

# Application Integrals

## Short Answer Type Questions

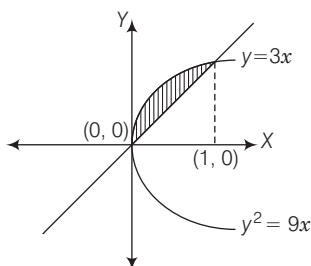
**Q. 1** Find the area of the region bounded by the curves  $y^2 = 9x$  and  $y = 3x$ .

**Thinking Process**

*On solving both the equation of curves, get the values of  $x$  and then at those values, find the area of the shaded region.*

**Sol.** We have,

$$\begin{aligned} y^2 &= 9x \text{ and } y = 3x \\ \Rightarrow (3x)^2 &= 9x \\ \Rightarrow 9x^2 - 9x &= 0 \\ \Rightarrow 9x(x - 1) &= 0 \\ \Rightarrow x &= 1, 0 \end{aligned}$$



$$\begin{aligned} \therefore \text{Required area, } A &= \int_0^1 \sqrt{9x} \, dx - \int_0^1 3x \, dx \\ &= 3 \int_0^1 x^{1/2} \, dx - 3 \int_0^1 x \, dx \\ &= 3 \left[ \frac{x^{3/2}}{3/2} \right]_0^1 - 3 \left[ \frac{x^2}{2} \right]_0^1 \\ &= 3 \left( \frac{2}{3} - 0 \right) - 3 \left( \frac{1}{2} - 0 \right) \\ &= 2 - \frac{3}{2} = \frac{1}{2} \text{ sq units} \end{aligned}$$

**Q. 2** Find the area of the region bounded by the parabola  $y^2 = 2px$   $x^2 = 2py$ .

**Sol.** We have,

$$y^2 = 2px \text{ and } x^2 = 2py$$

$\therefore$

$$y = \sqrt{2px}$$

$\Rightarrow$

$$x^2 = 2p \cdot \sqrt{2px}$$

$\Rightarrow$

$$x^4 = 4p^2 \cdot (2px)$$

$\Rightarrow$

$$x^4 = 8p^3x$$

$\Rightarrow$

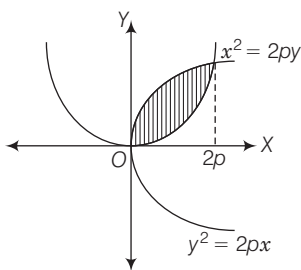
$$x^4 - 8p^3x = 0$$

$\Rightarrow$

$$x^3(x - 8p^3) = 0$$

$\Rightarrow$

$$x = 0, 2p$$

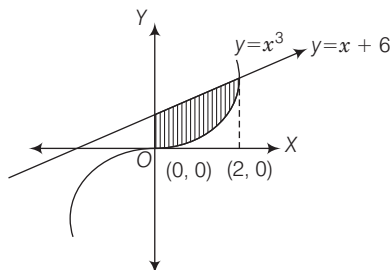


$\therefore$

$$\begin{aligned} \text{Required area} &= \int_0^{2p} \sqrt{2px} \, dx - \int_0^{2p} \frac{x^2}{2p} \, dx \\ &= \sqrt{2p} \int_0^{2p} x^{1/2} \, dx - \frac{1}{2p} \int_0^{2p} x^2 \, dx \\ &= \sqrt{2p} \left[ \frac{2(x)^{3/2}}{3} \right]_0^{2p} - \frac{1}{2p} \left[ \frac{x^3}{3} \right]_0^{2p} \\ &= \sqrt{2p} \left[ \frac{2}{3} \cdot (2p)^{3/2} - 0 \right] - \frac{1}{2p} \left[ \frac{1}{3} (2p)^3 - 0 \right] \\ &= \sqrt{2p} \left( \frac{2}{3} \cdot 2\sqrt{2}p^{3/2} \right) - \frac{1}{2p} \left( \frac{1}{3} 8p^3 \right) \\ &= \sqrt{2p} \left( \frac{4\sqrt{2}}{3} p^{3/2} \right) - \frac{1}{2p} \left( \frac{8}{3} p^3 \right) \\ &= \frac{4\sqrt{2}}{3} \cdot \sqrt{2}p^2 - \frac{8}{6}p^2 \\ &= \frac{(16 - 8)p^2}{6} = \frac{8p^2}{6} \\ &= \frac{4p^2}{3} \text{ sq units} \end{aligned}$$

**Q. 3** Find the area of the region bounded by the curve  $y = x^3$ ,  $y = x + 6$  and  $x = 0$ .

**Sol.** We have,  $y = x^3$ ,  $y = x + 6$  and  $x = 0$



$$\begin{aligned}
 \therefore & \quad x^3 = x + 6 \\
 \Rightarrow & \quad x^3 - x = 6 \\
 \Rightarrow & \quad x^3 - x - 6 = 0 \\
 \Rightarrow & \quad x^2(x - 2) + 2x(x - 2) + 3(x - 2) = 0 \\
 \Rightarrow & \quad (x - 2)(x^2 + 2x + 3) = 0 \\
 \Rightarrow & \quad x = 2, \text{ with two imaginary points}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ Required area of shaded region} &= \int_0^2 (x + 6 - x^3) dx \\
 &= \left[ \frac{x^2}{2} + 6x - \frac{x^4}{4} \right]_0^2 \\
 &= \left[ \frac{4}{2} + 12 - \frac{16}{4} - 0 \right] \\
 &= [2 + 12 - 4] = 10 \text{ sq units}
 \end{aligned}$$

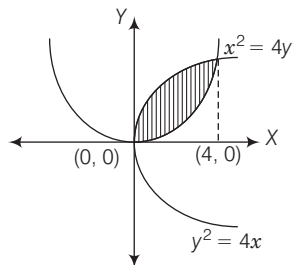
**Q. 4** Find the area of the region bounded by the curve  $y^2 = 4x$  and  $x^2 = 4y$ .

**Thinking Process**

First, by using both the equation get the values of  $x$  and then find the shaded region by using these value of  $x$  in the equation of curve in  $x$  only.

**Sol.** Given equation of curves are

$$\begin{aligned}
 & y^2 = 4x && \dots(i) \\
 \text{and} & x^2 = 4y && \dots(ii) \\
 \Rightarrow & \left( \frac{x^2}{4} \right)^2 = 4x \\
 \Rightarrow & \frac{x^4}{4 \cdot 4} = 4x \\
 \Rightarrow & x^4 = 64x \\
 \Rightarrow & x^4 - 64x = 0 \\
 \Rightarrow & x(x^3 - 4^3) = 0 \\
 \Rightarrow & x = 4, 0
 \end{aligned}$$

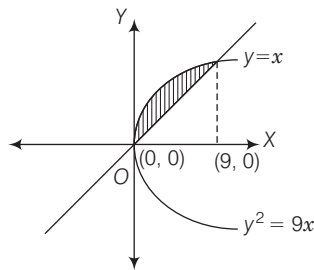


$$\begin{aligned}
 \therefore \text{Area of shaded region, } A &= \int_0^4 \left( \sqrt{4x} - \frac{x^2}{4} \right) dx \\
 &= \int_0^4 \left( 2\sqrt{x} - \frac{x^2}{4} \right) dx = \left[ \frac{2x^{3/2} \cdot 2}{3} - \frac{1}{4} \cdot \frac{x^3}{3} \right]_0^4 \\
 &= \frac{2 \cdot 2}{3} \cdot 8 - \frac{1}{4} \cdot \frac{64}{3} - 0 = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq units}
 \end{aligned}$$

**Q. 5** Find the area of the region included between  $y^2 = 9x$  and  $y = x$ .

**Sol.** We have,  $y^2 = 9x$  and  $y = x$

$$\begin{aligned}
 \Rightarrow & x^2 = 9x \\
 \Rightarrow & x^2 - 9x = 0 \\
 \Rightarrow & x(x - 9) = 0 \\
 \Rightarrow & x = 0, 9
 \end{aligned}$$

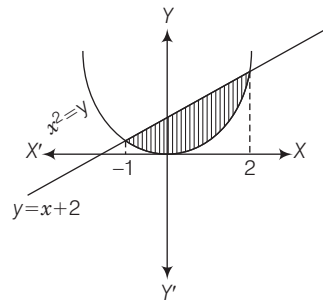


$$\begin{aligned}
 \therefore \text{Area of shaded region, } A &= \int_0^9 (\sqrt{9x} - x) dx = \int_0^9 3x^{1/2} dx - \int_0^9 x dx \\
 &= \left[ 3 \cdot \frac{x^{3/2}}{3} \cdot 2 \right]_0^9 - \left[ \frac{x^2}{2} \right]_0^9 \\
 &= \left[ \frac{3 \cdot 3^{\frac{3}{2} \times 2}}{3} \cdot 2 - 0 \right] - \left[ \frac{81}{2} - 0 \right] \\
 &= 54 - \frac{81}{2} = \frac{108 - 81}{2} = \frac{27}{2} \text{ sq units}
 \end{aligned}$$

**Q. 6** Find the area of the region enclosed by the parabola  $x^2 = y$  and the line  $y = x + 2$ .

**Sol.** We have,  $x^2 = y$  and  $y = x + 2$

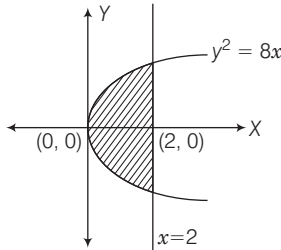
$$\begin{aligned}
 \Rightarrow & x^2 = x + 2 \\
 \Rightarrow & x^2 - x - 2 = 0 \\
 \Rightarrow & x^2 - 2x + x - 2 = 0 \\
 \Rightarrow & x(x - 2) + 1(x - 2) = 0 \\
 \Rightarrow & (x + 1)(x - 2) = 0 \\
 \Rightarrow & x = -1, 2
 \end{aligned}$$



$$\begin{aligned}
 \therefore \text{ Required area of shaded region} &= \int_{-1}^2 (x + 2 - x^2) dx = \left[ \frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 \\
 &= \left[ \frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} \right] \\
 &= 6 + \frac{3}{2} - \frac{9}{3} = \frac{36 + 9 - 18}{6} = \frac{27}{6} = \frac{9}{2} \text{ sq units}
 \end{aligned}$$

**Q. 7** Find the area of the region bounded by line  $x = 2$  and parabola  $y^2 = 8x$ .

**Sol.** We have,  $y^2 = 8x$  and  $x = 2$

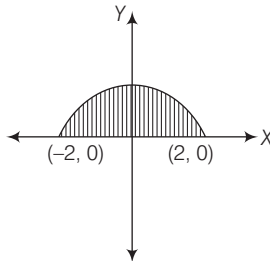


$$\begin{aligned}
 \therefore \text{ Area of shaded region, } A &= 2 \int_0^2 \sqrt{8x} dx = 2 \cdot 2\sqrt{2} \int_0^2 x^{1/2} dx \\
 &= 4 \cdot \sqrt{2} \cdot \left[ 2 \cdot \frac{x^{3/2}}{3} \right]_0^2 = 4\sqrt{2} \left[ \frac{2}{3} \cdot 2\sqrt{2} - 0 \right] \\
 &= \frac{32}{3} \text{ sq units}
 \end{aligned}$$

**Q. 8** Sketch the region  $\{(x, 0) : y = \sqrt{4 - x^2}\}$  and X-axis. Find the area of the region using integration.

**Sol.** Given region is  $\{(x, 0) : y = \sqrt{4 - x^2}\}$  and X-axis.

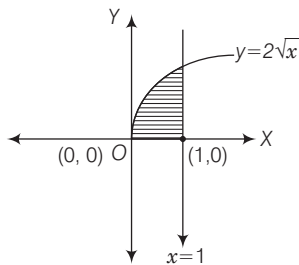
We have,  $y = \sqrt{4 - x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$



$$\begin{aligned}
 \therefore \text{ Area of shaded region, } A &= \int_{-2}^2 \sqrt{4 - x^2} dx = \int_{-2}^2 \sqrt{2^2 - x^2} dx \\
 &= \left[ \frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \cdot \sin^{-1} \frac{x}{2} \right]_{-2}^2 \\
 &= \frac{2}{2} \cdot 0 + 2 \cdot \frac{\pi}{2} + \frac{2}{2} \cdot 0 - 2 \sin^{-1}(-1) = 2 \cdot \frac{\pi}{2} + 2 \cdot \frac{\pi}{2} \\
 &= 2\pi \text{ sq units}
 \end{aligned}$$

**Q. 9** Calculate the area under the curve  $y = 2\sqrt{x}$  included between the lines  $x = 0$  and  $x = 1$ .

**Sol.** We have,  $y = 2\sqrt{x}$ ,  $x = 0$  and  $x = 1$

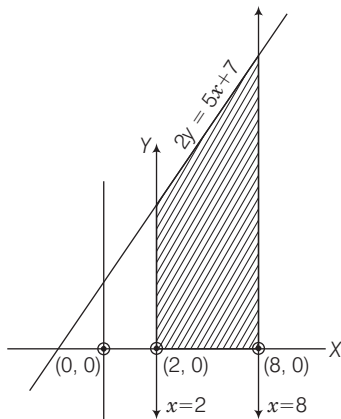


$$\begin{aligned}\therefore \text{Area of shaded region, } A &= \int_0^1 (2\sqrt{x}) dx \\ &= 2 \cdot \left[ \frac{x^{3/2}}{3} \cdot 2 \right]_0^1 \\ &= 2 \left( \frac{2}{3} \cdot 1 - 0 \right) = \frac{4}{3} \text{ sq units}\end{aligned}$$

**Q. 10** Using integration, find the area of the region bounded by the line  $2y = 5x + 7$ ,  $X$ -axis and the lines  $x = 2$  and  $x = 8$ .

**Sol.** We have,

$$\begin{aligned}2y &= 5x + 7 \\ \Rightarrow y &= \frac{5x}{2} + \frac{7}{2}\end{aligned}$$



$$\begin{aligned}\therefore \text{Area of shaded region} &= \frac{1}{2} \int_2^8 (5x + 7) dx = \frac{1}{2} \left[ 5 \cdot \frac{x^2}{2} + 7x \right]_2^8 \\ &= \frac{1}{2} [5 \cdot 32 + 7 \cdot 8 - 10 - 14] = \frac{1}{2} [160 + 56 - 24] \\ &= \frac{192}{2} = 96 \text{ sq units}\end{aligned}$$

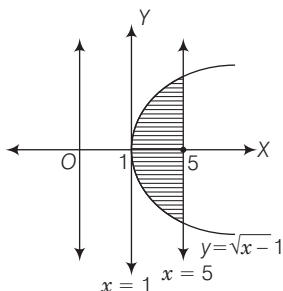
**Q. 11** Draw a rough sketch of the curve  $y = \sqrt{x-1}$  in the interval  $[1, 5]$ .

Find the area under the curve and between the lines  $x = 1$  and  $x = 5$ .

**Sol.** Given equation of the curve is  $y = \sqrt{x-1}$ .

$\Rightarrow$

$$y^2 = x - 1$$



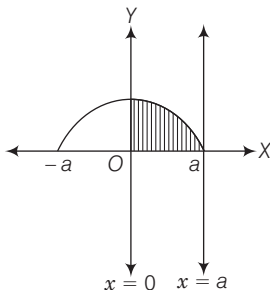
$$\begin{aligned} \therefore \text{Area of shaded region, } A &= \int_1^5 (x-1)^{1/2} dx = \left[ \frac{2 \cdot (x-1)^{3/2}}{3} \right]_1^5 \\ &= \left[ \frac{2}{3} \cdot (5-1)^{3/2} - 0 \right] = \frac{16}{3} \text{ sq units} \end{aligned}$$

**Q. 12** Determine the area under the curve  $y = \sqrt{a^2 - x^2}$  included between the lines  $x = 0$  and  $x = a$ .

**Sol.** Given equation of the curve is  $y = \sqrt{a^2 - x^2}$ .

$\Rightarrow$

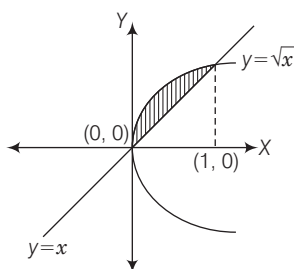
$$y^2 = a^2 - x^2 \Rightarrow y^2 + x^2 = a^2$$



$$\begin{aligned} \therefore \text{Required area of shaded region, } A &= \int_0^a \sqrt{a^2 - x^2} dx \\ &= \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \\ &= \left[ 0 + \frac{a^2}{2} \sin^{-1}(1) - 0 - \frac{a^2}{2} \sin^{-1} 0 \right] \\ &= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{\pi a^2}{4} \text{ sq units} \end{aligned}$$

**Q. 13** Find the area of the region bounded by  $y = \sqrt{x}$  and  $y = x$ .

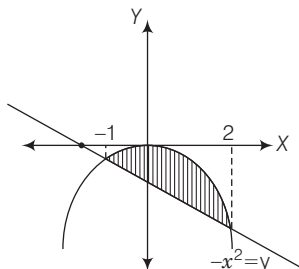
**Sol.** Given equation of curves are  $y = \sqrt{x}$  and  $y = x$ .  
 $\Rightarrow x = \sqrt{x} \Rightarrow x^2 = x$   
 $\Rightarrow x^2 - x = 0 \Rightarrow x(x - 1) = 0$   
 $\Rightarrow x = 0, 1$



$$\begin{aligned} \therefore \text{ Required area of shaded region, } A &= \int_0^1 (\sqrt{x}) dx - \int_0^1 x dx \\ &= \left[ 2 \cdot \frac{x^{3/2}}{3} \right]_0^1 - \left[ \frac{x^2}{2} \right]_0^1 \\ &= \frac{2}{3} \cdot 1 - \frac{1}{2} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ sq units} \end{aligned}$$

**Q. 14** Find the area enclosed by the curve  $y = -x^2$  and the straight line  $x + y + 2 = 0$ .

**Sol.** We have,  $y = -x^2$  and  $x + y + 2 = 0$



$$\begin{aligned} \Rightarrow -x - 2 &= -x^2 \Rightarrow x^2 - x - 2 = 0 \\ \Rightarrow x^2 + x - 2x - 2 &= 0 \Rightarrow x(x + 1) - 2(x + 1) = 0 \\ \Rightarrow (x - 2)(x + 1) &= 0 \Rightarrow x = 2, -1 \\ \therefore \text{ Area of shaded region, } A &= \left| \int_{-1}^2 (-x - 2 + x^2) dx \right| = \left| \int_{-1}^2 (x^2 - x - 2) dx \right| \\ &= \left| \left[ \frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-1}^2 \right| = \left| \left[ \frac{8}{3} - \frac{4}{2} - 4 + \frac{1}{3} + \frac{1}{2} - 2 \right] \right| \\ &= \left| \frac{16 - 12 - 24 + 2 + 3 - 12}{6} \right| = \left| -\frac{27}{6} \right| = \frac{9}{2} \text{ sq units} \end{aligned}$$

**Q. 15** Find the area bounded by the curve  $y = \sqrt{x}$ ,  $x = 2y + 3$  in the first quadrant and  $X$ -axis.

**Sol.** Given equation of the curves are  $y = \sqrt{x}$  and  $x = 2y + 3$  in the first quadrant.

On solving both the equations for  $y$ , we get

$$\begin{aligned} y &= \sqrt{2y + 3} \\ y^2 &= 2y + 3 \end{aligned}$$

$$\Rightarrow y^2 - 2y - 3 = 0$$

$$\Rightarrow y^2 - 3y + y - 3 = 0$$

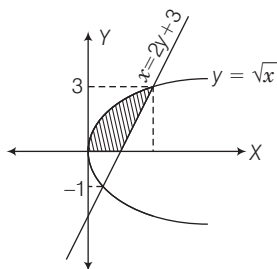
$$\Rightarrow y(y - 3) + 1(y - 3) = 0$$

$$\Rightarrow (y + 1)(y - 3) = 0$$

$$\Rightarrow y = -1, 3$$

$\therefore$  Required area of shaded region,

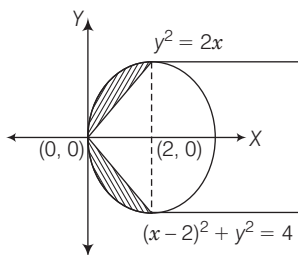
$$\begin{aligned} A &= \int_0^3 (2y + 3 - y^2) dy = \left[ \frac{2y^2}{2} + 3y - \frac{y^3}{3} \right]_0^3 \\ &= \left[ \frac{18}{2} + 9 - 9 - 0 \right] = 9 \text{ sq units} \end{aligned}$$



## Long Answer Type Questions

**Q. 16** Find the area of the region bounded by the curve  $y^2 = 2x$  and  $x^2 + y^2 = 4x$ .

**Sol.** We have,  $y^2 = 2x$  and  $x^2 + y^2 = 4x$



$$\Rightarrow x^2 + 2x = 4x$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0, 2$$

$$\text{Also, } x^2 + y^2 = 4x$$

$$\Rightarrow x^2 - 4x = -y^2$$

$$\Rightarrow x^2 - 4x + 4 = -y^2 + 4$$

$$\Rightarrow (x - 2)^2 - 2^2 = -y^2$$

$$\begin{aligned}
 \therefore \text{ Required area} &= 2 \cdot \int_0^2 \left[ \sqrt{2^2 - (x-2)^2} - \sqrt{2x} \right] dx \\
 &= 2 \left[ \left[ \frac{x-2}{2} \cdot \sqrt{2^2 - (x-2)^2} + \frac{2^2}{2} \sin^{-1} \left( \frac{x-2}{2} \right) \right]_0^2 - \left[ \sqrt{2} \cdot \frac{x^{3/2}}{3/2} \right]_0^2 \right] \\
 &= 2 \left[ \left( 0 + 0 - 1 \cdot 0 + 2 \cdot \frac{\pi}{2} \right) - \frac{2\sqrt{2}}{3} (2^{3/2} - 0) \right] \\
 &= \frac{4\pi}{2} - \frac{8 \cdot 2}{3} = 2\pi - \frac{16}{3} = 2 \left( \pi - \frac{8}{3} \right) \text{ sq units}
 \end{aligned}$$

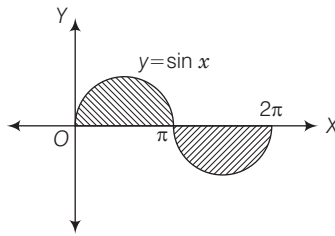
**Q. 17** Find the area bounded by the curve  $y = \sin x$  between  $x = 0$  and  $x = 2\pi$ .

**Thinking Process**

We know that,  $\sin x$  curve has positive region from  $[0, \pi]$  and negative region in  $[\pi, 2\pi]$ .

**Sol.** Required area  $= \int_0^{2\pi} \sin x \, dx = \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right|$

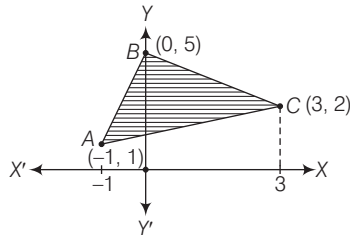
$$\begin{aligned}
 &= -[\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right| \\
 &= -[\cos \pi - \cos 0] + \left| -[\cos 2\pi - \cos \pi] \right|
 \end{aligned}$$



$$\begin{aligned}
 &= -[-1 - 1] + \left| -(1 + 1) \right| \\
 &= 2 + 2 = 4 \text{ sq units}
 \end{aligned}$$

**Q. 18** Find the area of region bounded by the triangle whose vertices are  $(-1, 1)$ ,  $(0, 5)$  and  $(3, 2)$ , using integration.

**Sol.** Let we have the vertices of a  $\triangle ABC$  as  $A(-1, 1)$ ,  $B(0, 5)$  and  $C(3, 2)$ .



$\therefore$  Equation of AB is  $y - 1 = \left( \frac{5-1}{0+1} \right) (x + 1)$

$\Rightarrow y - 1 = 4x + 4$

$\Rightarrow y = 4x + 5$

...(i)

and equation of BC is  $y - 5 = \left( \frac{2-5}{3-0} \right) (x - 0)$

$$\Rightarrow y - 5 = \frac{-3}{3}(x)$$

$$\Rightarrow y = 5 - x \quad \dots(ii)$$

Similarly, equation of AC is  $y - 1 = \left(\frac{2-1}{3+1}\right)(x+1)$

$$\Rightarrow y - 1 = \frac{1}{4}(x+1)$$

$$\Rightarrow 4y = x + 5 \quad \dots(iii)$$

$$\begin{aligned} \therefore \text{Area of shaded region} &= \int_{-1}^0 (y_1 - y_2) dx + \int_0^3 (y_1 - y_2) dx \\ &= \int_{-1}^0 \left[ 4x + 5 - \frac{x+5}{4} \right] dx + \int_0^3 \left[ 5 - x - \frac{x+5}{4} \right] dx \\ &= \left[ \frac{4x^2}{2} + 5x - \frac{x^2}{8} - \frac{5x}{4} \right]_{-1}^0 + \left[ 5x - \frac{x^2}{2} - \frac{x^2}{8} - \frac{5x}{4} \right]_0^3 \\ &= \left[ 0 - \left( 4 \cdot \frac{1}{2} + 5(-1) - \frac{1}{8} + \frac{5}{4} \right) \right] + \left[ \left( 15 - \frac{9}{2} - \frac{9}{8} - \frac{15}{4} \right) - 0 \right] \\ &= \left[ -2 + 5 + \frac{1}{8} - \frac{5}{4} + 15 - \frac{9}{2} - \frac{9}{8} - \frac{15}{4} \right] \\ &= 18 + \left( \frac{1 - 10 - 36 - 9 - 30}{8} \right) \\ &= 18 + \left( -\frac{84}{8} \right) = 18 - \frac{21}{2} = \frac{15}{2} \text{ sq units} \end{aligned}$$

**Q. 19** Draw a rough sketch of the region  $\{(x, y) : y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2\}$ . Also, find the area of the region sketched using method of integration.

**Sol.** We have,

$$y^2 = 6ax \text{ and } x^2 + y^2 = 16a^2$$

$$\Rightarrow x^2 + 6ax = 16a^2$$

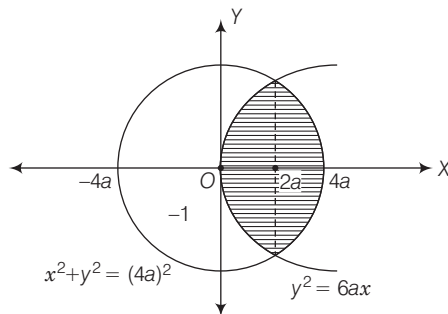
$$\Rightarrow x^2 + 6ax - 16a^2 = 0$$

$$\Rightarrow x^2 + 8ax - 2ax - 16a^2 = 0$$

$$\Rightarrow x(x + 8a) - 2a(x + 8a) = 0$$

$$\Rightarrow (x - 2a)(x + 8a) = 0$$

$$\Rightarrow x = 2a, -8a$$



$$\begin{aligned}
 \therefore \text{Area of required region} &= 2 \left[ \int_0^{2a} \sqrt{6ax} \, dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2} \, dx \right] \\
 &= 2 \left[ \int_0^{2a} \sqrt{6a} \, x^{1/2} \, dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2} \, dx \right] \\
 &= 2 \left[ \sqrt{6a} \left[ \frac{x^{3/2}}{3/2} \right]_0^{2a} + \left( \frac{x}{2} \sqrt{(4a)^2 - x^2} + \frac{(4a)^2}{2} \sin^{-1} \frac{x}{4a} \right) \Big|_{2a}^{4a} \right] \\
 &= 2 \left[ \sqrt{6a} \cdot \frac{2}{3} ((2a)^{3/2} - 0) + \frac{4a}{2} \cdot 0 + \frac{16a^2}{2} \cdot \frac{\pi}{2} - \frac{2a}{2} \sqrt{16a^2 - 4a^2} - \frac{16a^2}{2} \cdot \sin^{-1} \frac{2a}{4a} \right] \\
 &= 2 \left[ \sqrt{6a} \cdot \frac{2}{3} \cdot 2\sqrt{2} \, a^{3/2} + 0 + 4\pi a^2 - \frac{2a}{2} \cdot 2\sqrt{3}a - 8a^2 \cdot \frac{\pi}{6} \right] \\
 &= 2 \left[ \sqrt{12} \cdot \frac{4}{3} a^2 + 4\pi a^2 - 2\sqrt{3}a^2 - \frac{4a^2\pi}{3} \right] \\
 &= 2 \left[ \frac{8\sqrt{3}a^2 + 12\pi a^2 - 6\sqrt{3}a^2 - 4a^2\pi}{3} \right] \\
 &= \frac{2}{3} a^2 [8\sqrt{3} + 12\pi - 6\sqrt{3} - 4\pi] \\
 &= \frac{2}{3} a^2 [2\sqrt{3} + 8\pi] = \frac{4}{3} a^2 [\sqrt{3} + 4\pi]
 \end{aligned}$$

**Q. 20** Compute the area bounded by the lines  $x + 2y = 2$ ,  $y - x = 1$  and  $2x + y = 7$ .

**Sol.** We have,

$$x + 2y = 2 \quad \dots(i)$$

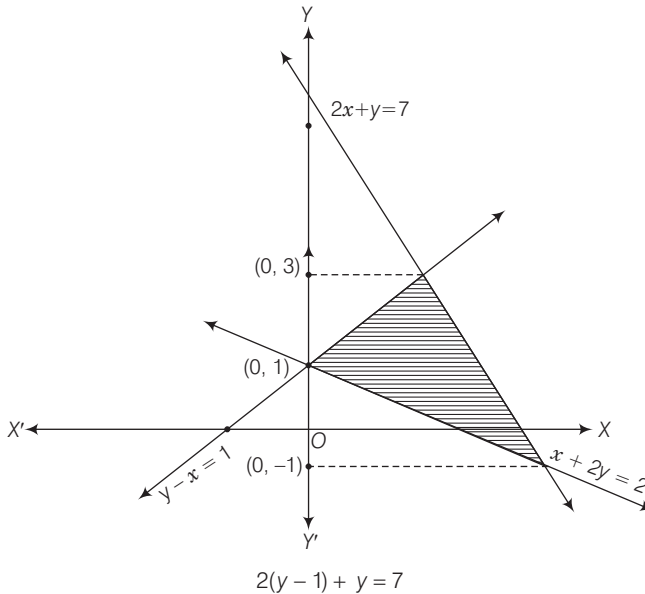
$$y - x = 1 \quad \dots(ii)$$

and

$$2x + y = 7 \quad \dots(iii)$$

On solving Eqs. (i) and (ii), we get

$$y - (2 - 2y) = 1 \Rightarrow 3y - 2 = 1 \Rightarrow y = 1$$



On solving Eqs. (ii) and (iii), we get

$$\Rightarrow 2y - 2 + y = 7$$

$$\Rightarrow y = 3$$

On solving Eqs. (i) and (iii), we get

$$\Rightarrow 2(2 - 2y) + y = 7$$

$$\Rightarrow 4 - 4y + y = 7$$

$$\Rightarrow -3y = 3$$

$$\Rightarrow y = -1$$

$$\therefore \text{ Required area} = \int_{-1}^1 (2 - 2y) dy + \int_{-1}^3 \frac{(7 - y)}{2} dy - \int_1^3 (y - 1) dy$$

$$= \left[ -2y + \frac{2y^2}{2} \right]_{-1}^1 + \left[ \frac{7y}{2} - \frac{y^2}{2 \cdot 2} \right]_{-1}^3 - \left[ \frac{y^2}{2} - y \right]_1^3$$

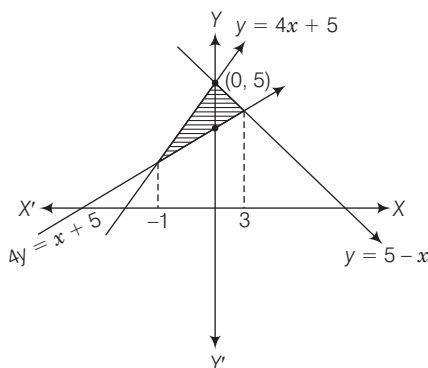
$$= \left[ -2 + \frac{2}{2} - 2 - \frac{2}{2} \right] + \left[ \frac{21}{2} - \frac{9}{4} + \frac{7}{2} + \frac{1}{4} \right] - \left[ \frac{9}{2} - 3 - \frac{1}{2} + 1 \right]$$

$$= [-4] + \left[ \frac{42 - 9 + 14 + 1}{4} \right] - \left[ \frac{9 - 6 - 1 + 2}{2} \right]$$

$$= -4 + 12 - 2 = 6 \text{ sq units}$$

**Q. 21** Find the area bounded by the lines  $y = 4x + 5$ ,  $y = 5 - x$  and  $4y = x + 5$ .

**Sol.**



Given equations of lines are

$$y = 4x + 5 \quad \dots(i)$$

$$y = 5 - x \quad \dots(ii)$$

and

$$4y = x + 5 \quad \dots(iii)$$

On solving Eqs. (i) and (ii), we get

$$4x + 5 = 5 - x$$

$$\Rightarrow x = 0$$

On solving Eqs. (i) and (iii), we get

$$4(4x + 5) = x + 5$$

$$\Rightarrow 16x + 20 = x + 5$$

$$\Rightarrow 15x = -15$$

$$\Rightarrow x = -1$$

On solving Eqs. (ii) and (iii), we get

$$4(5 - x) = x + 5$$

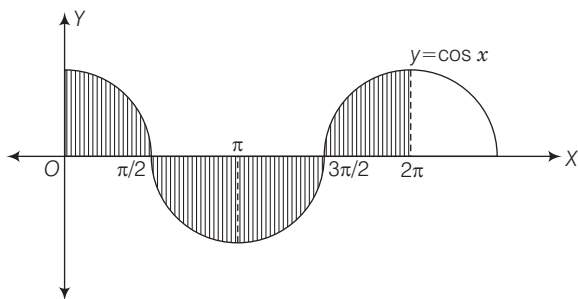
$$\Rightarrow 20 - 4x = x + 5$$

$$\begin{aligned}
 \Rightarrow \quad & x = 3 \\
 \therefore \text{ Required area} &= \int_{-1}^0 (4x + 5) dx + \int_0^3 (5 - x) dx - \frac{1}{4} \int_{-1}^3 (x + 5) dx \\
 &= \left[ \frac{4x^2}{2} + 5x \right]_{-1}^0 + \left[ 5x - \frac{x^2}{2} \right]_0^3 - \frac{1}{4} \left[ \frac{x^2}{2} + 5x \right]_{-1}^3 \\
 &= [0 - 2 + 5] + \left[ 15 - \frac{9}{2} - 0 \right] - \frac{1}{4} \left[ \frac{9}{2} + 15 - \frac{1}{2} + 5 \right] \\
 &= 3 + \frac{21}{2} - \frac{1}{4} \cdot 24 \\
 &= -3 + \frac{21}{2} = \frac{15}{2} \text{ sq units}
 \end{aligned}$$

**Q. 22** Find the area bounded by the curve  $y = 2\cos x$  and the  $X$ -axis from  $x = 0$  to  $x = 2\pi$ .

**Sol.** Required area of shaded region  $= \int_0^{2\pi} 2\cos x dx$

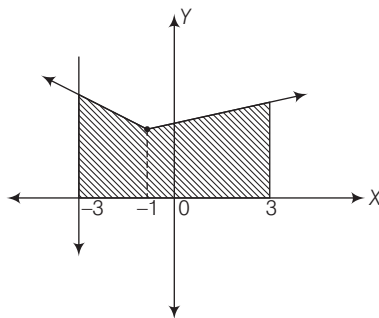
$$= \int_0^{\pi/2} 2\cos x dx + \left| \int_{\pi/2}^{3\pi/2} 2\cos x dx \right| + \int_{3\pi/2}^{2\pi} 2\cos x dx$$



$$\begin{aligned}
 &= 2[\sin x]_0^{\pi/2} + \left| 2(\sin x)_{\pi/2}^{3\pi/2} \right| + 2[\sin x]_{3\pi/2}^{2\pi} \\
 &= 2 + 4 + 2 = 8 \text{ sq units}
 \end{aligned}$$

**Q. 23** Draw a rough sketch of the given curve  $y = 1 + |x + 1|$ ,  $x = -3$ ,  $x = 3$ ,  $y = 0$  and find the area of the region bounded by them, using integration.

**Sol.** We have,  $y = 1 + |x + 1|$ ,  $x = -3$ ,  $x = 3$  and  $y = 0$



$$\begin{aligned}
 \therefore y &= \begin{cases} -x, & \text{if } x < -1 \\ x+2, & \text{if } x \geq -1 \end{cases} \\
 \therefore \text{Area of shaded region, } A &= \int_{-3}^{-1} -x \, dx + \int_{-1}^3 (x+2) \, dx \\
 &= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^3 \\
 &= -\left[\frac{1}{2} - \frac{9}{2}\right] + \left[\frac{9}{2} + 6 - \frac{1}{2} + 2\right] \\
 &= -[-4] + [8 + 4] \\
 &= 12 + 4 = 16 \text{ sq units}
 \end{aligned}$$

## Objective Type Questions

**Q. 24** The area of the region bounded by the Y-axis  $y = \cos x$  and  $y = \sin x$ , where  $0 \leq x \leq \frac{\pi}{2}$ , is

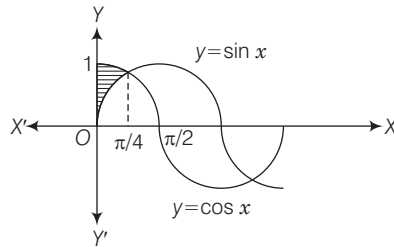
(a)  $\sqrt{2}$  sq units

(b)  $(\sqrt{2} + 1)$  sq units

(c)  $(\sqrt{2} - 1)$  sq units

(d)  $(2\sqrt{2} - 1)$  sq units

**Sol. (c)** We have, Y-axis i.e.,  $x = 0$ ,  $y = \cos x$  and  $y = \sin x$ , where  $0 \leq x \leq \frac{\pi}{2}$ .



$$\begin{aligned}
 \therefore \text{Required area} &= \int_0^{\pi/4} (\cos x - \sin x) \, dx \\
 &= [\sin x]_0^{\pi/4} + [\cos x]_0^{\pi/4} \\
 &= \left(\sin \frac{\pi}{4} - \sin 0\right) + \left(\cos \frac{\pi}{4} - \cos 0\right) \\
 &= \left(\frac{1}{\sqrt{2}} - 0\right) + \left(\frac{1}{\sqrt{2}} - 1\right) \\
 &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\
 &= -1 + \frac{2}{\sqrt{2}} = \frac{-\sqrt{2} + 2}{\sqrt{2}} \\
 &= \frac{-2 + 2\sqrt{2}}{2} = (\sqrt{2} - 1) \text{ sq units}
 \end{aligned}$$

**Q. 25** The area of the region bounded by the curve  $x^2 = 4y$  and the straight line  $x = 4y - 2$  is

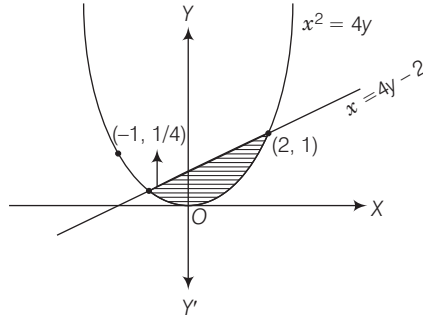
(a)  $\frac{3}{8}$  sq unit

(b)  $\frac{5}{8}$  sq unit

(c)  $\frac{7}{8}$  sq unit

(d)  $\frac{9}{8}$  sq units

**Sol. (d)** Given equation of curve is  $x^2 = 4y$  and the straight line  $x = 4y - 2$ .



For intersection point, put  $x = 4y - 2$  in equation of curve, we get

$$(4y - 2)^2 = 4y$$

$$\Rightarrow 16y^2 + 4 - 16y = 4y$$

$$\Rightarrow 16y^2 - 20y + 4 = 0$$

$$\Rightarrow 4y^2 - 5y + 1 = 0$$

$$\Rightarrow 4y^2 - 4y - y + 1 = 0$$

$$\Rightarrow 4y(y - 1) - 1(y - 1) = 0$$

$$\Rightarrow (4y - 1)(y - 1) = 0$$

$$\therefore y = 1, \frac{1}{4}$$

For  $y = 1$ ,  $x = \sqrt{4 \cdot 1} = 2$  [since, negative value does not satisfy the equation of line]

For  $y = \frac{1}{4}$ ,  $x = \sqrt{4 \cdot \frac{1}{4}} = -1$  [positive value does not satisfy the equation of line]

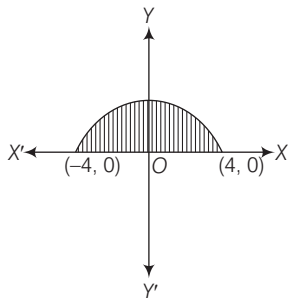
So, the intersection points are  $(2, 1)$  and  $\left(-1, \frac{1}{4}\right)$ .

$$\begin{aligned} \therefore \text{Area of shaded region} &= \int_{-1}^2 \left( \frac{x+2}{4} \right) dx - \int_{-1}^2 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_{-1}^2 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_{-1}^2 \\ &= -\frac{1}{4} \left[ \frac{4}{2} + 4 - \frac{1}{2} + 2 \right] - \frac{1}{4} \left[ \frac{8}{3} + \frac{1}{3} \right] \\ &= \frac{1}{4} \cdot \frac{15}{2} - \frac{1}{4} \cdot \frac{9}{3} = \frac{45-18}{24} \\ &= \frac{27}{24} = \frac{9}{8} \text{ sq units} \end{aligned}$$

**Q. 26** The area of the region bounded by the curve  $y = \sqrt{16 - x^2}$  and X-axis is

- (a)  $8\pi$  sq units (b)  $20\pi$  sq units  
(c)  $16\pi$  sq units (d)  $256\pi$  sq units

**Sol. (a)** Given equation of curve is  $y = \sqrt{16 - x^2}$  and the equation of line is X-axis i.e.,  $y = 0$ .



$$\therefore \sqrt{16 - x^2} = 0 \quad \dots(i)$$

$$\Rightarrow 16 - x^2 = 0$$

$$\Rightarrow x^2 = 16$$

$$\Rightarrow x = \pm 4$$

So, the intersection points are  $(4, 0)$  and  $(-4, 0)$ .

$$\therefore \text{Area of curve, } A = \int_{-4}^4 (16 - x^2)^{1/2} dx$$

$$= \int_{-4}^4 \sqrt{4^2 - x^2} dx$$

$$= \left[ \frac{x}{2} \sqrt{4^2 - x^2} + \frac{4^2}{2} \sin^{-1} \frac{x}{4} \right]_{-4}^4$$

$$= \left[ \frac{4}{2} \sqrt{4^2 - 4^2} + 8 \sin^{-1} \frac{4}{4} \right] - \left[ -\frac{4}{2} \sqrt{4^2 - (-4)^2} + 8 \sin^{-1} \left( -\frac{4}{4} \right) \right]$$

$$= \left[ 2 \cdot 0 + 8 \cdot \frac{\pi}{2} - 0 + 8 \cdot \frac{\pi}{2} \right] = 8\pi \text{ sq units}$$

**Q. 27** Area of the region in the first quadrant enclosed by the X-axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$  is

- (a)  $16\pi$  sq units (b)  $4\pi$  sq units (c)  $32\pi$  sq units (d)  $24\pi$  sq units

**Sol. (b)** We have, area enclosed by X-axis i.e.,  $y = 0$ ,  $y = x$  and the circle  $x^2 + y^2 = 32$  in first quadrant.

$$\text{Since, } x^2 + (x)^2 = 32 \quad [\because y = x]$$

$$\Rightarrow 2x^2 = 32$$

$$\Rightarrow x = \pm 4$$

So, the intersection point of circle  $x^2 + y^2 = 32$  and line  $y = x$  are  $(4, 4)$  or  $(-4, 4)$ .

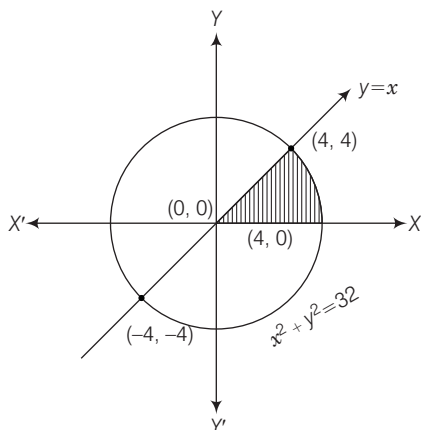
$$\text{and } x^2 + y^2 = (4\sqrt{2})^2$$

$$\text{Since, } y = 0$$

$$\therefore x^2 + (0)^2 = 32$$

$$\Rightarrow x = \pm 4\sqrt{2}$$

So, the circle intersects the  $X$ -axis at  $(\pm 4\sqrt{2}, 0)$ .

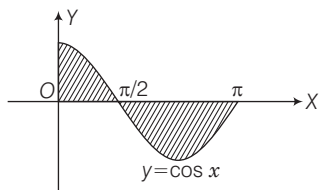


$$\begin{aligned}
 \text{Area of shaded region} &= \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2} dx \\
 &= \left[ \frac{x^2}{2} \right]_0^4 + \left[ \frac{x}{2} \sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2} \sin^{-1} \frac{x}{4\sqrt{2}} \right]_4^{4\sqrt{2}} \\
 &= \frac{16}{2} + \left[ \frac{4\sqrt{2}}{2} \cdot 0 + 16 \sin^{-1} \frac{(4\sqrt{2})}{(4\sqrt{2})} - \frac{4}{2} \sqrt{(4\sqrt{2})^2 - 16} - 16 \sin^{-1} \frac{4}{4\sqrt{2}} \right] \\
 &= 8 + \left[ 16 \cdot \frac{\pi}{2} - 2 \cdot \sqrt{16} - 16 \cdot \frac{\pi}{4} \right] \\
 &= 8 + [8\pi - 8 - 4\pi] = 4\pi \text{ sq units}
 \end{aligned}$$

**Q. 28** Area of the region bounded by the curve  $y = \cos x$  between  $x = 0$  and  $x = \pi$  is

- (a) 2 sq units (b) 4 sq units  
(c) 3 sq units (d) 1 sq unit

**Sol. (a)** Required area enclosed by the curve  $y = \cos x$ ,  $x = 0$  and  $x = \pi$  is

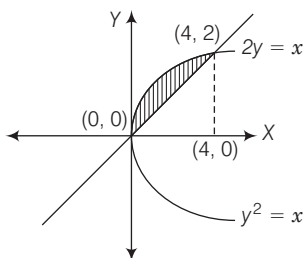


$$\begin{aligned}
 A &= \int_0^{\pi/2} \cos x dx + \left| \int_{\pi/2}^{\pi} \cos x dx \right| \\
 &= \left[ \sin \frac{\pi}{2} - \sin 0 \right] + \left| \sin \frac{\pi}{2} - \sin \pi \right| \\
 &= 1 + 1 = 2 \text{ sq units}
 \end{aligned}$$

**Q. 29** The area of the region bounded by parabola  $y^2 = x$  and the straight line  $2y = x$  is

- (a)  $\frac{4}{3}$  sq units      (b) 1 sq unit      (c)  $\frac{2}{3}$  sq unit      (d)  $\frac{1}{3}$  sq unit

**Sol. (a)** We have to find the area enclosed by parabola  $y^2 = x$  and the straight line  $2y = x$ .



$$\begin{aligned} \therefore \left(\frac{x}{2}\right)^2 &= x \\ \Rightarrow x^2 &= 4x \Rightarrow x(x-4) = 0 \\ \Rightarrow x &= 4 \Rightarrow y = 2 \text{ and } x = 0 \Rightarrow y = 0 \end{aligned}$$

So, the intersection points are (0, 0) and (4, 2).

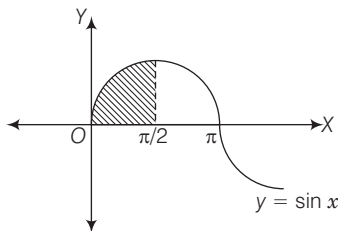
Area enclosed by shaded region,

$$\begin{aligned} A &= \int_0^4 \left[ \sqrt{x} - \frac{x}{2} \right] dx \\ &= \left[ \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{1}{2} \cdot \frac{x^2}{2} \right]_0^4 = \left[ 2 \cdot \frac{x^{3/2}}{3} - \frac{x^2}{4} \right]_0^4 \\ &= \frac{2}{3} 4^{3/2} - \frac{16}{4} - \frac{2}{3} \cdot 0 + \frac{1}{4} \cdot 0 \\ &= \frac{16}{3} - \frac{16}{4} = \frac{64 - 48}{12} = \frac{16}{12} = \frac{4}{3} \text{ sq units} \end{aligned}$$

**Q. 30** The area of the region bounded by the curve  $y = \sin x$  between the ordinates  $x = 0$ ,  $x = \frac{\pi}{2}$  and the X-axis is

- (a) 2 sq units      (b) 4 sq units      (c) 3 sq units      (d) 1 sq unit

**Sol. (d)** Area of the region bounded by the curve  $y = \sin x$  between the ordinates  $x = 0$ ,  $x = \frac{\pi}{2}$  and the X-axis is



$$\begin{aligned}
 A &= \int_0^{\pi/2} \sin x dx \\
 &= -[\cos x]_0^{\pi/2} = -\left[\cos \frac{\pi}{2} - \cos 0\right] \\
 &= -[0 - 1] = 1 \text{ sq unit}
 \end{aligned}$$

**Q. 31** The area of the region bounded by the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  is

- (a)  $20\pi$  sq units      (b)  $20\pi^2$  sq units      (c)  $16\pi^2$  sq units      (d)  $25\pi$  sq units

**Sol. (a)** We have,  $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$

Here,

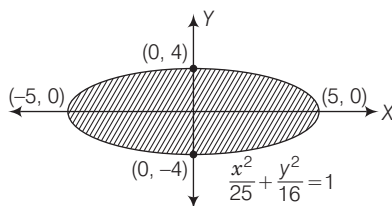
$$a = \pm 5 \text{ and } b = \pm 4$$

and

$$\frac{y^2}{4^2} = 1 - \frac{x^2}{5^2}$$

$\Rightarrow$

$$y^2 = 16 \left( 1 - \frac{x^2}{25} \right)$$



$\Rightarrow$

$$y = \sqrt{\frac{16}{25}(25 - x^2)}$$

$\Rightarrow$

$$y = \frac{4}{5} \sqrt{5^2 - x^2}$$

$$\therefore \text{Area enclosed by ellipse, } A = 2 \cdot \frac{4}{5} \int_{-5}^5 \sqrt{5^2 - x^2} dx$$

$$= 2 \cdot \frac{8}{5} \int_0^5 \sqrt{5^2 - x^2} dx$$

$$= 2 \cdot \frac{8}{5} \left[ \frac{x}{2} \sqrt{5^2 - x^2} + \frac{5^2}{2} \sin^{-1} \frac{x}{5} \right]_0^5$$

$$= 2 \cdot \frac{8}{5} \left[ \frac{5}{2} \sqrt{5^2 - 5^2} + \frac{5^2}{2} \sin^{-1} \frac{5}{5} - 0 - \frac{25}{2} \cdot 0 \right]$$

$$= 2 \cdot \frac{8}{5} \left[ \frac{25}{2} \cdot \frac{\pi}{2} \right]$$

$$= \frac{16}{5} \cdot \frac{25\pi}{4}$$

$$= 20\pi \text{ sq units}$$

**Q. 32** The area of the region bounded by the circle  $x^2 + y^2 = 1$  is

- (a)  $2\pi$  sq units (b)  $\pi$  sq units  
(c)  $3\pi$  sq units (d)  $4\pi$  sq units

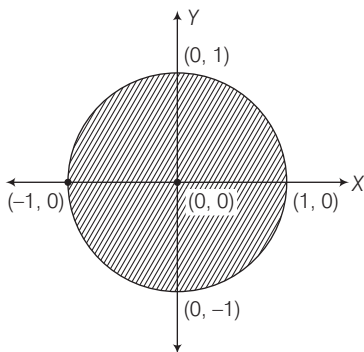
**Sol. (b)** We have,

$$x^2 + y^2 = 1^2$$

$[\because r = \pm 1]$

$\Rightarrow$

$$y^2 = 1 - x^2 \Rightarrow y = \sqrt{1 - x^2}$$



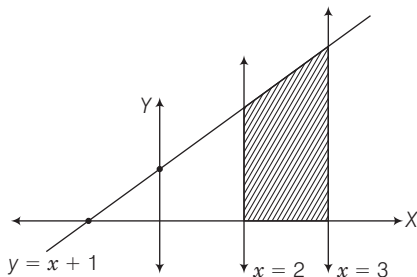
$$\begin{aligned} \therefore \text{Area enclosed by circle} &= 2 \int_{-1}^1 \sqrt{1^2 - x^2} dx = 2 \cdot 2 \int_0^1 \sqrt{1^2 - x^2} dx \\ &= 2 \cdot 2 \left[ \frac{x}{2} \sqrt{1^2 - x^2} + \frac{1^2}{2} \sin^{-1} \frac{x}{1} \right]_0^1 \\ &= 4 \left[ \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{\pi}{2} - 0 - \frac{1}{2} \cdot 0 \right] \\ &= 4 \cdot \frac{\pi}{4} = \pi \text{ sq units} \end{aligned}$$

**Q. 33** The area of the region bounded by the curve  $y = x + 1$  and the lines  $x = 2$ ,  $x = 3$ , is

- (a)  $\frac{7}{2}$  sq units (b)  $\frac{9}{2}$  sq units (c)  $\frac{11}{2}$  sq units (d)  $\frac{13}{2}$  sq units

**Sol. (a)** Required area,  $A = \int_2^3 (x + 1) dx = \left[ \frac{x^2}{2} + x \right]_2^3$

$$= \left[ \frac{9}{2} + 3 - \frac{4}{2} - 2 \right] = \left[ \frac{5}{2} + 1 \right] = \frac{7}{2} \text{ sq units}$$



**Q. 34** The area of the region bounded by the curve  $x = 2y + 3$  and the lines  $y = 1$ ,  $y = -1$  is

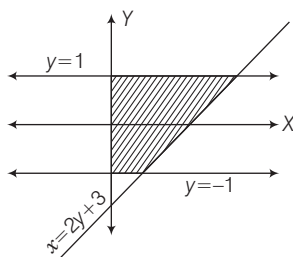
(a) 4 sq units

(b)  $\frac{3}{2}$  sq units

(c) 6 sq units

(d) 8 sq units

**Sol. (c)** Required area,  $A = \int_{-1}^1 (2y + 3) dy$



$$\begin{aligned}
 &= \left[ \frac{2y^2}{2} + 3y \right]_{-1}^1 \\
 &= [y^2 + 3y]_{-1}^1 \\
 &= [1 + 3 - 1 + 3] = 6 \text{ sq units}
 \end{aligned}$$