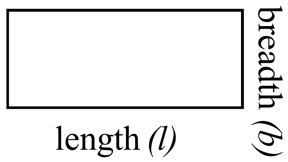
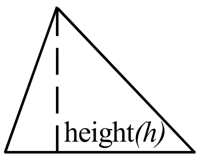
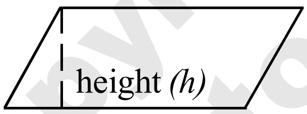
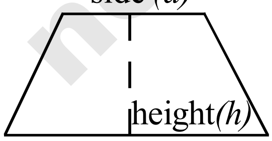
Chapter - 11

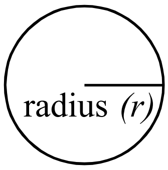
Mensuration

B7A4R5

Let us rewind our knowledge how to find the areas of triangle and quadrilateral. Let us recall the areas of different shapes from the following table.

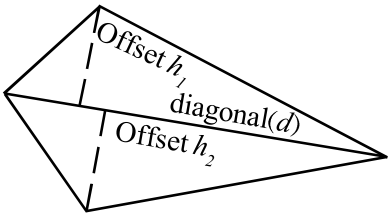
Shape	**Name of the shape**	**Area**
	Rectangle	length × breadth $= (l \times b)$
	Square	side × side $= (\text{side})^2$ $= a \times a = a^2$
	Triangle	$= \frac{1}{2} \times \text{base} \times \text{height}$ $= \frac{1}{2} \times b \times h$
	Parallelogram	= base × height $= b \times h$
	Trapezium	$= \frac{1}{2}$ (sum of the parallel sides)× (Perpendicular distance between parallel sides) $= \frac{1}{2} \times (a + b) \times h$

Shape **Name of the shape** **Area**



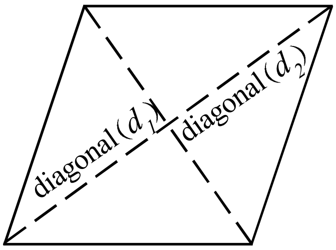
Circle $= \pi \times (\text{radius})^2$

$= \pi r^2$



Quadrilateral $= \frac{1}{2}$ (sum of the offsets) × diagonal

$= \frac{1}{2} (h_1 + h_2)\, d$



Rhombus $= \frac{1}{2} \times$ (product of the diagonals)

$= \frac{1}{2} \times d_1 \times d_2$

The above figures are two-dimensional. If we measure the dimensions in cm then the unit of areas will be sq. cm or cm^2 and if the measurements are in meters then the unit of area will be sq. m or m^2

Example 1 : The length of a square shaped garden is 40 m. The length and breadth of another rectangular garden are respectively 50 m and 30 m. Find the perimeter of both gardens. Which garden has more area and by how much?

Solution : Length of square garden = 40 m

Perimeter = 4 × length of the side

= 4 × a

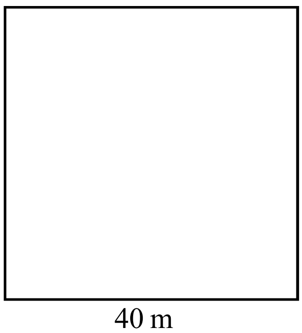
= 4 × 40 m

= 160 m

and area = (side)^2

= (40 m)^2

= 1600 sq. m



For the rectangular garden

length (l) = 50 m

breadth (b) = 30 m

$\therefore$ Area $= l \times b$

$= 50 \text{ m} \times 30 \text{ m}$

$= 1500$ sq. m

$\therefore$ Perimeter $= 2 \times (l + b)$

$= 2 \times (50 + 30)$ m

$= 2 \times 80$ m

$= 160$ m

$\therefore$ The area of the square garden is $(1600 - 1500) = 100$ sq. m more than the area of the rectangular garden.

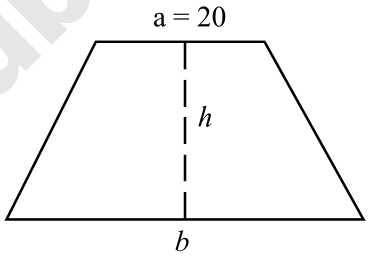
Observe that **two different shapes having equal perimeter may be of different areas**.

Example 2 : The area of a trapezium is 200 sq. m. The smaller of the two parallel sides is of length 20 m. and the perpendicular distance between them is 8 m. Find the length of the other parallel side.

Solution : Let the length of other parallel side = b

Heigth (h) = 8 m

Area of the trapezium = 200 sq. m



$\therefore \quad \frac{1}{2}(a + b)h = 200$

or, $\frac{1}{2}(20 + b).8 = 200$

or, $4(20 + b) = 200$

or, $20 + b = 50$

or, $b = 50 - 20$

$\therefore \quad b = 30$

$\therefore$ length of the other side = 30 m

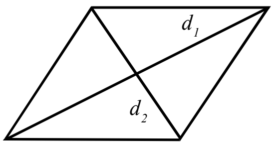
Example 2 : The length of diagonals of a rhombus are 10 cm and 14 cm. Find the area of the rhombus.

Solution : The length of diagonals $(d_1) = 10$ cm and $(d_2) = 14$ cm

$\therefore$ Area of the rhombus $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 10 \times 14$ sq. cm

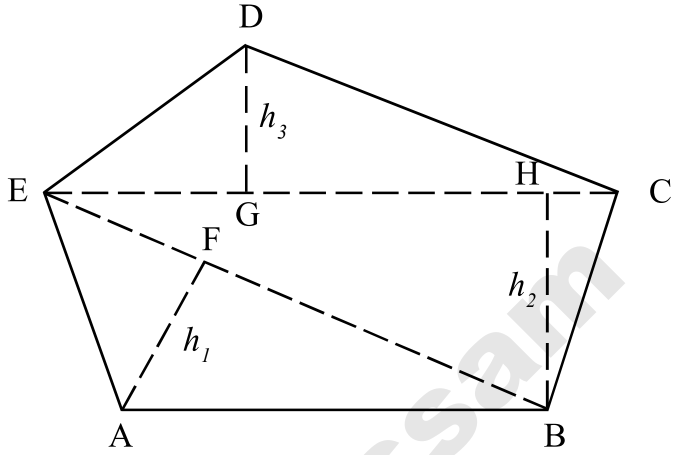
$= 70$ sq cm



11.1 Area of a Polygon

ABCDE is a pentagon. We have to determine the area of the pentagon. Draw two diagonals EB and EC to divide the pentagon into three triangles. The sum of the areas of these three triangles is the required area of the pentagon.

Now think how to find the areas of these three triangles.



We know that area of a triangle = $\frac{1}{2}$ × base × height.

By ascertaining the base and height of the respective triangle, we can find the area of all the triangles.

Considering diagonals EB and EC as the bases of the triangles. Then the perpendiculars from the points opposite to these diagonals will be AF, BH and DG respectively.

∴ Area of the pentagon ABCDE = Area of Δ ABE + Area of Δ BCE + Area of ΔCDE

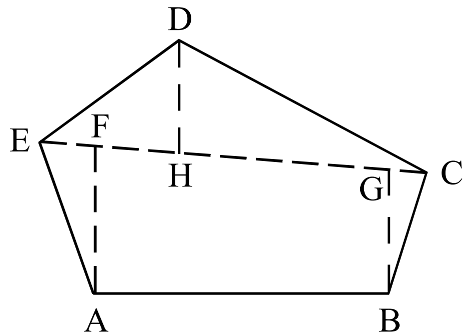
$$= \frac{1}{2}\text{EB.AF} + \frac{1}{2}\text{EC.BH} + \frac{1}{2}\text{EC.DG}$$

$$= \frac{1}{2}\text{EB.}h_1 + \frac{1}{2}\text{EC.}h_2 + \frac{1}{2}\text{EC.}h_3$$

Alternative Method :

Draw a diagonal EC and two perpendiculars AF and BG on it, the pentagon is divided into four parts, one trapezium ABGE and three triangle AFE, BGC and ECD. Draw perpendicular DH from D on EC.

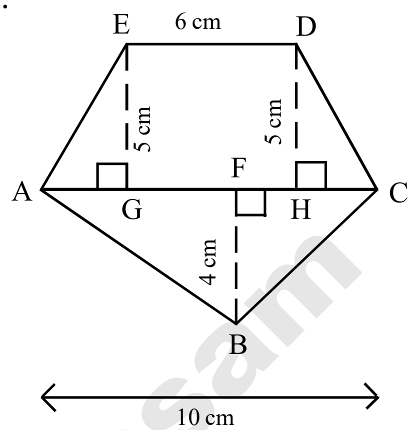
Now, the area of the Pentagon ABCDE= Area of right angle triangle AFE + Area of the trapezium ABGF+ Area of the right angle triangle BGC+Area of traingle ECD



$$= \frac{1}{2}\text{EF.AF} + \frac{1}{2}(\text{AF+BG}).\text{FG} + \frac{1}{2}\text{CG.BG} + \frac{1}{2}\text{EC.DH}$$

We see that *to find the area of a pentagon we can divide the pentagon into different plane figures such as triangle, square, rectangle, trapezium etc. The sum of those divided parts is the area of the given pentagon.*

Example 2.3.1: Find the area of the polygon as given in fig.



Solution :

Area of ΔABC $= \frac{1}{2} AC \times BF$

$= \frac{1}{2} \times 10 \text{ cm} \times 4 \text{ cm}$

$= 20$ sq. cm

Area of the trapezium ACDE $= \frac{1}{2}(AC + ED) \times EG$

$= \frac{1}{2}(10 + 6) \times 5$ sq cm

$= \frac{1}{2} \times 16 \times 5$ sq cm

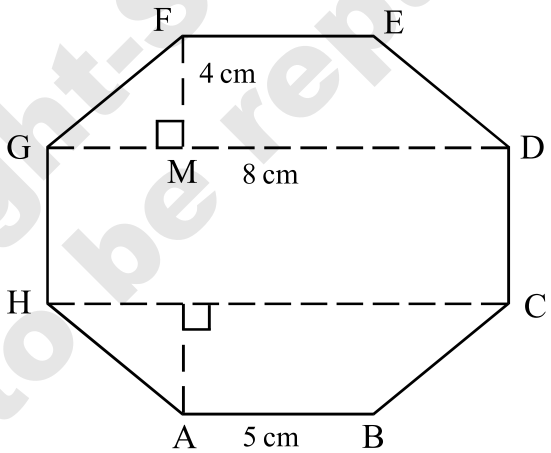
$= 40$ sq cm

$\therefore$ Area of the polygon ABCDE = Area of ΔABC + Area of ACDE

$= (20 + 40)$ sq cm

$= 60$ sq cm

Example 4 : In the given figure ABCDEFGH a regular octagon with given dimensions. Find its area.



Solution : Here AB = BC = CD = DE = EF = FG = GH = HA = 5 cm

Given that, FM = 4 cm

GD = 8 cm

Area of the trapezium GDEF $= \frac{1}{2}(GD + EF) \times FM$

$= \frac{1}{2}(8+5) \times 4$ sq. cm

= 26 sq. cm

Again, the area of the rectangle HCDG = length × breadth

= GD × GH

= 8×5 sq. cm

= 40 sq. cm

Since ABCDEFGH is a regular octagon.

Therefore area of the trapezium ABCH = Area of the trapezium GDEF= 26 sq. cm

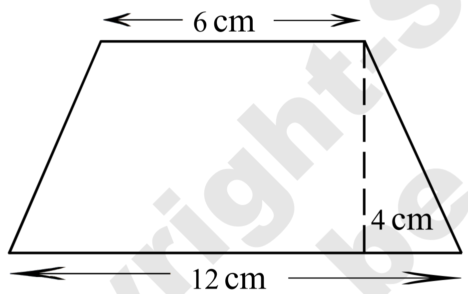
Hence the area of the octagon = Area of the trapezium GDEF + Area of the rectangle HCDG + Area of the trapezium ABCH.

= (26 + 40 + 26) sq. cm

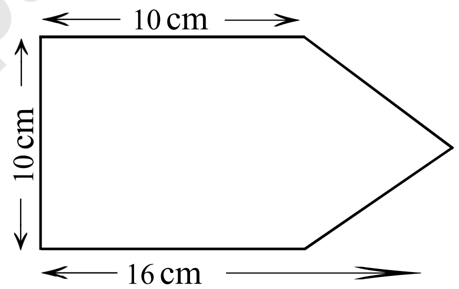
= 92 sq. cm

Exercise 11.1

1. Find the area of the following figures.



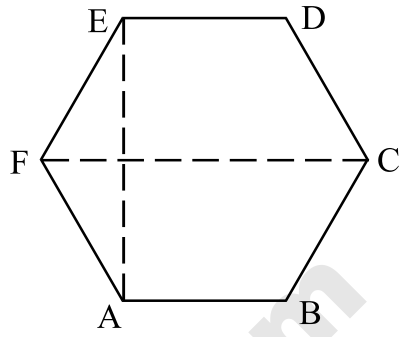
(a)



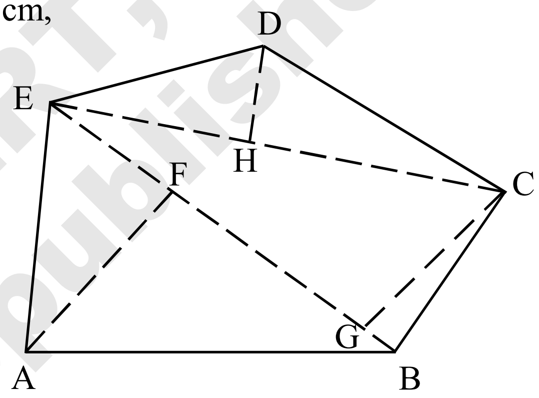
(b)

2. The area of a trapezium is 34 sq cm. If the length of one of the parallel sides is 10 cm and the perpedicular distance between them is 4 cm, find the other parallel side.
3. The length of a diagonal of a quadrilateral is 20m. If the perpendiculars upon it from the opposite points are of lengths 8.5 m and 11 m, find the area of the quadrilateral.
4. Find the area of a rhombus whose diagonals are of length 10 cm and 14 cm.
5. The area of a quadrilateral is 100 sq. cm. If the offsets are of length 6 cm and 4 cm find the length of the diagonal.

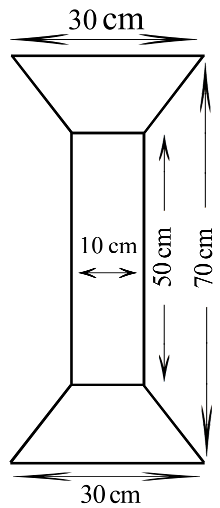
6. The length of each side of a regular hexagon is 6cm, If CF = 10cm AE = 8 cm, find the area of the hexagon. (You can try different methods)



7. A rhombus and a triangle are equal in areas. If the base and height of the triangle are 24.8 cm and 5.5 cm respectively and the length of one diagonal of the rhombus is 22 cm, find the length of other diagonal of the rhombus.
8. The area of a trapezium shaped garden is 480 sq. m and its height is 15m. If length of one of the parallel sides is 20m, find the length of the other parallel side.
9. In the polygon ABCDE, BE = 8 cm, CE = 10 cm, AF = 5 cm, CG = 4 cm, DH = 3 cm. Find the area of the polygon.

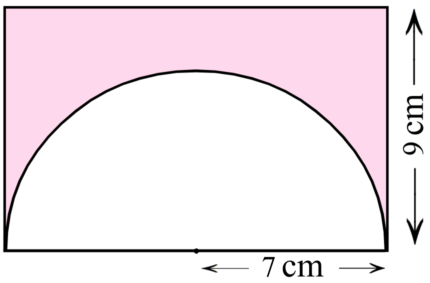


10. The area of a trapezium is 68 sq. cm and the length of the parallel sides are respectively 13 cm and 21 cm. Find the perpendicular distance between the parallel sides.
11. Find the area of the given figure,

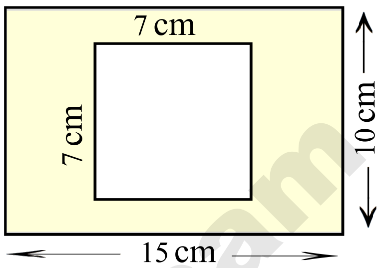


12. Find the area of shaded portion of the following figures. $\left(\pi = \frac{22}{7}\right)$

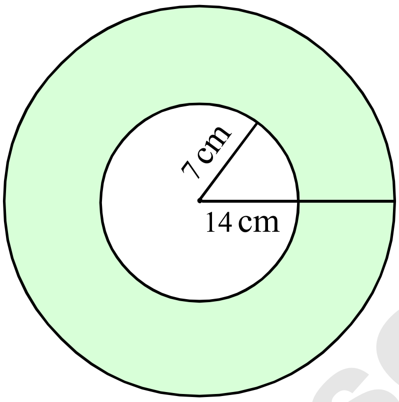
(a)



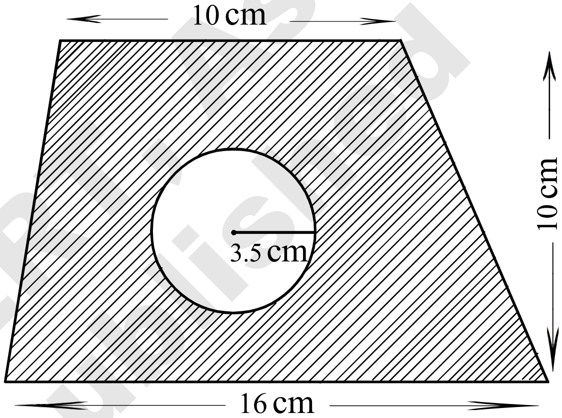
(b)



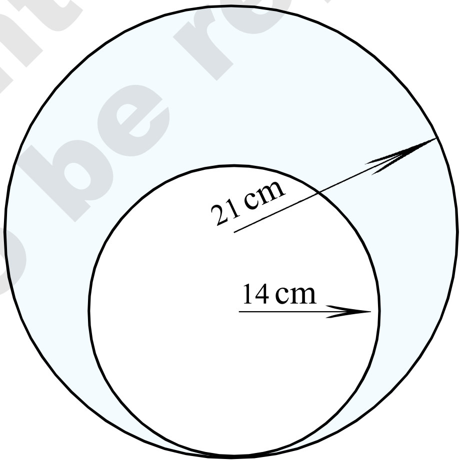
(c)



(d)

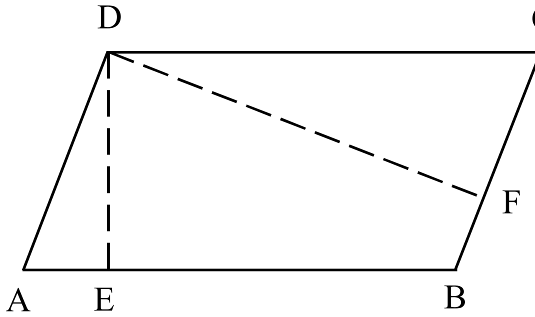


(d)



13. If the lengths of two parallel sides of a trapezium are 7cm and 5cm and they are 4 cm apart, find the area of the trapezium.
14. The lengths of a side and a diagonal of a rhombus are 13cm and 24cm respectively, find the area of the rhombus.

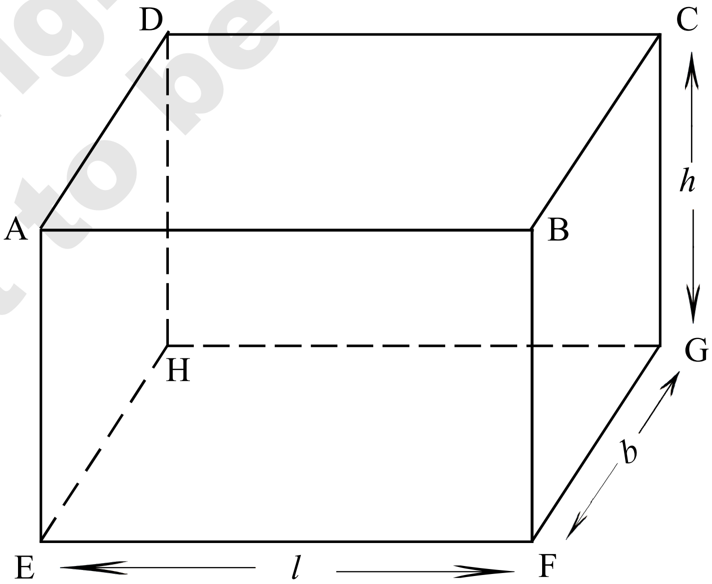
15. In the parallelogram ABCD, DE and DF are perpendiculars. If AB= 12 cm, DE = 7 cm, DF = 14 cm, find the length of BC.

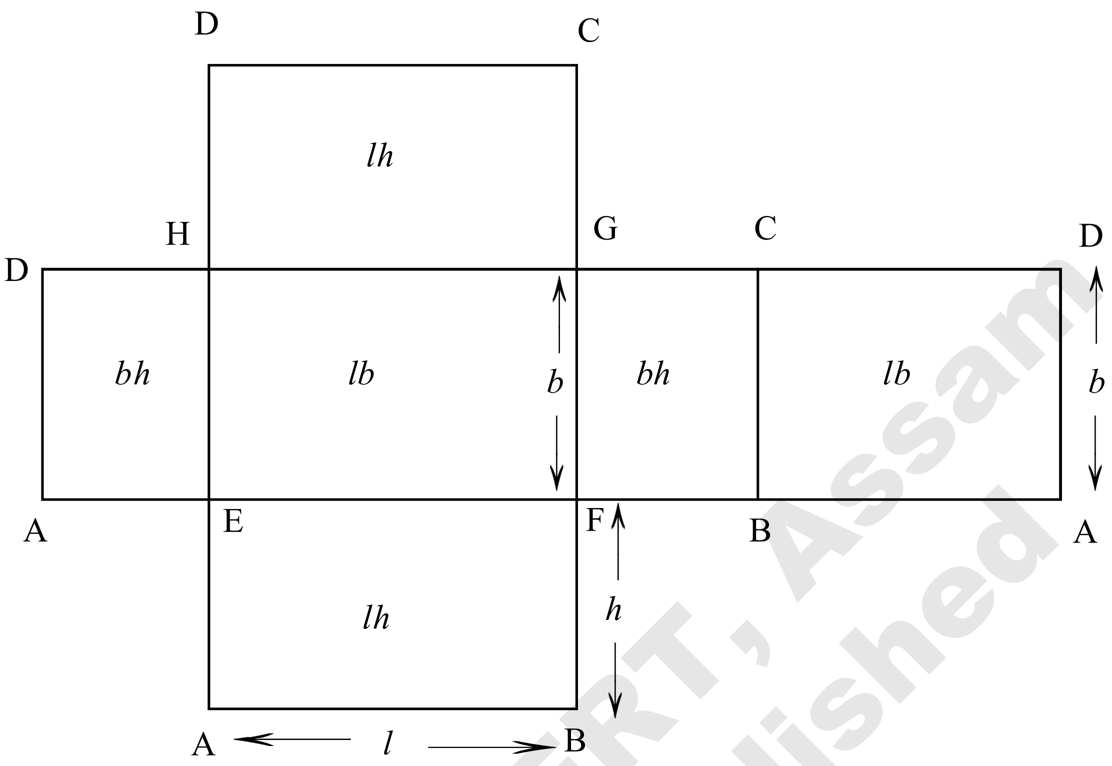


16. The parallel sides of a trapezium are in the ratio 1:2. If the distance between the parallel sides is 12 cm and the area of the trapezium is 180 sq. cm, find the lengths of the parallel sides.

11.2 Surface Area of a Cuboid

A chalk box (as shown in fig) in given. Open it by scissors through the edges to get 6 surfaces. Definitely you will get three pairs of equal surfaces. The sum of areas of all these surfaces will given the total surface area of the cuboid.





Form the figure, the 6 surface are AEHD, EFGH, FBAE, HGCD, FBCG and ABCD.

In the Cuboid, ABCD and EFGH are top and bottom surfaces while lateral surfaces are AEHD, FBAE, HGCD and FBCG.

Given l, b, h be the length, breadth and height respectvely of the cuboid.

Area of the surface AEHD $= bh$

Area of the surface EFGH $= lb$

Area of the surface FBAE $= lh$

Area of the surface HGCD $= lh$

Area of the surface FBCG $= bh$

Area of the surface ABCD $= lb$

$\therefore$ Total surface area of the cuboid = sum of the areas of the six surfaces

$= bh + lb + lh + lh + bh + lb$

$= 2\,lb + 2\,bh + 2\,lh$

$= 2(lb + bh + lh)$

Remember : The total surface area of a cuboid $= 2(lb + bh + lh)$

11.2.1 Lateral Surface : In any 3-D shaped (except sphere) solids the surfaces excluding top and bottom faces are called its Lateral Surface.

The lateral surface of the cuboid $= lh + bh + lh + bh$

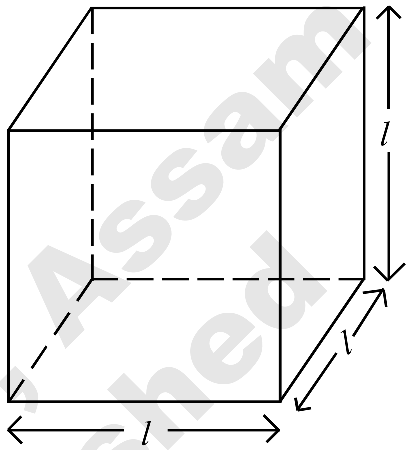
$= 2lh + 2bh$

$= 2h(l + b)$

$= 2(l + b) \times h$

$=$ perimeter of base $\times$ height

11.3 Surface Area of a Cube



You have already learnt how to find the surface area of a cuboid. We know that in a cube, the length, breadth and height are equal. Hence to find the surface area of a cube, we put $l = b = h$ in the formula for surface area of a cuboid.

The surface area of a cube $= 2(l^2 + l^2 + l^2) = 6l^2$

The four surfaces excluding top and bottom are the lateral surfaces of the cube.

Area of lateral surface area of a cube $= 4 \times l^2$

Activity : Collect three match boxes of same dimensions and place the boxes as shown in the following figures.

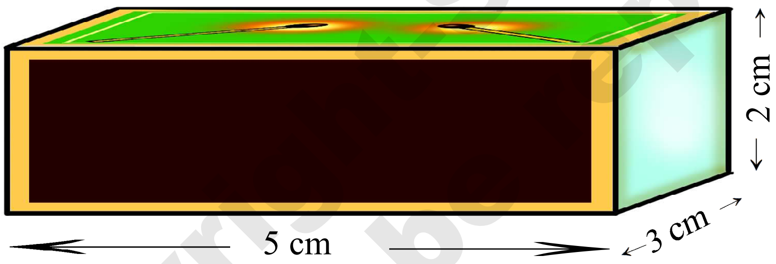


Figure (a)

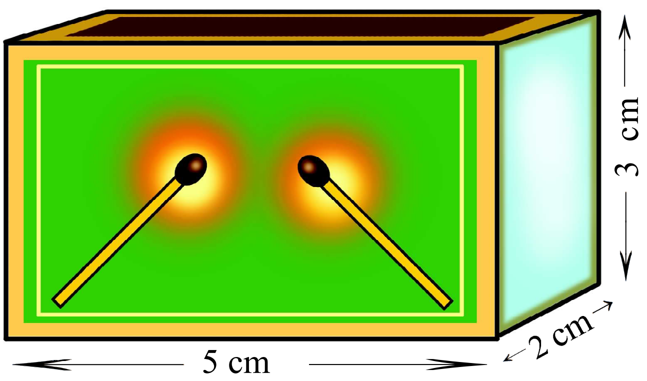


Figure (b)

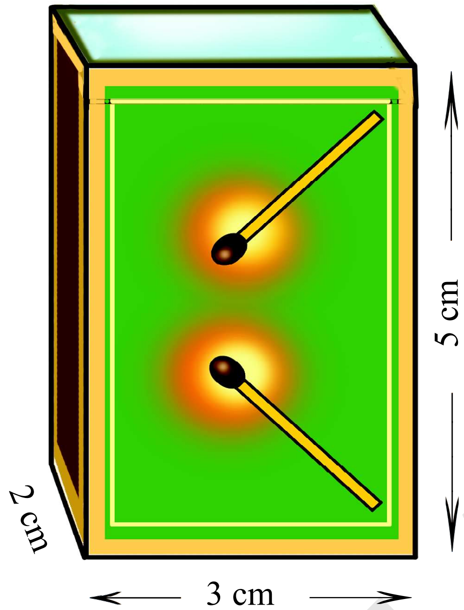


Figure (c)

Considering the match box of length *(l)* = 5 cm, breadth *(b)* = 3 cm and height *(h)* = 2 cm find the lateral surface areas of fig *(a)*, *(b)* & *(c)*.

Are all the results same? If not why? Take help from your teacher. Collect different cuboidal objects like toothpaste box, shoe box, medicine box etc. and find the lateral surface areas by changing the bases as done in the above example of match box.

Now, collect three ludo dice and do the same activity. Find the lateral surface area in each case. Are the results same ?

Example 1 : The length, breadth & height of a chalk box are 16 cm, 8 cm, 6 cm respectively Find the total surface area and lateral surface area of the box.

Solution : Chalk box is of cubiodal shape

Given $l = 16$ cm, $b = 8$ cm and $h = 6$ cm

$\therefore$ Total surface area of the box $= 2\,(lb + bh + lh)$

$= 2(16 \times 8 + 8 \times 6 + 16 \times 6)$ sq. cm

$= 544$ sq. cm

Lateral surface area of the box $= 2\,(lh + bh)$

$= 2\,(16 \times 6 + 8 \times 6)$ sq. cm

$= 288$ sq. cm

Example 2 : Find the length of each side of a cube, if its total surface area is 384 sq m.

Solution : Let the length of each side of the cube = l .

Given, total surface area of cube = 384 sq. m.

$\therefore \quad 6l^2 = 384$

or, $l^2 = \frac{384}{6} = 64 = 8^2$

or, $l = 8$

Length of each side = 8 m.

Example 3 : The internal measures of a cuboidal room is 16 m × 10 m × 8 m. Find the total cost of white washing all four walls of the room, if the cost of white washing is Rs. 10 per sq m.

Solution : Given length of the room (l) = 16 m

Breadth of the room (b) = 10 m

Height of the room (h) = 8 m

$\therefore$ The total area of the four walls $= 2(lh + bh)$

$= 2(l + b).h$

$= 2(16 + 10).8$ sq m

$= 2 \times 26 \times 8$ sq m

$= 416$ sq. m

Cost of white washing for 1 sq. m = Rs.10

$\therefore$ Cost of while washing for 416 sq. m = Rs.10 × 416 = Rs.4160

Example 4 : If the surface area of the face of a cube is 25 sq. cm, what will be its total surface area?

Solution : Area of one surface of the cube $(l)^2 = 25$ sq. cm.

$\therefore$ Total surface area of the cube $= 6 \times l^2$

$= 6 \times 25$ sq. cm

$= 150$ sq. cm

Example 5 : A certain paint of two litres can paint an area of 9.375 sq. m. How many bricks of dimension 22.5 cm × 10 cm × 7.5 cm can be painted with the tin of 2 litres of paint.

Solution : Given, length of the brick (l) = 22.5 cm, breadth (b) = 10 cm

thickness (h) = 7.5 cm,

$\therefore$ Total surface area of a brick $= 2(lb + bh + lh)$

$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)$ sq.cm

$= 2(225 + 75 + 168.75)$ sq.cm

$= 937.5$ sq.cm

2 litres of colour can paint an area

$= 9.375$ Sq.m.

$= 9.375 \times 10000$ sq.cm

$= 93750$ sq.cm

1m = 100 cm

1 sq. m $= 100 \times 100$ sq. cm

$= 10000$ sq. cm

$\therefore$ The number of bricks that can be painted $= \dfrac{93750}{937.5} = \dfrac{937500}{9375} = 100$

Example 6 : The dimensions of cuboidal tin box are 30 cm $\times$ 40 cm $\times$ 50 cm. Find the cost of the tin sheets required for making 20 such tin boxes if the cost of tin sheet is Rs. 20 per square metre.

Solution : Given the length $(l) = 30$ cm, breadth $(b) = 40$ cm, height $(h) = 50$ cm

$\therefore$ Total surface area of the box $= 2(lb + bh + lh)$

$= 2(30 \times 40 + 40 \times 50 + 30 \times 50)$ sq.cm

$= 2(1200 + 2000 + 1500)$ sq.cm

$= 2 \times 4700$ sq.cm

$= 9400$ sq.cm

$\therefore$ Total surface area of 20 tin boxes $= 20 \times 9400$ sq.cm

$= 188000$ sq.cm

$= \dfrac{188000}{10000}$ sq.m

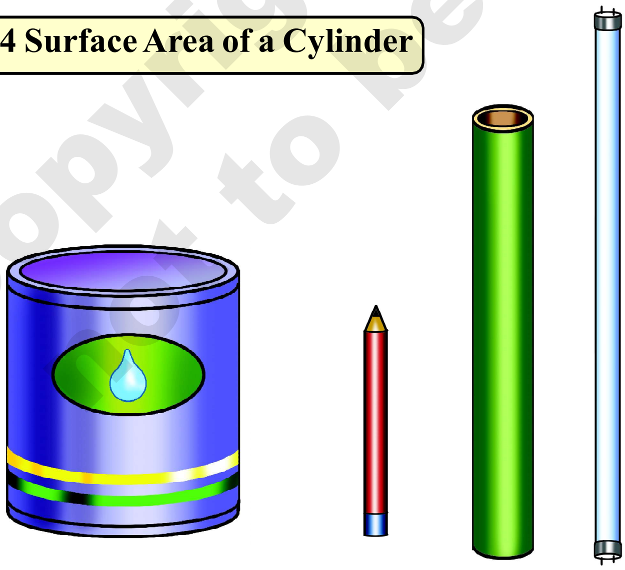
$= 18.8$ sq.m

Cost of 1 sq.m tin sheet = Rs.20

$\therefore$ Cost of 18.8 sq.m tin = Rs.(20×18.8) = Rs. 376

$\therefore$ Total cost of the tin sheets = Rs.376

11. 4 Surface Area of a Cylinder



Look at the objects. All are of *Cylindrical Shape*. You will find many such objects in your surroundings which are cylindrical.

When you observe carefully such cylindrical objects, you will see -

(i) One lateral surface

(ii) Two circular faces

Take a cylindrical can and wrap a paper around the can such that it just fits around the can and remove the excess paper by cutting with a scissors. The area of this piece of paper fitted with the can is the curved surface area of the cylinder. The shape of the paper that goes around the can will be rectangular in shape. The length of this rectangular strip is equal to the circumference of circular face of the can and breadth is the height of the can.

Area of the curved surface = Area of the rectangular strip of paper

= Circumference of base × height

= $2\ \pi r \times h$

= $2\ \pi r.h$

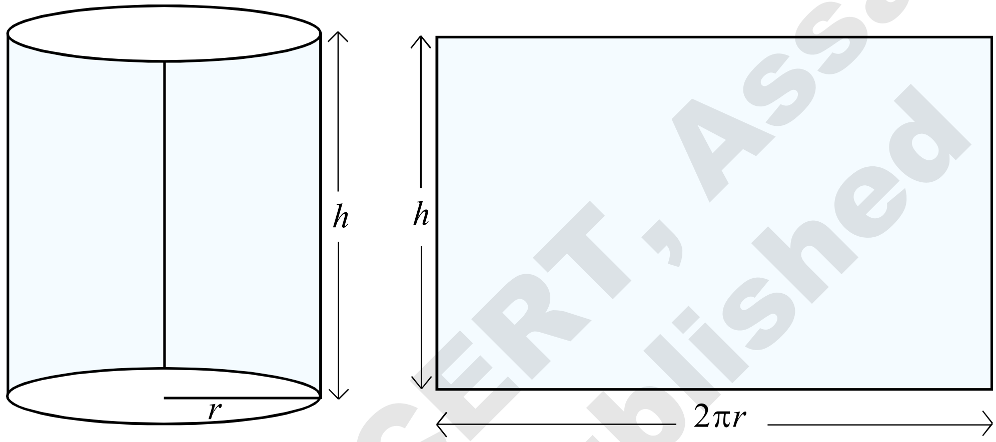


Fig (i)

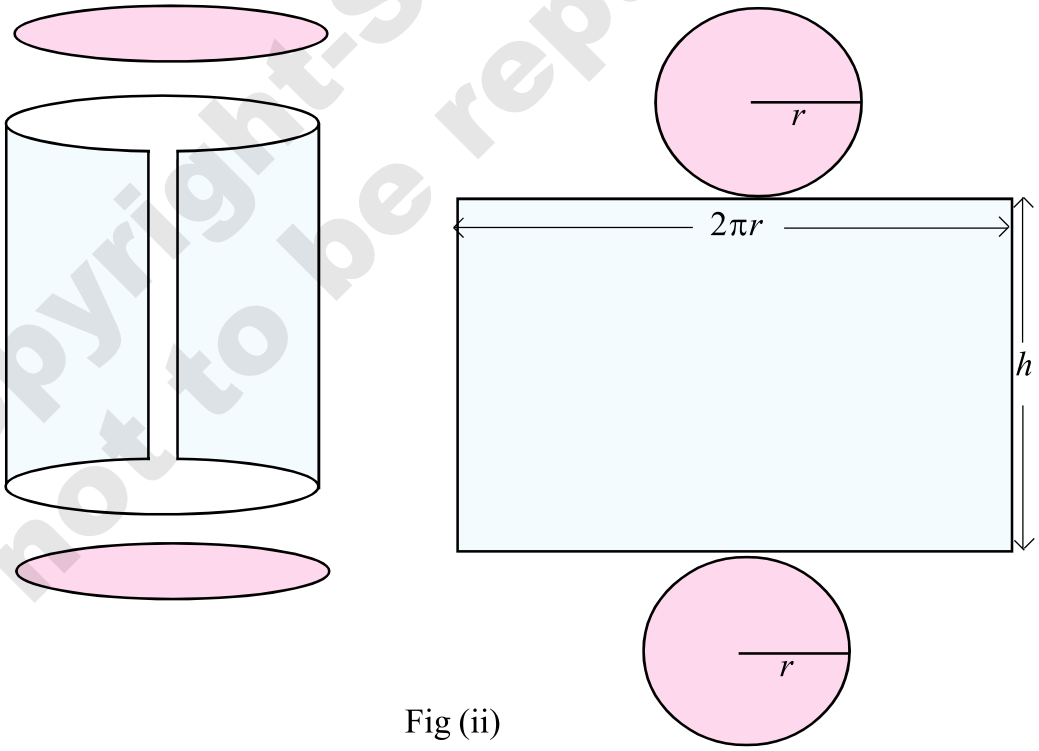


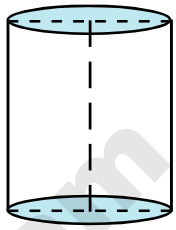
Fig (ii)

The total area of top and bottom circular surfaces $= 2 \times \pi r^2$ (Fig ii)

$\therefore$ The total surface area of the can = Curved surface area + area of top and bottom surfaces

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\ (r + h)$



Remember : Most of the cylinders we observe are right circular cylinders because line segment (axis) joining the centres of top and bottom of circular bases is perpendicular to both the circular bases.

Surface area of a cylinder with one open end :

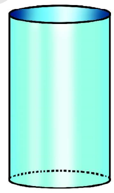
In an one open end cylinder, the cylinder will consist of its lateral surface and one circular base

Therefore, total surface area = $2\pi rh + \pi r^2$

Surface area of a cylinder with both open ends (top and bottom) = $2\pi rh$

In case of cylinder with top and bottom ends open, the surface area = $2\pi rh$

Points to remember : In case of a right circular cylinder



(i) Total surface area $= 2\pi r\ (r + h)$

(ii) Total surface area with one end open = $2\pi rh + \pi r^2$

(iii) Total surface area with both end open $= 2\pi rh$

Example 7 : A cylindrical milk powder container is of height 15 cm and has radius 7 cm. Find the lateral and total surface area of the container ($\pi = \frac{22}{7}$).

Solution : Radius of the base of the container (r) = 7 cm

Height of the cylinder (h) = 15 cm

Lateral surface area $= 2\pi rh$

$= 2\times\frac{22}{7} \times 7 \times 15$ sq.cm

$= 660$ sq.cm

Total surface area $= 2\pi r\ (r + h)$

$= 2\times\frac{22}{7} \times 7(7 + 15)$ sq.cm

$= 2 \times 22 \times 22$ sq.cm

$= 968$ sq.cm

Example 8 : The surface area and radius of the base of a metallic closed cylinder are 968 sq. cm and 7 cm respectively. Find the height of the cylinder.

Solution : Radius of closed cylinder (r) = 7 cm

Total surface area of the cylinder = 968 sq. cm

$$\text{or, } 2\pi r (r + h) = 968$$

$$\text{or, } 2 \times \frac{22}{7} \times 7 (7 + h) = 968$$

$$\text{or, } 44 \times (7 + h) = 968$$

$$\text{or, } 7 + h = \frac{968}{44}$$

$$\text{or, } 7 + h = 22$$

$$\text{or, } h = 22 - 7$$

$$\therefore \quad h = 15$$

Height of the cylinder = 15 cm

Example 9 : The height and curved surface area of a cylinder are 14cm and 88 sq.cm respectively. Find the diameter of the base of the cylinder.

Solution : Height of cylinder (h) = 14 cm

Curved surface area = 88 sq.cm

According to question, $2\pi rh = 88$ (the curved surface of a cylinder = $2\pi rh$)

$$\text{or, } 2 \times \frac{22}{7} \times 14 \times r = 88$$

$$\text{or, } 88r = 88$$

$$\therefore \quad r = \frac{88}{88} = 1 \text{ cm}$$

The diameter of the base = $2r$

= 2×1 cm

= 2 cm

Example 10 : Find the cost of painting a iron water pipe of length 10 m and cross sectional diameter 28cm at the rate of Rs. 120 per sq.m.

Solution : A water pipe is a cylinder with both ends open

$\therefore$ Surface area of the pipe = $2\pi rh$

= $\pi(2r)h$ where, $2r = 28$ cm

$= \frac{28}{100}$ m.

Here, h = height (length of the pipe) = 10 m.

$\therefore$ The surface area of the pipe $= \pi \times (2r) \times h$

$$= \frac{22}{7} \times \frac{28}{100} \times 10 \text{ sq.m} = \frac{88}{10} \text{ sq.m}$$

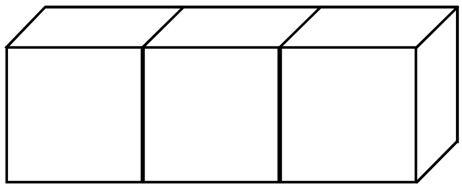
Given, cost of painting of 1 sq.m area is Rs.120

$\therefore$ Cost of painting of $\frac{88}{10}$ sq.m of area = Rs.120 $\times \frac{88}{10}$ = Rs.88 $\times$ 12 = Rs.1056

$\therefore$ Cost of painting the pipe = Rs.1056

Exercise 11.2

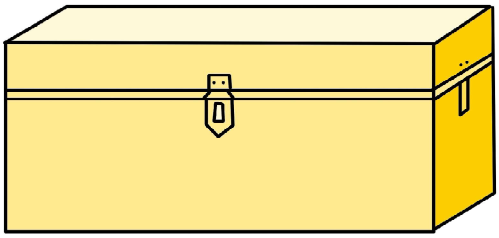
1. The length of each edge of a cube is 27 cm. Find the surface area of the cube.
2. The perimeter of the floor of a room is 30 meter and the height of the room is $\frac{1}{10}$ th of its perimeter. Find the area of four walls of the room.
3. Find the area of the base surface of a cuboid whose total surface area and lateral surface areas are 50 sq meter and 30 sq meter respectively.
4. A box of dimensions 80 cm $\times$ 48 cm $\times$ 24 cm is to be covered with a cloth. How many meter of cloth of width 96 cm is required to cover 100 such boxes?
5. A roller takes 500 complete revolutions to level a playground. Find the cost of levelling the ground at the rate of 75 paise per sq.meter if the diameter and length of the roller are 84 cm and 120 cm. respectively.
6. Find the curved surface area of a cylindrical container of curd with one end open whose height and radius are 30 cm and 14 cm respectively.
7. 3 cubes each of side length 5 cm are joined end to end. Find the total surface area of the cuboid so formed.



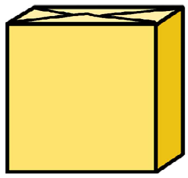
8. Find the diameter of the base of a cylinder of height 14 cm and lateral surface area 88 sq.cm.
9. There are 25 right circular cylindrical posts in a temple. Each post is of height 4 m and radius 28 cm. Find the cost of painting the curved surface of the posts at the rate of Rs.8 per square metre.
10. Find the lateral surface area and area of circular base of a cylindrical glass of height 12 cm and diameter 7 cm.
11. Cross sectional diameter and length of a hume pipe are 1400 mm and 2500 mm respectively. Find the cost of painting exterior side of 22 such pipes at the rate of Rs. 8 per square metre.

11.5 Volume of a Cuboid

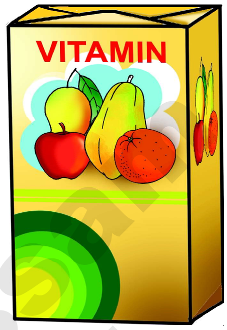
Volume of a 3- dimensional solid object is the measure of the space occupied by it. Following are few examples of cuboids.



Box

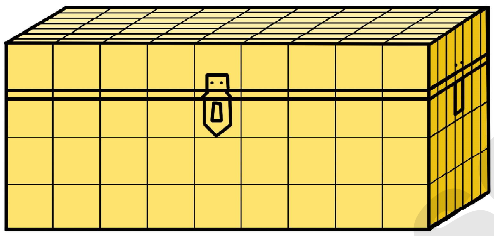


Chalk Box

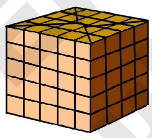


Medicine Box

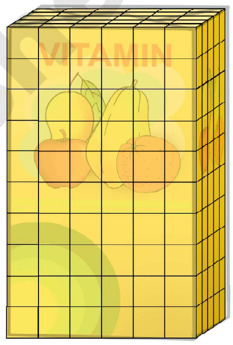
Let us make few cuboids by joining cubes of equal dimension 1 cm × 1 cm × 1 cm (called unit cube)



(i) (9 × 4 × 7)



(ii) (5 × 5 × 4)



(iii) (10 × 6 × 5)

Number of unit cubes :

(i) along the length = 9 Nos.
along the breadth = 7 Nos.
along the height = 4 Nos.
Total = 252 Nos.

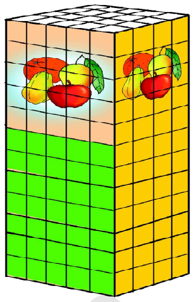
(ii) along the length = 5 Nos.
along the breadth = 4 Nos.
along the height = 5 Nos.
Total = 100 Nos.

(iii) along length = 6 Nos.
along breadth = 5 Nos.
along height =10 Nos.
Total = 300 Nos.

Observe that the volume of a cuboid can be measured by counting the number of unit cubes required to form the cuboid.

If we use cubes with dimension 1 cm × 1 cm × 1cm or 1 cubic cm then the volumes of the above cuboid will be 252 cubic cm, 100 cubic cm and 300 cubic cm respectively.

Note that *the unit for the volume of cuboid is cubic cm or* cm^3 *etc.*



Look at the medicine box. If the length, breadth, height of the box are 5cm, 4cm and 12cm respectively, then how many number of unit cubes will be required to form the box?

$5 \times 4 \times 12 = 240$, Isn't it?

i.e the volume of the cuboidal medicine box = 240 cubic cm

Remember : To represent the size of a cuboid we express its dimension as lenght × breadth × height.

The volume of a cuboid is the product of these three measures expressed is cubic unit.

Complete the following table.

Length *(l)* cm	Breadth *(b)* cm	Height *(h)* cm	Volume = $l \times b \times h$ cubic cm (cc)
10	7	6	420
8	9	12	....
3	6	11	....
4	15	20	....
12	8	6	....
9	7	12	....

Example 11 : Find the volume of a cuboid whose length, breadth and height measures are 15cm, 10cm and 8 cm respectively.

Solution : Length of the cuboid *(l)* = 15 cm

Breadth of the cuboid *(b)* = 10 cm

Height of the cuboid *(h)* = 8 cm

∴ Volume of the cuboid = length × breadth × height

= 15 cm × 10 cm × 8 cm

= 1200 cu cm

Remember : We use different units for solids having different sizes in terms of volumes, For example, to measure the volume of a big water reservoir we can use cubic metre instead of cu cm.

1 cubic metre = 1 m × 1 m × 1 m

= 100 cm × 100 cm × 100 cm

= 1000000 cu. cm

= 10^6 cu cm

1 cubic. m = 10^6 cu cm

Example 12 : The volume of a cuboid is 440 cu cm. Find the height of the cuboid if its length and breadth are 22 cm and 4 cm respectively.

Solution : Volume of the cuboid = 440 cu cm

Length (l) = 22 cm

Breadth (b) = 4 cm

i.e., $l \times b \times h = 440$ (h = height of the cuboid)

or, $22 \times 4 \times h = 440$

or, $88\,h = 440$

$$\therefore h = \frac{440}{88} = 5$$

Height of the cuboid = 5 cm

Group Activity :

Collect 24 pieces of Rubik cube or Ludo dice. Arrange these Rubik cubes or Ludo dice to form cuboids of different dimensions and find the volumes and surface areas of these cuboids.

11.6 Volume of a Cube :

The cube is a special case of a cuboid where $l = b = h$. Thus, the volume of the cube is $l \times l \times l = l^3$, where l is the length of edge of cube.

In the given figure a cube is formed by the 64 unit cube

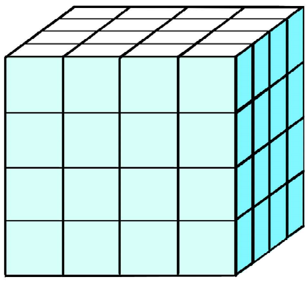
Observe that in the cubes

length = breadth = height = 4 unit

Volume of the cube = 64 no of unit cubes

i.e. Volume of the cube = $4 \times 4 \times 4$ cube unit

= 64 cube units



Example 13 : Find the volume of a cube whose length of edge is 7 cm.

Solution : Length of side of the cube = 7 cm

Volume of the cube = $(\text{length})^3 = (7\text{ cm})^3$

= 343 cubic cm

Example 14 : How many cubes of side 5 cm can be cut from a cube of side 20 cm?

Solution : Length of the big cube = 20 cm

$\therefore$ Volume of the big cube (v) = $(20 \text{ cm})^3$

= 8000 cu. cm

Again

Length of the small cube = 5 cm

Volume of the small cube (v) = $(5 \text{ cm})^3$

= 125 cu. cm

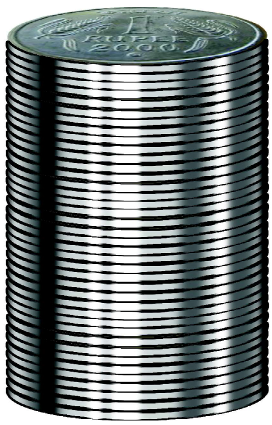
$\therefore$ Number of cubes = $\dfrac{\text{Volume of Bigger cube}}{\text{Volume of Smaller cube}} = \dfrac{8000}{125} = 64$

11.7 Volume of a Cylinder

Collect some one rupee coins and place them one above the other. The solid so formed will be a cylinder. The total volume of these coins will be the volume of the cylinder. If we assume the thickness (height) of a single one rupee coin as 1 unit then the height of the cylinder will be sum of heights of these coins. If the total height of the coins is *h* (units) then the height of the cylinder is also *h*(units)

Volume of the cylinder = Surface area of a coin × height

$= \pi r^2 \times h \qquad [\pi = \frac{22}{7}]$



Here, radius of the coin = r

Sum of height of coins = Height of the cylinder = h

Volume of the cylinder = $\pi r^2 h$

(Complete the following table):

height (h) cm	diameter (d) cm	radius (r) cm	area of the base πr^2 sq.cm	volume of cylinder (v)cu.cm
15	14	?	?	?
10	?	?	44	?
20	?	0.7	?	?
?	?	?	250	500

11.8 Volume and Capacity

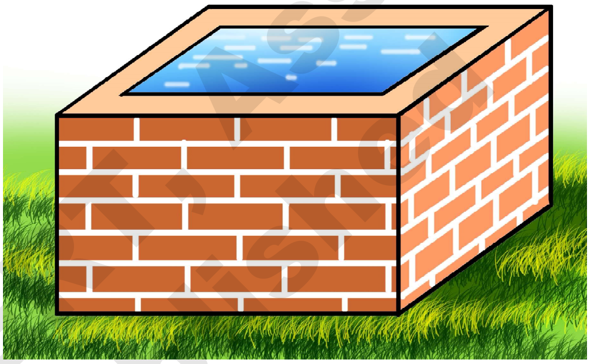
The fundamental difference between volume and capacity-

(i) Volume refers to the amount of space occupied by an object.

(ii) Capacity refers to the quantity that a container hold.

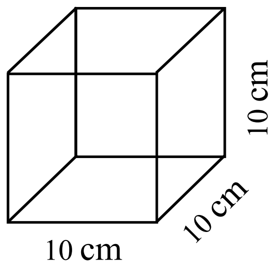
Let us understand the basic difference between the words volume and capacity from the following discussion-

As shown in the figure when we multiply the external dimensions (length, breadth, height) of a cistern we get its volume. On the other hand, if we multiply the inner dimensions (length, breadth, height) of the cistern we get the capacity of the cistern. Thus capacity of the cistern is the amount of water the cistern can hold, In this case, volume of the cistern is more than its capacity.



If the thickness of four walls of the cistern is highly negligible then its volume and capacity may be considered as equal. In this chapter we will consider container with such case, of negligible in thickness of walls and hence the volume of container will refer to capacity and vice versa.

For example : The inner volume of a container whose inner length, breadth & height are each 10 cm = 10 cm × 10 cm × 10 cm
= 1000 cu.cm
= 1000 ml. (1 cu. cm = 1 mililitre = 1 ml)
= 1 *l* (litre)



Therefore, capacity of the container = 1 Litre.

Remember :

1 Cu. cm = 1 ml
1000 cu.cm = 1 Litre
1 cu. m = 1000 Litre
or 1000000 cu. cm = 1000 Litre

Example 15 : Find the amount of water that a cuboid of demension 2 m × 0.5 m × 2 m can hold.

Solution : Volume of the cuboid $= \text{length} \times \text{breadth} \times \text{height}$

$= 2\text{ m} \times 0.5\text{ m} \times 2\text{ m}$

$= 2 \times \frac{1}{2} \times 2$ cu.m

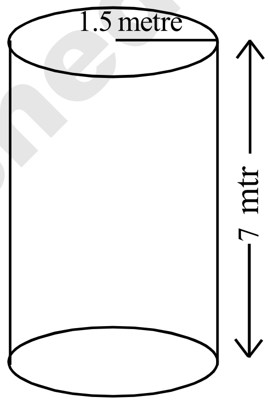
$= 2$ cu. m

Capacity of 1 Cubic metre volume of water = 1000 litre

Capacity of 2 Cubic metre volume of water = 2×1000 litre

$\therefore$ Capacity of the container = 2000 litre.

Example 16 : Find the amount of milk that a cylindrical milk container hold whose inner diameter is 1.5 m and height 7 m



Solution : Radius of the container *(r)* = 1.5 m

Height of the container *(h)* = 7 m

Volume of the container *(v)* $= \pi r^2 h$

$= \frac{22}{7} \times (1.5)^2 \times 7$ cu. m

$= 22 \times 2.25$ cu. m

$= 49.5$ cu. m

$\because$ 1 cu. m = 1000 Litre

$\therefore$ 49.5 cu. m = (49.5×1000)Litre

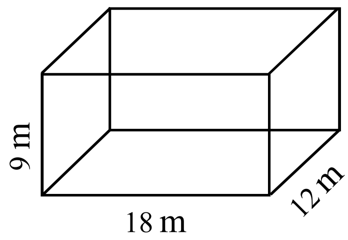
= 49500 Litre

$\therefore$ Amount of milk in the container = 49500 Litre

Example : How many cubes of edge 3m can be cut from a cuboid of dimension $18\text{m} \times 12\text{m} \times 9\text{m}$?

Solution : Volume of cuboid $= 18 \times 12 \times 9$ cu m.

$= 1944$ cu. m



Edge of the cube = 3 m

Volume of cube $= (\text{length})^3$

$= 3^3$ cu. m

$= 27$ cu. m

Number of cubes $= \dfrac{\text{Volume of cuboid}}{\text{Volume of a cube}}$

$$= \frac{1944}{27}$$

$$= 72$$

Example 18 : Find the volume of a cube whose surface area is 96 sq.cm.

Solution : Surface area of the cube = 96 cm^2, Surface area of the cube = $(\text{length})^2$

$$\therefore \; 6 \times (\text{length})^2 = 96 \text{ cm}^2$$

$$\text{or } (\text{length})^2 = \frac{96}{6} = 16 \text{ cm}^2$$

$$\therefore \; \text{length} = \sqrt{16} \text{ cm} = 4 \text{ cm}$$

$$\therefore \; \text{Volume of the cube} = \text{length}^3$$

$$= 4^3 \text{ cu. cm}$$

$$= 64 \text{ cu. cm}$$

Example 19 : A right circular cylinder of height 10 cm and radius 4.5 cm is formed by melting some coins of thickness 0.2 cm and diameter 1.5 cm. Find the number of coins required.

Solution : Diameter of a coin (d_1) = 1.5 cm

$$\text{Radius of a coin } (r_1) = \frac{1.5}{2} = 0.75 \text{ cm}$$

Thickness or length (h_1) = 0.2 cm

$$\therefore \; \text{Volume of a coin} = \pi r_1^2 h$$

$$= \pi \times (0.75)^2 \times 0.2 \text{ cu.cm}$$

Again diameter of cylinder (d_2) = 4.5 cm

$$\text{Radius of cylinder } (r_2) = \frac{4.5}{2} = 2.25 \text{ cm}$$

Height of cylinder (h_2) = 10 cm

$$\therefore \; \text{Volume of cylinder} = \pi r_2^2 h_2$$

$$= \pi \times (2.25)^2 \times 10 \text{ cu.cm}$$

$$\therefore \; \text{Number of coin required} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$$

$$= \frac{\pi \times r_1^2 \times h_1}{\pi \times r_2^2 \times h_2} = \frac{(2.25)^2 \times 10}{(0.75)^2 \times 0.2}$$

$$= \frac{2.25 \times 2.25 \times 10}{0.75 \times 0.75 \times 0.2} = 3 \times 3 \times 50 = 450$$

Example 20 : Under a certain temperature mass of an ice block of 1 cubic metre is 900 kilogram. Find the mass of an identical cubical block of length 50 cm.

Solution : Side length of ice cube = 50 cm

$$= \frac{50}{100} \text{ m} = \frac{1}{2} \text{ m}$$

$\therefore$ Volume of the cube = $(\frac{1}{2})^3$ cu. m = $\frac{1}{8}$ cu m

mass of 1 cu m ice block = 900 kilogram

$\therefore$ mass of $\frac{1}{8}$ cu m ice cube = $\frac{1}{8} \times 900$ Kilogram

= 112.5 Kilogram

Exercise 11.3

1. A cuboidal container can hold 105 litres of water. If the base is of dimensions 15m × 3.5m, find the height of the container.
2. A metallic cube of side measuring 12 cm is melted to form three smaller cubes. If the edges of two smaller cubes are of length 6 cm and 8 cm respectively, find the edge of the third cube.
3. Find the height of a cuboid whose volume and area of the base are 900 cu cm and 180 sq cm respectively.
4. How many cubes of side length 6 cm can be put inside a cuboid, open at one end of dimensions 60 cm × 54 cm × 30 cm?
5. What is the amount of soil which is dug out from a circular pit of 21 metres deep and 6 metre diameter?
6. Water is filled at the rate of 40 litres per minute into a cubrical tank. Find the total time to fill the tank if its volume is 54 cu m.
7. A metalic solid of volume 2200 cu cm is melted to form a uniform wire of cross sectional diameter 0.5 cm. Find the length of the wire.
8. Find the height of a cuboid whose volume and area of base are 440 cu cm and 88 sq cm respectively.
9. Find the height of a cuboid whose volume and area of the base are 160 cu m and 2800 sq cm.
10. Find the amount of water required to fill a rectangular water reservior if the inner dimensions of the reservior is 6 m × 2 m × 1m.

11. Find the volume of a cylinder whose height is 25 cm and the circumference of the base is 132 cm.
12. The heights of two cylinders having equal volumes are in the ratio 1:4. Find the ratio of radii of their bases.
13. A cuboidal water tank measures internally 4.2 m length, 300 cm breadth and 1.8 m height. Find the capacity of the tank in litres.
14. The total surface area of a cylindrical pillar is 924 square cm. If its curved surface area is two-third of its total surface area, find the radius of its base and volume.

What we have learnt

1. Area of a trapezium = $\frac{1}{2}$(sum of the parallel sides) $\times$ (perpendiculer distance between them).

2. Area of a rhombus = $\frac{1}{2}$ (Product of the diagonals)

3. We can divide a polygon into different plane figures such as triangle, square, rectangle, Trapezium etc to find the area of the polygon which is equal to sum of areas of the plane figures.

4. Total surface area of a solid is the sum of areas of its all surfaces
   - (i) Surface area of a cuboid = $2(lb + bh + lh)$
   - (ii) Surface area of a cube = $6l^2$
   - (iii) Surface area of a cylinder having closed ends = $2\pi r(r + h)$
   - (iv) Surface area of a cylinder with both end open = $2\pi rh$
   - (v) Surface area of a cylinder with one end open = $2\pi rh + \pi r^2$

5. Volume
   - (i) Volume of a cuboid = $l \times b \times h$
   - (ii) Volume of a cube = l^3
   - (iii) Volume of a cylinder = $\pi r^2 h$

6. (i) 1 cu. cm= 1 milli litre (ml)
   - (ii) 1 litre = 1000 m = 1000 cu. cm
   - (iii) 1 cu m = 100 cm $\times$ 100 cm $\times$ 100 cm
     = 1000000 cu. cm
     = 1000000 ml
     = 1000 litre.

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