Probablity

Introduction

In class previous classes we have studied about the probability as a measure of uncertainity of various phenomenon in our daily life. Normally we obtain the probability of an events as the ratio of the number of favourable outcomes to that of the total possible outcomes. This is called classical theory of probability. Up to class IXth we have studied statistical theory of probability. But the drawback of these theory is that it cannot be applied to the activities which have infinite numbers of outcomes. In this chapter we will study about this approach which is called axiomatic approach of probability. To understand this we have to understand the basic concept.

Random Experiments

A experiment is said to be a random experiment if it has more than one possible outcome and it is not possible to predict the outcome in advance. For example, throwing a dice or tossing a coin is a random experiment.

Outcomes

The possible results of a random experiment is called outcomes. For example, throwing a dice and getting 1, 2, 3, 4, 5, 6 are the possible out comes.

Sample Space

The set of all possible outcome of a random experiment is called sample space.
Each element of the sample space is called sample point.
For example, when we throw a dice we get $\{1, 2 - - - 6\}$, then it is called sample space.

Events

The subset E of a sample space is called events. There are different types of events which is defined below:

Impossible Events

The events which will never occur is called impossible events. The empty set is also called impossible events.

Illustrative EXAMPLE

Throwing a dice and getting 7 is an impossible events

Sure Events

The events which will definitely occur is called sure events.

Illustrative EXAMPLE

Throwing a dice and getting 1, 2, 3, 4, 5, or 6 is a sure event

Simple Events

The events having only one points of the sample space is called simple events.
For example, in the experiment of tossing two coins, a sample space is S = {HH, HT, TH, TT}. There are four simple events in this sample space.

Compound Events

The events having more than one sample points is called compound events.
For example, in the experiment of tossing a coin thrice, the events of getting exactly one head, getting at least one head, at most one head are the compound events.
{HTT, THT, TTH,}, {HTT, THT, TTH, HHT, HTH, THH, HHH}, {HTT, TTH, TTT}

Complementary Events

For a event A, the event corresponding to the set of elements excluding the elements of the set A is called complementary events. It is denoted by A" or A7.

Mutually Exclusive Events

Two events A and B are said to be mutually exclusive if the occurrence of any one of them excludes the occurrence of the other events or we can say that they cannot occur simultaneously and are considered to be disjoint sets.

For example, A = {HTT, THT, TTH, TTT} and B = {HHT, THH, HTH, HHH} are disjoint sets.

Exhaustive Events

The set of events $E_1, E_2, E_3 ----E_n$ is said to be exhaustive if the sample space is equal to $S = \bigcup_{k=1}^{n} E_k$.

For example. If A = {HTT, THT, TTH, TTT} and B = {HHT, THH, HTH, HHH}, then $A \cup B$ = {HHT, THH, HTH, HHH, HTT, THT, TTH, TTT}, which is the entire sample space for three toss of a coin.

Event 'A or B'

Two events A or B is equivalent to the union of the two events. A or B = $A \cup B$

Event 'A and B'

Two events A and B is equivalent to the intersection of the two events.

A and B = $A \cap B$

Event' A but not B':

Two events A but not B is denoted as the difference of A and B' denoted by A but not B = $A \cap B'$ =A-B

Probability of Events

For any two events A and B, the probability of union of the two sets is given by:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are disjoints then $A \cap B = \phi$, then

$P(A \cup B) = P(A) + P(B)$

If A and B are independent then,

$P(A \cap B) = P(A) \times P(B)$

Thus, $P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

Probability of Event 'not A'

$P(not) = 1 - P(A)$

Probability (A but not B)

$P(A but\, not B) = P(A \cap B') = P(A) - P(A \cap B)$

Probability Neither A nor B

$P(neither\ A nor\ B) = P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

Commonly Asked QUESTIONS

Thomas has 9 ball in his bag of which 4 are green, 3 are blue and 2 are yellow. The ball are similar in shape and size. A ball is drawn at a random from the bag and is found to be either green or blue. Find the probability of the events to occur.

(a) $\frac{7}{9}$ (b) $\frac{4}{9}$

(c) $\frac{3}{9}$ (d) $\frac{5}{9}$

(e) None of these

Answer: (a)

Explanation

Let the event of green ball = G and that of the blue ball = B

Then the event either green or blue is defined as the = G or B

Since the events of green or blue ball is mutually exclusive, therefore, P(G or B) = P (GUB) = P (G)

$$= \frac{4}{9} + \frac{3}{9} = \frac{7}{9}$$

Two students James and Harry appears in an examination. The probability that James will qualify the examination is 0.05 and that Harry will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. The probability only one of them will qualify the examination is:

(a) $\frac{63}{100}$ (b) $\frac{87}{100}$

(c) $\frac{49}{50}$ (d) $\frac{11}{100}$

(e) None of these

Answer: (d)

Explanation

Let the event that James will qualify = A

The event that Harry will qualify = B

Then the probability that only one of them will qualify the examination is given by

$$P(A \cap B') + P(A' \cap B)$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= 0.05 - 0.02 + 0.10 - 0.02$$

$$= 0.11 = \frac{11}{100}$$

A board has to select two individuals for two different posts in the board of directors. There were two men and two women candidate for the post. Find the probability that no men is selected for the required post.

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

(e) None of these

Answer: (b)

Explanation

Let the event of selecting the men = M

The event of selecting women = F

There are total four individuals for the said post and two are required to be selected. Thus the total number of ways in which two person out of four person can be selected is given by ${}^4C_2$ ways

Since no men is to be selected for the post, so both should be the women.

Thus selection of two women can be done in ${}^2C_2 = 1$ ways.

Therefore required probability $= (nomen) = \frac{{}^2C_2}{{}^4C_2} = \frac{1\times2\times1}{4\times3} = \frac{1}{6}$

In a school the students of class XIIth, of which 40% like English, 30% like Humanity, 10% like both English and Humanity. If one of the student is selected at a random, the probability that he or she like neither English nor Humanity is___.

(a) 0.6 (b) 0.8

(c) 0.4 (d) 0.2

(e) None of these

Answer: (c)

In a company there are 100 employee working in two different departments of the company. They are in a group of 60 and 40 employes in the different departments. There are two friends working in the same company in the same post, find the probability that both of them are working in the different departments.

(a) $\frac{16}{33}$ (b) $\frac{17}{33}$

(c) $\frac{18}{33}$ (d) $\frac{19}{33}$

(e) None of these

Answer: (a)

Sample Space for the Throw of Pair of dice

When two dice are thrown then the sample space is { (1,1), (1,2) (13) (14) (15) (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6); (4;!); (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

Drawing a Card from a Well - Shuffled of 52 Cards

We know that there are 52 cards in pack of card. It has four types, which is Spade, Club, Heart and Diamond. All are equally divided. It means that 13.spades, 13 clubs, 13 hearts and 13 diamonds. In 52 cards, 26 red card and 26 black card.' 1111

Face Cards

Kings, queens and jack are called face cards, the total number of face cards is 12: In which 6 is Red face cards and 6 is black face cards.

Equally Likely Events

A given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

Probability of Occurrence of an Event

Let event is denoted by E, the probability of occurrence is the ratio of favourrable outcomes to the total possible outcomes of the sample space. Mathematically, if number of favourable outcomes be denoted by n (a) and total number of possible out comes be denoted by n(S) then, $P(E)=\frac{P(A)}{P(S)}$.

Sure Event

The event which will definitely occur is called sure events.

Illustrative EXAMPLE

In a single throw of dice, what will be the probability of getting a number which is less than 7?

Solution:

Sample space = {1, 2, 3, 4, 5, 6}

$\therefore n(E)=6$

$\therefore p(E)=\frac{n(E)}{n(S)}=\frac{6}{6}=1$. S o, it is sure event.

Impossible Event

Impossible event is the event whose probability is 0 (zero).

Illustrative EXAMPLE

On tossing a coin, the probability of getting 1?

Solution:

We know that when a coin is tossed then only head or tail will comes up. Hence the probability of occurrence of one is not there, therefore probability is zero.

NOTE: From above calculation we can say that probability of any event is $0 \le P(E) \le 1$. It means that maximum probability = 1, and minimum probability =0. Also, probability can never be negative or greater than one.

Complementary event

Let E be an event, then e' (not E) is called complementary event.

$\therefore P(E')+P(E)=1$

$P(E')=1-P(E)$

Illustrative EXAMPLE

The Probability that it will rain today is 0.84. What is the probability that it will not rain today?

(a) 0.12 (b) 0.1

(c) 0.3 (d) 0.4

(e) None of these

Answer: (e) = 0.84,

we know that P(E') + P(e) = 1

$\Rightarrow 0.84 + P(E') + P(E) = 1$

$\Rightarrow (E') = 1 - 0.84 = 0.16$

Illustrative EXAMPLE

What is the probability that an ordinary year will have 53 Sunday?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{7}$

(e) None of these

Answer: (d)

An ordinary year has 365 days, i.e., 52 week and 1 day,

The probability that this day will be Sunday $= \frac{1}{7}$

Commonly Asked QUESTIONS

What is the probability of a sure event?

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) $\frac{3}{2}$

(e) None of these

Answer: (c)

The probability of a sure event is 1.

What is the probability of an impossible event?

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{2}{1}$

(e) None of these

Answer: (a)

The probability of an impossible event is 0.

One card is drawn from a well shuffled deck of 52 cards. What is the probability of getting a black face card?

(a) $\frac{1}{26}$ (b) $\frac{3}{14}$

(c) $\frac{3}{26}$ (d) $\frac{3}{13}$

(e) None of these

Answer: (c)

Number of black face card = (2 king + 2 queen + 2 jack) = 6.

$\therefore \ P(E) = \frac{6}{52} = \frac{3}{26}$

There are 20 tickets numbered from 1, 2, 3, —, 20 respectively. One ticket is drawn at random, what is the probability that the number on the ticket is a multiple of 3 or 5?

(a) $\frac{1}{4}$ (b) $\frac{9}{20}$

(c) $\frac{2}{5}$ (d) $\frac{4}{10}$

(e) None of these

Answer: (b)

Number of all possible number's = 20

Number of multiples of 5 are A = 5, 10, 15, 20

Number of multiples of 3 are B = 3, 6, 9,12,15,18

Number of multiples of both 3 and 5 are $A \cap B$ = 15

$P(A) = \frac{4}{20}, P(B) = \frac{6}{20} \, and \, P(A \cap B) = \frac{1}{20}$

Now, $P(A \cap B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{20} + \frac{6}{20} - \frac{1}{20}$

$= \frac{9}{20}$

$\therefore P(E) = \frac{9}{20}$

A bag contains 8 red balls, 2 black balls and 5 white balls. One ball is drawn at random from the bag, what is the probability that the ball drawn is not black?

(a) $\frac{2}{15}$ (b) $\frac{13}{15}$

(c) $\frac{8}{15}$ (d) $\frac{1}{3}$

(e) None of these

Answer: (b)
Total number of balls = 15
Number of non black balls = 13

$$\therefore P(E) = \frac{13}{15}$$

A bag contains 3 white marbles, 4 red marbles, and 5 black marbles. One marble is drawn at random from the bag, what is the probability that the marble drawn is neither black nor white?

(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
(d) $\frac{3}{4}$
(e) None of these

Answer: (c)
Total number of marbles = 12
Number of favourable cases = 4

Required probability $\frac{4}{12} = \frac{1}{3}$

What is the probability that an ordinary year will have 53 Monday?

(a) $\frac{2}{7}$
(b) $\frac{1}{7}$
(c) $\frac{7}{23}$
(d) $\frac{7}{52}$
(e) None of these

Answer: (b)
An ordinary year has 365 days, i.e 52 weeks and 1 day
This day can be any of 7 days of the week

$$\therefore P(E) = \frac{1}{7}$$

Thomas purchases 24 lottery tickets which contains 8 prizes. What is the probability of winning the prize?

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{2}{3}$
(d) $\frac{4}{5}$
(e) None of these

Answer: (b)

Total number of tickets = 24
Number of prizes = 8

$\therefore\ P(E3) = \frac{8}{24} = \frac{1}{3}$

There are 25 cards numbered from 1 to 25. One card is drawn at random, what is the probability that the number on this card is not divisible by 4?

(a) $\frac{4}{25}$ (b) $\frac{21}{25}$

(c) $\frac{6}{25}$ (d) $\frac{19}{25}$

(e) None of these

Answer: (d)

You Must KNOW

- If a class has 23 students in it then the probability that at least two of the students share a birthday is about 0.5. Surprised? If there are 50 students in a class then it's virtually certain that two will share the same birthday.
- Geologists can, calculate the probability of future earthquakes based on past activity.
- Great earthquakes that cause complete destruction near their epicenters occur once every 5 to 10 years (8.0-8.9).
- The largest earthquakes, such as the 1964 Good Friday quake in Alaska, will take place once or twice within a century (9.0 and up).

SUMMARY

- The probability of an event is the ratio of total favourable outcome to the total possible outcome of the events.
- The set of all possible outcome of a random experiment is called sample space.
- Two events are mutually exclusive if $A \cap B = \phi$.
- The sum of all probabilities of a sample space is always 1.
- The probability of a events cannot be negative and greater than 1.

Self Evaluation TEST

Duration
10 Minutes

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1 **A company sends performance letters to its employee at the end of the year with some advice of improvement if required for the employee. In the process the company sends three letters to three of its employee addressed to each of them. The clerk of the company inserted the letters into the envelopes at random so that each envelopes contains one letter without taking care of the address on the envelope. Find the probability that at least one letter reaches the correct address.**

(a) $\frac{5}{6}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

(e) None of these

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2 **In a number game a child has to choose six different natural numbers form the set of first 20 natural numbers. If his six numbers matches the number in the given sequence of six number then he wins the prize. The probability that a child does not win the prize in that game is____.**

(a) $\frac{1}{38760}$ (b) $\frac{38759}{38760}$

(c) $\frac{38769}{38770}$ (d) $\frac{1}{38770}$

(e) None of these

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3 **If the letters of the word 'UNIVERSITY' are arranged randomly, then find the probability that no two 'I' comes together.**

(a) $\frac{4}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

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4 **Four cards are drawn from the pack of 52 cards. The probability of drawing the four cards of same suits from the pack of 52 cards is____.**

(a) $\frac{11}{4165}$

(b) $\frac{44}{4165}$

(c) $\frac{53}{4165}$

(d) $\frac{67}{4165}$

(e) None of these

5 **Mary plays card game with her friend Micheal. Micheal asked her to draw two cards from the pack such that both the cards are either red or king. Find the probability of her success.**

(a) $\frac{325}{1326}$ (b) $\frac{43}{221}$

(c) $\frac{55}{221}$ (d) $\frac{1}{221}$

(e) None of these

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6 **Three people Smith, Harish and Robert went for a job interview in a company. The probability that Smith is selected is twice that of Harish and the probability of Harish being selected is thrice that of Robert. The probability that Smith is selected for the job in the interview is______.**

(a) $\frac{3}{5}$ (b) $\frac{3}{10}$

(c) $\frac{1}{10}$ (d) $\frac{7}{10}$

(e) None of these

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7 **A cycic person starts moving in such a way that the probability that he takes steps forward is 0.4 and he takes steps backward is 0.6. The probability that at the end of 11th steps he is one step away from the starting point is_____.**

(a) $\frac{92512}{9765625}$ (b) $\frac{592512}{9765625}$

(c) $\frac{3592512}{9765625}$ (d) $\frac{2512}{9765625}$

(e) None of these

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8 **Three dice are rolled together. Find the probability of getting total atleast 6.**

(a) $\frac{103}{108}$ (b) $\frac{102}{108}$

(c) $\frac{102}{108}$ (d) $\frac{97}{108}$

(e) None of these

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9 **Three friends Jack, Robert and Mary tries to solve a problem. The probabilities that each of them solve the problem are $\frac{1}{3},\frac{2}{7},\frac{3}{8}$ respectively. Find the probabilities that exactly one of them solves the problem if all of them try to solve the problem.**

(a) $\frac{33}{56}$ (b) $\frac{31}{56}$

(c) $\frac{25}{56}$ (d) $\frac{23}{56}$

(e) None of these

10 **Robert speaks truth in 60% of cases and Mary speaks truth in 90% of cases. If both of them are asked to give a statement/then the probability that they contradict each other's statements is:**

(a) $\frac{19}{50}$ (b) $\frac{29}{50}$

(c) $\frac{31}{50}$ (d) $\frac{21}{50}$

(e) None of these

Answers – Self Evaluation Test																			
1.	C	**2.**	B	**3.**	D	**4.**	B	**5.**	C	**6.**	A	**7.**	C	**8.**	A	**9.**	C	**10.**	D

Self Evaluation Test SOLUTIONS

1. The probability that at least one letter is in proper envelope = 1 - Probability that no letters are in proper envelopes

$$=1-\frac{2}{6}=\frac{2}{3}$$

2. Since there is only one way that a child can win the prize in the game.

Thus the probability that the child wins the prize is $\frac{1}{38760}$

The probability that the child does not wins the prize $=1-\frac{1}{38760}=\frac{38759}{38760}$

3. The total number of permutations of the letters of the word is given by $\frac{10!}{2!}$

If we consider two I's as one then the total number of letters in the word is 9. so the total permutations of the letters of the word = 9!

Hence the required probability is $=\frac{9!2!}{10!}=\frac{1}{5}$