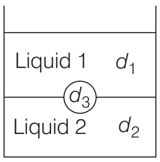
DAY SEVENTEEN

Unit Test 3

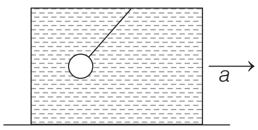
(General Properties of Matter)

1 A jar is filled with two non-mixing liquids 1 and 2 having densities d_1 and d_2 respectively. A solid ball, made of a material of density d_3, is dropped in the jar. It comes to equilibrium in the position as shown in the figure. Which of the following is true for d_1, d_2 and d_3?



(a) $d_1 > d_3 > d_2$ (b) $d_1 < d_2 < d_3$
(c) $d_1 < d_3 < d_2$ (d) $d_3 < d_1 < d_2$

2 A spherical body of volume V and density σ is suspended from a string, the other end of the string is connected to the roof of a sealed container filled with an ideal fluid of density ρ.



If the container accelerates towards right with a constant acceleration a, then the force exerted by the liquid on the body when it is in equilibrium w.r.t. fluid, is

(a) $V\rho\sqrt{a^2 + g^2} + V\sigma a$ (b) $V\sigma a$
(c) $\sqrt{[V\rho(g + a)]^2 + [V\sigma a]^2}$ (d) $V\rho\sqrt{g^2 + a^2}$

3 A hot metallic sphere of radius r radiates heat. Its rate of cooling is

(a) independent of r
(b) proportional to r
(c) proportional to r^2
(d) proportional to $1/r$

4 A certain ideal gas undergoes a polytropic process pV^n = constant such that the molar specific heat during the process is negative. If the ratio of the specific heat of the gas be γ, then the range of values of n will be

(a) $0 < n < \gamma$
(b) $1 < n < \gamma$
(c) $n = \gamma$
(d) $n > \gamma$

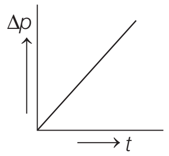
5 Pressure p, volume V and temperature T for a certain material are related by

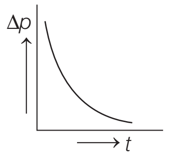
$$p = \frac{AT - BT^2}{V}$$

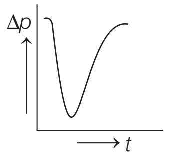
where, A and B are constants. Find an expression for the work done by the material if the temperature changes from T_1 to T_2 reduce while the pressure remains constant.

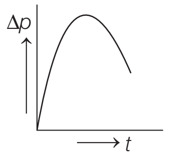
(a) $W = A(T_2 - T_1) - B(T_2^3 - T_1^3)$
(b) $W = A(T_2^2 - T_1^2) - B(T_2 - T_1)$
(c) $W = A(T_2 - T_1) - B\left(T_2 - \frac{1}{2}T_1\right)$
(d) $W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$

6 A soap bubble is very slowly blown on the end of a glass tube by a mechanical pump which supplies a fixed volume of air every minute whatever be the pressure against which it is pumping. The excess pressure Δp inside the bubble varies with time is shown by which of the graph?

(a) 

(b) 

(c) 

(d) 

7 A small electric immersion heater is used to heat 100 g of water for a cup of instant coffee. The heater is labelled "200 W," which means that it converts electrical energy to thermal energy at this rate. Calculate the time required to bring all this water from 23°C to 100°C, ignoring any heat losses. [c = 4190 J kg^{-1} K^{-1}]

(a) 100 s (b) 200 s (c) 190 s (d) 161 s

8 The average depth of Indian ocean is about 3000 m. Calculate the fractional compression, $\Delta V/V$, of water at the bottom of the ocean. Given that the bulk modulus of water is 2.2×10^9 Nm^{-2}.

(Take, $g = 10\,ms^{-2}$)

(a) 1.36×10^{-2}
(b) 3×10^{-3}
(c) 1.5×10^{-2}
(d) 1.36×10^{-6}

9 The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K, $2K$ and thickness x, $4x$, respectively are T_2 and T_1 $(T_2 > T_1)$. The rate of heat transfer through the slab in a steady state is

$\left[\frac{A(T_2 - T_1)K}{x}\right] f$ with f which is equal to

(a) 1
(b) 1/2
(c) 2/3
(d) 1/3

10 An aeroplane has a mass of 1.60×10^4 kg and each wing has an area of $40\,m^2$. During level flight, the pressure on the wings's lower surface is 7×10^4 Pa. The pressure on the upper surface of the wing is

(Take, $p_0 = 10^5$ Pa and assume the pressure difference is only on wings and not on body)

(a) 10^5 Pa
(b) 6.8×10^4 Pa
(c) 7×10^4 Pa
(d) 6.6×10^4 Pa

11 The coefficients of thermal conductivity of copper, mercury and glass are K_c, K_m and K_g, respectively, such that $K_c > k_m > K_g$. If the same quantity of heat is to flow per second per unit area of each and corresponding temperature gradients are, X_c, X_m and X_g, respectively, then

(a) $X_c = X_m = X_g$
(b) $X_c > X_m > X_g$
(c) $X_c < X_m < X_g$
(d) $X_m < X_c < X_g$

12 The temperature of the source of a Carnot's heat engine is 1000°C. Its efficiency could be 100% only if the temperature of the sink is

(a) 1000°C
(b) 0°C
(c) equal to triple of water
(d) – 273.16°C

13 A steel rod is 3.00 cm in diameter at 25°C. A brass ring has an interior diameter of 2.992 cm at 25°C. At what common temperature will the ring just slide onto the rod? (take, $\alpha_s = 11 \times 10^{-6}\,°C^{-1}, \alpha_b = 19 \times 10^{-6}\,°C^{-1}$)

(a) 460°C (b) 260°C (c) 500°C (d) 360°C

14 A diver is hunting for a fish with a water gun. He accidentally fires the gun, so that bullet punctures the side of the ship. The hole is located at a depth of 10 m below the water surface. The speed with which water enter in the ship is

(a) 18 ms^{-1}
(b) 14 ms^{-1}
(c) 25 ms^{-1}
(d) Cannot be determined

15 A material has a Poisson's ratio 0.3. If a uniform rod of it suffers longitudinal strain $4 \cdot 5 \times 10^{-3}$, then calculate the percentage change in its volume.

(a) 0.15% (b) 0.25% (c) 0.18% (d) 0.5%

16 Compute the number of moles and in 1.00 cm^3 of an ideal gas at a pressure of 100 Pa and at a temperature of 220 K.

(a) 3.35×10^{-8}mol
(b) 4.57×10^{-7}mol
(c) 5.47×10^{-8}mol
(d) 2.75×10^{-8}mol

17 A slab consists of two parallel layers of copper and brass of the same thickness same area of cross-section and having thermal conductivities in the ratio 1 : 4. If the free face of brass is at 100°C and that of copper is at 0°C, the temperature of the interface is

(a) 80°C (b) 20°C (c) 60°C (d) 40°C

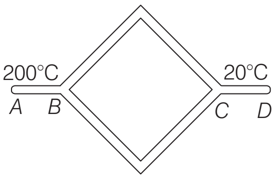
18 On applying a stress of x Nm^{-2}, the length of wire of some material becomes double. Value of the Young's modulus for the material of the wire in Nm^{-2}, is [Assume Hooke's law to be valid] "Go for approx results"

(a) x
(b) $2x$
(c) $x/2$
(d) Insufficient information

19 743 J of heat energy is added to raise the temperature of 5 mole of an ideal gas by 2 K at constant pressure. How much heat energy is required to raise the temperature of the same mass of the gas by 2K at constant volume? (Take, R = 8.3 J/K-mol)

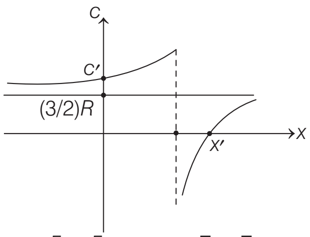
(a) 826 J (b) 743 J (c) 660 J (d) 620 J

20 Six identical conducting rods are joined as shown in figure given below. Points A and D are maintained at temperatures 200 °C and 20°C, respectively. The temperature of junction B will be



(a) 120°C (b) 100°C (c) 140 °C (d) 80°C

21 One mole of an ideal gas is taken along the process in which pV^x = constant. The graph shown represent the variation of molar heat capacity of such a gas with respect to x. The values of c' and x', respectively, are given by



(a) $\frac{5}{2}R, \frac{5}{2}$ (b) $\frac{5}{2}R, \frac{5}{3}$ (c) $\frac{7}{2}R, \frac{7}{2}$ (d) $\frac{5}{2}R, \frac{7}{5}$

22 A wire of length L and radius r is fixed at one end. When a stretching force F is applied at the free end, the elongation in the wire is l. When another wire of the same material but of length $2L$ and radius $2r$, also fixed at one end is stretched by a force $2F$ applied at the free end, then elongation in the second wire will be

(a) $l/2$ (b) l (c) $2l$ (d) $l/4$

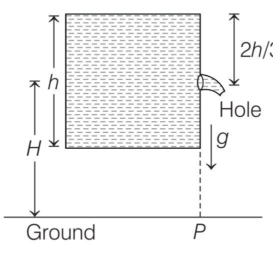
23 A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by 75.0°C. What are the temperatures of the two reservoirs?

(a) 58°C, 10°C (b) 78°C, –5°C
(c) 68°C, –7°C (d) 50°C, 0°C

24 A material has a Poisson's ratio 0.50. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(a) 0.6 (b) 0.4 (c) 0.2 (d) zero

25 An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel, as shown in the figure.



The water which comes out from the hole at the instant when the hole is at height H above the ground, strikes the ground at a distance x from P. Which of the following is correct for the situation described?

(a) The value of x is $2\sqrt{\frac{2hH}{3}}$ (b) The value of x is $\sqrt{\frac{4hH}{3}}$
(c) The value of x cannot be computed from the information provided
(d) The question is irrelevant as no water comes out from the hole

26 Water flows through a horizontal pipe of varying cross-section at the rate of 20 litres per minute. Then the velocity of water at a point where diameter is 4 cm, is

(a) $0.25\ \text{ms}^{-1}$ (b) $0.26\ \text{ms}^{-1}$ (c) $0.22\ \text{ms}^{-1}$ (d) $0.4\ \text{ms}^{-1}$

27 Oxygen gas having a volume of 1000 cm^3 at 40.0°C and 1.01×10^5 Pa expands until its volume is 1500 cm^3 and its pressure is 1.06×10^5 Pa. Find the final temperature of the sample.

(a) 197°C (b) 220 K (c) 300°C (d) 300 K

28 A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal sizes. The energy spent in this process is [Surface tension of mercury is equal to $32 \times 10^{-2}\ \text{Nm}^{-1}$]

(a) 3.98×10^{-4} J (b) 8.46×10^{-4} J
(c) 3.98×10^{-2} J (d) 8.46×10^{-2} J

29 A chef, on finding his stove out of order, decides to boil the water for his wife's coffee by shaking it in a thermos flask. Suppose that he uses tap water at 15°C and that the water falls 30 cm in each shake, the chef making 30 shakes each minute. Neglecting any loss of thermal energy by the flask, how long must he shake the flask until the water reaches 100°C ?

(a) 2.25×10^3 min (b) 3.97×10^3 min
(c) 4.03×10^3 min (d) 5.25×10^3 min

30 The maximum amount of heat which may be lost per second by radiation by a sphere 14 cm in diameter at a temperature of 227°C, when placed in an enclosure at 27°C. Given, Stefan's constant $= 5.7 \times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$

(a) 45.48 cal/s (b) 40 cal/s
(c) 42.5 cal/s (d) 40.5 cal/s

31 Four moles of an ideal gas undergo a reversible isothermal expansion from volume V_1 to volume $V_2 = 2V_1$ at temperature $T = 400$ K. Find the entropy change of the gas.

(a) $8.22 \times 10^3\ \text{JK}^{-1}$ (b) $8.22 \times 10^2\ \text{JK}^{-1}$
(c) $23.1\ \text{JK}^{-1}$ (d) $10.00 \times 10^3\ \text{JK}^{-1}$

32 A swimmer of mass m rests on top of a styrofoam slab, which has a thickness h and density ρ_s. The area of the slab if it floats in water with its upper surface just awash is (take, density of water to be ρ_w)

(a) $\frac{m}{h(\rho_s + \rho_w)}$ (b) $\frac{m}{h\rho_w}$ (c) $\frac{m}{h(\rho_s - \rho_w)}$ (d) $\frac{m}{h(\rho_w - \rho_s)}$

33 An ice-berg of density $900\,kg\text{-}m^{-3}$ is floating in water of density $1000\,kg\text{-}m^{-3}$. The percentage of volume of ice-berg outside the water is

(a) 20% (b) 35% (c) 10% (d) 11%

34 A uniform capillary tube of length l and inner radius r with its upper end sealed is submerged vertically into water. The outside pressure is p_0 and surface tension of water is γ. When a length x of the capillary is submerged into water, it is found that the water level inside and outside the capillary coincide. The value of x is

(a) $\dfrac{l}{\left(1+\dfrac{p_0 r}{4\gamma}\right)}$ (b) $l\left(1-\dfrac{p_0 r}{4\gamma}\right)$ (c) $l\left(1-\dfrac{p_0 r}{2\gamma}\right)$ (d) $\dfrac{l}{\left(1+\dfrac{p_0 r}{2\gamma}\right)}$

35 2 moles of an ideal monoatomic gas is carried from a state (p_0, V_0) to state $(2p_0, 2V_0)$ along a straight line path in a p-V diagram. The amount of heat absorbed by the gas in the process is given by

(a) $3p_0V_0$ (b) $\dfrac{9}{2}p_0V_0$

(c) $6p_0V_0$ (d) $\dfrac{3}{2}p_0V_0$

36 The stress along the length of a rod (with rectangular cross-section) is 1% of the Young's modulus of its materials. What is the approximate percentage of change of its volume? (Poisson's ratio of the material of the rod is 0.3)

(a) 3% (b) 1%

(c) 0.7% (d) 0.4%

Direction (Q. Nos. 37-40) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

37 **Statement I** A ship floats higher in water on a high pressure day than on a low pressure day.

Statement II Floating of ship in the water is possible because of the buoyant force which is present due to the pressure difference.

38 **Statement I** More is the cohesive force, more is the surface tension.

Statement II More cohesive force leads to more shrinking of the liquid surface.

39 **Statement I** Water expands both when heated or cooled from 4°C.

Statement II Density of water is minimum at 4°C.

40 **Statement I** If the temperature of a star is doubled, then the rate of loss of heat from it becomes 16 times.

Statement II Specific heat varies with temperature.

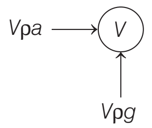
ANSWERS

1. (c)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (b)
11. (c)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (c)
21. (b)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (a)
31. (c)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (c)	**36.** (d)	**37.** (d)	**38.** (b)	**39.** (c)	**40.** (b)

Hints and Explanations

1 d_3 floats in d_2 and sinks in $d_1 \Rightarrow d_1 < d_3 < d_2$.

2 The forces exerted by liquid on body is shown in figure, when body is in equilibrium w.r.t. fluid.



Forces exerted by liquid

So, required force, $F = \rho V \sqrt{a^2 + g^2}$

3 Rate of cooling

$$R_c = \frac{d\theta}{dt} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$$

$$\Rightarrow \frac{d\theta}{dt} \propto \frac{A}{V} \propto \frac{r^2}{r^3} \Rightarrow \frac{d\theta}{dt} \propto \frac{1}{r}$$

4 Since, pV^n = constant and also $pV = RT$, taking 1 mol of the gas for simplicity $dU = C_V dt$

where, $C_V \to$ molar specific heat at constant volume

Now, the molar specific heat in a polytropic process

pV^n = constant is given by

$$C_V = \left(\frac{R}{\gamma - 1}\right) - \left(\frac{R}{n-1}\right) = \frac{(n-\gamma)R}{(n-1)(\gamma-1)} \quad \ldots\text{(i)}$$

From this equation, we see that C_V will be negative when $n < \gamma$ and $n > 1$, simultaneously, i.e. $1 < n < \gamma$. Since, γ for all ideal gases is greater than 1, if $n > \gamma$ or $n < 1$, then C_V will be positive.

5 Work $W = p(V_2 - V_1)$ at constant p

Initial volume, $V_1 = (AT_1 - BT_1^2)/p$

Final volume is $V_2 = (AT_2 - BT_2^2)/p$

$\Rightarrow W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$

6 $\Delta p = \dfrac{2T}{R}$

As, the number of moles of air increases. Radius increases and Δp decreases.

7 Heat required to raise the temperature of water must be equal to the power output of the heater P, multiplied by time t.

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P}$$

$$= \frac{(4190)\ (0.100)\ (100° - 23°)}{200} = 161 \text{ s}$$

8 Here, $h = 3000$ m,

$B = 2.2 \times 10^9 \text{ Nm}^{-2}$

$p = h\rho g = 3000 \times 10^3 \times 10$

$= 3 \times 10^7 \text{ Nm}^{-2}$

$\therefore$ Compressional strain $= \dfrac{\Delta V}{V} = \dfrac{p}{B}$

$$= \frac{3 \times 10^7}{2.2 \times 10^9} = 1.36 \times 10^{-2}$$

9 Equation of thermal conductivity of the given combination $K_{\text{eq}} = \dfrac{l_1 + l_2}{\dfrac{l_1}{K_1} + \dfrac{l_2}{K_2}}$

$$= \frac{x + 4x}{\dfrac{x}{K} + \dfrac{4x}{2K}} = \frac{5}{3}K$$

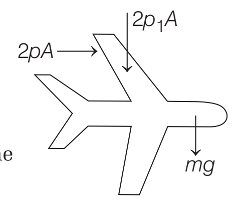
Hence, rate of flow of heat through the given combination is

$$\frac{\theta}{t} = \frac{K_{\text{eq}} A(T_2 - T_1)}{(x + 4x)} = \frac{5/3KA(T_2 - T_1)}{5x}$$

$$= \frac{1/3KA(T_2 - T_1)}{x}$$

On comparing it with given equations, we get $f = \dfrac{1}{3}$.

10 Let p_1 be the pressure on the upper wing surface, then for the vertical equilibrium of the plane



$2p_1 A + mg = 2pA$

$$\Rightarrow \quad p_1 = p - \frac{mg}{2A}$$

$$= 7 \times 10^4 - \frac{1.6 \times 10^4 \times 10}{2 \times 40}$$

$= 6.8 \times 10^4$ Pa

11 $\dfrac{dQ/dt}{A} = K\left(\dfrac{\Delta\theta}{\Delta x}\right)$

Rate of flow of heat per unit area = Thermal conductivity × Temperature gradient

Temperature gradient (X) $\propto \dfrac{1}{\text{Thermal conductivity (K)}}$

$\left(\text{As}, \dfrac{dQ/dt}{A} = \text{constant}\right)$

As, $K_c > K_m > K_g$, therefore $X_c < X_m < X_g$

12 Efficiency of Carnot's heat engine = 100%

$$\frac{T_1 - T_2}{T_1} \times 100 = 100$$

$\Rightarrow \quad T_1 - T_2 = T_1$

$\Rightarrow \quad T_2 = 0\text{ K} = -273.16°\text{ C}$

13 If $D_s = D_b$,

then $D_{s0} + \alpha_s D_{s0}\Delta T = D_{b0} + \alpha_b D_{b0}\Delta T$

So, $\quad \Delta T = \dfrac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}}$

$$= \frac{3.000 - 2.992}{(19 \times 10^{-6})(2.992) - (11 \times 10^{-6})(3.00)}$$

$= 335°$C

The temperature is

$T = 25°\text{C} + 335°\text{C} = 360°\text{C}$

14 Applying the Bernoulli's theorem for any two convenient points, let us say we are applying just at the water surface and just inside the hole.

$$p_0 + 0 + 0 = p_0 + \frac{\rho v^2}{2} + \rho g(-h)$$

where, v is required speed and water surface is taken as the reference level.

$\Rightarrow \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10}$

$= 14 \text{ ms}^{-1}$

15 Here, $\sigma = 0.3$, $\dfrac{\Delta l}{l} = 4.5 \times 10^{-3}$

$$\sigma = -\frac{\Delta R / R}{\Delta l / l} \Rightarrow \frac{\Delta R}{R} = -\sigma\frac{\Delta l}{l}$$

$\Rightarrow \dfrac{\Delta R}{R} = -0.3 \times (4.5 \times 10^{-3}$

$= -1.35 \times 10^{-3}$

Volume, $V = \pi R^2 l$

$$\frac{\Delta V}{V} = \frac{2\Delta R}{R} + \frac{\Delta l}{l}$$

Percentage change in volume $= \dfrac{\Delta V}{V} \times 100$

$= \left(\dfrac{2\Delta R}{R} + \dfrac{\Delta l}{l}\right) \times 100$

$= [2 \times (-1.35) + 4.5] \times 10^{-3} \times 100$

$= 1.8 \times 10^{-3} \times 100 = 0.18\%$

16 From the ideal gas law, $pV = nRT$

$$n = \frac{pV}{RT} = \frac{(100)(1.0 \times 10^{-6})}{(8.31)(220)}$$

$= 5.47 \times 10^{-8}$ mol

17 Temperature of the interface,

$$\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$$

$\left(\because \dfrac{K_1}{K_2} = \dfrac{1}{4} \Rightarrow \text{If } K_1 = K, \text{ then } K_2 = 4K\right)$

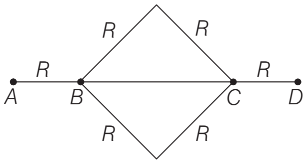
$$\Rightarrow \quad \theta = \frac{K \times 0 + 4K \times 100}{5K} = 80°\text{C}$$

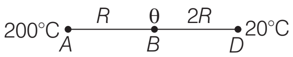
18 $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{x}{\frac{2l - l}{l}} = \dfrac{x}{1} = x$

In actual, the above expression is not exact for this much elongation.

19 For constant pressure process,
$Q_1 = nC_p\Delta T = 743$ J
For constant volume process,
$$Q_2 = nC_V dT = n(C_p - R)dT$$
$$= nC_p dT - nRdT$$
$$Q_2 = 743 - 5 \times 8.3 \times 2 = 660 \text{ J}$$

20 Let the thermal resistance of each rod be R.
The two resistances connected along two paths from B to C are equivalent to $2R$ each and their parallel combination is R.
Effective thermal resistance between B and $D = 2R$





Temperature of interface,
$$\theta = \frac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$$
$$\theta = \frac{R \times 20 + 2R \times 200}{R + 2R} = \frac{420}{3} = 140°\text{C}$$

21 At $x = \infty, C = \frac{3}{2}R$, from pV^x = constant

$\Rightarrow p^{1/x}V$ = another constant, so at $x = \infty, V$ = constant

hence $C = C_v = \frac{3}{2}R$ and then
$$C_p = C_V + R = \frac{5}{2}R$$
at $x = 0$, P = constant and $C = C'$

hence, $C' = C_p = \frac{5}{2}R$

at $x = x', C = 0$, so the process is adiabatic,hence
$$x' = \frac{C_p}{C_V} = \frac{5}{3}$$

22 $Y = \dfrac{F/A}{\Delta l/l}$

For the first wire, $Y = \dfrac{F \times L}{\pi r^2 \times l}$

For the second wire, $Y = \dfrac{2F \times 2L}{\pi(2r)^2 \times l'}$

From the above two equations, $l' = l$

23 For an ideal engine, the efficiency is related to the reservoir temperatures by $\varepsilon = (T_H - T_C)/T_H$. Thus,
$$T_H = (T_H - T_C)/\varepsilon$$
$$= (75\text{ K})/(0.22) = 341\text{ K}$$
$$= 68°\text{C}.$$
The temperature of the cold reservoir is
$$T_C = T_H - 75 = 341\text{K} - 75\text{K}$$
$$= 266\text{K} = -7°\text{C}.$$

24 $\dfrac{dV}{V} = (1 + 2\sigma)\dfrac{dl}{l}$
$$= 2 \times 2 \times 10^{-3} = 4 \times 10^{-3}$$
$$\left[\because \sigma = 0.5 = \frac{1}{2}\right]$$
$\therefore$ Percentage change in volume
$$= 4 \times 10^{-1} = 0.4\%$$

25 As vessel is falling freely under gravity, the pressure at all points within the liquid remains the same as the atmospheric pressure. If we apply Bernoulli's theorem just inside and outside the hole, then
$$p_{\text{inside}} + \frac{\rho v^2_{\text{inside}}}{2} + \rho g H$$
$$= p_{\text{outside}} + \frac{\rho v^2_{\text{outside}}}{2} + \rho g H$$
$v_{\text{inside}} = 0, p_{\text{inside}} = p_{\text{outside}} = p$ [atmospheric pressure]

Therefore, $v_{\text{outside}} = 0$

i.e. no water comes out from the hole.

26 $V = 20 \text{ litres/min} = \dfrac{20 \times 1000}{60 \times (100)^3}\ \text{m}^3\text{s}^{-1}$
$$= \frac{1}{3} \times 10^{-3}\ \text{m}^3\text{s}^{-1}$$
Radius, $r = \frac{4}{2} = 2$ cm $= 0.02$ m

Area of cross-section,
$$a = \pi r^2 = \frac{22}{7} \times (0.02)^2\ \text{m}^2$$
Let v be the velocity of the flow of water at the given point, then
$$V = av$$
$$\Rightarrow \frac{1}{3} \times 10^{-3} = \frac{22}{7} \times (0.02)^2 \times v$$
$$\Rightarrow \quad v = \frac{7 \times 10^{-3}}{3 \times 22 \times (0.02)^2}$$
$$= 0.2639 \simeq 0.26\ \text{ms}^{-1}$$

27 $n = \dfrac{PV}{RT} = \dfrac{1.06 \times 10^5 \times 1000 \times 10^{-6}}{8.31 \times 313}$
$$= 4.07 \times 10^{-2}$$
Using $pV = nRT$
$$T = \frac{pV}{nR} = \frac{(1.06 \times 10^5)(1500 \times 10^{-6})}{(4.07 \times 10^{-2})(8.31)}$$
$$= 470\text{ K}$$
$$= 197°\text{C}$$

28 Let r be the radius of one droplet.
Now, $\frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3, r = \frac{R}{100}$
$$= \frac{1}{100}\text{cm} = 10^{-4}\text{ m}$$
$$A_i = 4\pi R^2$$
$$A_f = 10^6 \times 4\pi r^2$$
Change in area,
$\Delta A = A_f - A_i = 4\pi \times 99 \times 10^{-4}$ m^2

Increase in surface energy
$= S\Delta A = 32 \times 10^{-2} \times 4\pi \times 99 \times 10^{-4}$ J
$= 3.98 \times 10^{-2}$ J

The increase in surface energy is at the expense of internal energy, so energy spent $= 3.98 \times 10^{-2}$ J

29 Time taken,
$$t = \frac{Q}{Rmgh} = \frac{cm(T_f - T_i)}{Rmgh} = \frac{c(T_f - T_i)}{Rgh}$$
$$= \frac{(4190)(100 - 15)}{(30)(9.8)(0.30)}$$
$$= 4.03 \times 10^3 \text{ min}$$

30 Temperature of sphere
$$T = 227°\text{C} = 500\text{ K}$$
Temperature of surroundings
$T_0 = 27°\text{C} = 300$ K
Radius, $r = 7$ cm $= 0.07$ m
Area of sphere,
$$A = 4\pi r^2 = 4 \times \frac{22}{7} \times (0.07)^2$$
$$= 6.16 \times 10^{-2}\ \text{m}^2$$
The energy lost from A,
$AE = A\sigma(T^4 - T_0^4)$
$6.16 \times 10^{-2} \times 5.7 \times 10^{-8} \times [(500)^4 - (300)^4]$
$= 6.16 \times 5.7 \times 10^{-10} \times 100^4\ (5^4 - 3^4)$
$= 6.16 \times 5.7 \times 10^{-2} \times 544\ \text{Js}^{-1}$

The heat lost per sec.
$$H = \frac{AE}{J} = \frac{6.16 \times 5.7 \times 10^{-2} \times 544}{4.2}$$
$$= 45.48 \text{ cal / s}$$

31 We have $p = nRT/V$. The work done by the gas during the isothermal expansion is
$$W = \int_{V_1}^{V_2} p\,dV = nRT\int_{V_1}^{V_2}\frac{dV}{V} = nRT \ln\frac{V_2}{V_1}$$
Substituting $V_2 = 2V_1$ to obtain $W = nRT$
$= (4.00)(8.314)(400)\ln 2$
$= 9.22 \times 10^3$ J

Since, the expansion is isothermal,
$\Delta E_{\text{int}} = 0$ and $Q = W$

Thus, $\Delta S = \dfrac{W}{T}$
$$= \frac{9.22 \times 10^3\text{ J}}{400\text{ K}} = 23.1\text{ J K}^{-1}$$

32 From equilibrium,

$mg + Ah\rho_s \times g = Ah\rho_w \times g$

where, A is the required cross-sectional area

$$\Rightarrow \quad A = \frac{m}{h(\rho_w - \rho_s)}$$

33 Let volume of ice-berg is V and its density is ρ. If V_{in} is volume inside the water, then

$$V_{\text{in}}\sigma g = V\rho g$$

where, σ = density of water

$$\Rightarrow \quad V_{\text{in}} = \left(\frac{\rho}{\sigma}\right)V$$

$$\Rightarrow \quad V_{\text{out}} = V - V_{\text{in}} = \left(\frac{\sigma - \rho}{\sigma}\right)V$$

$$= \left(\frac{1000 - 900}{1000}\right)V$$

$$= \frac{V}{10}$$

$$\Rightarrow \quad \frac{V_{\text{out}}}{V} = 0.1 = 10\%.$$

34 The pressure inside tube changes when it is submerged in water. Thus

$$p_1V_1 = p_2V_2$$

$$p_0(lA) = p'(l - x)A$$

$$\therefore \quad p' = \frac{p_0 l}{l - x}$$

As level of water is same inside and outside of capillary tube

$$\therefore \quad p' - p_0 = \frac{2\gamma}{r}$$

$$\text{or} \quad \frac{p_0 l}{l - x} - p_0 = \frac{2\gamma}{r}$$

$$\Rightarrow \quad x = \frac{l}{1 + \frac{p_0 r}{2\gamma}}$$

35 The internal energy, $\Delta U = nC_V\Delta T$

C_V = specific heat of gas at constant volume

$$\Rightarrow \quad \Delta U = n\cdot\frac{3R}{2}\left(\frac{4p_0V_0}{nR} - \frac{p_0V_0}{nR}\right)$$

$$= n\cdot\frac{3R}{2}\cdot\frac{3p_0V_0}{nR} = \frac{9}{2}p_0V_0$$

Work done by the gas,

$$W = (2p_0 + p_0)\frac{V_0}{2} = \frac{3p_0V_0}{2}$$

From first law of thermodynamics,

$$\Delta Q = dW + dU$$

$$= \frac{3p_0V_0}{2} + \frac{9}{2}p_0V_0$$

$$= \frac{12p_0V_0}{2} = 6p_0V_0$$

36 Stress $= \dfrac{F}{\Delta A} = 1\%$ of $Y = \dfrac{Y}{100}$

$$\text{Also,} \quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/\Delta A}{\Delta l/l} = \frac{\frac{Y}{100}}{\Delta l/l}$$

$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{1}{100}$$

Poisson's ratio, $\sigma = \dfrac{-\Delta r/r}{\Delta l/l}$

$$\Rightarrow \frac{\Delta r}{r} = -\sigma\frac{\Delta l}{l} = -\frac{0.3}{100}$$

$$\therefore \frac{\Delta V}{V}\times 100 = \left(\frac{2\Delta r}{r} + \frac{\Delta l}{l}\right)\times 100$$

$$= \left(2\times\frac{-0.3}{100} + \frac{1}{100}\right)\times 100 = 0.4\%$$

37 A body of weight $w = mg = V\rho g$ float in a liquid as a upthrust $F = V\sigma g$ acts vertically upwards through the centre of gravity of displaced liquid also called the centre of buoyancy. It is independence of atmospheric pressure.

38 Surface tension, $S = \dfrac{\text{Force}}{\text{Length}} = \dfrac{F}{l}$

$\therefore S \propto F$

Thus, more the force, the surface tension is more. Also, this force tends to have the least possible surface area.

39 At 4°C, the volume of water is minimum. When it is cooled below 4°C or heated above 4°C, then it expands or its volume increases. As volume at 4°C is minimum, thus its density $\left(= \dfrac{\text{mass}}{\text{volume}}\right)$ will be maximum.

40 From Stefan's law, $E = \sigma T^4$ or $E \propto T^4$

$$\therefore \quad \frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{T}{2T}\right)^4$$

$$\text{or} \quad \frac{E_1}{E_2} = \frac{1}{16}$$

$$\text{or} \quad E_2 = 16E_1 = 16 \text{ times}$$

Specific heat too varies with temperature. As a matter of fact, specific heat is zero at 0K for all the materials.