = 90^\circ, BC = EF$ and $AC = DF$.

To prove : $\triangle ABC \cong \triangle DEF$.

Construction : Produce DE to G such that $GE = AB$. Join GF.



Proof : In $\triangle ABC$ and $\triangle GEF$, we have :

$AB = GE$ (construction),

$BC = EF$ (given), $\angle B = \angle FEG = 90^\circ$

$\therefore \triangle ABC \cong \triangle GEF$ (SAS-criteria)

$\therefore \angle A = \angle G$ and $AC = GF$ (c.p.c.t.)

Now, $AC = GF$ and $AC = DF \Rightarrow GF = DF$

$\Rightarrow \angle G = \angle D \Rightarrow \angle A = \angle D$ [$\because \angle G = \angle A$]

Now, $\angle A = \angle D, \angle B = \angle E \Rightarrow 3^{rd} \angle C = 3^{rd} \angle F$.

Thus, in $\triangle ABC$ and $\triangle DEF$, we have:

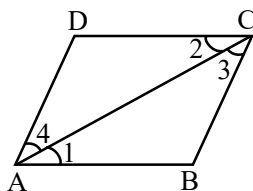
$BC = EF, AC = DF$ and $\angle C = \angle F$.

$\therefore \triangle ABC \cong \triangle DEF$ (SAS-criteria).

❖ **EXAMPLES** ❖

Ex.1 Prove that diagonal of a parallelogram divides it into two congruent triangles.

Sol. Let ABCD is a parallelogram and AC is a diagonal.



(By SSS) : In $\triangle ABC$ and $\triangle ADC$

$AB = CD$ (opp. sides of \parallel^{gm})

$BC = AD$ (opp. sides of \parallel^{gm})

$AC = AC$ (common)

\therefore By SSS, $\triangle ABC \cong \triangle CDA$ proved

{other results : $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle B = \angle D$
(c.p.c.t.)}

(By ASA) : In $\triangle ABC$ and $\triangle ADC$

$\angle 1 = \angle 2$ (alternate)

$AC = AC$ (common)

$\angle 3 = \angle 4$ (alternate)

\therefore By ASA, $\triangle ABC \cong \triangle CDA$

{other results : $\angle B = \angle D$, $AB = CD$, $BC = AD$
(c.p.c.t.)}

(By AAS) : In $\triangle ABC$ and $\triangle ADC$

$\angle 1 = \angle 2$ (alternate)

$\angle 3 = \angle 4$ (alternate)

$BC = AD$ (opp. sides)

$\therefore \triangle ABC \cong \triangle CDA$

{other results : $AB = CD$, $\angle B = \angle D$, $AC = AC$
(c.p.c.t.)}

(By SAS) : In $\triangle ABC$ and $\triangle ADC$

$AB = CD$ (opp. sides of \parallel^{gm})

$\angle 1 = \angle 2$ (alternate)

$AC = AC$ (common)

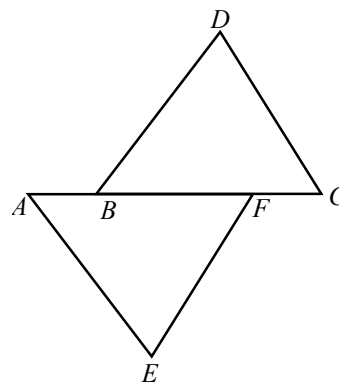
$\therefore \triangle ABC \cong \triangle CDA$

{other results: $\angle 3 = \angle 4$, $BC = AD$, $\angle B = \angle D$
(c.p.c.t.)}

We can not use 'RHS' for this proof.

Note : ASS or SSA criteria for congruency is not valid.

Ex.2 In Fig. it is given that $AB = CF$, $EF = BD$ and $\angle AFE = \angle DBC$. Prove that $\triangle AFE \cong \triangle CBD$.



Sol. We have, $AB = CF$

$\Rightarrow AB + BF = CF + BF$

$\Rightarrow AF = CB$ (i)

In $\triangle AFE$ and $\triangle CBD$, we have

$AF = CB$ [From (i)]

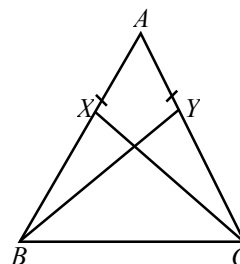
$\angle AFE = \angle DBC$ [Given]

and $EF = BD$ [Given]

So, by SAS criterion of congruence, we have

$\triangle AFE \cong \triangle CBD$

Ex.3 In Fig. X and Y are two points on equal sides AB and AC of a $\triangle ABC$ such that $AX = AY$. Prove that $XC = YB$.



Sol. In $\triangle AXC$ and $\triangle AYB$, we have

$AX = AY$ [Given]

$\angle A = \angle A$ [Common angle]

$AC = AB$ [Given]

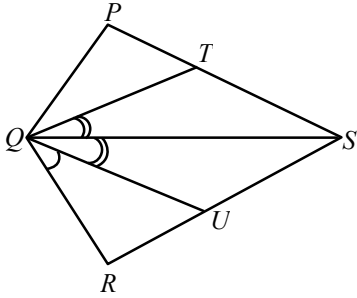
So, by SAS criterion of congruence

$\triangle AXC \cong \triangle AYB$

$\Rightarrow XC = YB$

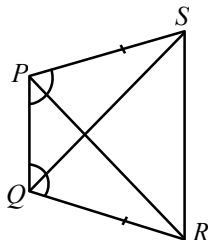
(c.p.c.t.)

Ex.4 In Fig. PQRS is a quadrilateral and T and U are respectively points on PS and RS such that $PQ = RQ$, $\angle PQT = \angle RQU$ and $\angle TQS = \angle UQS$. Prove that $QT = QU$.



Sol. We have,
 $\angle PQT = \angle RQU$
 and, $\angle TQS = \angle UQS$
 $\therefore \angle PQT + \angle TQS = \angle RQU + \angle UQS$
 $\Rightarrow \angle PQS = \angle RQS$ (i)
 Thus, in triangles PQS and RQS, we have
 $PQ = RQ$ [Given]
 $\angle PQS = \angle RQS$
 [From (i)]
 and, $QS = QS$ [Common side]
 Therefore, by SAS congruence criterion, we have
 $\triangle PQS \cong \triangle RQS$
 $\Rightarrow \angle QPS = \angle QRS$
 (c.p.c.t.)
 $\Rightarrow \angle QPT = \angle QRU$ (ii)
 Now, consider triangles QPT and QRS. In these two triangles, we have
 $QP = QR$ [Given]
 $\angle PQT = \angle RQU$ [Given]
 $\angle QPT = \angle QRU$ [From (ii)]
 Therefore, by ASA congruence criterion, we get
 $\triangle QPT \cong \triangle QRU$
 $\Rightarrow QT = QU$.

Ex.5 In Fig. $PS = QR$ and $\angle SPQ = \angle RQP$.



Prove that $PR = QS$ and $\angle QPR = \angle PQS$.

Sol. In $\triangle SPQ$ and $\triangle RQP$, we have
 $PS = QR$ [Given]
 $\angle SPQ = \angle RQP$ [Given]
 $PQ = PQ$ [Common]

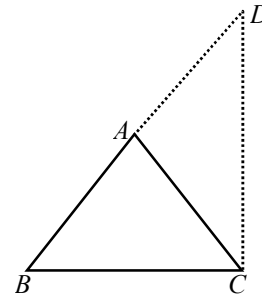
Therefore, by SAS criterion of congruence, we have

$$\triangle SPQ \cong \triangle RQP \Rightarrow SQ = RP \text{ and}$$

$$\angle QPR = \angle PQS$$

Ex.6 $\triangle ABC$ is an isosceles triangle with $AB = AC$. Side BA is produced to D such that $AB = AD$. Prove that $\angle BCD$ is a right angle.

Sol. Given : A $\triangle ABC$ such that $AB = AC$. Side BA is produced to D such that $AB = AD$.



Construction : Join CD .

To prove : $\angle BCD = 90^\circ$

Proof : In $\triangle ABC$, we have $AB = AC$

$$\Rightarrow \angle ACB = \angle ABC$$
(i)

[\ominus Angles opp. to equal sides are equal]

Now, $AB = AD$ [Given]

$$\therefore AD = AC$$
 [$\because AB = AC$]

Thus, in $\triangle ADC$, we have

$$AD = AC$$

$$\Rightarrow \angle ACD = \angle ADC$$
(ii)

[\ominus Angles opp. to equal sides are equal]

Adding (i) and (ii), we get

$$\angle ACB + \angle ACD = \angle ABC + \angle ADC$$

$$\Rightarrow \angle BCD = \angle ABC + \angle BDC$$

$$[\ominus \angle ADC = \angle BDC, \angle ABC = \angle DBC]$$

$$\Rightarrow \angle BCD + \angle BCD = \angle DBC + \angle BCD + \angle BDC$$

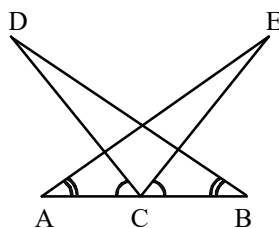
[Adding $\angle BCD$ on both side]

$$\Rightarrow 2 \angle BCD = 180^\circ$$

[\ominus Sum of the angles of a \triangle is 180°]

Hence, $\angle BCD$ is a right angle.

Ex.7 In Fig. $AC = BC$, $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$.



Prove that triangles DBC and EAC are congruent, and hence $DC = EC$.

Sol. We have,

$$\begin{aligned}\angle DCA &= \angle ECB \\ \Rightarrow \angle DCA + \angle ECD &= \angle ECB + \angle ECD \\ \Rightarrow \angle ECA &= \angle DCB \quad \dots (i)\end{aligned}$$

Now, in Δ s DBC and EAC, we have

$$\angle DCB = \angle ECA \quad [\text{From (i)}]$$

$$BC = AC \quad [\text{Given}]$$

$$\text{and } \angle DBC = \angle EAC \quad [\text{Given}]$$

So, by ASA criterion of congruence

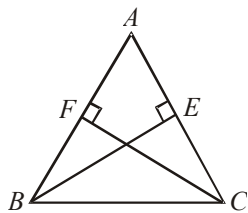
$$\Delta DBC \cong \Delta EAC$$

$$\Rightarrow DC = EC$$

(c.p.c.t.)

Ex.8 If the altitudes from two vertices of a triangle to the opposite sides are equal, prove that the triangle is isosceles.

Sol. Given : A ΔABC in which altitudes BE and CF from B and C respectively on AC and AB are equal.



To prove : ΔABC is isosceles i.e. $AB = AC$

Proof : In Δ s ABC and ACF, we have

$$\angle AEB = \angle AFC \quad [\text{Each equal to } 90^\circ]$$

$$\angle BAE = \angle CAF \quad [\text{Common angle}]$$

$$\text{and, } BE = CF \quad [\text{Given}]$$

So, by AAS criterion of congruence, we have

$$\Delta ABE \cong \Delta ACF$$

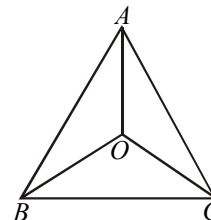
$$\Rightarrow AB = AC \quad \left[\begin{array}{l} \Theta \text{ Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

Hence, ΔABC is isosceles.

Ex.9 In ΔABC , $AB = AC$ and the bisectors of angles B and C intersect at point O. Prove that $BO = CO$ and the ray AO is the bisector of angle BAC.

Sol. In ΔABC , we have

$$AB = AC$$



$$\Rightarrow \angle B = \angle C \quad \left[\begin{array}{l} \Theta \text{ Angles opposite to} \\ \text{equal sides are equal} \end{array} \right]$$

$$\Rightarrow \frac{1}{2} \angle B = \frac{1}{2} \angle C$$

$$\Rightarrow \angle OBC = \angle OCB \quad \dots (i)$$

$$\left[\begin{array}{l} \Theta \text{ OB and OC are bisectors of } \angle \text{s B and C} \\ \text{respectively } \therefore \angle OBC = \frac{1}{2} \angle B \text{ \& } \angle OCB = \frac{1}{2} \angle C \end{array} \right]$$

$$\Rightarrow OB = OC \quad \dots (ii)$$

[Θ Sides opp. to equal \angle s are equal]

Now, in ΔABO and ΔACO , we have

$$AB = AC \quad [\text{Given}]$$

$$\angle OBC = \angle OCB \quad [\text{From (i)}]$$

$$OB = OC \quad [\text{From (ii)}]$$

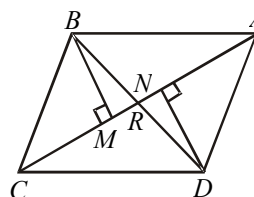
So, by SAS criterion of congruence

$$\Delta ABO \cong \Delta ACO$$

$$\Rightarrow \angle BAO = \angle CAO \quad [\Theta \text{ Corresponding parts of congruent triangles are equal}]$$

\Rightarrow AO is the bisector of $\angle BAC$.

Ex.10 In Fig. BM and DN are both perpendiculars to the segments AC and BM = DN.



Prove that AC bisects BD.

Sol. In Δ s BMR and DNR, we have

$$\angle BMR = \angle DNR$$

[Each equal to 90° Θ $BM \perp AC$ and $DN \perp AC$]

$$\angle BRM = \angle DRN \quad [\text{Vert. opp. angles}]$$

and, $BM = DN$ [Given]
 So, by AAS criterion of congruence

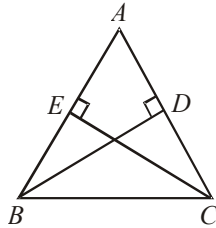
$$\triangle BMR \cong \triangle DNR$$

$$\Rightarrow BR = DR \left[\begin{array}{l} \ominus \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

$\Rightarrow R$ is the mid-point of BD .

Hence, AC bisects BD .

Ex.11 In Fig. BD and CE are two altitudes of a $\triangle ABC$ such that $BD = CE$.



Prove that $\triangle ABC$ is isosceles.

Sol. In $\triangle ABD$ and $\triangle ACE$, we have

$$\angle ADB = \angle AEC = 90^\circ \quad [\text{Given}]$$

$$\angle BAD = \angle CAE \quad [\text{Common}]$$

$$\text{and, } BD = CE \quad [\text{Given}]$$

So, by AAS congruence criterion, we have

$$\triangle ABD \cong \triangle ACE$$

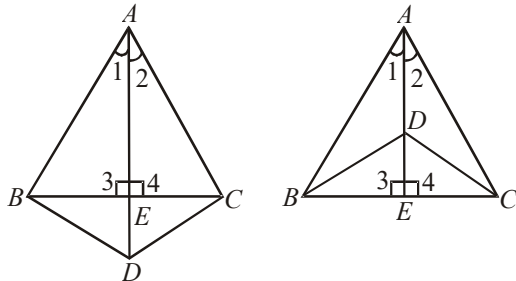
$$\Rightarrow AB = AC \left[\begin{array}{l} \ominus \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

Hence, $\triangle ABC$ is isosceles.

Ex.12 If two isosceles triangles have a common base, the line joining their vertices bisects them at right angles.

Sol. **Given :** Two isosceles triangles ABC and DBC having the common base BC such that $AB = AC$ and $DB = DC$.

To prove : AD (or AD produced) bisects BC at right angle.



Proof : In $\triangle ABD$ and $\triangle ACD$, we have

$$AB = AC \quad [\text{Given}]$$

$$BD = CD \quad [\text{Given}]$$

$$AD = AD \quad [\text{Common side}]$$

So, by SSS criterion of congruence

$$\triangle ABD \cong \triangle ACD$$

$$\Rightarrow \angle 1 = \angle 2 \quad \dots (i)$$

$$\left[\begin{array}{l} \ominus \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

Now, in $\triangle ABE$ and $\triangle ACE$, we have

$$AB = AC \quad [\text{Given}]$$

$$\angle 1 = \angle 2 \quad [\text{From (i)}]$$

$$\text{and, } AE = AE \quad [\text{Common side}]$$

So, by SAS criterion of congruence,

$$\triangle ABE \cong \triangle ACE$$

$$\Rightarrow BE = CE \left[\begin{array}{l} \ominus \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

$$\text{and, } \angle 3 = \angle 4$$

$$\text{But, } \angle 3 + \angle 4 = 180^\circ$$

$$[\ominus \text{Sum of the angles of a linear pair is } 180^\circ]$$

$$\Rightarrow 2\angle 3 = 180^\circ \quad [\ominus \angle 3 = \angle 4]$$

$$\Rightarrow \angle 3 = 90^\circ$$

$$\therefore \angle 3 = \angle 4 = 90^\circ$$

Hence, AD bisects BC at right angles.

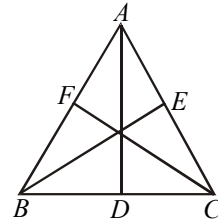
Ex.13 AD , BE and CF , the altitudes of $\triangle ABC$ are equal. Prove that $\triangle ABC$ is an equilateral triangle

Sol. In right triangles BCE and BFC , we have

$$\text{Hyp. } BC = \text{Hyp. } BC$$

$$BE = CF \quad [\text{Given}]$$

So, by RHS criterion of congruence,



$$\triangle BCE \cong \triangle BFC.$$

$$\Rightarrow \angle B = \angle C \left[\begin{array}{l} \ominus \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

$$\Rightarrow AC = AB \quad \dots (i)$$

$$[\ominus \text{Sides opposite to equal angles are equal}]$$

Similarly, $\triangle ABD \cong \triangle ABE$

$$\Rightarrow \angle B = \angle A$$

$$[\text{Corresponding parts of congruent triangles are equal}]$$

$$\Rightarrow AC = BC \quad \dots (ii)$$

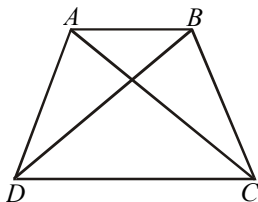
[Sides opposite to equal angles are equal]

From (i) and (ii), we get

$$AB = BC = AC$$

Hence, $\triangle ABC$ is an equilateral triangle.

Ex.14 In Fig. $AD = BC$ and $BD = CA$.



Prove that $\angle ADB = \angle BCA$ and

$$\angle DAB = \angle CBA.$$

Sol. In triangles ABD and ABC, we have

$$AD = BC \quad [\text{Given}]$$

$$BD = CA \quad [\text{Given}]$$

and $AB = AB$ [Common]

So, by SSS congruence criterion, we have

$$\triangle ABD \cong \triangle BCA \Rightarrow \angle DAB = \angle ABC$$

$$\left[\begin{array}{l} \Theta \text{ corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

$$\Rightarrow \angle DAB = \angle CBA$$

Ex.15 Line-segment AB is parallel to another line-segment CD. O is the mid-point of AD (see figure). Show that (i) $\triangle AOB \cong \triangle DOC$ (ii) O is also the mid point of BC.

Sol. (i) Consider $\triangle AOB$ and $\triangle DOC$

$$\angle ABO = \angle DCO$$

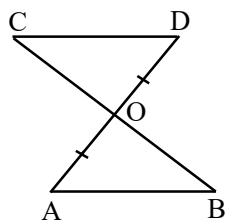
(Alternate angles as $AB \parallel CD$
and BC is the transversal)

$$\angle AOB = \angle DOC$$

(Vertically opposite angles)

$$OA = OD \quad (\text{Given})$$

Therefore, $\triangle AOB \cong \triangle DOC$ (AAS rule)

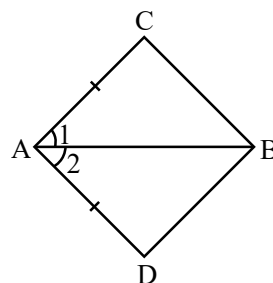


$$(ii) \quad OB = OC \quad (\text{c.p.c.t.})$$

So, O is the mid-point of BC.

Ex.16 In quadrilateral ABCD,

$AC = AD$ and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD ? [NCERT]



Sol. In $\triangle ABC$ & $\triangle ABD$

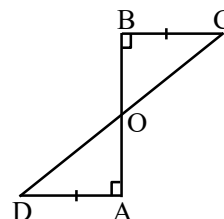
$$AB = AB \text{ (common)}$$

$$\angle 1 = \angle 2 \quad \{\Theta \text{ AB is bisector of } \angle A\}$$

$$AC = AD \text{ (Given)}$$

\therefore By SAS, $\triangle ABC \cong \triangle ABD$ Proved
also $BC = BD$ (c.p.c.t.)

Ex.17 AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB. [NCERT]



Sol. To show CD bisect AB i.e. $AO = OB$

$$\therefore \text{ in } \triangle OAD \text{ and } \triangle OBC$$

$$\angle O = \angle O \text{ (vertically opposite angles)}$$

$$\angle A = \angle B = 90^\circ \text{ (Given)}$$

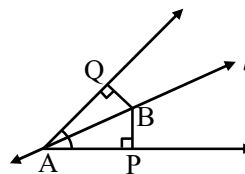
$$AD = BC \text{ (Given)}$$

\therefore By AAS, $\triangle OAD \cong \triangle OBC$

$$\therefore OA = OB \text{ (c.p.c.t.)}$$

\therefore CD, bisects AB. Proved

Ex.18 Line l is the bisector of an angle $\angle A$ and B is any point on l . BP and BQ are perpendiculars from B to the arms of $\angle A$ (see figure). Show that : [NCERT]



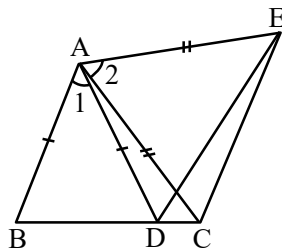
$$(i) \quad \triangle APB \cong \triangle AQB$$

(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

- Sol.** (i) In $\triangle APB$ and $\triangle AQB$
 $\angle P = \angle Q = 90^\circ$ (Given)
 $\angle PAB = \angle QAB$ (Given that 'I' bisect $\angle A$)
 $AB = AB$ (Common)
 \therefore By AAS, $\triangle APB \cong \triangle AQB$. Proved
 (ii) $BP = BQ$ (c.p.c.t.) Proved.

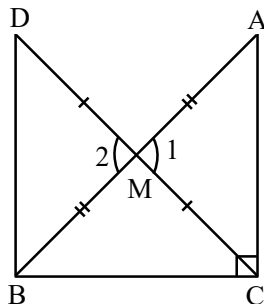
Ex.19 In given figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.

[NCERT]



- Sol.** In $\triangle ABC$ and $\triangle ADE$
 $AB = AD$ (Given)
 $\angle BAC = \angle DAE \left\{ \begin{array}{l} \angle 1 = \angle 2 \quad \text{Given} \\ \angle 1 + \angle DAC = \angle 2 + \angle DAC \end{array} \right.$
 $AC = AE$ (Given)
 \therefore By SAS, $\triangle ABC \cong \triangle ADE$
 $\therefore BC = DE$ (c.p.c.t.) Proved.

Ex.20 In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B (see figure). Show that: [NCERT]



- (i) $\triangle AMC \cong \triangle BMD$
 (ii) $\angle DBC$ is a right angle
 (iii) $\triangle DBC \cong \triangle ACB$
 (iv) $CM = \frac{1}{2} AB$

- Sol.** (i) In $\triangle AMC$ and $\triangle BMD$
 $AM = MB$ (M is mid point of AB)
 $\angle 1 = \angle 2$ (vertically opposite angles)

$CM = MD$ (given)

\therefore By SAS, $\triangle AMC \cong \triangle BMD$ Proved.

(ii) $\angle ACM = \angle MDB$ (c.p.c.t. of (i))

These are alternate angles

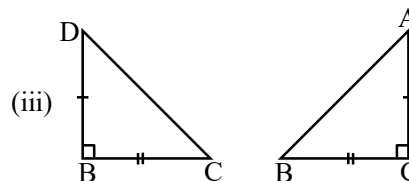
$\therefore DB \parallel AC$

So $\angle DBC + \angle ACB = 180^\circ$

(Cointerior angles)

$\Rightarrow \angle DBC + 90^\circ = 180^\circ$

$\Rightarrow \angle DBC = 90^\circ$ Proved.



In $\triangle DBC$ & $\triangle ACB$

$BC = BC$ (common)

$\angle DBC = \angle ACB = 90^\circ$

$DB = AC$ (c.p.c.t. of part (i))

\therefore By SAS, $\triangle DBC \cong \triangle ACB$. Proved

(iv) $DC = AB$ (c.p.c.t. of part (iii))

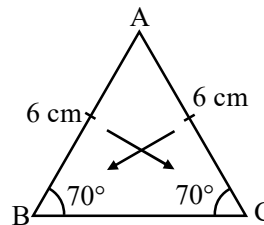
But $CM = \frac{1}{2} DC$ (given)

$\therefore CM = \frac{1}{2} AB$ Proved.



ISOSCELES TRIANGLE

A triangle in which two sides are equal & opposite angles of these two sides are also equal.



$AB = AC = 6 \text{ cm}$, $\angle B = \angle C = 70^\circ$

Ex.21 Find $\angle BAC$ of an isosceles triangle in which $AB = AC$ and $\angle B = \frac{1}{3}$ of right angle.

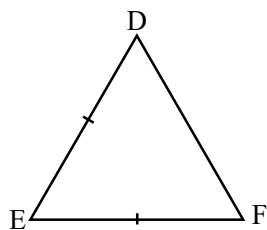
Sol. $\angle B = \angle C = \frac{1}{3}(90) = 30^\circ$

$\therefore \angle A + \angle B + \angle C = 180^\circ$ (A.P.)

$\angle A + 30^\circ + 30^\circ = 180^\circ \Rightarrow \angle A = 120^\circ$.

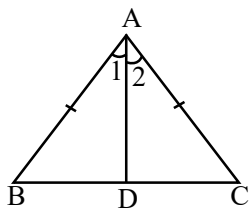
Ex.22 In isosceles triangle DEF, $DE = EF$ and $\angle E = 70^\circ$ then find other two angles.

Sol.



$$\begin{aligned} \text{Let } \angle D &= \angle F = x \\ \therefore \angle D + \angle E + \angle F &= 180^\circ \\ &\quad (\text{angle sum property}) \\ \Rightarrow x + 70^\circ + x &= 180^\circ \\ \Rightarrow 2x &= 110^\circ \\ \Rightarrow x &= 55^\circ \\ \Rightarrow \angle D &= \angle F = 55^\circ. \end{aligned}$$

Theorem (2) : Angles opposite to equal sides of an isosceles triangle are equal.



Given : In $\triangle ABC$, $AB = AC$

To prove : $\angle B = \angle C$

Construction : Draw AD, bisector of $\angle A$

$$\therefore \angle 1 = \angle 2$$

Proof : In $\triangle ADB$ & $\triangle ADC$

$$AD = AD \text{ (Common)}$$

$$\angle 1 = \angle 2 \text{ (by construction)}$$

$$AB = AC$$

By SAS, $\triangle ADB \cong \triangle ADC$

$$\therefore \angle B = \angle C \text{ (c.p.c.t.) Proved.}$$

Note : Other result : $\angle ADB = \angle ADC$ (c.p.c.t.)

But $\angle ADB + \angle ADC = 180^\circ$ (linear pair)

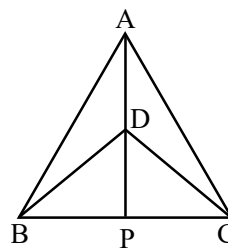
$$\therefore \angle ADB = \angle ADC = 90^\circ \Rightarrow AD \perp BC$$

and $BD = DC$ (c.p.c.t.) $\Rightarrow AD$ is median

\therefore we can say AD is perpendicular bisector of BC or we can say in isosceles Δ , median is angle bisector and perpendicular to base also.

Ex.23 $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see fig.). If AD is extended to intersect BC at P. Show that

[NCERT]



- (i) $\triangle ABD \cong \triangle ACD$
- (ii) $\triangle ABP \cong \triangle ACP$
- (iii) AP bisects $\angle A$ as well as $\angle D$
- (iv) AP is the perpendicular bisector of BC.

Sol.

- (i) In $\triangle ABD$ & $\triangle ACD$

$$AB = AC \text{ } (\triangle ABC \text{ is isosceles } \Delta)$$

$$AD = AD \text{ (Common)}$$

$$BD = DC \text{ } (\triangle DBC \text{ is isosceles } \Delta)$$

$$\therefore \text{By SSS, } \triangle ABD \cong \triangle ACD \text{ Proved.}$$

- (ii) In $\triangle ABP$ & $\triangle ACP$

$$AB = AC \text{ } (\triangle ABC \text{ is isosceles } \Delta)$$

$$\angle ABP = \angle ACP \text{ } \{\triangle ABC \text{ is isosceles } \Delta\}$$

$$AP = AP \text{ (common)}$$

$$\therefore \text{By SAS, } \triangle ABP \cong \triangle ACP \text{ Proved.}$$

- (iii) $\angle BAP = \angle CAP$ (c.p.c.t. of part (ii))

$$\therefore \angle A \text{ is bisected by AP}$$

$$\text{and } \angle ADB = \angle ADC \text{ (c.p.c.t. of part (ii))}$$

$$\therefore \angle D \text{ is bisected by AP.}$$

- (iv) $\angle APB = \angle APC$ (c.p.c.t. of part (ii))

$$\text{but } \angle APB + \angle APC = 180^\circ \text{ (linear pair)}$$

$$\therefore \angle APB + \angle APB = 180^\circ$$

$$2\angle APB = 180^\circ$$

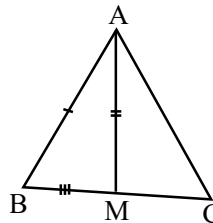
$$\angle APB = 90^\circ = \angle APC$$

$$\text{also } PB = PC \text{ (c.p.c.t. of part (ii))}$$

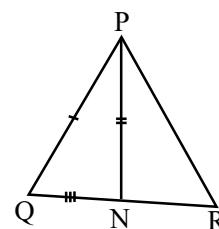
$$\therefore \text{AP is perpendicular bisector of BC.}$$

Proved.

Ex.24 Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\triangle PQR$ (see figure). Show that:



$$(i) \triangle ABM \cong \triangle PQN$$



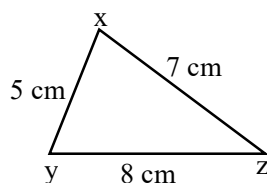
$$(ii) \triangle ABC \cong \triangle PQR$$

- Sol.** (i) In $\triangle ABM$ & $\triangle PQN$
 $AB = PQ$ (given)
 $AM = PN$ (given)
 $BM = QN$ ($\because BC = QR$
 $\therefore \frac{BC}{2} = \frac{QR}{2}$)
 \therefore By SSS, $\triangle ABM \cong \triangle PQN$ Proved.
- (ii) In $\triangle ABC$ & $\triangle PQR$
 $AB = PQ$ (given)
 $\angle B = \angle Q$ (c.p.c.t. of part (i))
 $BC = QR$ (given)
 \therefore By SAS, $\triangle ABC \cong \triangle PQR$ Proved.

➤ SOME MORE RESULTS BASED ON CONGRUENT TRIANGLES

- (1) If two sides of a triangle are unequal, then the longer side has the greater angle opposite to it.
- (2) In a triangle, the greater angle has the longer side opposite to it.
- (3) Of all the line segments that can be drawn to a given line, from a point not lying on it, the perpendicular line segment is the shortest.
- (4) The sum of any two sides of a triangle is greater than its third side.
- (5) The difference between any two sides of a triangle is less than its third side.
- (6) Exterior angle is greater than one opposite interior angle.

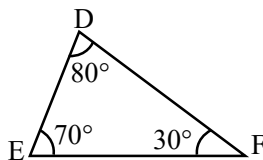
Ex.25 Find the relation between angles in figure.



Sol. $\because yz > xz > xy$
 $\therefore \angle x > \angle y > \angle z$.

(\because Angle opposite to longer side is greater)

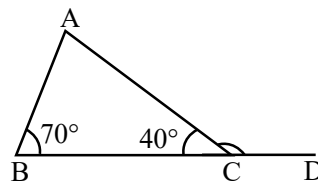
Ex.26 Find the relation between the sides of triangle in figure .



Sol. $\because \angle D > \angle E > \angle F$
 $\therefore EF > DF > DE$

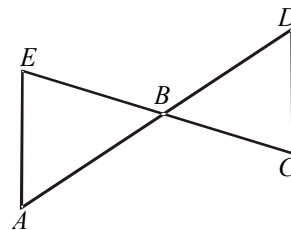
{ \because side opposite to greater angle is longer}

Ex.27 Find $\angle ACD$ then what is the relation between
 (i) $\angle ACD$, $\angle ABC$ (ii) $\angle ACD$ & $\angle A$



Sol. $\angle ACD + 40^\circ = 180^\circ$ (linear pair)
 $\angle ACD = 140^\circ$ **Ans.**
 also $\angle A + \angle B = \angle ACD$
 (exterior angle = sum of opp. interior angles)
 $\Rightarrow \angle A + 70^\circ = 140^\circ \Rightarrow \angle A = 140^\circ - 70^\circ$
 $\Rightarrow \angle A = 70^\circ$
 Now $\angle ACD > \angle B$ **Ans.**
 $\angle ACD > \angle A$ **Ans.**

Ex.28 In Fig. $\angle E > \angle A$ and $\angle C > \angle D$.



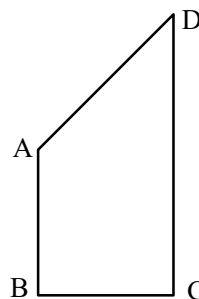
Prove that $AD > EC$.

Sol. In $\triangle ABE$, it is given that
 $\angle E > \angle A$
 $\Rightarrow AB > BE$ (i)
 In $\triangle BCD$, it is given that
 $\angle C > \angle D$
 $\Rightarrow BD > BC$ (ii)

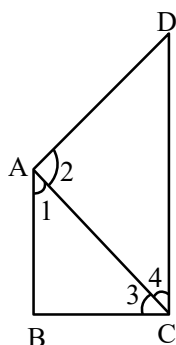
Adding (i) and (ii), we get

$$AB + BD > BE + BC \Rightarrow AD > EC$$

Ex.29 AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see figure). Show that $\angle A > \angle C$. [NCERT]



Sol. Draw diagonal AC.



In $\triangle ABC$, $AB < BC$ { \because AB is smallest}

$$\Rightarrow \angle 3 < \angle 1 \quad \dots\dots(1)$$

{angle opp. to longer side is larger}

Also in $\triangle ADC$

$AD < CD$ \because CD is longest

$$\Rightarrow \angle 4 < \angle 2 \quad \dots\dots(2)$$

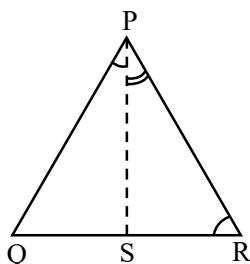
adding equation (1) & (2)

$$\angle 3 + \angle 4 < \angle 1 + \angle 2$$

$$\angle C < \angle A$$

or $\angle A > \angle C$ Proved.

Ex.30 In given figure, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$. [NCERT]

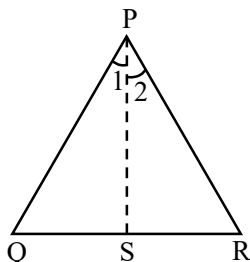


Sol. In $\triangle PQR$, $PR > PQ$

$$\Rightarrow \angle Q > \angle R \quad \dots\dots(1)$$

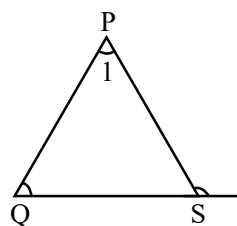
{angle opposite to longer side is greater}

$$\text{and } \angle 1 = \angle 2 \quad (\because PS \text{ is } \angle \text{ bisector}) \quad \dots\dots(2)$$



$$\text{Now for } \triangle PQS, \angle PSR = \angle Q + \angle 1 \quad \dots\dots(3)$$

{exterior angle = sum of opposite interior angle}



$$\& \text{ for } \triangle PSR, \angle PSQ = \angle R + \angle 2 \quad \dots\dots(4)$$

By equation (1), (2), (3), (4), $\angle PSR > \angle PSQ$

Proved.

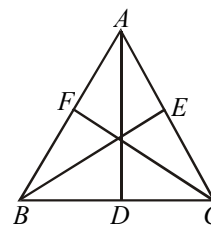
Ex.31 AD , BE and CF , the altitudes of $\triangle ABC$ are equal. Prove that $\triangle ABC$ is an equilateral triangle

Sol. In right triangles BCE and BFC , we have

$$\text{Hyp. } BC = \text{Hyp. } BC$$

$$BE = CF \quad [\text{Given}]$$

So, by RHS criterion of congruence,



$$\triangle BCE \cong \triangle BFC.$$

$$\Rightarrow \angle B = \angle C \quad \left[\begin{array}{l} \because \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

$$\Rightarrow AC = AB \quad \dots\dots(i)$$

[\because Sides opposite to equal angles are equal]

Similarly, $\triangle ABD \cong \triangle ABE$

$$\Rightarrow \angle B = \angle A$$

[Corresponding parts of congruent triangles are equal]

$$\Rightarrow AC = BC \quad \dots\dots(ii)$$

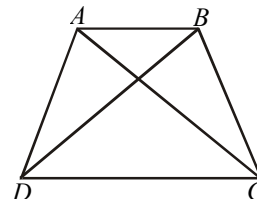
[Sides opposite to equal angles are equal]

From (i) and (ii), we get

$$AB = BC = AC$$

Hence, $\triangle ABC$ is an equilateral triangle.

Ex.32 In Fig. $AD = BC$ and $BD = CA$.



Prove that $\angle ADB = \angle BCA$ and

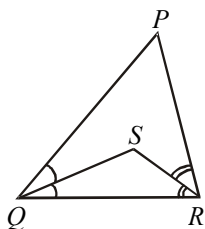
$$\angle DAB = \angle CBA.$$

Sol. In triangles ABD and ABC, we have
 $AD = BC$ [Given]
 $BD = CA$ [Given]
and $AB = AB$ [Common]
So, by SSS congruence criterion, we have
 $\triangle ABD \cong \triangle CBA \Rightarrow \angle DAB = \angle ABC$

[\ominus corresponding parts of
congruent triangles are equal]

$$\Rightarrow \angle DAB = \angle CBA$$

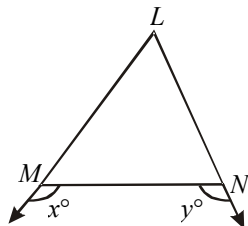
Ex.33 In Fig. $PQ > PR$. QS and RS are the bisectors of $\angle Q$ and $\angle R$ respectively.



Prove that $SQ > SR$.

Sol. In $\triangle PQR$, we have
 $PQ > PR$ [Given]
 $\Rightarrow \angle PRQ > \angle PQR$ [Angle opp. to larger side
of a triangle is greater]
 $\Rightarrow \frac{1}{2} \angle PRQ > \frac{1}{2} \angle PQR$
 $\Rightarrow \angle SRQ > \angle SQR$
[\ominus RS and QS are bisectors of
 $\angle PRQ$ and $\angle PQR$ respectively]
 $\Rightarrow SQ > SR$
[\ominus Side opp. to greater angle is larger]

Ex.34 In Fig. [NCERT]

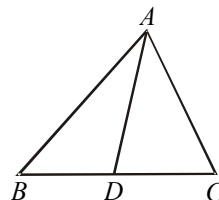


if $x > y$, show that $\angle M > \angle N$.

Sol. We have,
 $\angle LMN + x^\circ = 180^\circ$ (i)
[Angles of a linear pair]
 $\Rightarrow \angle LMN + y^\circ = 180^\circ$ (ii)
[Angles of a linear pair]
 $\therefore \angle LMN + x^\circ = \angle LMN + y^\circ$
But $x > y$. Therefore,

$\angle LMN < \angle LNM$
 $\Rightarrow \angle LNM > \angle LMN$
 $\Rightarrow LM > LN$
[\ominus Side opp. to greater angle is larger]

Ex.35 In Fig. $AB > AC$. Show that $AB > AD$.



Sol. In $\triangle ABC$, we have
 $AB > AC$ [Given]
 $\Rightarrow \angle ACB > \angle ABC$ (i)
[\ominus Angle opp. to larger side is greater]
Now, in $\triangle ACD$, CD is produced to B,
forming an ext $\angle ADB$.
 $\therefore \angle ADB > \angle ACD$

[\ominus Exterior angle of \triangle is greater
than each of interior opp. angle]

$$\Rightarrow \angle ADB > \angle ACB \quad \dots (ii)$$

$$[\because \angle ACD = \angle ACB]$$

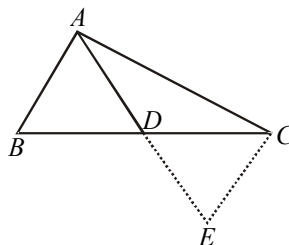
From (i) and (ii), we get

$$\begin{aligned} \angle ADB &> \angle ABC \\ \Rightarrow \angle ADB &> \angle ABD [\because \angle ABC = \angle ABD] \\ \Rightarrow AB &> AD \end{aligned}$$

[\ominus Side opp. to greater angle is larger]

Ex.36 Prove that any two sides of a triangle are together greater than twice the median drawn to the third side.

Sol. **Given :** A $\triangle ABC$ in which AD is a median.



To prove : $AB + AC > 2 AD$

Construction : Produce AD to E such that
 $AD = DE$. Join EC.

Proof : In $\triangle ADB$ and EDC , we have

$AD = DE$ [By construction]
 $BD = DC$ [∵ D is the mid point of BC]
 and, $\angle ADB = \angle EDC$ [Vertically opp. angles]
 So, by SAS criterion of congruence

$$\triangle ADB \cong \triangle EDC$$

$$\Rightarrow AB = EC \quad \left[\begin{array}{l} \text{Corresponding parts of} \\ \text{congruent triangles are equal} \end{array} \right]$$

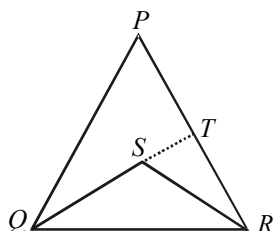
Now in $\triangle AEC$, we have

$AC + EC > AE$ [∵ Sum of any two sides of a \triangle is greater than the third]

$$\Rightarrow AC + AB > 2 AD$$

$$[\because AD = DE \therefore AE = AD + DE = 2AD \text{ and } EC = AB]$$

Ex.37 In Fig. PQR is a triangle and S is any point in its interior, show that $SQ + SR < PQ + PR$.



Sol. Given : S is any point in the interior of $\triangle PQR$.

To Prove : $SQ + SR < PQ + PR$

Construction : Produce QS to meet PR in T.

Proof : In $\triangle PQT$, we have

$$PQ + PT > QT \quad \left[\begin{array}{l} \text{Sum of the two sides of a } \triangle \\ \text{is greater than the third side} \end{array} \right]$$

$$\Rightarrow PQ + PT > QS + ST \quad \dots (i)$$

$$[\because QT = QS + ST]$$

In $\triangle RST$, we have

$$ST + TR > SR \quad \dots (ii)$$

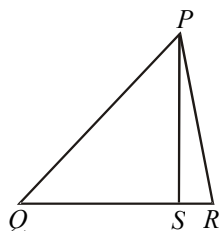
Adding (i) and (ii), we get

$$PQ + PT + ST + TR > SQ + ST + SR$$

$$\Rightarrow PQ + (PT + TR) > SQ + SR$$

$$\Rightarrow PQ + PR > SQ + SR \Rightarrow SQ + SR < PQ + PR.$$

Ex.38 In $\triangle PQR$ S is any point on the side QR. Show that $PQ + QR + RP > 2 PS$.



Sol. In $\triangle PQS$, we have

$$PQ + QS > PS \quad \dots (i)$$

[∵ Sum of the two sides of a \triangle is greater than the third side]

Similarly, in $\triangle PRS$, we have

$$RP + RS > PS \quad \dots (ii)$$

Adding (i) and (ii), we get

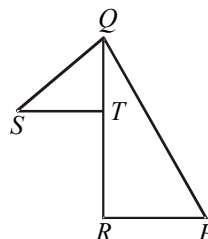
$$(PQ + QS) + (RP + RS) > PS + PS$$

$$\Rightarrow PQ + (QS + RS) + RP > 2 PS$$

$$\Rightarrow PQ + QR + RP > 2 PS$$

$$[\because QS + RS = QR]$$

Ex.39 In Fig. T is a point on side QR of $\triangle PQR$ and S is a point such that $RT = ST$.



Prove that $PQ + PR > QS$.

Sol. In $\triangle PQR$, we have

$$PQ + PR > QR$$

$$\Rightarrow PQ + PR > QT + RT \quad [\because QR = QT + RT]$$

$$\Rightarrow PQ + PR > QT + ST \quad \dots (i)$$

$$[\because RT = ST \text{ (Given)}]$$

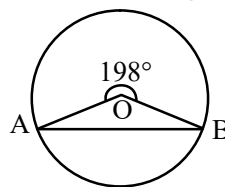
In $\triangle QST$, we have

$$QT + ST > QS \quad \dots (ii)$$

From (i) and (ii), we get

$$PQ + PR > QS.$$

Ex.40 Find $\angle OBA$ in given figure



Sol.

$$\because \angle AOB + 198^\circ = 360^\circ$$

$$\angle AOB = 360^\circ - 198^\circ = 162^\circ$$

and $OA = OB = \text{radius of circle}$

$$\angle A = \angle B = x \text{ (let)}$$

$$\therefore x + x + 162^\circ = 180^\circ \text{ (a.s.p.)}$$

$$2x + 162^\circ$$

$$x = 9^\circ$$

$$\therefore \angle OBA = 9^\circ.$$

IMPORTANT POINTS TO BE REMEMBERED

1. A plane figure bounded by three lines in a plane is called a triangle.
2. A triangle, no two of whose sides are equal is called a scalene triangle.
3. A triangle whose two sides are equal is called an isosceles triangle.
4. A triangle whose sides are equal is also called an equilateral triangle.
5. A triangle with one angle a right angle is called a right angled triangle.
6. The sum of the three angles of a triangle is 180° .
7. If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.
8. If two triangles ABC and DEF are congruent under the correspondence $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, then we write $\triangle ABC \cong \triangle DEF$ or $\triangle ABC \leftrightarrow \triangle DEF$.
9. Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle (SAS congruence criterion).
10. Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle (ASA congruence criterion).
11. If any two angles and non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the triangles are congruent (AAS congruence criterion).
12. If three sides of one triangle are equal to three of the other triangle, then the two triangles are congruent (SSS congruence criterion).
13. If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and one side of other triangle, then the two triangles are congruent (RHS congruence criterion).
14. Angles opposite to equal sides of a triangle are equal.
15. If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is isosceles.
16. In an isosceles triangle altitude from the vertex bisects the base.
17. If the bisector of the vertical angle of a triangle bisects the opposite side, then the triangle is isosceles.
18. If the altitudes of a triangle are equal, then it is equilateral.
19. In a triangle, side opposite to the larger angle is longer.
20. Sum of any two sides of a triangle is greater than the third side.