293

III

Total No. of Questions - 24
Total No. of Printed Pages - 4

Regd. No. | | | | | | | | | | |

Part - III

MATHEMATICS, Paper - II (B)

(Coordinate Geometry and Calculus)

(English Version)

Time : 3 Hours *Max. Marks : 75*

Note : This question paper consists of **three** sections *A*, *B* and *C*.

SECTION A 10 × 2 = 20

I. Very Short Answer Type Questions.

i) Attempt **all** questions.

ii) Each question carries **two** marks.

1. If $x^2 + y^2 + 2gx + 2fy = 0$ represents a circle with centre $(-4, -3)$, then find g, f and the radius of the circle.

2. If the length of the tangent from $(2, 5)$ to the circle $x^2 + y^2 - 5x + 4y + k = 0$ is $\sqrt{37}$, then find the value of k.

3. Find the angle between the circles $x^2 + y^2 + 4x - 14y + 28 = 0$, $x^2 + y^2 + 4x - 5 = 0$.

4. Find the equation of the parabola whose vertex is $(3, -2)$ and focus is $(3, 1)$.

5. If the angle between the asymtotes is 30^0, then find the eccentricity of the hyperbola.

6. Evaluate $\int Sec^2x \cdot Cosec^2x \, dx$.

7. Evaluate $\int \frac{e^x(1+x)}{Cos^2(xe^x)} dx$.

8. Evaluate $\int_0^4 |2-x| dx$.

9. Find the value of $\int_0^{2\pi} Sin^4x \cdot Cos^6x \, dx$

10. Form the differential equation of the curve $y = a\,Cos(nx+b)$ (where a, b are parameters).

SECTION B

$5 \times 4 = 20$

II. Short Answer Type Questions.

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. Show that $x + y + 1 = 0$ touches the circle $x^2 + y^2 - 3x + 7y + 14 = 0$ and find its point of contact.

12. If $x+y=3$ is the equation of the chord AB of the circle $x^2+y^2-2x+4y-8=0$. Find the equation of the circle having AB as diameter.

13. Find the equation of the ellipse in the standard form such that distance between foci is 8 and distance between directrices is 32.

14. Find the equation of the tangents to the ellipse $2x^2 + y^2 = 8$, which are (i) Parallel to $x-2y-4=0$ and (ii) Perpendicular to $x+y+2=0$.

15. Find the centre, foci, eccentricity and length of the latus rectum of the hyperbola $16y^2 - 9x^2 = 144$.

16. Evaluate $\int_0^{\pi/2} \frac{a\,Sin\,x + b\,Cos\,x}{Sin\,x + Cos\,x}\,dx$.

17. Solve the differential equation $\frac{dy}{dx} + \frac{4x}{1+x^2}y = \frac{1}{\left(1+x^2\right)^2}$.

SECTION C

5 × 7 = 35

III. Long Answer Type Questions.

i) Attempt **any five** questions.

ii) Each question carries **seven** marks.

18. Show that the four points $(1, 1), (-6, 0), (-2, 2), (-2, -8)$ are concyclic and find the equation of the circle on which they lie.

19. Show that the circles $x^2 + y^2 - 6x - 2y + 1 = 0$, $x^2 + y^2 + 2x - 8y + 13 = 0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.

20. Show that the equation of common tangents to the circle $x^2+y^2=2a^2$ and the parabola $y^2=8ax$ are $y=\pm(x+2a)$.

21. Evaluate $\int \frac{2x+5}{\sqrt{x^2-2x+10}}\,dx$.

22. Obtain the reduction formula for $I_n=\int Sec^n x\,dx$, n is a positive integer, $n\geq 2$ and deduce the value of $\int Sec^5 x\,dx$.

23. Show that the area of the region bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (ellipse) is πab. Also deduce the area of the circle $x^2+y^2=a^2$.

24. Solve the differential equation $(x^2-y^2)dx-xy\,dy=0$.