# Optical Instruments

## Exercise Solutions

**Question 1:** A person looks at different trees in an open space with the following details. Arrange the trees in decreasing order of their apparent sizes.

| Tree | Height(m) | Distance from the eye(m) |
|---|---|---|
| A | 2.0 | 50 |
| B | 2.5 | 80 |
| C | 1.8 | 70 |
| D | 2.8 | 100 |

**Solution:**

Let θ be the angle formed by the eyes while looking at tree.

θ = Height/Distance

For trees, A,B,C and D, the value of angle θ becomes:

$\theta_A = 2/50 = 0.04$
$\theta_B = 2.5/80 = 0.031$
$\theta_C = 1.8/70 = 0.0257$ and
$\theta_D = 2.8/100 = 0.028$

Since, $\theta_A > \theta_B > \theta_C > \theta_D$

The arrangement in decreasing order is given by A, B, D and C.

**Question 2:** An object is to be seen through a simple microscope of focal length 12 cm. Where should the object the placed so as to produce maximum angular magnification? The least distance for clear vision is 25 cm.

**Solution:**
For maximum angular magnification,
Image distance = Least distance of clear vision
$v = d = -25$ cm

Where d is the distance of clear vision.

Using len's formula-

$1/f = 1/v - 1/u$

Here $f = 12$ and $v = -25$

=> $u = -1.8$ cm

The object should be placed 8.1 cm away from the lens.

**Question 3:** A simple microscope has a magnifying power of 3.0 when the image is formed at the near point (25 cm) of a normal eye.
(a) What is its focal length?
(b) What will be its magnifying power if the image is formed at infinity?

**Solution:**

(a) $m = 1 + D/f$

=> $3 = 1 + 25/f$

=> $f = 12.5$ cm

Focal length when the image is formed at 25cm is 12.5cm

(b) When the image is formed at infinity

$m = D/f = 25/12.5 = 2$
Where m = magnifying power

**Question 4:** A child has near point at 10 cm. What is the maximum angular magnification the child can have with a convex lens of focal length 10 cm?

**Solution:**

Here D = 10 cm and f = 10 cm
The maximum angular magnification of the child when the image is formed at distance D=10cm

=> m = 1 + D/f

= 1 + 10/10 =2

Angular magnification is 2.0 when the near point is 10 cm.

**Question 5:** A simple microscope is rated 5 X for a normal relaxed eye. What will be its magnifying power for a relaxed farsighted eye whose near point is 40 cm?

**Solution:**
Here D = 40 cm, m = 5 and

Least distance of distinct vision for a normal eye = 25 cm

For relaxed eyes, the image will be formed at infinity.

So, m = D/f = 25/f

=> f = 40 cm

And, m = D/f = 40/5 = 8 cm

The magnifying power of simple microscope for far sighted eye will be 8x.

**Question 6:** Find the maximum magnifying power of a compound microscope having a 25 dioptre lens as the objective, a 5 diopter lens as the eyepiece and the separation 30 cm between the two lenses. The least distance form clear vision is 25 cm.

**Solution:**

For eyepiece:

Power of the eyepiece = 5 D

The focal length of the eyepiece: $f_e = 1/5 = 0.2$ cm = 20 cm

and $v_e = -25$

Using lens formula,

$1/f = 1/v - 1/u$

$\Rightarrow 1/u_e = 1/-25 - 1/20$

$\Rightarrow u_e = 11.11$ cm

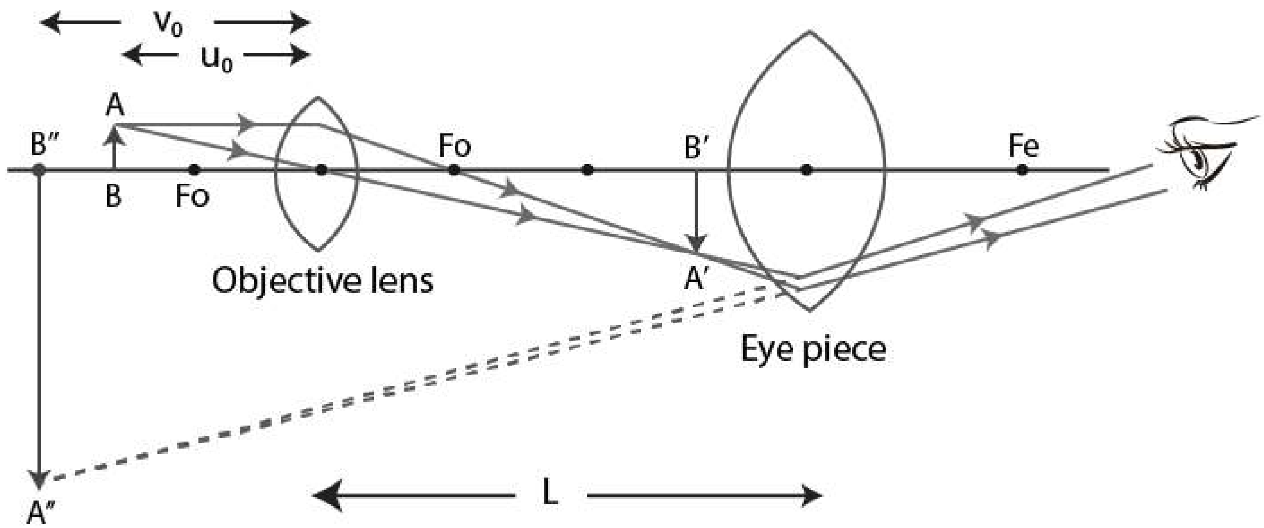
For Objective lens:

Power of the objective lens = 25 D

The focal length of the objective lens is $m = 1/f_0$

$\Rightarrow f_0 = 1/25 = 0.04$ m =4 cm

Now,




L = length of the compound microscope
$u_0$ = object distance and
$v_0$ = image distance for the objective lens.

So, image distance for the objective lens: $v_0 = L - u_0$

=> $v_0 = 30 - 11.11 = 18.89$ cm

[Since the image formed is real and inverted]

Using lens formula,
$1/f_0 = 1/v_0 - 1/u_0$

=> $1/u_0 = 1/18.89 - 1/4$

or $u_0 = -5.07$ cm

Now, maximum magnifying power of the compound microscope, m :

$m = (v_0/u_0)(1 + D/f_e)$

$= (-18.89/-5.07)[1 + 25/20]$

= 8.3831

Therefore, the maximum magnifying power of the compound microscope is 8.3831.

**Question 7:** The separation between the objective and the eyepiece of a compound microscope can be adjusted between 9.8 cm to 11.8 cm. If the focal lengths of the objective and the eyepiece are 1.0 cm and 6 cm respectively, find the range of the magnifying power if the image is always needed at 24 cm from the eye.

**Solution:**

For the given compound microscope

$f_0$ = 1 cm, D= 24 cm and $f_e$ = 6 cm

For eye piece, $v_e$ = -24 cm and $f_e$ = 6 cm

Now, using mirror formula,

$1/v_e - 1/u_e = 1/f_e$

=> $1/u_e = 1/(-24) - 1/6 = -5/24$

=> $u_e = -4.8$ cm

(a) When the separation between the objective and the eyepiece = 9.8 cm

for the objective lens, the image will be at a distance

$v_o = 9.8 - 4.8 = 5$ cm

using mirror formula, $1/f_o = 1/v_o - 1/u_o$

=> $1/u_o = 1/5 - 1 = -4/5$

or $u_o = -1.25$ cm

Now, Magnifying power of the compound microscope:

$m = (v_o/u_o) (1 + D/f_e)$

$= (5/1.25)[1 + 24/6]$

= 20

(b) When the separation between the objective and the eyepiece is 11.8 cm

$v_o = 11.8 - 4.8 = 7$ cm

Again from mirror formula,

$1/7 - 1/u_o = 1$

=> $u_o = -1.16$ cm

Now, Magnifying power of the compound microscope is

$m = (v_o/u_o)(1 + D/f_e)$

$= (7/1.16)[1 + 24/6]$

= 30

Hence, magnification lies in the range of 20-30.

**Question 8:** An eye can distinguish between two points of an object if they are separated by more than 0.22mm when the object is placed at 25 cm from the eye. The object is now seen by a compound microscope having a 20 D objective and 10 D eyepiece separated by a distance of 20 cm. The final image is formed at 25 cm from the eye. What is the minimum separation between two points of the object which can now be distinguished?

**Solution:**
The focal length of the eyepiece : $f_e = 1/10 = 10$ cm
D = 25 cm
Separation between objective and eyepiece = 20 cm

For eyepiece:
Power of the eyepiece = 10 D

To distinguish the two points having minimum separation, the magnifying power should be maximum-Image distance,

$v_e$ = -25 cm and focal length = $f_e$ = 10 cm

$1/u_e = 1/v_e - 1/f_e = 1/-25 - 1/10 = -7/50$ cm

=> $u_e = -50/7$ cm

The image distance for the objective lens should be

$v_o = 20 - 50/7 = 90/7$ cm

For Objective lens:

The focal length of the objective lens: $f_o = 1/5$

$1/u_o = 1/v_o - 1/f_o = 7/90 - 1/5 = -11/90$ cm

$u_o = -90/11$ cm

Now, max magnification of the compound microscope :

$m = (-v_o/u_o)(1 + D/f_e)$

= 11/7 x 3.5

= 5.5

Hence, minimum separation eye can distinguish [ 0.22/5.5 = 0.04] = 0.04 mm.

**Question 9 :** A compound microscope has a magnifying power of 100 when the image is formed at infinity. the objective has a focal length of 0.5 cm and the tube length is 6.5 cm. Find the focal length of the eyepiece.

**Solution:**
For the given compound microscope: $f_o$ = 0.5 cm and tube length = 6.5 cm

m (Magnifying power) = 100

When the image is formed at infinity, image formed by the objective lens is real and it should lie on the focus point of the eyepiece.

$v_o + f_e = 6.5$ cm ...(1)

For normal adjustment, the magnifying power

$m = v_o/u_o (D/f_e)$

[As, $v_o/u_o = 1 - v_o/f_o$]

$\Rightarrow m = -1 - (v_o/f_o)(D/f_e)$

$\Rightarrow 100 = -1 - (v_o/0.5)(25/f_e)$

$\Rightarrow 2v_o - 4 f_e = 1$ .....(2)

Solving (1) and (2), we get

$f_e = 2$ cm and $v_o = 4.5$

The focal length of the eyepiece is 2 cm when the image formed is at infinity.

**Question 10:** A compound microscope consists of an objective of focal length 1 cm and an eyepiece of focal length 5 cm. An object is placed at a distance of 0.5 cm from the objective. What should be the separation between the lenses so that the microscope projects an inverted real image of the object on a screen 30 cm behind the eyepiece?

**Solution:**
Here, $f_o$ = 1.0 cm, $f_e$ = 5 cm, $u_o$ = 0.5 cm and $v_e$ = 30 cm

Using lens formula for objective lens:

$1/f_o = 1/v_o - 1/u_o$

on substituting the values,

=> $v_o = -1$ cm

Objective lens forms a virtual image at the side same as that of the object at a distance of 1 cm from the objective lens.
This image acts as a virtual object for the eyepiece.

For eyepiece,
$1/f_e = 1/v_e - 1/u_e$

Here $v_e = 30$ and $f_e = 5$ (given)

=> $u_e = -6$ cm

Separation between the objective and the eyepiece is (6 − 1) 5 cm

**Question 11:** An optical instrument used for angular magnification has a 25 D objective and a 20 D eyepiece.
The tube length is 25 cm when the eye is least strained.
(a) Whether it is a microscope or a telescope?
(b) What is the angular magnification produced?

**Solution:**
The focal length of the objective lens :
$f_o = 1/25$ D = 0.04 m = 4 cm and

$f_e = 1/20$ D = 0.05 m = 5 cm

(a) Instrument must be a microscope.
As, focal length of objective is less than the eyepiece, $f_o < f_e$

(b) length of the Tube, l = 25 cm

Distance between image and eye lens, $v_e$ = 25 cm

We know, length of the tube, $l = v_o + f_e$

=> $v_o$ = 20 cm

Now, using lens formula:

$1/f_o = 1/v_o - 1/u_o$

on substituting the values,

=> $u_o$ = 5 cm

Now, For the microscope, the angular magnification m:

$m = (v_o/u_o)(1 + D/f_e) = 20/5 \times 25/5 = 20$ cm

**Question 12:** An astronomical telescope is to be designed to have a magnifying power of 50 in normal adjustment. If the length of the tube is 102 cm, find the powers of the objective and the eyepiece.

**Solution:**
For the astronomical telescope in normal adjustment. m = 50, L = 102 cm
Where m is magnifying power and L is length of the tube

Let $f_o$ and $f_e$ focal length of objective and eyepiece respectively,

$m = f_o/f_e$

=> $f_o = 50 f_e$ ...(1) and
$L = f_o + f_e = 102$ cm ...(2)

Solving (1) and (2) we get
$f_e$ = 2 cm = 0.02 m and $f_o$ = 100 cm = 1m

Now,

Power of objective lens = $1/f_o = 1$ D and

Power of eye piece lens = $1/f_e = 1/0.02 = 50$ D

**Question 13:** The eyepiece of an astronomical telescope has a focal length of 10 cm. The telescope is focussed for normal vision of distant objects when the tube length is 1.0 m. Find the focal length of the objective and the magnifying power of the telescope.

**Solution:**
Focal length of the objective: $f_o + f_e = L$

Where L = Length of the tube and $f_e$ = Focal length of the eyepiece
Here L = 100 cm and $f_o$ = 10 cm (given)

=> $f_o = L - f_e = 90$ cm

Now, Magnifying power of the telescope: $m = f_o/f_e = 9$

**Question 14:** A Galilean telescope is 27 cm long when focussed to form an image at infinity. If the objective has a focal length of 30 cm, what is the focal length of the eyepiece?

**Solution:**

Focal length of the objective: $f_o - f_e = L$
[The image will be formed at infinity]

Where L = Length of the tube and $f_o$ = Focal length of the objective lens of the telescope
Here L = 27 cm and $f_o$ = 30 cm (given)

=> $f_o = L - f_e = 90$ cm

As concave eyepiece lens used in Galilean telescope => $f_e = f_o - L = 30-27 = 3$ cm

**Question 15:** A far sighted person cannot see objects placed closer to 50 cm. Find the power of the lens needed to see the objects at 20 cm.

**Solution:**
For the far sighted person, u = -20 cm, v = -50 cm

=> 1/f = 1/v - 1/u

=> 1/f = 3/100

=> f = 100/3 cm = 1/3 m

The power of the lens is 3D

**Question 16**: A nearsighted person cannot clearly see beyond 200 cm. Find the power of the lens needed to see objects at large distances.

Solution:
For the near sighted person, so, u = infinity, v = -200 cm= -2 m

=> 1/f = 1/v - 1/u

=> 1/f = -1/2 = -0.5

=> f = 100/3 cm = 1/3 m

The power of the lens is -0.5 D.

**Question 17:** A person wears glasses of power – 2.5 D. Is the person farsighted or near sighted? What is the far point of the person without the glasses?

**Solution:**
A person wears glasses of power − 25 D.
So, person must be near sighted, so, u = infinity, v = far point

we know, m = 1/f => f= 1/m

or f = 1/(-2.5) = -0.4 m = -40 cm

using lens formula:

=> $1/f = 1/v - 1/u$

=> $1/v = 1/(-40)$

=> $v = -40$

The far point for the near-sighted person is 40 cm.

**Question 18:** A professor read a greeting card received on his 50th birthday with + 2.5 D glasses keeping the card 25 cm away. Ten years later, he reads his farewell letter with the same glasses but he has to keep the letter 50 cm away. What power of lens should he now use?

**Solution:**

A professor read a greeting card received on his 50th birthday with + 2.5 D glasses keeping the card 25 cm away. (Given)

After 10 years,
$u = -50$ cm, $f = (1/2.5)$ D = 40 cm and v = near point

Now, $1/f = 1/v - 1/u$

=> $1/v = 1/200$

=> near point is 200 cm

To read the farewell letter at a distance of 25 cm,
$u = -25$ cm, $v = 200$cm

Using lens formula:

$f = 2/9$ m

So, required power of the lens = $p = 1/f = 4.5$ D

**Question 19:** A normal eye has retina 2 cm behind the eye-lens when the eye is
(a) fully relaxed,
(b) most strained?

**Solution:**

We know, lens formula, $1/f = 1/v - 1/u$

(a) When the eye lens is fully relaxed, u = infinity
The retina is 2 cm behind the eye-lens, so v = 2 cm = 0.02 m

Using lens formula: f = 0.02

Now, power of the eye lens = p = 1/f = 50 D

(b)When the eye lens is strained:

Distance of object, u = – 25 cm = – 0.25 m
and v = 2 cm = 0.02 m
Using lens formula:
f = 1/54

Now, power of the eye lens = p = 1/f = 54 D

**Question 20:** The near point and the far point of a child are at 10 cm and 100 cm. If the retina is 2.0 cm behind the eye-lens, what is the range of the power of the eye-lens?

**Solution:**

We know, lens formula, $1/f = 1/v - 1/u$

Near point of the child = u = 10 cm = 0.1
Far point of the child = 100 cm
Distance of the retina from the eye lens= v = 2cm = 0.02 m

Using lens formula: f = 1/60 m

Now, Power of the lens when the near sight is 10cm = p = 1/f = 60 D

When, the near point of the child is 100cm = 1m, so, u = -1m

Using lens formula: 1/f = 1/(0.02) - 1/-1 = 51

=> f = 1/51

Now, Power of the lens when near point 1m is p = 1/f = 51 D

So, the range of power of the eyes is +60D to +51D

**Question 21:** A near-sighted person cannot see beyond 25 cm. Assuming that the separation of the glass from the eye is 1 cm, find the power of lens needed to see distant objects.

**Solution:** The separation of the glass from the eye is 1 cm
For the near-sighted person, u = infinity

Distance of the image from glass is v = Distance between image and eye − distance between the glass and the eye
= 25 − 1 = 24 cm = 0.24 m

For glass, u = infinity and v = -24 cm = -0.24 m

Using lens formula, 1/f = 1/v - 1/u = 1/(-0/24) = -4.2 D

**Question 22:** A person has near point at 100 cm. What power of lens is needed to read at 20 cm if he/she uses
(a) contact lens,
(b) spectacles having glasses 2.0 cm separated from the eyes?

**Solution:**
Near point of the person = 100 cm
It is required to read at 20 cm in the following two cases:

(a) When the contact lens is used,
u = – 20 cm = – 0.2 m and v = – 100 cm = – 1 m

Using lens formula, 1/f = 1/v - 1/u

=> 1/f = 1/-1 - 1/(-0.2) = 4

Now, power of the contact lens = p = 1/f = 4D

(b)When the person uses spectacles at 2 cm from the eyes:

u = – (20 – 2) = – 18 cm = – 0.18 m and v = – 100 cm = –1 m
Using lens formula, 1/f = 1/-1 - 1/(-0.18) = 9/41

Now, power of spectacles = p = 1/f = 4.55 D

**Question 23:** A lady uses + 1.5 D glasses to have normal vision from 25 cm onwards. She uses a 20 D lens as a simple microscope to see an object. Find the maximum magnifying power if she uses the microscope
(a) together with her glass
(b) without the glass. Do the answers suggest that an object can be more clearly seen through a microscope without using the correcting glasses?

**Solution:**
Focal lens of the glasses = f = 1/p = (1/1.5) m
As,
Power of the glasses to have normal vision from 25 cm onwards, p = +1.5 D. (given)

and u = -25 cm = -0.25 m

Using lens formula . 1/f = 1/v - 1/u

=> 1/v = 1/f + 1/u =1/1.5 + 1/(-0.25) = 2.5

=> v = 0.4 m = 40 cm

When she is not wearing glasses, the near point is 40 cm. The focal length of magnifying glass at that point becomes: $f = 1/20 = 0.05$ m = 5 cm

(a) The maximum magnifying power with glass:

$m = 1 + D/f$

[Given D = 25 cm]

$= 1 + 25/5$

$= 6$

(b) Without the glasses, D = 40 cm

$m = 1 + D/f = 1 + 40/5 = 9$

**Question 24:** A lady cannot see objects closer than 40 cm from the left eye and closer than 100 cm from the right eye. While on a mountaineering trip, she is lost from her team. She tries to make an astronomical telescope from her reading glasses to look for her teammates.
(a) Which glass should she uses as the eyepiece?
(b) What magnification can she get with relaxed eye?

**Solution:**

A lady cannot see objects closer than 40 cm from the left eye and closer than 100 cm from the right eye.

For the left glass lens:
Least distance of clear vision, $u = -25$ cm
The lady cannot see objects closer than 40cm i.e. $v = -40$ cm

Using lens formula, $1/f = 1/v - 1/u$

$\Rightarrow 1/f = 3/200$

$\Rightarrow f = 66.6$ cm

Hence, the focal length of the left eye glass lens is 66.6 cm

For the right eye glass:
The lady cannot see objects closer than 40cm = v = – 100 cm
Least distance of clear vision =u = – 25 cm

Using lens formula:
$1/f = 1/v - 1/u$

$\Rightarrow 1/f = 3/100$

$\Rightarrow f = 33.3$ cm

Hence, the focal length of the right eye glass lens is 33.3 cm

(a) For an astronomical telescope, eye piece lens should have smaller focal length. so she should use right lens, f = 33.3 cm, as the eye piece lens.

(b) With relaxed eye,
$f_o = 200/3$ cm and $f_e = 100/3$ cm

So, magnification = $m = f_o/f_e = 2$