# The Circle

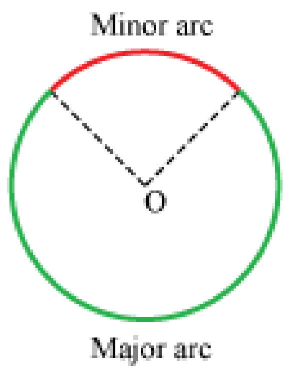
## Circle and Its Attributes

A circle exhibits various interesting properties which make it a special geometric figure.

Let us discuss the same.

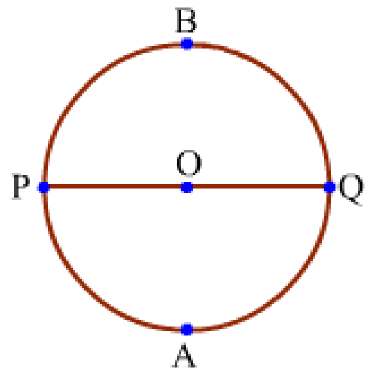
**Minor and major arc:**

An arc less than one-half of the entire arc of a circle is called the minor arc of the circle, while an arc greater than one-half of the entire arc of a circle is called the major arc of the circle.




**Semicircular arc:**

Diameter of a circle divides it into two congruent arcs. Each of these arcs is known as semicircular arc.




In the above figure, PQ is diameter which formed semicircular arcs PBQ and PAQ.

**Finding radius of a circle when its diameter is given:**

We know that the radius of a circle is half of its diameter.

Let *r* be the radius and *d* be the diameter of a circle, then we have $r = \frac{d}{2}$.
Using this formula, we can find the radius of the circle if its diameter is given.

Let us take a look at some examples.

We have to find the radius of the circle when diameter is given.

(i) *d* = 12 cm

$$r = \frac{d}{2}$$
$$\Rightarrow r = \frac{12}{2}$$
$$\Rightarrow r = 6 \text{ cm}$$

(ii) *d* = 25 cm

$$r = \frac{d}{2}$$
$$\Rightarrow r = \frac{25}{2}$$
$$\Rightarrow r = 12.5 \text{ cm}$$

**Finding diameter of a circle when its radius is given:**

We know that the diameter of a circle is twice its radius.
∴ *d* = 2*r*

Using this formula, we can find the diameter of the circle when its radius is given.

Let us take a look at some examples.
We have to find the radius of the circle when diameter is given.

(i) *r* = 15.5 cm

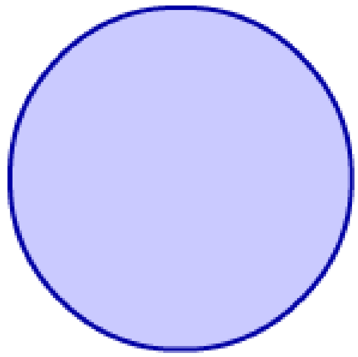
*d* = 2*r*
*d* = 2 × 15.5
*d* = 31 cm

(ii) *r* = 13 cm
*d* = 2*r*

$d = 2 \times 13$
$d = 26$ cm

Let us discuss some more concepts related to circles.

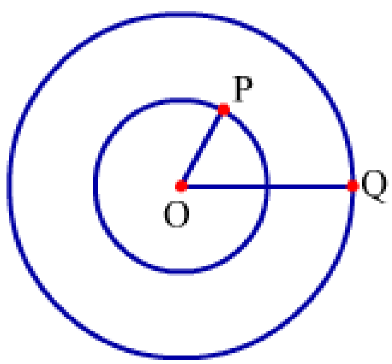
**Circular region:** Look at the following circle.



The whole shaded part is the region of this circle.

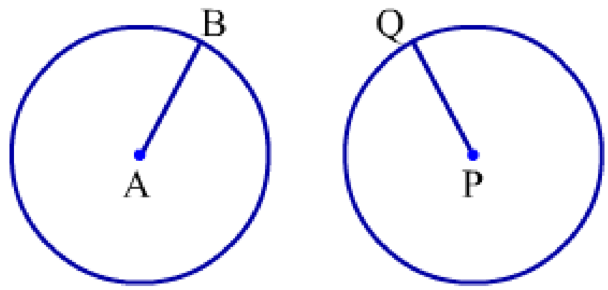
Thus, the interior and boundary together make the region of the circle.

**Concentric circles:** Circles of different radii but having the same centre are known as concentric circles.



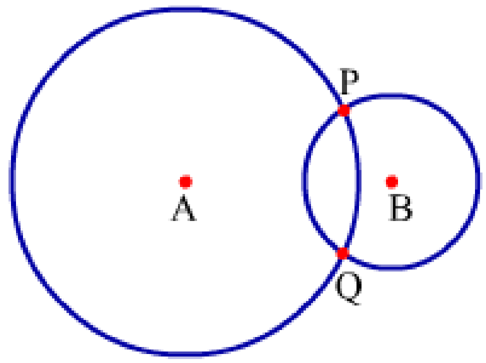

In the above figure, two circles have the same centre O but the different radii OP and OQ such that OQ > OP. These circles are concentric circles.

**Congruent circles:** If the radii of two or more circles are equal, then the circles are said to be congruent to each other.



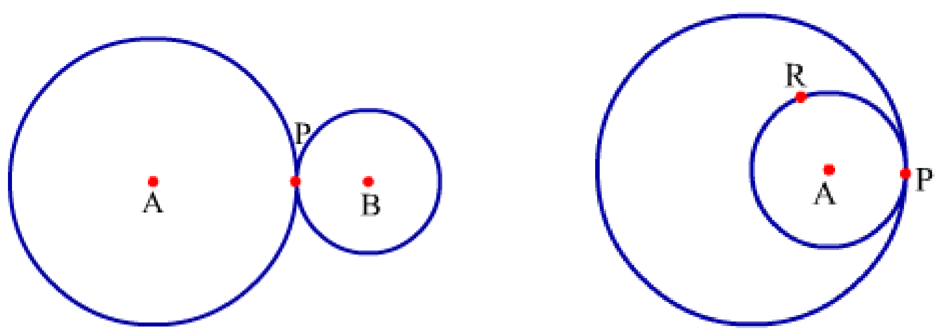

In the above figure, AB and PQ are the radii of the circles such that AB = PQ. Thus, these circles are congruent to each other.

**Intersecting circles:** Two coplanar circles (circles in the same plane) which intersect each other at two distinct points are known as intersecting circles.



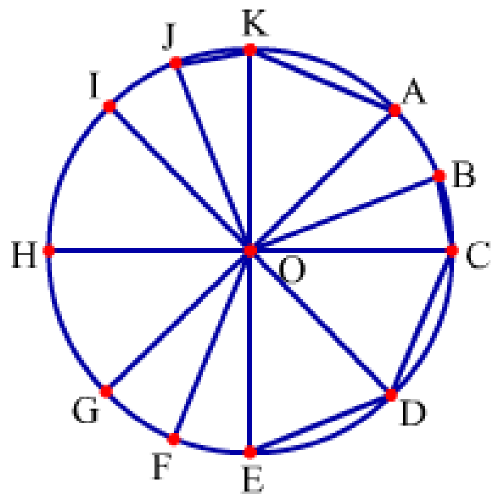

In the above figure, circles with centres A and B intersect each other at two distinct points P and Q. Thus, these are intersecting circles.

If two coplanar circles intersect each other at only one point, then the circles are known as touching circles.




In each of both the above figure, circles touch each other at only one point P. Thus, circles in each figure are touching circles.

Now, observe the following figure.




Here, OA, OB, OC, ..., OK are all radii of the circle. Similarly, we can draw many more radii of this circle.

So, it can be said that **a circle has innumerable radii.**

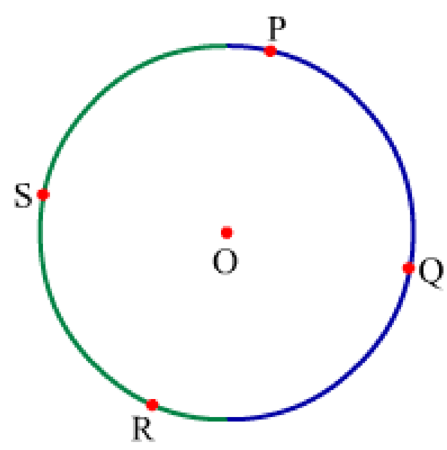
It can be seen that AG, CH, DI and EK all are diameters of the circle. Similarly, we can draw many more diameters of this circle.

So, it can be said that **a circle has innumerable diameters.**

Also, BC, CD, DE, JK and KA are the chords of the circle. Similarly, many more chords of this circle can be drawn.

Thus, it can be said that **a circle has innumerable chords.**

Now, observe the following circle.



It can be seen that points P and R divide this circle into two parts or arcs which are coloured differently. The name "arc PR" does not explain that which of two arcs we are talking about. So, we marked a point on each arc to clarify this. It can be seen that point S is marked on the green arc and point Q is marked on the blue arc.

Now, we can give a three letters name to each arc. Thus, green arc can be named as arc PSR or arc RSP whereas blue arc can be named as arc PQR or arc RQP.

Similarly, we can denote any arc by three letters.

Let us discuss some examples to understand this concept better.

**Example 1:**

**With respect to the figure drawn below, name**

**(a) the centre**

**(b) the diameter**

**(c) any two radii**

**(d) a chord**

**(e) a point lying in the interior of the circle**

**(f) a point lying in the exterior of the circle**

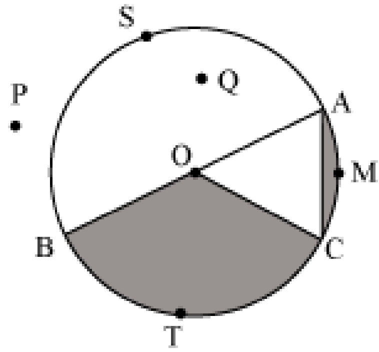
**(g) a sector**

**(h) a segment**

**(i) a point lying on the circle**

**(j) two semi-circles**

**(k) any two arcs**




**Solution:**

**(a)** O is the centre of the circle.

**(b)** $\overline{AB}$ is the diameter of the circle.

**(c)** Two radii of the circle are $\overline{OB}$ and $\overline{OC}$.

**(d)** $\overline{AC}$ is a chord of the circle.

**(e)** Q is a point that lies in the interior of the circle.

**(f)** P is a point that lies in the exterior of the circle.

**(g)** BOC is a sector of the circle.

**(h)** AMC is a segment of the circle.

**(i)** S is a point that lies on the boundary of the circle (or simply, on the circle).

**(j)** The semi-circles in the given figure are ASB and ATB.

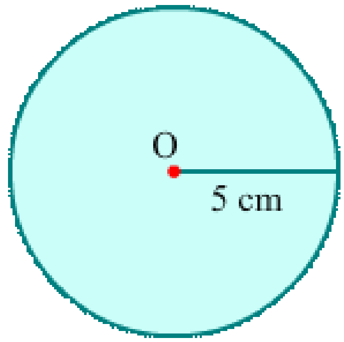
**(k)** BTC and AMC are two arcs of the circle

**Example 2:**

**Using ruler and compass, draw circle of radius 5 cm. Mark its centre and draw the radius.**

**Solution:**

On using a ruler, first we draw the radius 5 cm of the circle and then assuming O as a centre we draw a circle of radius 5 cm by using a compass. Thus, we get a circle of radius 5 cm as shown below.




## Some More Attributes of Circle

Circle is a simple closed curve and it can be defined as follows:

**A circle is the locus of points in a plane which are equidistant from a fixed point in the same plane.**

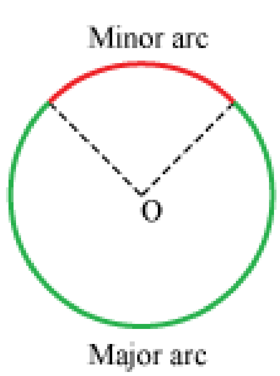
The fixed point is called **centre** of the circle and distance (constant distance) of each point from centre is called **radius** of the circle.

A circle exhibits various interesting properties which make it a special geometric figure.

Let us discuss the same.

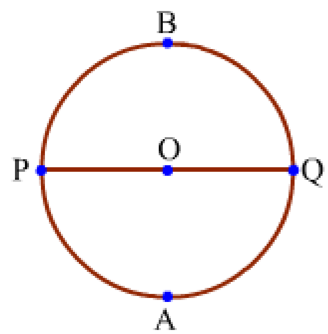
**Minor and major arc:**

An arc less than one-half of the entire arc of a circle is called the minor arc of the circle, while an arc greater than one-half of the entire arc of a circle is called the major arc of the circle.



**Semicircular arc:**

Diameter of a circle divides it into two congruent arcs. Each of these arcs is known as semicircular arc.

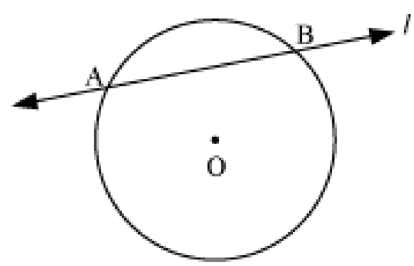



In the above figure, PQ is diameter which formed semicircular arcs PBQ and PAQ.

**Secant:**

**A line that meets a circle at two points is called the secant of the circle.**

Look at the following figure.

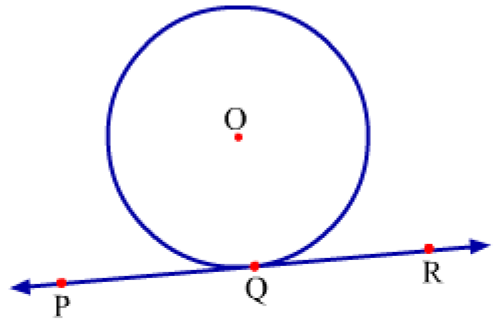



In the figure, a line *l* intersects the circle at two points i.e., A and B. This line is called the **secant** to the circle.

**Tangent:**

**A line that meets a circle at one and only one point is called a tangent to the circle. The point where the tangent touches the circle is called the point of contact.**

Consider the following figure.



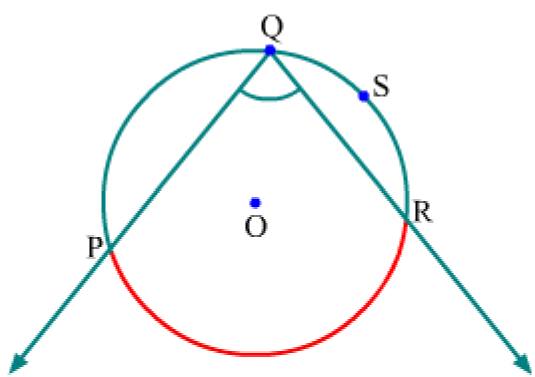

**What do you observe in this figure?**

Here, line PR touches the circle at a point Q. So, line PR is the tangent to the circle and Q is the point of contact.

**Inscribed angle:**

**If an angle is inscribed in the arc of a circle such that the vertex of the angle lies on the arc other than its end points and end points of the arc lie on the arms of the angle, then the angle is called inscribed angle.**

Observe the following figure.

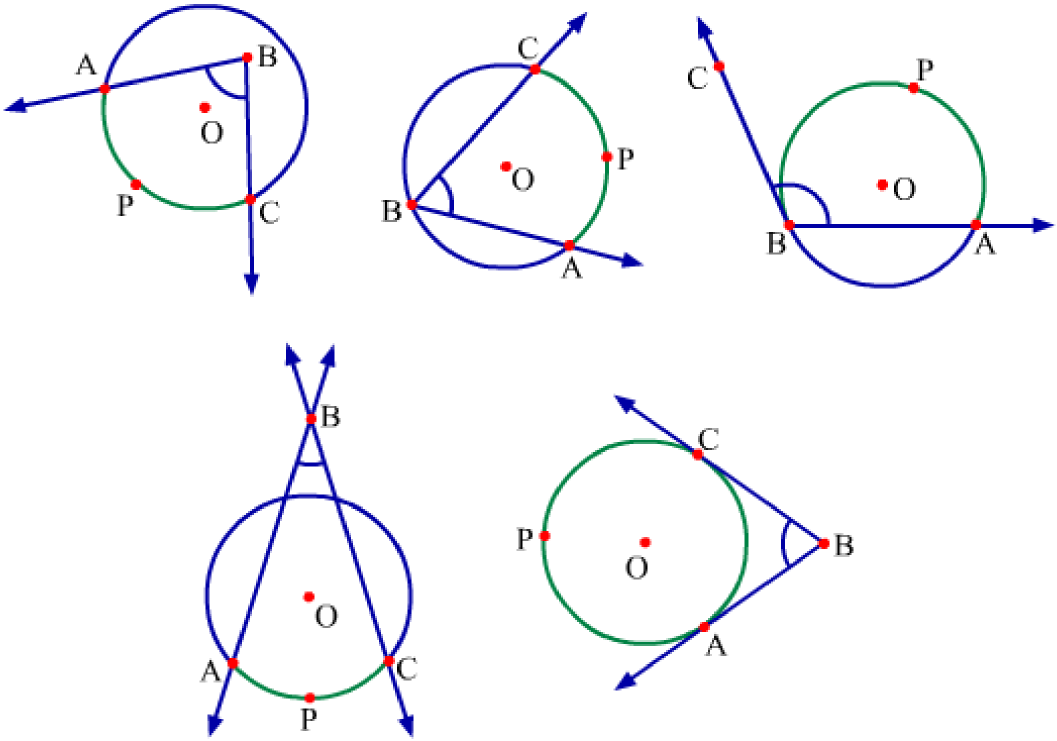



In this figure, vertex Q of ∠PQR lies on the arc PSR. So, ∠PQR is inscribed in the arc PSR or arc PQR.

**Intercepted arc:**

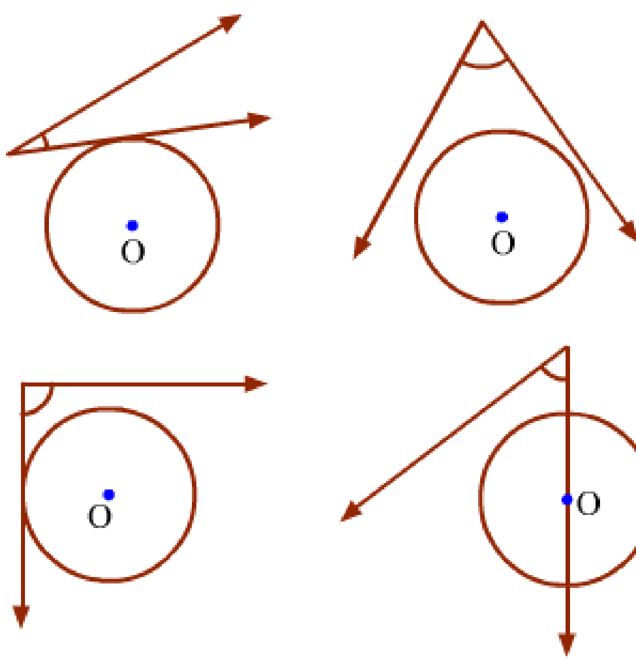
**If an angle and an arc of a circle are given such that each arm of the angle contains an end point of the arc and all points of the arc except the end points lies in the interior of the angle, then the arc is said to be intercepted by the angle.**

Look at the figures given below:




In each figure, the intercepted arc is coloured green.

Now, observe the following figures.

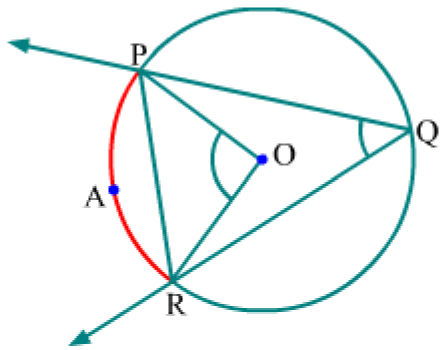


In these figures, angle and arc do not satisfy the conditions given in the definition, so there is no intercepted arc.

**Angle subtended by an arc and central angle:**

**If an angle has its vertex on a circle and both of its arms intersect the circle at points other than vertex, then it is said that the angle is subtended by its intercepted arc.**

Let us consider the following figure.




Here, ∠PQR intercepts the arc PAR. Thus, it can be said that ∠PQR is subtended by arc PAR at a point Q on the circle.

It can be observed that ∠PQR is inscribed in the arc PQR.

So it can be said that:

**The angle subtended by intercepted arc to any point on the circle acts as the inscribed angle for the arc formed by the remaining part of the circle.**

Also, the arc PAR subtends ∠POR to the centre.

**The angle subtended by an arc to the centre is called central angle.**

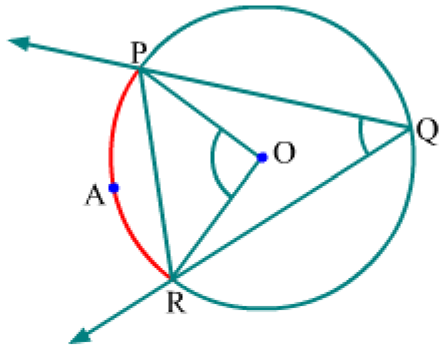
Thus, ∠POR is the central angle in the above figure.

**Measures of minor and major arcs:**

The measure of the central angle corresponding to the minor arc is also the measure of minor arc.

Measure of major arc = 360° – Measure of corresponding minor arc

Observe the following figure.



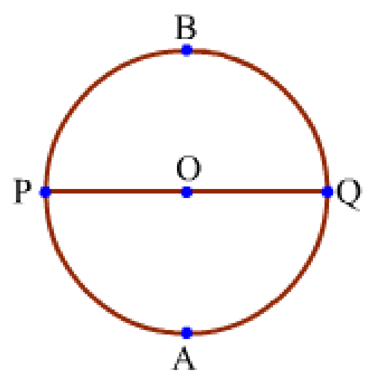

According to the above figure, we have

Measure of minor arc PAR = Central angle = ∠AOB

Measure of major arc PQR = 360° – Measure of corresponding minor arc = 360° – ∠AOB

**Measure of semicircular arc:**

The measure of a semicircular arc is always 180°.

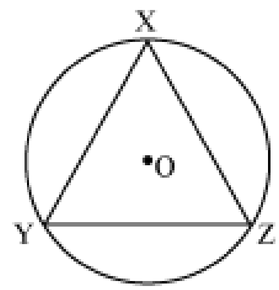



In the above figure, measure of both the semicircular arcs PBQ and PAQ is 180°.

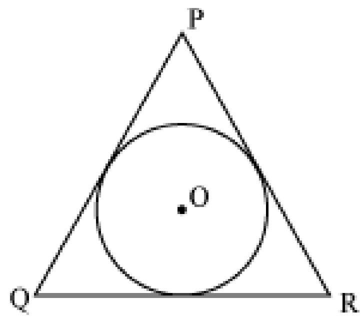
We will now study two terms which relate circles and triangles:

1. Circumcircle of a triangle
2. Incircle of a triangle

A circle which passes through all the three vertices of a triangle is called the **circumcircle** of the triangle.



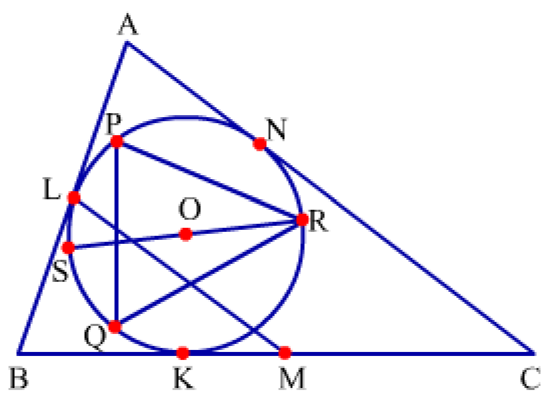

A circle (drawn inside a triangle) which touches all the three sides of the triangle is called the **incircle** of the triangle.




Let us solve some problems to understand these concepts better.

**Example 1:**

**Observe the following figure.**




**With respect to the given figure, name the**

1. **Triangle for which the given circle is an incircle.**
2. **Triangle for which the given circle is a circumcircle.**
3. **Chord(s) of the given circle and their respective major and minor arcs.**
4. **Tangent(s) to the given circle.**
5. **Secant(s) to the given circle.**

6. **Semicircular arcs**

   **Solution:**

1. The given circle is an Incircle for ΔABC.
2. The given circle is a circumcircle for ΔPQR.
3. Chords of the given circle and their respective major and minor arcs are as follows:

   Chord PQ; major arc PRQ and minor arc PLQ

   Chord QR; major arc QPR and minor arc QKR

   Chord RP; major arc RQP and minor arc RNP

4. Tangents to the given circle are AB, BC and CA.

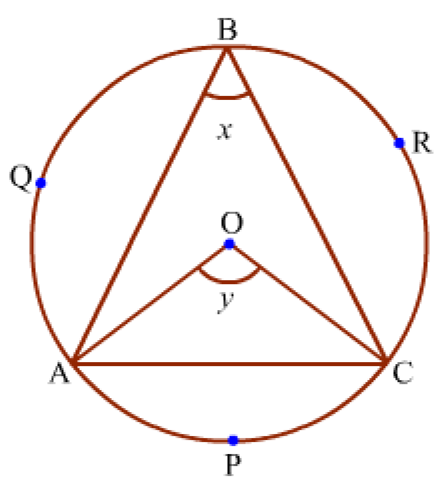
5. Secant to the given circle is LM.

6. RS is the diameter which forms semicircular arcs SPR (can also be named as SLR and SNR) and SKR

   (can also be named as SQR).

**Example 2:**

**Observe the following figure.**




**Find the following attributes from the figure.**

1. **Angle subtended by arc BRC to the circle.**
2. **Angle subtended by arc APC to the centre.**

3. **Arc intercepted by ∠BCA.**
4. **Angle inscribed in arc BQC.**
5. **Measure of minor arc APC.**
6. **Measure of major arc ABC.**

Solution:

1. Angle subtended by arc BRC to the circle = ∠BAC
2. Angle subtended by arc APC to the circle = ∠ABC = $x$
3. Arc intercepted by ∠BCA = arc BQA
4. Angle inscribed in arc BQC = ∠BAC
5. Measure of minor arc APC = Measure of ∠AOC = $y$
6. Measure of major arc ABC = 360° – Measure of ∠AOC = 360° – $y$

## Construction of Circumcircle of a Triangle

We know that a circle which passes through all the vertices of ΔABC is called circumcircle of ΔABC.

Now, suppose the sides of a ΔABC are given as AB = 5 cm, BC = 5.4 cm, and CA = 6 cm and we are required to construct a circumcircle of ΔABC.

Let us now look at one more example to understand the construction of circumcircle more clearly.

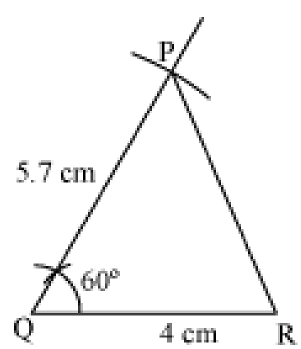
**Example:**

**Construct a ΔPQR such that ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm. Also, construct the circumcircle of this triangle.**

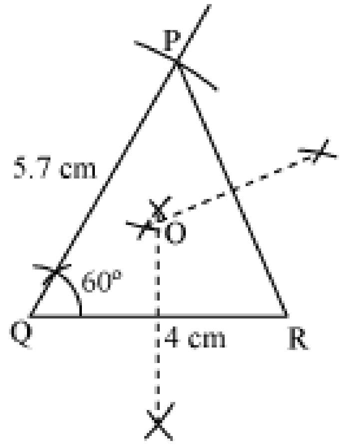
**Solution:**

The steps of construction are as follows:

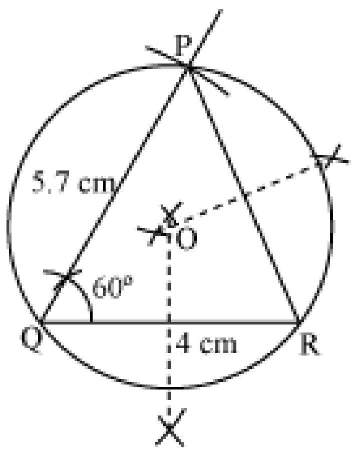
(1) Draw a triangle PQR with ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm



(2) Draw perpendicular bisector of any two sides, say QR and PR. Let these perpendicular bisectors meet at O.




(3) With O as centre and radius equal to OP, draw a circle. The circle so drawn passes through the points P, Q, and R, and is the required circumcircle of ΔPQR.




## Construction of Incircle of a Triangle

Suppose we have to construct an Incircle of a triangle ABC and the following information about the ΔABC is given:

∠B = 60°, ∠C = 60°, and BC = 5.5 cm

Let us now look at one more example to understand this concept better.

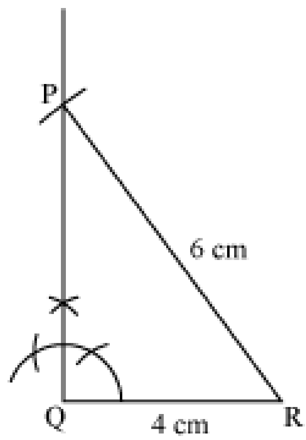
**Example:**

**Construct a right triangle ΔPQR, right angled at Q, such that QR = 4 cm and PR = 6 cm. Also construct the incircle of ΔPQR.**

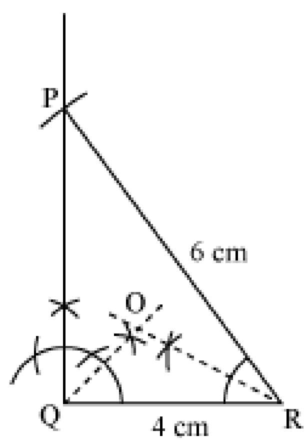
**Solution:**

The steps of construction are as follows:

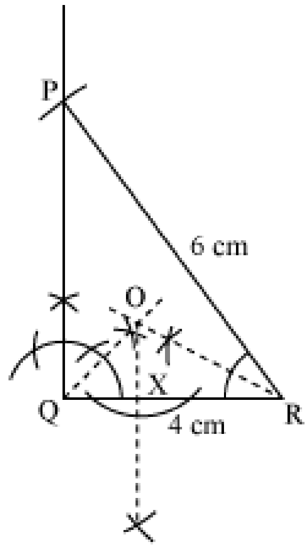
(1) Draw a ΔPQR right-angled at Q with QR = 4 cm and PR = 6 cm




(2) Draw bisectors of ∠Q and ∠R. Let these bisectors meet at the point O.




(3) From O, draw OX perpendicular to the side QR.




(4) With O as centre and radius equal to OX, draw a circle. The circle so drawn touches all the sides of ΔPQR and is the required incircle of ΔPQR.