Chapter - 1

Electric Field

Electrostatical phenomena can be observed in many ways. It is a common experience for us that when a glass rod is rubbed by a silk cloth it acquires a property of attracting tiny bits of paper. Also, if an air filled baloon after being rubbed by cloth put into contact with a wall it remains cling to the wall for quite a long time. All such phenomena result from the forces between charges at rest. This chapter and next two chapters are devoted to electrostatics which is the study of effects of interactions between charges at rest.

In this chapter we are going to study about electric charges and their properities, force between two charged objects and concepts related with electric field and electric dipole. The study of electrostatics is important not only from concenptual aspects but it has a number of applications which includes, photocopying mechine, computer printer, electrostatic memory and seismograph.

1.1 Electric charge

According to history, Thales of Miletus in Greece is said to have discovered around 600 BC that when amber was rubbed with woolen cloth it would attract tiny pieces of straw, feathers etc. The greek word for amber is electron and from this root word comes the word electricity . Similar effects were observed on rubbing a glass rod with silk or an eboniterod with cat skin. Substances in such states are said to be electrified or electrically charged. In examples cited above the objects were charged (electrified) due to friction and thus the effect is termed as frictional electricity. However, as we will see shortly that there are other ways also to charge a given object.

An object when electrified beheves somewhat different than when it is uncharged, and it can be said that the object has acquired a characteristic property (charge). This characteristic property of an electrified object is termed as electric charge.

Charge is an intrisic property of elementary particles which constitutes the matter, i.e. it is a property that comes automatically with such particles wherever they exist. Although a formal definition of charge can not be given and it can be understood in terms of it effects. However it can be said that "charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects."

1.1.1 Types of Charges

From a number of experiments it was found that there are two kinds of elcetric charges, which are given names positive and negative charges. To determine the type of charge we perfrom the following experiment, experimental setup for which is depicted in Figure 1.1.

Consider a glass rod that has been rubbed on silk is suspended by a thread. If we bring a second, similarly charged glass rod near by, the two rods repel each other; that is, each rod experiences a force directed away from the other rod. Like wise if we suspend an ebonite rod that has been rubbed on catskin and bring a second similarly charged ebonite rod near by, again the two rods repel each other. However if we rub an ebonite rod with catskin and suspend it using a thread and then bring a glass rod rubbed on silk near by, the two rods attract each other.

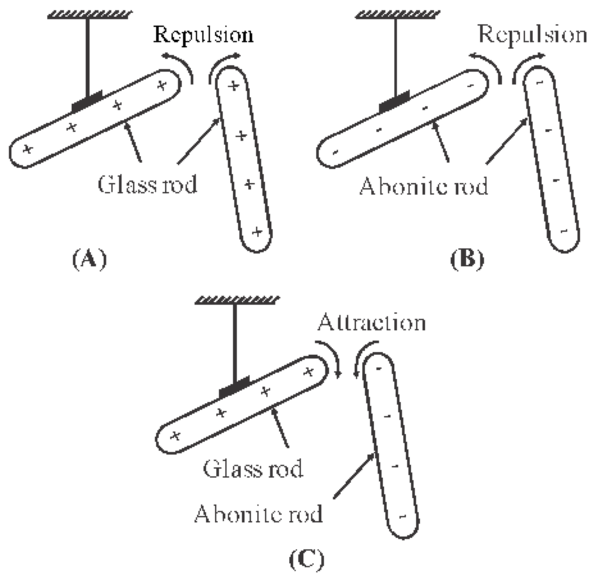


Fig. 1.1 Experimental setup for determining the types of charges

From above experimental observations we can conclude that two glass rods which have been rubed on silk have same type (sign) of charge and hence the charge of same type (sign) repel each other. Likewise, the two ebonite rods which have been rubbed on cat skin have same type of charge and repel each other. However, the type of charge on glass rod and that on ebonite rod are of opposite signs indicated by the fact that there is attraction between them.

The 'positive' and 'negative' names and signs for electric charges were given by Benjamin Franklin. Franklin arbitrarily chosed the type of charge on glass rod rubbed on silk as positive. From experimental observations the conclusion is that "charges with same sign repel each other and the charges with opposite signs attract each other." Thus all other charged objects which are repelled by such a positively charged glass rod must have positive charge and all such charged objects which are attracted to a positively charged glass rod must have negative charge.

According to modern view all matter is composed of atoms. Every atom consits of a nucleus (composed of neutrons and proptons) and electrons. Protons are positively charged, electrons are negatively charged and neutrons are electrically neutral. In atoms, the number of electrons is equal to the number of protons and atoms are neutral or uncharged. As the matter is composed of atoms the same is also true for matter. If in an object there is excess of electrons over it neutral configuration it is said to be negatively charged and if there is a deficiecny of electrons it is said to be positively charged.

Materials throught which charge (generally electrons) can flow freely are called conductors e.g. copper. Materials through which charge can not flow are called insulators or dielectrics e.g. glass, plastic and ebonite. Now let us discuss in brief the methods of charging various objects.

1.1.2 (a) Charging by friction

We have seen the process of charging by frication in experiment described earlier (Fig 1.1). When the two bodies are rubbed together the electricity so produced is called as frictional electricity. In this process a transfer of electrons take place from one body to another. The body from which electrons have been transferred is left with a deficiency of electrons so it gets positively charged and the body which recieves electrons becomes negatively charged. For example, when a glass rod is rubed with silk its gets positively charged while the piece of silk gets equal negative charge. This happens due to transfer of electrons from glass to silk piece at the point of contact. In the process of rubbing though the number of contact points increases, there by amount of charge transferred increases, however it is worth noting that amount of charge transferred in the process is quite small.

In the table presented below on rubbing objects mentioned in column I with objects mentioned in column II, object mentioned in column I gets positively charged and the object belonging to column II gets negatively charged.

Table 1.1

I (+)	II (-)
Glass rod	Silk cloth
Catskin	(i) plastic rod
	(ii) ebonite rod
Woolen cloth	(i) amber
	(ii) plastic
	(iii) ebonite
	(iv) rubber

If in place of a glass rod we take a copper rod in hand and rub it with some woolen cloth then the charge tranferred from woolen cloth to the rod flows through our body to the ground and the conducting rod does not get charged. However if we hold the conducting rod using an insulating handle and then rub it with woolen cloth the conducting rod can be charged. Here the insulating handle does not allow charge to flow through the body to the ground.

1.1.2 (b) Charging by Conduction (contact)

As we here mentioned, conductors are materials in which electric charge moves quite freely. When some charge is given to a conductor, it quickly redistributes it self over the entire outer surface of the conductor, however it is not so for insulators. If some charge is given to an insulator it remains at the place where it was given. This difference in beheviour of conductor and insulator will be explained in next chapter.

The direct transfer of charge from one object to another object in contact is called charging by contact. Conduction from a charged object involves transfer of like charges. Consider two conductors, one charged and another uncharged as shown in Fig. 1.2 . Bring the conductors in contact with each other. The charge (whether positive or negative) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with same sign. This is called as charging by conduction (through contact).

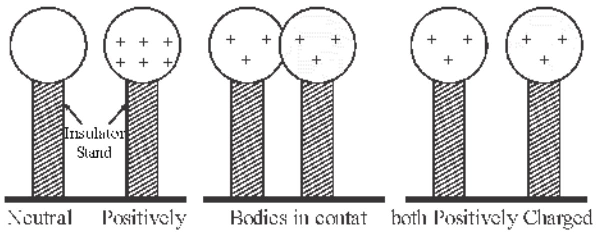


Fig. 1.2 Charging by Conduction

1.1.2 (c) Charging by Induction

The process under which a charged object induces an opposite type of charge on another object without coming into contact with it is called charging by electrostatic induction.

Fig. 1.3 shows an example of charging by induction. An uncharged metal ball is supported on an insulating stand [Fig. 1.3 (a)]. When we bring a negatively charged rod near it, without actually touching it [Fig. 1.3 (b)], the free electrons in the metal balls are repelled by the negative charge on the rod and they shift toward the right, away from the rod. They can not escape the ball because the supporting stand is insulator. So we get excess negative charge at the right surface of the ball and a deficiency of negative charge (electrons) i.e. a net positive charge at the left surface. These excess charges are called induced charges. However, the ball is still electrically neutral.

When you contact one end of a conducting wire to the right surface of the ball and the other end to the earth [Fig 1.3 (a)] the negative charge (electrons) flows through the wire to the earth. Now suppose we disconnect the wire [Fig 1.3 (d)] and then remove the rod [Fig 1.3 (e)] a net positive charge is left on the ball. The charge on the negatively charged rod has not changed in this process. The earth acquires a negative charge that is equal in magnitude to the induced positive charge remaining on the ball.

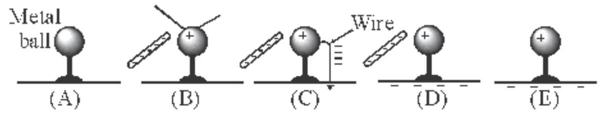


Fig. 1.3 Charging a metal ball by induction

On bringing a positively charged rod near the metal ball and repeating above steps the ball can be negatively charged.

How a charged object (positive or negative) attracts an uncharged object:

When an uncharged object is brought near the charged object electrostatic induction takes place. As a result, the near end of the uncharged object acquires opposite type of charge and hence attraction takes place between two unlike charged objects. The other (far) end of uncharged object acquires similar charge, hence there is force of repulsion between two like charges, but this force is weak (compared to the attractive force) because of larger distance. Thus the net force between a charged and uncharged objects is attractive. As an example we can note that, after combing dry hair with a plastic comb and if we take it near the tiny bits of paper they are attracted by it.

Following points are worth noting regarding charging an object

(1) In charging, the mass of body changes. Consider two identical metallic spheres of excetly the same mass. One is given a positive and the other an equal negative charge. Their masses after charging are different with negative charged sphere having greater mass (in principle). This is because the negatively charged sphere has gained additional electrons so its mass is increased while the positively charged sphere has lost some electrons causing a decrease in mass. However, this increase or decrease in mass is negligibly small owning to the very small mass of electrons.

(2) The true test of electrification is repulsion and not attraction as attraction may also takes place between a charged and an uncharged object.

(3) Charge can be detected or measured with the help of gold leaf electroscape, elecrometer, or ballastic

golvameter.

(4) When X-rays (electromagnetic waves having wavelength between 0.1A to 10 A) are incident on a metal surface eletrons are ejected. Thus the surface becomes positively charged.

1.1.3 Electroscape

A simple apparatus to detect charge on an object is the gold leaf electroscape. It is a very sensitive apparatus.

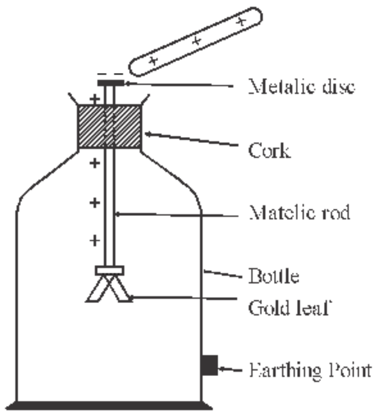


Fig. 1.4 Gold-leaf electroscape

As depicted in Fig. 1.4 in a gold-leaf electroscape a vertical metal rod is enclosed in a glass jar with two thin gold leaves attached to its lower end. The upper end of the rod is connected to a conducting disc. When a charged object touches the metal disc charge flows on to the gold leaves which then spread apart because of electrical repulsion between their charges. The degree of divergence is an indicator of the amount of charge.

If a charged object is brought near a charged electroscape the leaves will further diverge if the charge on the object is similar to that on the electroscape and will usually converge if opposite. In this menner we can determine the nature of charge on an object.

1.1.4 Unit of charge

In SI units current (I) is assumed to be a fundamental quantity with ampere (A) as unit. Since I = charge/time so charge is a derived quantity. In SI unit, the unit for charge is coulomb and is denoted by C.

$1\ C = 1\ As$

and dimensions $[Q] = M^{0}L^{0}T^{1}A^{1}$

Since coulomb is a relatively large unit so follwoing units are also used for charge.

$1\ \mu C = 10^{\ 6}\ C$

$1\ nC = 10^{-9}\ C$

$1\ pC = 10^{\ 12}\ C$

In CGS unit the charge is expressed in stat coulomb (esu) also called franklin.

$1\ C = 3 \times 10^{9}\ esu$

A practical unit of charge is Faraday (and not farad)

1 Faraday = 96500 C

1.2 Properties of Charge

We have seen that the charges are of two types positive and negative and they tend to cancel each other. Here, we are discussing some other important properities of electric charge.

1.2.1 Additivity of Electric charges

Charge is a scalar quantity. Electric charge is additive and the net charge in a system is given by the algebraic sum of the charges present within. Special care must be taken regarding the sign of charges while adding. For exapmle if three chages +3q, -4q and +5q are given to an object the net charge on the object is +4q. If the sum of the charges on a object is zero the object is said to be electrically neutral. Here it is worth noting that mass is a scalar quantity but it can have positive values only.

1.2.2 Invarience of Electric Charge

Electric charge is independent of the choice of the frame of reference. In other words the charge on an object is independent of the speed of the object or the observer. i.e. the value of charge (q) on a particle is independent of the velocity of object. Charge on object at rest = charge on this object in motion i.e.

$q_{rest} = q_{motion}$

This property is worth mentioning as in contrast to charge, mass of a body depends on its speed. According to Einstein's special theory of realtivity (about which you will learn in higher classes) at speeds comparable to the speed of light ($v \approx c$) the mass of a particle becomes many times larger than its rest mass but charge does not change. The ratio q/m of charge q and mass m of a particle is called it specific charge, this depends on speed and at high speeds its value decreases.

1.2.3 Conservation of Electric charge :

According to this "the net charge on an isolated system is always conserved and it does not change even if some interaction or process is being completed in the system. In other words" charge can neither be created nor destroyed, it can only be transferred.

Illustrations

(i) In the example of frictional electricity, both the glass rod and silk cloth are uncharged (neutral) before rubbing them together. When they are rubbed together a positive charge appears on the rod and a negative charge of equal magenitude appears on the silk. In this process few electrons are transferred from the glass rod to the silk so silk cloth gets negatively charged while glass rod is positively charged by the same amount. Here the glass rod and the silk cloth forms a composite uncharged (neutral) system. Intially both are neutral, after they are rubbed together the charges of equal magnitudes but opposite sign appear on these two objects so the net charge on the system is still zero.

(ii) When an electron (whose charge is -e) and its antiparticle, the position (whose charge is +e), under go an **annihiliation** process they transform into two **gamma rays** (high frequecny electromagnetic wave) which are neutral. In this process total charge before annihiliation was (-e) + e = 0 and after the process is zero again. Thus the charge is conserved. This process is written as -

$$\underset{\text{(Electron)}}{e} + \underset{\text{(Positron)}}{e} \longrightarrow \gamma(1.02\,\text{MeV})$$

In pair production, the converse of annihilation charge is also conserved. In this process a gamma ray transforms into a positive and an electron.

$$\gamma(1.02\,\text{MeV}) \longrightarrow \underset{\text{(Electron)}}{e} + \underset{\text{(Positron)}}{e'}$$

(iii) Charge is also conserved in radioactive decay and nuclear reactions. You will learn more about these in a later chapter. Few examples are cited below

Radioactive decay

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}(\alpha\text{-particle})$$

$$Q_i = 92e \qquad Q_f = 90e + 2e = 92e$$

Nuclear reaction:

$${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{18}_{9}\text{F} \rightarrow {}^{17}_{8}O + {}^{1}_{1}\text{H}$$

$$7e + 2e \rightarrow 9e \rightarrow 8e + e$$

$$\text{Qi} = 9e \qquad Qs = 9e$$

The hypothesis of conservation of charge first put forward by Benjamin Frankilin is empirical with no known exceptions so far.

1.2.4 Quantization of charge

When a physical quantity can have only distrete values rather than any value, the quantity is said to be quantised. The minimum value that this quantity can have is called as the quantum of that quantity.

When two insulators are rubbed together these get charged due to exchange of electrons. The exchage of electrons always takes place in whole numbers. The minimum number of electrons that can be exchanged is unity therefore the total charge on an object must be an integral multiple of electronic charge. This was established experimentally by Millikan by his famous oil drop experiment.

From the observations of Millikan oil drop experiments that the smallest charge that can exists in nature is the charge of an electron which is equal to 1.6021×10^{-19} C. It is common to consider its value to be 1.6×10^{-19} C for the purpose of calculations. If the charge on an electron e is taken as the elementary unit i.e. quantum of charge than charge on any object can be expressed as -

$$q = \pm ne \text{ with } n = 1, 2, \text{-------}$$

and charge on an object can never be $\pm$ 1.2 e, $\pm$ 1.6e, $\pm$ 2.3 e, etc.

The quantum of charge is so small that when electricity is studied on a macroscopic scale the grainness of electricity does not show up and charges appears to be continuous. For explanation of internal structures of protons and neutrons these are assumed to be composed of particles (called quarks) having charges $\pm 2/3$ (e) and $\pm 1/3$ (e). However quarks do not exist in free state, the quantum of charge is still e.

Some important facts regarding the charge are as follows -

(i) Charge is always associated with mass, i.e, charge cannot exist without mass though mass can exist without charge. Photon is a particle which is both massless and chargeless while neutron has no charge but a finite mass. In general each charged particle has some mass.

(ii) A stationary charge produces only electric field in its surrounding space. If a charge particle is moving at a uniform velocity it produces both electric and magnetic fields but does not radiate evergy. An accelerated charge not only produces electric and magentic fields but also radiate energy in the form of electromagentic waves in spece surrounding it.

Example 1.1 - How many electrons are to be removed from a metallic sphere in order to positively charge it with 1 C charge.

Solution : Use q = ne

Here $q = 1$ C

$$n = \frac{q}{e}$$

$$n = \frac{1}{1.6\times10^{-19}} = 6.25\times10^{18} \text{ electrons}$$

Example 1.2 : A body is charged such that its mass increases by 9.1 ng then

(i) How many electrons were given to the body

(ii) Determine the value of charge and its nature

Solution : Here, the change in mass

$$\Delta M = 9.1\times10^{-9}g = 9.1\times10^{-12}\,kg$$

and mass of electron

$$m_e = 9.1\times10^{\ 31}\,kg$$

(i) As $\Delta M = nm_e$

$$n = \frac{\Delta M}{m_e}$$

$$n = \frac{9.1\times10^{-12}}{9.1\times10^{\ 31}} = 10^{19} \text{ electrons}$$

(ii) Value of charge $q = ne$

$$q = 10^{19}\times1.6\times10^{-19}$$

$$q = 1.6C$$

As the body is receving electrons it must be negatively charged.

Example 1.3 - Calculate the amount of positive and negative charges present in a cup of water (250gm)

Solution -Here the mass of water m = 250 gm

Molecular mass of water M = 18 gm

Number of Molecular in one cup of water

$$N = \frac{m}{M}\times N_A$$

(Here N_A is Avagaro Number)

$$N = \frac{250}{18}\times6.023\times10^{23}$$

As one molecule water consits of two hydrogen and one oxygen atom so one molecule of water contains 10 protons and 10 electrons. Electrons and protons have equal but opposite charge.

So amount of positive (or negative) charge in one cup of water -

$$q = N\times10e$$

$$q = \frac{250}{18}\times6.023\times10^{23}\times10\times1.6\times10^{-19}$$

$$q = 1.337\times10^{7}\,C$$

1.3 Coulomb's law

Based on experiments in 1875 Coulomb put forward a law regarding the electric forces acting betweem two point charges at rest. This law is known as Coulomb's law and according to it " the magnitude of the electric force (of repulsion or attraction) acting between two point charges at rest is directly proportional to the product of magenitudes of the charges and inversely proporional to the square of the distance between them. This force acts along the line joining the two charges and depends on the nature of medium between the charges. This law is also termed as Coulomb's inverse square law.

If two poitns charges q_1 and q_2 are at a distance r apart then.

$$F \propto q_1 q_2 \qquad \ldots (1.1)$$

and $$F \propto \frac{1}{r^2} \qquad \ldots (1.2)$$

i.e. $$F \propto \frac{q_1 q_2}{r^2} \qquad \ldots$$

or $$F = k\frac{q_1 q_2}{r^2} \qquad \ldots (1.3)$$

Where k is a proportionality constant whose numerical value depends on the system of units used and the nature of the medium present between the charges.

For vacuum (free space) or air medium in SI units.

$$k = \frac{1}{4\pi \in_0} = 9\times 10^9 \frac{Nm^2}{C^2}$$

Where $\in_0$ (epsilon - knot or epsilon zero) called the permitivity of free space.

$$\in_0 = 8.854\times 10^{-12} C^2 / Nm^2$$

Hence for free space or air

$$F = \frac{1}{4\pi \in_0}\cdot\frac{q_1 q_2}{r^2} \qquad \ldots (1.4)$$

Dimensions of $\in_0$

$$\in_0 = \frac{[Q]^2}{[F][Length]^2} = \frac{T^2A^2}{MLT^{-2}L^2}$$

$$\therefore \quad [\in_0] = M^{-1}L^{-3}T^{4}A^2$$

If $q_1 = q_2 = 1$ C

and $r = 1$ m

Then $F = \dfrac{1}{4\pi \in_0} = 9\times 10^9$ N

Thus, if the force acting between two equal charges placed in free space or air at a distance of 1m apart is 9×10^9 N then the magnitude of each charge is 1 Coulomb.

For a medium other than freespace (or air)

$$F_{medium} = \frac{1}{4\pi \in}\cdot\frac{q_1 q_2}{r^2} \qquad \ldots (1.5)$$

Here $\in$ is called permitivity of the medium.

1.3.1 Dielectric Constant (Relative permitivity)

It has been found from experiments that if two charges are kept in different media at a given separation then the force acting between them changes with the change in medium. The force is maximum for free space or air as medium while for insulator medium it is relatively small, in presence of a conducting medium the force reduces to zero.

Hence in presence of a medium the factor by which the force is reduced compared to its value in free space is termed as relative permitivity, dielectric constant or specific inductive capacity of medium. It is denated by $\in_r$.

$$\in_r = \frac{\text{Force between the charges in vacuum } (F)}{\text{Force between the charges in that medium } (F_m)}$$

$$\in_r = \frac{\frac{1}{4\pi \in_0}\cdot\frac{q_1 q_2}{r^2}}{\frac{1}{4\pi \in}\cdot\frac{q_1 q_2}{r^2}} = \frac{\in}{\in_0}$$

$\in_r = \dfrac{\in}{\in_0}$ is a dimensionless quantity

Many times symbol K is used in place of $\in_r$ Insulators are also called dielectrics.

Values of relative permitivity for some common dielectric materials are shown in table 1.2

Table 1.2 : Realtive permitivities of some dielectrics (at 20° C)

Medium	Dielectric Constent	Medium	Dielectric Constent
Air	1.00059	Glycerene	42.5
Glass	5 to 10	Rubber	7
Mica	3 to 6	Oxygen	1.00053
Parefinn wax	2 to 2.5	Conductor	-----(∞)
Distilledwater	80		
Free space	1		

1.3.2 Vector Form of Coulomb's law

As force is a vector quantity it is useful to write Coulomb's law in vector form. For this, let us consider that realtive to some arbitarily chosen origin the position vectors of point charges q_1 and q_2 placed in free space are r_1 and r_2 respectively. According to Fig (1.5) the position vectorof charge q_2 relative to charge q_1 is then.

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

and $|\vec{r}_{21}| = |\vec{r}_2 - \vec{r}_1|$

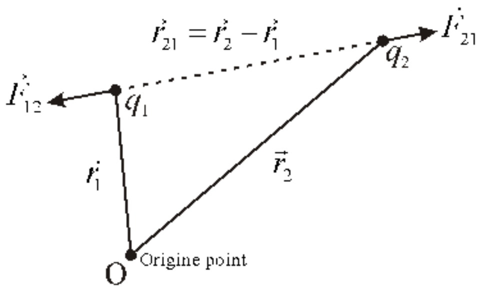


Fig. 1.5 Vector representation of Coulomb's law

From Coulomb's law the electric force on charge q_2 due to charge q_1

$$\vec{F}_{21} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{|\vec{r}_{21}|^2} \hat{r}_{21}$$

Where $\hat{r}_{21}$ is a unit vector directed from q_1 to q_2.

Accordingly $\vec{F}_{21} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{|\vec{r}_{21}|^3} \vec{r}_{21}$

or $\vec{F}_{21} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$ ...(1.6)

Likewise the force on charge q_1 due to charge q_2

$$\vec{F}_{12} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{|\vec{r}_{12}|^3} \vec{r}_{12}$$

Where $\hat{r}_{12}$ is a unit vector directed from q_2 to q_1. Accordingly

$$\vec{F}_{12} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_1 - \vec{r}_2) \quad \text{...(1.7)}$$

as $\hat{r}_{12}$ and $\hat{r}_{21}$ are directed opposite to each other, therefore $\vec{F}_{21} = -\vec{F}_{12}$

Thus it is clear that for two point charges the force which one charge exerts on the other is equal and opposite to the force which the other charge exerts on the first (whatever the signs of charges may be). Thus the force is an action-reaction pair or Coulomb's law is consistent with Newton's third law. This force acts along the line joining the two charges i.e. electrostatic force is a central force in nature.

Important facts

1. Strictly specking Coulomb's law is valid for point charges at rest. If the point charges are in motion the Coulomb's law can not account for the force acting between them as now in addition to electric force, magnetic force also acts between the charges.
2. When the charges are sperated by 10^{-15} m or less the Coulomb's law is not appicable as now nuclear force also acts between the charges.
3. The force between two charges is not affected by the presence of other charges, therefore the Coulomb force is a two body interaction. Therefore the principle of linear superposition is applicable for Coulomb forces (see section 1.4).
4. Coulomb's law is inverse square law and Coulomb force is conservative in nature.

Example 1.4 - In hydrogen atom the separation between the electron and proton is 5.3×10^{-11} m. Calculate the force of attraction between them. Compare this force with the gravitational force acting between them $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$, electronic charge $e = 1.6 \times 10^{-19} C$ mass of electron $m_e = 9.1 \times 10^{-31}$ kg, mass proton $m_p = 1.67 \times 10^{-27}$ kg.

Solution : Electrostatic force of attraction between electron and proton

$$F_e = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$$

$$F_e = 9 \times 10^9 \times \frac{\left(1.6 \times 10^{-19}\right)\left(1.6 \times 10^{-19}\right)}{\left(5.3 \times 10^{-11}\right)^2}$$

$$F_e = 8.2 \times 10^{\ 8} N$$

Gravitational force between electron and proton

$$F_g = G \frac{m_e m_p}{r^2}$$

$$F_g = \frac{6.67 \times 10^{-11} \times \left(9.1 \times 10^{-31}\right)\left(1.67 \times 10^{-27}\right)}{\left(5.3 \times 10^{-11}\right)^2}$$

$$F_g = \frac{101.36}{28.09} \times 10^{-47} = 3.6 \times 10^{-47}\ N$$

$$\frac{F_e}{F_g} = \frac{8.2 \times 10^{-8}}{3.6 \times 10^{-47}} = 2.27 \times 10^{39}$$

Therefore the electrostatic force is 2.27×10^{39} time large than the gravitational force.

Example 1.5 - Two positive ions of same charge repel each other by a force of $3.7 \times 10^{-9}\ N$ when they are 5Å apart. How many electrons are less on each ion compared to their neutral atom state.

Solution : Let the charge on each ion = q

Here $r = 5\text{Å} = 5 \times 10^{-10}\ m$

Force $F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$ Here $q_1 = q_2 = q$

$$F = \frac{1}{4\pi \in_0} \frac{q^2}{r^2}$$

or $3.7 \times 10^{-9} = 9 \times 10^9 \frac{q^2}{\left(5 \times 10^{-10}\right)^2}$

$$q = \sqrt{\frac{25 \times 3.7}{9} \times 10^{-38}}$$

$$q = \frac{5}{3} \times 1.92 \times 10^{-19}$$

$$= \frac{9.6}{3} \times 10^{\ 19} = 3.2 \times 10^{\ 19}\ C$$

$q - ne$ (n = number of electron)

Example 1.6 - The force between two point charges placed in free space is 18N. Keeping the same seperation between them if these charge are placed in glass medium of dielectric constant 6. Calculate the force acting between them.

Solution : $F_m = \frac{F}{\in_r}$

Here $F = 18\,N$ $\in_r = 6$

$$F_m = \frac{18}{6} = 3N$$

Example 1.7 - A point charge $q_1 = 2\mu C,\ (2m,\ 1m)$ is located at (2m, 1m) and another point charge $q_2 = -5\mu C,\ (-2m,\ 4m)$ is located at (-2m, 4m). Determine the force on q_2 due to q_1

Solution : As per question

$q_1 = 2\mu C,\ q_2 = -5\mu C$, $\vec{r}_1 = 2\hat{i} + 1\hat{j}\,m$ and

$\vec{r}_2 = -2\hat{i} + 4\hat{j}\,m$.

As the charges are of opposite signs so force on q_2 due to q_1 is of attractive nature and acts towards q_1 i.e. from point (-2, 4) to (2, 1) along $\hat{r}_{12}$

$$\vec{F}_{21} = \frac{k|q_1||q_2|}{r^2}\hat{r}_{12} = \frac{k|q_1||q_2|}{|\vec{r}_1 - \vec{r}_2|^2}(\hat{r}_{12})$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 5 \times 10^{-6}}{\left|\left(2\hat{i} + 1\hat{j}\right) - \left(-2\hat{i} + 4\hat{j}\right)\right|^3}\left[\left(2\hat{i} + 1\hat{j}\right) - \left(-2\hat{i} + 4\hat{j}\right)\right]$$

$$= \frac{90\times10^{3}}{\left|4\hat{i}-3\hat{j}\right|^{3}}\left(4\hat{i}-3\hat{j}\right) = \frac{90\times10^{3}}{125}\left(4\hat{i}-3\hat{j}\right)$$

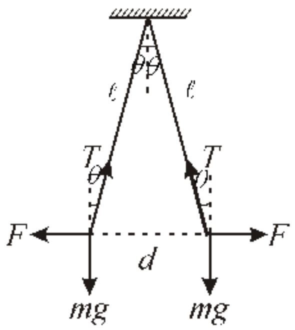
$$= 7.2\times10^{4}\left(4\hat{i}-3\hat{j}\right)\text{N}$$

Example 1.8 - Two small point like spheres, each having a mass of 200 g are suspended from a common point by insulating strings of length 40 cm each. The spheres are identically charged and the separation between them at equilibrium is found to be 4 cm. Find the charge on each sphere.

Soultion : The forces acting on two spheres are shown in figure. As each sphere is in equilibrium the net force on each is zero. So

$$T\cos\theta = mg \qquad \ldots\text{(i)}$$

and $$T\sin\theta = F = \frac{kq^2}{d^2} \qquad \ldots\text{(ii)}$$



From equations (i) and (ii)

$$\tan\theta = \frac{kq^2}{mgd^2}$$

As $d(=4cm),\ \ell(=40cm)$

$$\tan\theta \approx \sin\theta = \frac{d/2}{\ell} = \frac{2}{40} = \frac{1}{20}$$

hus $$q = \sqrt{\frac{mgd^2\sin\theta}{k}} = \sqrt{\frac{0.2\times10\times16\times10^{4}}{9\times10^{9}}\times\frac{1}{20}}$$

$$= \frac{4}{3}\times10^{-7}\text{ C}$$

1.4 Force among Many Charges and Superposition Principle

The electric force acting between two point charges does not affect by presence of other charges near by, therefore the force on a point charge at rest due to two or more stationary point charges is obtained by the supper position principle. According to this pricniple "the force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to other charges taken one at a time.

If the force acting on charge under question due to other n charges are $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots \vec{F}_n$ respectively then the net force acting on it is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

Consider a system of n charges at rest placed in free space. Let the charges be $q_1, q_2 \ldots q_n$ respectively and we wish to determine the net force due to this system of charges on a charge q_o. Let the position vector of $q_1, q_2 \ldots q_n$ etc and relative distance from q_o to are $\vec{r}_{01}, \vec{r}_{02}, \vec{r}_{02} \ldots \vec{r}_{0n}$ respectively (see Fig. 1.6). If the force on q_0 due to q_1 is represented by $\vec{F}_{01}$ then

$$\vec{F}_{01} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_0}{\left|\vec{r}_{01}\right|^2}\hat{r}_{01}$$

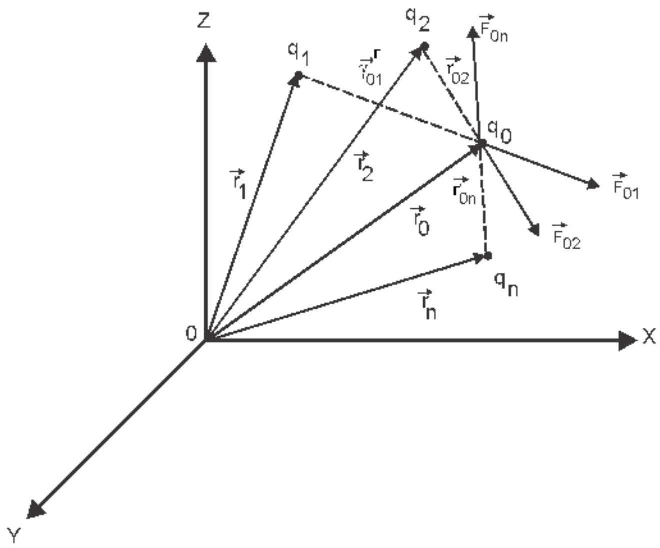


Fig 1.6 : Force on q_0 due to a number of point charges

Where $\hat{r}_{01}$ is a unit vector directed from q_1 to q_0. Similary if the forces acting on q_0 due to other charges are $\vec{F}_{02}$, $\vec{F}_{03}...\vec{F}_{0n}$ respectively then from the principle of superposition, the net force acting on q_0

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ... + \vec{F}_{0n}$$

$$\vec{F}_0 = \frac{1}{4\pi \in_0} \frac{q_1 q_0}{|\vec{r}_{01}|^2} \hat{r}_{01} + \frac{1}{4\pi \in_0} \frac{q_2 q_0}{|\vec{r}_{02}|^2} \hat{r}_{02} + ... + \frac{1}{4\pi \in_0} \frac{q_n q_0}{|\vec{r}_{0n}|^2} \hat{r}_0$$

$$\vec{F}_0 = \frac{1}{4\pi \in_0} q_0 \left[\frac{q_1}{|\vec{r}_{01}|^2} \hat{r}_{01} + \frac{q_2}{|\vec{r}_{02}|^2} \hat{r}_{02} + ... + \frac{q_n}{|\vec{r}_{0n}|^2} \hat{r}_{0n} \right]$$

$$\vec{F}_0 = \frac{1}{4\pi \in_0} q_0 \sum_{i=1}^{n} \frac{q_i}{r_{0i}^{\ 2}} \hat{r}_{0i} \qquad ...(1.8)$$

To determine the net force, paralleogram law of forces or polygon law of forces may be used according to situation.

Example 1.9 Two point charges of 9e and 4e are at a distance *r* apart. Where on the line joining the two charges another charge *q* is placed such that it ramins in equilibrium?

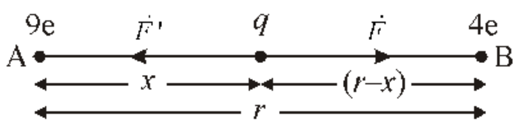
Solution : The pictorial representation of the problem is shown in figure shown below. For equilibrium of q, net force on it must be zero.

$$\vec{F} + \vec{F}^1 = 0$$

$$\vec{F} = -\vec{F}^1$$

However in magnitude

$$F = F'$$



$$\frac{kq9e}{x^2} = \frac{kq4e}{(r-x)^2} \Rightarrow \frac{9}{x^2} = \frac{4}{(r-x)^2}$$

on taking square root

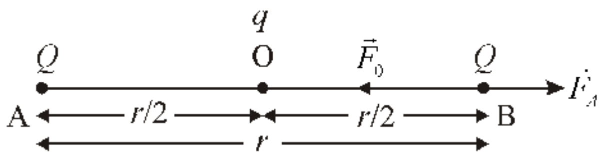
$$\frac{3}{x} = \frac{2}{(r-x)} \quad \text{or} \quad 2x = 3r - 3x$$

$$5x = 3r \Rightarrow x = \frac{3}{5} r$$

It should be placed at 3/5 *r* from charge 9*e*.

Example 1.10 Two identical charges Q are placed r distance apart. At the mid point on the line joining them another charge q is placed. What should be its magnitude and sign so that the entire system is in equilibrium?

Solution :



As the charge q is on the mid point on the line joining the two charges Q each, due to symmetry force on q is always zero. For the equilibrium of entire system it is essential that the force on remining charges a each at A and B must be zero for charge Q at B to be in equilibrium to be a equilibrium

$$\vec{F}_A + \vec{F}_0 = 0$$

$$\vec{F}_A = -\vec{F}_0$$

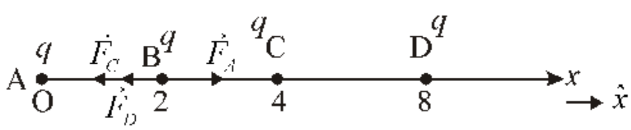
$$\text{or} \quad \frac{kQQ}{r^2} = -\frac{kqQ}{(r/2)^2}$$

$$\frac{Q}{r^2} = -\frac{4q}{r^2} \Rightarrow Q = -4q$$

$$\Rightarrow \qquad q = -Q/4$$

Example 1.11 Four identical point charges each 2 µC are placed on axis at positions $x = 0, 2, 4, 8$ cm respectively. Determine the resultant force acting on the charge placed at $x = 2$ cm.

Solution : According to question



Force on charge at B due to charge at A

$$\vec{F}_A = \frac{kqq}{(2\times10^{-2})^2}\hat{i} = \frac{9\times10^9\times2\times10^{-6}\times2\times10^{-6}}{4\times10^{-4}}\hat{i}$$

$\vec{F}_A = 90\hat{i}$ N

force on charge at B due to charge at C

$$\vec{F}_C = \frac{9\times10^9 \times 2\times10^{-6} \times 2\times10^{-6}}{4\times10^{-4}}\left(-\hat{i}\right) = -90\hat{i} \text{ N}$$

force on charge at B due to charge at D

$$\vec{F}_D = \frac{9\times10^9 \times 2\times10^{-6} \times 2\times10^{-6}}{\left(6\times10^{-2}\right)^2}\left(-\hat{i}\right) = -10\hat{i} \text{ N}$$

Therefore the net force on charge at B

$$\vec{F}_B = \vec{F}_A + \vec{F}_C + \vec{F}_D$$

$$= 90\hat{i} + \left(-90\hat{i}\right) + \left(-10\hat{i}\right) = -10\hat{i} \text{ N}$$

$$\vec{F}_B = 10\ (-\hat{i}\ ) \text{ N}$$

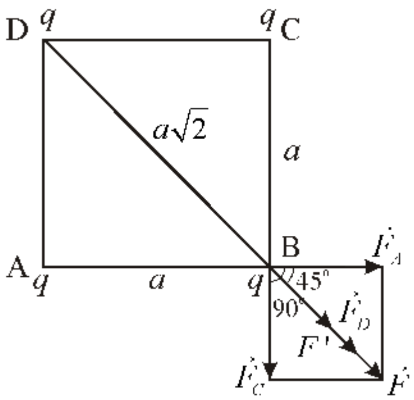
So the net force is 10N in negative x direction.

Note : [In this problem from symmetry it is obvious that force on B due to charges at A and C cancel out so the net force on B is due to charge at D only]

Example 1.12 : Four identical charges q each are placed at the four verticles of a square of side a. Determine the magnitude of net force on each charge due to remaining charges.

Soultion : The situation pertaining to the question is depicted in adjoining figure. Here we determine the force on charge at B due to remaining charges. From symmetry forces acting on charges placed at other points due to remaining charges will be equal in magnitude but differ in directions. From fig.

$$BD = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$



also $\quad \left|\vec{F}_A\right| = \left|\vec{F}_C\right| = \dfrac{kq^2}{a^2}$

and as shown in fig $\vec{F}_A$ and $\vec{F}_C$ are mutually perpendicular. So their resultant F' makes angle of 45° with both $\vec{F}_A$ and $\vec{F}_C$

$$\vec{F}' = \sqrt{F_A^2 + F_C^2} = \sqrt{2F_A^2} = F_A\sqrt{2}$$

$$\vec{F}' = \frac{kq^2}{a^2}\sqrt{2}$$

Next, force on charge at B due to charge at D

$$F_D = \frac{kq^2}{\left(a\sqrt{2}\right)^2} = \frac{kq^2}{2a^2}$$

as $\vec{F}'$ and $\vec{F}_D$ are in same direction (fig) so net force on charge at B

$$\vec{F} = \vec{F}' + \vec{F}_D$$

$$F = \frac{kq^2}{a^2}\sqrt{2} + \frac{kq^2}{2a^2}$$

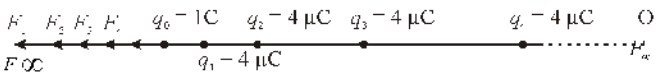
$$F = \frac{kq^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

and is directed along DB

Example 1.13 : Infinite number of point charges 4 μC each are placed on x axis at 1m, 2m, 4m, 8m.......... respectively. Determine the force due to these at a 1C charge placed at origin.

Solution : Here

$$q_1 = q_2 = q_3 = q_4 = 4\mu C = 4\times10^{-6}\, C$$



$$q_0 = 1C$$

Net force on q_0

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \ldots + \vec{F}_\infty$$

$$\vec{F}_0 = \frac{kqq_0}{r_1^2}\left(-\hat{i}\right) + \frac{kqq_0}{r_2^2}\left(-\hat{i}\right) + \frac{kqq_0}{r_3^2}\left(-\hat{i}\right) + \ldots\infty$$

$$\vec{F}_0 = kqq_0\left[\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+...\infty\right](-\hat{i})$$

$$\vec{F}_0 = 9\times10^9\times4\times10^{-6}\times1\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+...\infty\right](-\hat{i})$$

The term within the bracket forms a geometrical progression with first term a = 1 and common ratio r = 1/4. The sum of such a series is given by S =

$$\vec{F}_0 = 36\times10^3\left[\frac{a}{1-r}\right](-\hat{i})$$

$$= 36\times10^3\left[\frac{1}{(1-1/4)}\right](-\hat{i})$$

$$\vec{F}_0 = 36\times10^3\times\frac{4}{3}(-\hat{i})$$

$$= 48\times10^3(-\hat{i})\text{N}$$

1.5 Electric Field :

When a point charge is brought near another charge, it experiences a force of attraction or repulsion. If we are interested only in determining the force acting between these charges then Coulomb's law is sufficient. Similarly for a system of point charges to determine force acting on a charge due to the remaining charges we use principle of superposition along with Coulomb's law. However a nagging question remains: how does a charged particle interacts with another charge kept at a distance as the charges are not touching each other? In other words how does one charge know about the presence of the other charge? To answer such a question the concept of electric field is very important. To explain the interaction between two charges, it can be imagined that a charge creates an electric field in its surrounding space. When another charge is placed in this eletric field then due to this electric field first charge does some action on the second charge whereby the second charge experieneces the presence of the first. Thus the space surrounding a charge or system of charges in which some other charged particle experiences a force of attraction or repulsion depending upon its nature is called electric field. A particle is considered to be in an electric field if it experiences electric force. The concept of electric field was but forward first of all by scientist Michael Faraday. Electric field is a vector field which is described mathematically in terms of the intensity of electric field.

1.5.1 Intensity of Electric Field

By definition intensity of electric field at some point in an electric field is equal to the force acting on a unit positive test charge placed at that point and its direction is same as the direction of force acting on this unit test charge. The test charge is assumed to be sufficiently small positive point charge such that it does not disturb the charge (or charges) that create the electric field. Thus the presence of test charge does not modify the original electric field. Electric field intensity is a vectory quantity. It is denoted by $\vec{E}$. Many times electric field intensity is referred as electric field.

If $\vec{F}$ is the force acting on some positive test charge q_0 placed at some point in a given electric field, then the intensity of electric field at that point is

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad ...(1.9)$$

Since q_0 should be sufficiently small so as not to disturb the electric field it is more proper to write

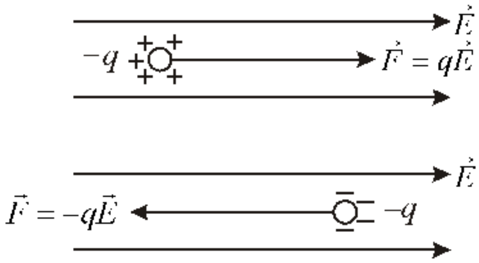
$$\vec{E} = \lim_{q_0\to0}\frac{\vec{F}}{q_0} \qquad ...(1.9(a))$$

The electric field intensity is expressed in units of N/C in SI system. In next chapter we will see that another unit for $\vec{E}$ is V/m. The dimensional formula for electric field intensity is

$$E = \frac{F}{q_0} = \frac{[M^1L^1T^{-2}]}{[A^1T^1]} = [M^1L^1T^{-3}A^{-1}]$$

$$= [MLT^{3}A^{1}]$$

If a charged particle having a charge of magnitude q is placed in electric field of intensity $\vec{E}$ then the force acting on the particle is given by



$\vec{F} = q\vec{E}$

If the particle is positively charged the direction of force $\vec{F}$ is same as that of $\vec{E}$ if the particle is negatively charged, the force $\vec{F}$ on it is directed opposite to field. (Fig 1.7)

Fig 1.7 : Force on a charged particle in electric field

Electric field due to a positive point charge or uniform spherical positive charge distribution is directed radially outward from it. If the point source charge is negative the electric field is directed radially toward it fig (1.8)

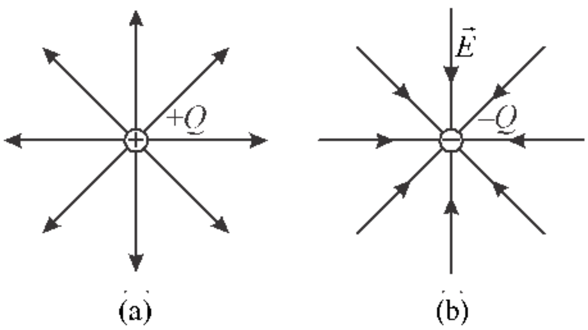


Fig 1.8 : Electric field due to
(a) positive (b) negative point charge

1.6 Electric field due to a point charge

Consider a point charge + Q situated at a point O (Fig 1.9) we are interested in electric field $\vec{E}$ at a point P at a distance r from O. Let us consider a test charge + q_0 imagined to be placed at P.

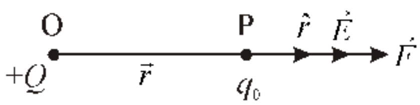


Fig 1.9 : Electric field due to a point charge

From Coulomb's law force exerted on test q_0 at P due to charge + Q is

$$\vec{F} = \frac{kQq_0}{r^2}\hat{r}$$

By definition $\vec{E} = \frac{\vec{F}}{q_0}$

$$\text{so } \vec{E} = \frac{1}{q_0} \cdot \frac{kQq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

Where is a unit vector directed from Q towards q_0. Electric field at point **P** is in direction $\vec{OP}$

If instead of + Q, a point charge - Q is placed at O then electric field at P is given as

$$\vec{E} = \frac{1}{4\pi \in_0} \cdot \frac{Q}{r^2}\hat{r} \qquad \ldots (1.10)$$

$$\vec{E} = \frac{1}{4\pi \in_0} \cdot \frac{-Q}{r^2}(-\hat{r}) \qquad \ldots (1.11)$$

Thus it is obvious that for a point charge i.e. the intensity of electric field is inversely proportional to the square of distance. For a point charge this variation is shown graphically in Fig 1.10

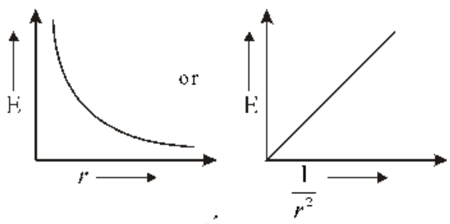


Fig 1.10 : Variation of $\vec{E}$ with r for a point charge

If the point charge is situated in a medium of dielectric constant $\in_r$ then

$$E_m = \frac{1}{4\pi \in} \cdot \frac{Q}{r^2}\hat{r} = \frac{1}{4\pi \in_r \in_0} \cdot \frac{Q}{r^2}\hat{r}$$

$$E_m = \frac{E}{\in_r} \Rightarrow E_m < E \qquad (\because \in_r > 1)$$

Thus for a given distance in a dielectric medium the electric field intensity decreases by $\in_r$ compared to that in free space.

Table 1.3 : Typical values of some electric fields present in various cases

System	Electric field
X-ray tube	$5\times10^{6}\ N/C$
Dielectric strength of air	$3\times10^{6}\ N/C$
Van-de Graft Generator	$2\times10^{6}\ N/C$
Atomsphere	100 N/C
Arround domestic electric wires	300 N/C

1.7 Electric Field due to a System of Charges:

To calculate the electric field intensity in a point due to a system of charges, the principle of super position of electric fields is employed.

According to this principle "the electric field at a point due to a system of point charges is equal to the vector sum of electric fields at that point due to each of the charges of the system".

If for a system of n point charges $q_1, q_2, q_3, ..., q_n$ the electric field at point P due to these charges are $\vec{E}_1, \vec{E}_2, \vec{E}_3, ..., \vec{E}_n$ respectively (Fig 1.11) then net electric field at P.

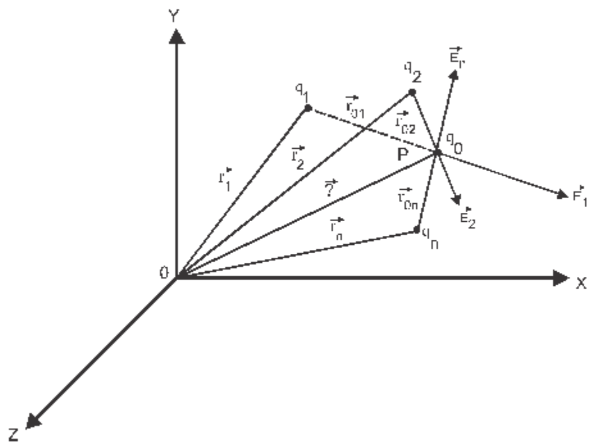


Fig 1.11 : Electric field due to system of point charges

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n$$

$$\vec{E} = \frac{1}{4\pi\in_0}\frac{q_1}{r_{01}^2}\hat{r}_{01} + \frac{1}{4\pi\in_0}\frac{q_2}{r_{02}^2}\hat{r}_{02} + ... + \frac{1}{4\pi\in_0}\frac{q_n}{r_{0n}^{\ 2}}\hat{r}_{0n}$$

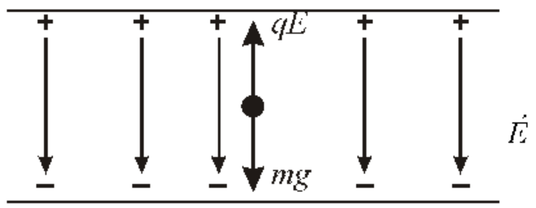
$$\because \quad \hat{r}_{01} = \frac{\vec{r}_{01}}{|\vec{r}_{01}|} = \frac{\vec{r}_{01}}{r_{01}}$$

$$\vec{E} = \frac{1}{4\pi\in_0}\frac{q_1}{r_{01}^3}\vec{r}_{01} + \frac{1}{4\pi\in_0}\frac{q_2}{r_{02}^3}\vec{r}_{02} + ... + \frac{1}{4\pi\in_0}\frac{q_n}{r_{0n}^3}\vec{r}_{0n}$$

$$\vec{E} = \frac{1}{4\pi\in_0}\sum_{i=1}^{n}\frac{q_i}{r_{0i}^3}\vec{r}_{0i} \qquad ...(1.12)$$

Example 1.14 An oil drop has a charge equal to that of 12 electrons and it remains in equilibrium in a constant electric field of 2.55×10^{4} N/C . It the density of oil is 1.26×10^{3} kg/m^3 then determine the radius of the drop.

Soultion : For equilibrium of charged drop in electric field its weight must be balanced by electric force.



i.e $\quad mg = qE$

but $\quad m = V\rho$

or $\quad m = \frac{4}{3}\pi r^3\rho$

Where and r is its radius, V is its volume and p is density of oil

then $\quad \frac{4}{3}\pi r^3\rho g = neE$

$$r = \left[\frac{3neE}{4\pi\rho g}\right]^{1/3}$$

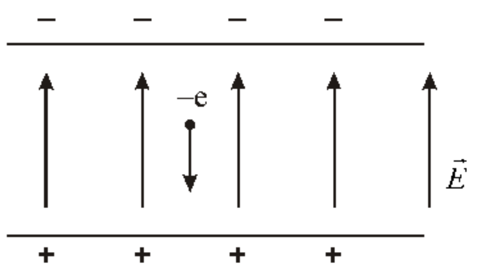
On substituting the values of relevent quantities

$$r = \left[\frac{3\times 12\times 1.6\times 10^{\;19}\times 2.55\times 10^{4}}{4\times 3.14\times 1.26\times 10^{3}\times 9.8}\right]^{1/3}$$

$$r = 9.8\times 10^{\;7}\,m$$

Example 1.15 An electron falls from rest in a constant electric field of $2.0\times 10^{4}\ N/C$ through 1.5 cm. Keeping the magnitude of electric field same now its direction is reversed and now a proton falls from rest in this field through the same distance. Determine the time of fall in both these cases. Compare this situation with "free fall under gravity".

Soultion : First case- As shown in Fig. (a) the electric field is acting vartically upward so force on negatively charged electron F_e = eE is directed downwards.



So the acceleration of electron

$$a_e = \frac{F_e}{m_e} = \frac{eE}{m_e}$$

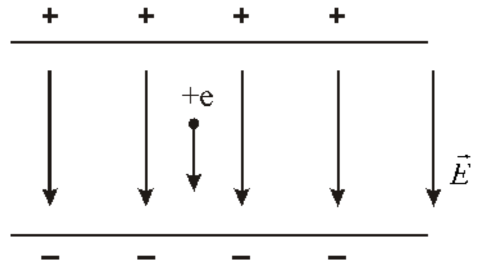
If starting from rest the electrons falls through a distance h in time t then

$$h = \frac{1}{2}a_e t_e^2$$

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$$

$$t_e = \sqrt{\frac{2\times 1.5\times 10^{\;2}\times 9.1\times 10^{\;31}}{1.6\times 10^{-19}\times 2.0\times 10^{4}}} = 2.9\times 10^{-9}\,s$$

Second case : As shown in Fig (b) the electric field acts vertically downward so force on proton (positive charge) also acts downward, hence acceleration of proton.



$$a_p = \frac{F_p}{m_p} = \frac{eE}{m_p}$$

For proton, time to fall through a distance h

$$t_p = \sqrt{\frac{2h}{a_p}} = \sqrt{\frac{2hm_p}{eE}}$$

$$t_p = \sqrt{\frac{2\times 1.5\times 10^{-2}\times 1.67\times 10^{-27}}{1.6\times 10^{\;19}\times 2.0\times 10^{4}}} = 1.3\times 10^{\;7}\,s$$

For both the cases accelerations a_e and a_p are much greater compared to acceleration due to gravity g hence gravitional effect is taken as negligible.

e.g. $$a_p = \frac{eE}{m_p} = \frac{\left(1.6\times 10^{\;19}\right)\times\left(2.0\times 10^{4}\right)}{1.67\times 10^{-27}}$$

$$= 1.9\times 10^{12}\ \text{m/s}^2$$

$$g\left(\sim 10\ \text{m/s}^2\right)\ \text{to}\ \sim 10^{11}$$

Which is nearly $\sim 10^{11}$ times more than $g\left(\sim 10\ \text{m/s}^2\right)$, acceleration of electron a_e is 183 times more than a_p

Time to fell freely under gravity is $t_g = \sqrt{\frac{2h}{g}}$ is independent of mass so it is same for both electron and proton.

Example 1.16 At some point a force of 2.25 N acts an a charge of $5\times10^{-4}C$. Determine the electric field intensity at that point.

Soultion : Here $q_0 = 5\times10^{-4}C$

$$F = 2.25\,N$$

$$E = \frac{F}{q_0} = \frac{2.25}{5\times10^{-4}} = 4.5\times10^3\,N/C$$

Example 1.17 In a rectangular coordinate system, two positive point charges 10^{-4} C each are fixed at points $x = +0.1m$, $y = 0$ and $x = -0.1m,\ y = 0$. Find the magnitude and direction of electric field at the following points.

(a) the origin (b) $x = 0.2m, y = 0$ (c) $x = 0,\ y = 0.1m$

Solution : For the system of charges, placed as shown in Fig (a)

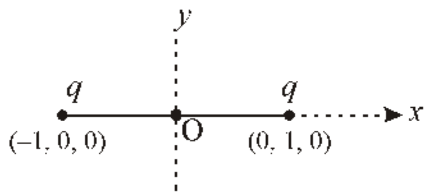


Fig (a)

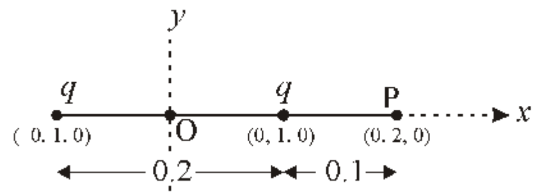
(a) electric field at origin must be zero as fields produced by individual charges at this location are equal and opposite.

(b) For a point such as P shown in Fig (b), electric field due to individual charges are in same direction (along + ve x axis) so net electric field at P

$$E = \frac{kq}{(0.1+0.2)^2} + \frac{kq}{(0.2-0.1)^2}$$

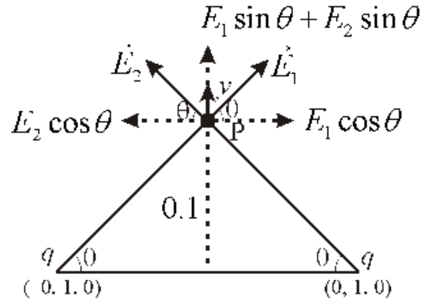
$$= \frac{kq}{0.09} + \frac{kq}{0.01} = \frac{kq}{0.09}[10]$$

$$= \frac{9\times10^9\times10^{-8}\times10}{0.09} = 1.0\times10^4\ \text{N}$$



(c) At point P (0, 0.1) the electric fields due to the given charges have equal magnitudes i.e. $E_1 = E_2$

In this case, the x components of $\vec{E}_1$ and $\vec{E}_2$ cancel while y components add [fig (c)] net electric field at P is then E



$$E = E_1\sin\theta + E_2\sin\theta\ \ \text{(along Y axis)}$$

$$\text{E} = 2E_1\sin\theta = 2E_1\sin 45^\circ$$

[From geometry of figure it can be seen that $\theta = 45^\circ$]

$$\therefore\ E = \frac{2kq}{r_1^2}\sin 45^\circ = \frac{2\times9\times10^9}{\left[(0.1)^2+(0.1)^2\right]}\times10^{-8}\times\frac{1}{\sqrt{2}}$$

$$= \frac{2\times9\times10}{\sqrt{2}\,[0.02]} = 6.36\times10^3\ \text{N/C}$$

1.8 Electric Field Lines

The concept of electric field lines is very useful in visulalizing electric field arround charge configurations graphically. An electric field line is an imaginary line or curve drawn through a region of space (where electric field exists) so that its tangent at any point is in the direction of electric field vector at that point. Fig. 1.12 depicts the basic idea.

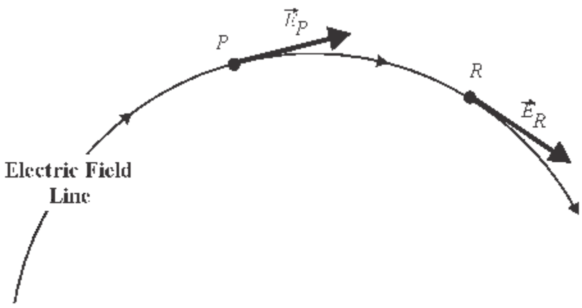
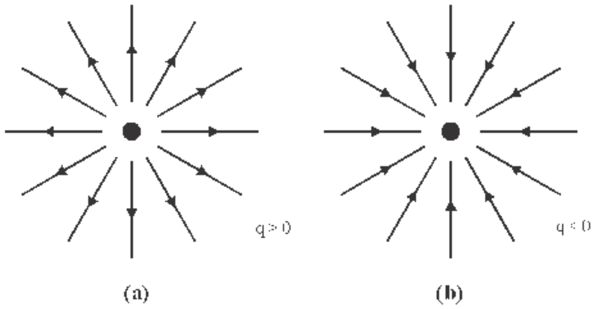


Fig 1.12 : The direction of electric field at any point is tangent to the field line through that point

The concept of field lines was given by British Scientist Michal Faraday (in the first half of 19th century) to develop an intutive non mathematical way of visualizing electric field arround charge configurations. Faraday called them "line of force" but the term "field line" is more appropriate.

Fig 1.13 depicts the field lines arround some simple charge configruations. The field lines are in three dimensional space, however the figure shown here exhibit them only in a plane. For a single isolated positive charge the field lines are radially outward while for a single negative charge field lines are radially inwards. The field lines surrounding a system of two point charges (q,q) depicts a vivid graphical representation of their mutual repulsion, while those surround a dipole (two equal and opposite charges) (q, -q) depicts clearly the mutual attraction between the charges.



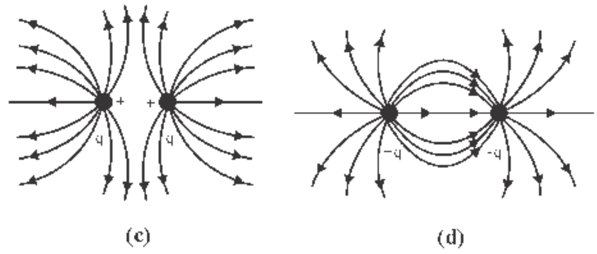


Fig 1.13 : Electric Field lines due to some simple change distributions

The electric field lines follow some general properties:

(i) Field lines always orignates from positive charges and terminates at negative charges. In case of a single charge the may start or terminates at infinity. For an isolated positive charge field lines are radially outwards while for a negative charge these are radially inwards [as in Fig 1.12 (a) and (b)]

(ii) In a charge free region field lines can be considered as continuous curves without any breaks. Tangent drawn at any point on an electric field line gives the direction of electric field at that point, thus it indicates the force acting on a unit positive charge placed at that point.

(iii) The number of field lines that starts from or end on, a charge is proportional to the magnitude of that charge.

(iv) Number of field lines per unit area normal to the area at a point is proporitional to the intensity of electric field at that point. Thus the electric field is strong when the field lines are crowded and weak when they are far apart. In fig. 1.14 field is maximum at A and minimum at C.

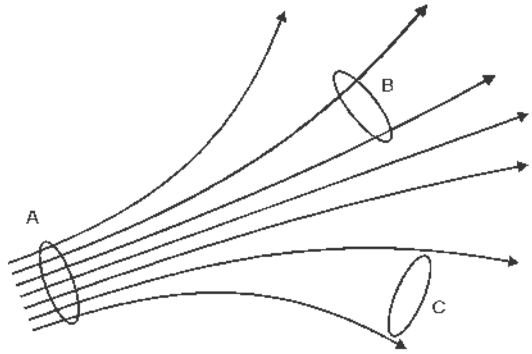


Fig 1.14 : The field strength is proportional to the number of lines that crosses unit area normal to the field

(v) Two electric field lines can never cross each other since it they cross at a point, intensity at that point will have two directions (corresponding to two tangents) which is meaningless.

(vi) In electrostatics, electric field lines can never be closed loops as a line cannot start and end at the same charge. This follows from the conservative nature of electrostatic field.

(vii) There exists a longitudinal tension in the field lines which explains attraction between two unlike charges. The field lines exert a lateral pressure on each other which explains for the repulsion between two like charges. (see figs 1.13 (c) and (d).

(viii) The field lines are perpendicular to an equipotential surface. (You will learn about equilpotential surface in next chapter). Since a charged conductor is an equipotential surface hence field lines are always normal to the conductor surface-

In Fig. 1.15, electric field lines are shown for different types of electric field. For uniform electric field, field lines are equispaced parallel lines as in fig. 1.15 (c).

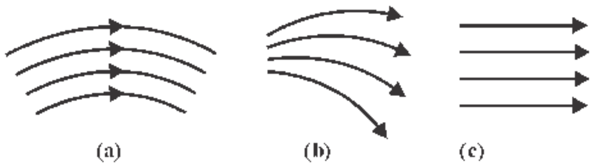


Fig 1.15 : (a) Direction is not constant
(b) magnitude and direction both are not constant
(c) both magnitude and direction are constant.

Electric field lines are not same as trajectories of charged particle. It is a common misconception that a charge particle of charge q in some electric field must move along an electric field line. As electric field $\vec{E}$ at any point is tangent to the field line that passes through that point, it is correct that the net force $\vec{F} = q\vec{E}$ and hence acceleration of the particle are tangent to the field line, however from our study of kinematics we know that when a particle moves on a curve its acceleration can not be tangent to the path. Thus, in general the path of a particle is not same as a field line.

A charged particle will move along a field line only if field line is straight and initially either it is at rest or its velocity is parallel or antiparallel to the field line.

1.9 Electric Dipole and Dipole Moment

"The arrangement of two equal and opposite charges separated by a small distance is called an electric dipole". In fig 1.16 an electric dipole is shown where the magnitude of each charge is q and their separation is 2a.

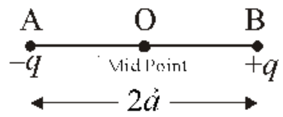


Fig 1.16 : Electric Dipole

The mid point between the charges is called 'centre' of dipole and the line joining the charges is called its axis. The line through centre and perpendicular to the axis is called equatorial line. The electrical behaviour of a dipole is described in terms of its dipole moment. It is a vector quantity denoted by $\vec{p}$.

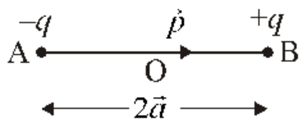


Fig 1.17

The magnitude of electric dipole moment is defined as the product of the magnitude of one of the charges and their separation. If the separation 2a between the charges is considered as a vector directed from the negative charge to the positive charge then by definition.

$$\vec{p} = 2\vec{a}q \qquad \ldots (1.13)$$

The SI unit for dipole moment is Coulomb x meter = C.m and it has dimensions of $M^{0}L^{1}T^{1}A^{1}$

There exists some molecules in nature where there is a finite separation between centre of positive and centre of negative charges. Such molecules are termed as polar molecules. Few examples are NaCl, H_2O, HCl etc.

There are also some molecules in which normally the centres of positive and negative charges coincides. However, in presence of external electric field the centre of negative charge gets shifted by a small amount relative to the centre of positive charge, there by creating a dipole moment. Such dipoles are termed as induced electric dipoles.

Example 1.18 In NaCl molecule the separation between Na^+ and Cl^- ions is 1.28 Å . Find the electric dipole moment of the molecule.

Solution : Here $q = 1.6\times10^{-19}\,C$

$$2a = 1.28\mathring{A} = 1.28\times10^{-10}\,m$$

$$p = q2a$$

$$p = 1.6\times10^{-19}\times1.28\times10^{-10}$$

$$= 2.048\times10^{-29}\,Cm$$

1.10 Electric Field due to a Dipole

The electric field due to a dipole at some point, is the vector sum of the electric field produced by the individual charges of the dipole at that point i.e. the principle of super position is used to calculate the electric field of a dipole. For the sake of simplicity, here, we will determine the electric field at axial and equatorial points of the dipole.

1.10.1 Electric field at a point on the axial line of an Electric Dipole

In fig 1.18 AB is an electric dipole consisting of charges a q and -q. We wish to determine electric field at a point P on its axis at a distance r from the centre.

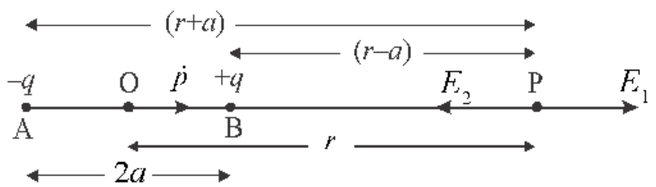


Fig 1.18 : Electric field at an axial point of a dipole

Electric field at point P due to charge +q at B

$$\vec{E}_1 = \frac{1}{4\pi\in_0}\frac{q}{(r-a)^2}\hat{p} \text{ (in direction BP)} \quad \ldots (1.14)$$

here, $\hat{p}$ is a unit vector in direction of dipole moment $\vec{p}$ Electric field at point P due to charge -q at A.

$$\vec{E}_2 = \frac{1}{4\pi\in_0}\frac{q}{(r+a)^2}(-\hat{p}) \text{ (in direction PB)} \ldots (1.15)$$

Hence net electric field at point P

$$\vec{E}_{axial} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}q\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}q\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}\cdot q\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}\frac{2pr}{(r^2-a^2)^2}\hat{p} \quad \ldots (1.16)$$

(Here $q\cdot 2a = p$)

If a is very small compared to r ($a << r$) then a^2 can be assumed negligible compared to r^2 . Then

$$\vec{E}_{axial} = \frac{1}{4\pi\in_0}\frac{2p}{r^3}\hat{p} \quad \ldots (1.17)$$

its magnitude is $E_{axial} = \dfrac{1}{4\pi\in_0}\dfrac{2p}{r^3} \quad \ldots (1.18)$

Fram the above result it is clear that the field intensity at axial point does not very as r^{-2} as for a single point charge rather it varies as r^{-3} . Thus electric field intensity decreases realtively more rapidly with distance compared to a single point charge.

The direction of electric field at axial line is in direction of dipole moment ($\vec{p}$).

1.10.2 Electric field at a point on Equatorial line of an Electric Dipole

Fig. 1.19 shows an electric dipole AB with charges +q and -q at B and A respectively, the displacement AB $= 2\vec{a}$. We wish to determine electric field at a point P at distance r from the centre O on equatorial line.

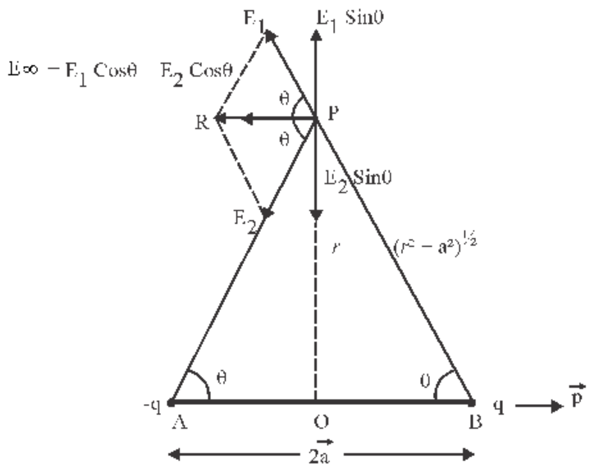


Fig. 1.19 : Electric field at a point on equatorial line of dipole

From Δ AOP and Δ BOP be

$$PA = PB = \left(r^2 + a^2\right)^{1/2}$$

$$(PA)^2 = (PB)^2 = \left(r^2 + a^2\right)$$

Electric field at P due to charge +q

$$\vec{E}_1 = \frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)}, \text{ (along direction BP)} \quad ...(1.19)$$

Electric field at P due to charge -q

$$\vec{E}_1 = \frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)}, \text{ (directed along PA)} \quad ...(1.20)$$

Thus $\vec{E}_1$ and $\vec{E}_2$ are of equal magnitudes but differ in directions.

$$\text{i.e. } \left|\vec{E}_1\right| = \left|\vec{E}_2\right| = \frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)} \quad ...(1.21)$$

It we resolve E_1 and E_2 along axial and equatorial lines then equatorial components $E_1 \sin\theta$ and $E_2 \sin\theta$ cancel out being equal and opposite. Axial components $E_1 \cos\theta$ and $E_2 \cos\theta$ add as their directions are same. Therefore

$$\vec{E}_{\text{equater}} = \left(E_1 \cos\theta + E_2 \cos\theta\right)\left(-\hat{p}\right)$$

Here, $-\hat{p}$ indicates that the electric field is opposite to the direction of dipole moment as can be seen from fig 1.19

$$\because \quad \left|\vec{E}_1\right| = \left|\vec{E}_2\right|$$

$$\vec{E}_{\text{equater}} = 2E_1 \cos\theta\left(-\hat{p}\right)$$

$$\vec{E}_{\text{equater}} = 2\frac{1}{4\pi \in_0} \frac{q}{\left(r^2 + a^2\right)} \cdot \frac{a}{\left(r^2 + a^2\right)^{1/2}}\left(-\hat{p}\right)$$

$$\because \quad \cos\theta = \frac{a}{\left(r^2 + a^2\right)^{1/2}}$$

$$\vec{E}_{\text{equater}} = \frac{1}{4\pi \in_0} \frac{p}{\left(r^2 + a^2\right)^{3/2}}\left(-\hat{p}\right) \quad ...(1.22)$$

$$\vec{E}_{\text{equater}} = -\frac{1}{4\pi \in_0} \frac{\vec{p}}{\left(r^2 + a^2\right)^{3/2}} \quad ...(1.23)$$

If a is much smaller than r $(a << r)$ then $a^2 << r^2$

$$\text{so } \vec{E}_{\text{equater}} = -\frac{1}{4\pi \in_0} \frac{\vec{p}}{r^3} \quad ...(1.24)$$

$$\left|\vec{E}_{\text{equater}}\right| = \frac{1}{4\pi \in_0} \cdot \frac{p}{r^3} \quad ...(1.25)$$

It is clear that for same distance r from centre O.

Therefore (i) the electric field at axial point is twice as that on eqautorial point at the same distance.

(ii) At axial points, the electric field is along the direction of dipole moment, whereas at equatorial points the direction of electric field is opposite to the dipole moment. From the above discussion it is clear that for both axial and equatorial positions the dipole electric field at large distances $(r >> 2a)$ varies as $E \propto 1/r^3$ (and not as $(E \propto 1/r^2)$ as in case of a single point charge) and it falls off more repidly compared to the electric field due to a single point charge. The physical reason for this rapid decreases in this electric field for a dipole is that from distant points a dipole look like two equal and opposite charges that almost (but not exactly) coincide. Thus their electric fields at distant points almost but not quite cancel each other.

Example 1.19 Two point charges 5 μC and –5 μC are 1cm apart. Calculate the electric field at a distance of 0.3 4 m from their centre at a point.

(i) on the axis (ii) on equatorial line

Solution : Here $q = 5\mu C = 5 \times 10^{-6} C$

$$2a = 1cm = 10^{-2} m$$

$r = 0.30m$ (thus r >> a)

Electric dipole moment $p = q.2a$

$$p = 5 \times 10^{-6} \times 10^{-2} = 5 \times 10^{-8} \, Cm$$

(i) At axial position $E_{axial} = \frac{1}{4\pi \in_0} \frac{2p}{r^3}$

$$E_{axial} = \frac{9 \times 10^9 \times 2 \times 5 \times 10^{-8}}{(0.30)^3}$$

$$= 3.33 \times 10^4 \, N/C$$

(ii) At equatorial point $E_{eq} = \frac{1}{4\pi \in_0} \frac{p}{r^3}$

$$E_{eq} = \frac{9 \times 10^9 \times 5 \times 10^{-8}}{(0.30)^3} = 1.67 \times 10^4 \, N/C$$

1.11 Torque on a Dipole in a Uniform Electric Field

Fig 1.20 (a) shows an electric dipole AB placed in a uniform electric field with its dipole moment oriented at angle θ with E. The force on charge +q, of the dipole is $F = q\vec{E}$, in direction of $\vec{E}$ and on -q is $F = q\vec{E}$, in direction opposite to electric field E. Hence the net force on dipole

$$\vec{F}_{net} = q\vec{E} + \left(-q\vec{E}\right) = 0$$

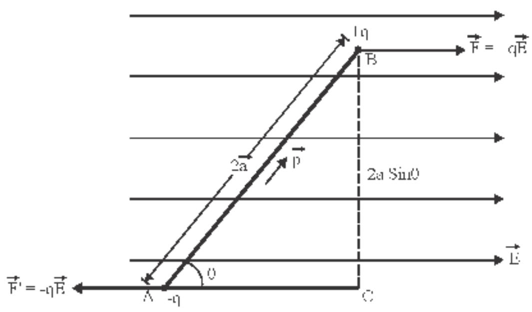


Fig 1.20 (a) : Dipole in Uniform Electric field

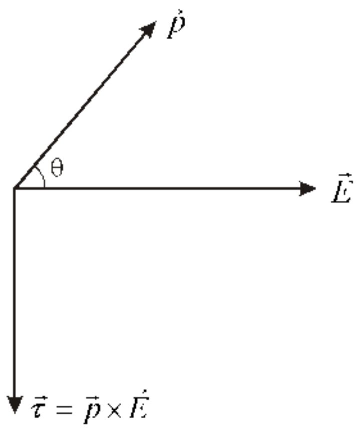


Fig 1.20 (b) : Direction of torque

Thus the net force on the dipole is zero so it will not have a translational motion. However as the two forces are not colinear they constitute a couple producing a net torque on the dipole. This torque tends to align the dipole in direction of electric field.

Magnitude of torque = (force on any of the charge) x perpendicular distance between lines of action of the forces

$$\tau = qE(BC)$$

From fig $\sin\theta = \frac{BC}{2a}$

$$BC = 2a\sin\theta$$

So $\tau = qE(2a\sin\theta)$ $\because q \cdot 2a = p$

$$\tau = pE\sin\theta \quad \ldots (1.26)$$

In vector notations $\vec{\tau} = \vec{p} \times \vec{E}$ (N m) . . . (1.27)

Direction of $\vec{\tau}$ is perpendicular to plane containing $\vec{p}$ and $\vec{E}$ in accordence with right hand screw rule.

Special cases

(i) (a) When $\theta = 0^o$

$$\sin\theta = 0$$

dipole is in stable equilibrium

(b) When $\theta = 180^o$

$$\tau = pE\sin 180^o = 0$$

dipole is in unstable equilibrium

(ii) When $\theta = 90^o$

then $\tau_{max} = pE\sin 90^o$

$$\tau_{max} = pE$$

Example 1.20 Two charges $\pm$ 1000 μC , 2 mm apart constitute an electric dipole. This dipole is placed in a uniform electric field of $15\times10^4\ N/C$ at 30° with field. Find the torque scting on the dipole.

Solution : Here $q = 1000\,\mu C = 10^{-3} C$

$$E = 15\times10^4 N/C$$

$$2a = 2mm = 2\times10^{-3} m$$

$$\theta = 30^o$$

Torque $\tau = pE\sin\theta$

$$\tau = q(2a)E\sin\theta$$

$$\tau = 10^{-3}\times2\times10^{-3}\times15\times10^4\sin 30^o$$

$$\tau = 15\times10^{-2}\ Nm$$

Important Points

1. An object can be charged in three ways (i) by friction (ii) by conduction (contact) (iii) by electrostatic induction.
2. Charge is not created in process of friction. Actually in process of friction transfer of a few electrons takes place from one object to another as a result of which one object gets positively charged and another negatively charged.
3. Like charges repel and unlike charges attract each other.
4. Electric charge is quantized Quantum of charge = electronic charge e = $1.602\times10^{-19} C$. Any charge can be written as $q = \pm ne \quad n = 1,2,3...$
5. For two points charges in free space, electrostatic force between them is given by Coulomb's law as

   $$F = \frac{1}{4\pi\in_0}\frac{q_1q_2}{r^2}N$$

   $\in_0$ is the permitivity of free space or air
6. In presence of some medium the force between two charges is smaller than the force between them in free space (for same separation) by a factor called relative permitivity or dielectric constant of medium.
7. The electric field intensity at any point is given by

   $\vec{E} = \lim_{q_0\to0}\frac{\vec{F}}{q_0}N/C$ where $\vec{F}$, is the force acting on test charge q_0 due to electric field.

8. Electric field due to a point charge is given by

$$\vec{E} = \frac{1}{4\pi \in_0} \frac{q}{r^2} \hat{r}$$

9. Electric field lines can never cross each other since if they cross at a point intensity at that point will have two directions which is absurd.

10. Net electric field due to a number of charges is the vector sum of electric fields due to individual charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots + \vec{E}_n$$

11. A system consiting of two equal but opposite charges kept at a small separation is called an electric dipole.

12. Electric field intensity at an axial point of a dipole

$$\vec{E}_{\text{axial}} = \frac{1}{4\pi \in_0} \frac{2p}{r^3} \hat{p} \quad (\text{for } a << r)$$

13. Electric fields intensity at an equatorial point of a dipole

$$\vec{E}_{\text{equater}} = -\frac{1}{4\pi \in_0} \frac{\vec{p}}{r^3} \quad (\text{for } a << r)$$

14. Torque on a dipole placed in a uniform electric field

$$\tau = pE \sin\theta$$

Questions For Practice

Multiple Choice Questions -

1. Two identical charges separated by a distance of 3 m experience a force of repulsion of 16 N, the magnitude of each charge is

(a) 2 μC (b) 4 μC

(c) 40 μC (d) 80 μC

2. The force acting between two charges is 8N. If the separation between them is trippled then the force acting them will be

(a) F (b) F/3

(c) F/9 (d) F/27

3. To give a charge of 5×10^{-19} C to some object, how many electrona are to be removed from ir?

(a) 3 (b) 5

(c) 7 (d) 9

4. Two point charges + 9e and +e are at a separation of 16 cm. Where on the line joining them another charge q must be put so it remains in equilibrium.

(a) 24 cm away from +9e

(b) 12 cm away from + 9e

(c) 24 cm away from +e

(d) 12 cm away from +e

5. Two identical spheres having unequal and opposite charges are 90 cm apart. They are now made to contant and then again separated by the same distance. Now they repel each other by a force of 0.025 N. Final charge on each sphere is

(a) 1.5 μC (b) 1.5 C

(c) 3 C (d) 3 μC

6. On putting a glass plate in between the two charges the electrostatic force between them as compared to earlier is

(a) more (b) less

(c) zero (d) infinite

7. The dipolemoment of HCl molecule is 3.4×10^{-30} cm the separation between its ions is -

(a) 2.12×10^{-11} m (b) zero

(c) 2 mm (d) 2 cm

8. For an electron and a proton kept in same uniform electric field the ratio of their accelerations is

(a) Zero (b) m_p / m_e

(c) 1 (d) m_e / m_p

9. Four equal and like charges are placed on the four vertices of a square. It the field intensity due to any of the charge at the centre is E then the net electric field intensity at the centre of square will be

(a) Zero (b) E

(c) E/4 (d) 4E

10. On placing a dipole in a uniform electric field it is acted upon by a

(a) Torque only

(b) Force only

(c) Both force and torque

(d) Neither force nor torque

11. For the torque to be maximum on a dipole placed in an electric field angle between $\vec{p}$ and $\vec{E}$ must be

(a) 0° (b) 180°

(c) 45° (d) 90°

12. An electron and a proton are apart. The dipole moment of the system is

(a) 3.2×10^{-29} Cm (b) 1.6×10^{-19} Cm

(c) 1.6×10^{-29} Cm (d) 3.2×10^{-19} Cm

13. For the same distance from centre of dipole the ratio of electric fields at longitudinal and transverse position is

(a) 1 : 2 (b) 2 : 1

(c) 1 : 4 (d) 4 : 1

14. The force of attraction between +5 μC and –5 μ charges kept at some distance apart is 9N. When the two charges are made to contact and then separted again by the same distance the force acting between them becomes.

(a) Infinite (b) $9\times10^{9}\,N$

(c) 1 N (d) Zero

15. Two equal but unlike charged objects are kept at some distance apart with a force F acting between them. If 75% charge of one of them is some how transferred to the other then the new force between them is -

(a) $\frac{F}{16}$ (b) $\frac{7F}{16}$

(c) $\frac{9F}{16}$ (d) $\frac{15}{16}F$

Very Short Answer Questions

1. Write the value of one quantum of a charge.

2. The electrostatic force between two protons separated by a distance r is F. It the protons are replaced by electrons then what the force is going to be?

3. The force exerted by one charge on other is F. In presence of a third charge what will be the force on second charge by the first charge.

4. If the dielectric constant of a medium is unity then what is its absolute permitivity.

5. For two point charges q_1 and q_2 product $q_1q_2<0$. What is the nature of the force between them.

6. For two point charges q_1 and q_2 the product $q_1q_2>0$. What the nature of force acting between them?

7. What is the force acting on a charge placed in electric field E.

8. What is the effect of speed of a charged particle on its charge and mass.

9. What is the magnitude of the intensity of electric field that can balance the weight of an electron? Given

$e = 1.6\times10^{-19}\,C$, $m_e = 9.1\times10^{\ 31}\,kg$

10. The force acting between two charges placed in free space is F. It a brass plate is now put in the region between the charges then what is the value of force?

11. Name the experiment with which the quantum nature of charge was established?

12. Give definition of electric dipole moment.

13. Write the condition for an ideal electric dipole.

14. Give the example of a particle which has zero he rest mass and zero charge.

15. On what the value of k depends in the expression $k = \dfrac{1}{4\pi \in_0}$ for Coulomb's law?

16. Write the charge on nucleus $_7N^{14}$ in coulomb.

17. On rubbing an ebonite rod with furr it gets negatively charged, why?

18. Write the CGS and SI unit of charge. What is the relation between them.

19. When an electric dipole is in stable equilibrium in a uniform electric field.

20. What is the net force on an electric dipole in a uniform electric field?

Short Answer Questions

1. What is meant by frictional electricity? Describe its origin.

2. State Coulomb's law for electrostatic force between two point charges at rest?

3. Explain quantisation of charge.

4. Write the law of superposition for forces?

5. The electric field at the mid point of the line joining the two charges is zero. What conculsion you can draw from it regarding the nature of charges.

6. A singley charge negative ion and an electron are allowed to move from rest in a uniform electric field E. Which of them will move faster and why?

7. What is meant by electric field lines? Write its two properties.

8. Explain law of conservation of charge.

9. Define the relative permitivity of a medium.

10. How can a metallic sphere be charged without touching it?

11. How will you show that electric charges are of two types?

12. What does $q_1 + q_2 = 0$ in reference to charges.

13. A dipole is kept in a uniform electric field. Show that it will not have a translatory motion.

14. A charged rod P attracts another charged rod R while the it repels another charged rod Q. What will be the nature of force developed between Q and R.

15. For determining the electric field due to a point charge, the test charge employed should be infintesimal. Why, describe.

16. A copper sphere of 2 gram contains 2×10^{22} atoms. Nucleus of each atom has a charge 29 e. What fraction of electrons should be removed from the sphere to give it a charge of 2 μC.

17. Consider two identical metallic spheres of exactly the same mass. One of them is given some negative charge and other is charged positively by the same amount, will there be any difference in the masses of sphere after charging? If yes, why?

18. On moving away from a point charge the electric field due to charge decreases. The same is true for an electric dipole. Does the electric field for both these cases decreases at the same rate?

19. Use conservation of charge to identify elements X in following nuclear reactions

(a) $_1H^1 + _4Be^9 \rightarrow X + _0n^1$

(b) $_6C^{12} + _1H^1 \rightarrow X$

(c) $_8N^{15} + _1H^1 \rightarrow X + _2He^4$

Essay Type Questions

1. Define Coulomb's law for the electrostatic force between two charges, and discuss its limitations. Using this law define 1 coulomb of charge.

2. Give definition of electric field. Derive expression for electric field due to a point charge. If another charge q_0 is brought in this field what will be the electric force acting on it?

3. What is meant by an electric dipole. Define electric dipole moment. Derive an expression for intensity of electric field at an axial point of an electric dipole.

4. Derive an expression for the intensity of electric field due to an electric dipole on a point situated on its equatorial line.

5. Derive expression for the torque acting on a dipole placed in a uniform electric field. When will its value be maximum?

Answer

Mutliple Choice Questions -

1. (c) 2. (c) 3. (A) 4. (B) 5. (A) 6. (B) 7. (A) 8. (B)9. (A) 10. (A) 11. (D) 12. (C) 13. (B) 14. (D) 15. (A)

Very Short Answer Questions

1. One quantum of charge = e $= 1.6\times10^{19}\ C$

2. F

3. F

4. $\in = \in_r \in_0 = 1\times8.85\times10^{12}$

   $= 8.85\times10^{12}\ C^2/Nm^2$

5. If $q_1q_2 < 0$ then one of the charge must be positive and other negative so there will be an attractive force acting between them.

6. If $q_1q_2 > 0$ then both charges must have same sign (either both positive or both negative) and a force of repulsion acts between them.

7. $\vec{F} = q\vec{E}$

8. If speed is of the order of the speed of light then mass increases with increase in speed however charge remains invariant (constant).

9. $eE = mg$

   $$E = \frac{mg}{e} = \frac{9.1\times10^{31}\times9.8}{1.6\times10^{-19}}$$

   $= 5.57\times10^{-11}\ N/C$

10. $F_m = \frac{F}{\in_r} = \frac{F}{\infty} = 0$

11. Milikens oil drop experiment

12. The product of magnitude of any charge of the dipole and separation between them is called electric dipole moment. It is a vector quanity $\vec{p} = q\cdot2\vec{a}$ directed from negative to positive charge.

13. The magnitude of charge q should be high and separation (2a) between them should be small such that product q (2a) is finite.

14. Photon

15. On nature of medium and system of units.

16. From $q = Ze$ Is

    $q = 7e = 7\times1.6\times10^{19}C = 11.2\times10^{19}C$

17. As electrons are more loosely bound in four compared to ebonite, on rubbing them fews electrons are transferred from fur to ebonite.

18. CGS unit is esu or stat coulomb and SI unit is coulomb (C) 1 C = 3 × 10^9 esu

19. When $\vec{p}$ and $\vec{E}$ parallel i.e. angle between them is 0.

20. Zero (0).

Numerial Problems

1. The charges of $2\times10^{-7}C$ and $3\times10^{-7}C$ respectively are present on two small spheres kept in air at a distance 30 cm apart. Find the force between them.

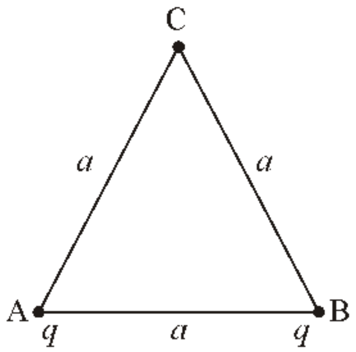
   (Ans: $6\times10^{-3}\ N$)

2. Two identical metallic spheres are charged with $+10\ \mu C$ and $-20\ \mu C$. If they are but into contact and then kept at the same separation as earlier then find the ratio of forces in final and initial situations.

   (Ans : 8:1)

3. Equal charges q each one placed at the vertices A and B of an equilateral triangle. Find the magnitude

of electric field at the vertex C.



(Ans : $E = \frac{1}{4\pi \in_0} \frac{\sqrt{3}q}{a}$)

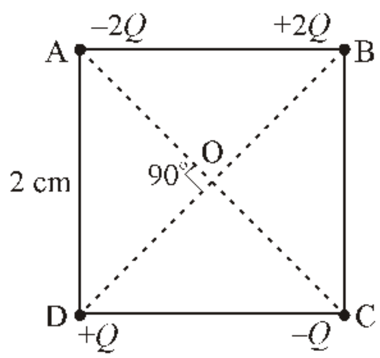
4. Two identical charges spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density $0.8\ g\ cm^{-3}$, the angle remains the same. What is the dielectric constant of liquid (density of material of sphere is $1.6\ g\ cm^{-3}$.

(Ans : $\in_r = 2$)

5. Two idensity spherical conductors B and C carry equal like charges and repel each other by a force F when placed at a certain distance apart. Another identical conductor which is uncharged now removed away from B and C with B then with C and removed away from B and C. Find the new force acting between B and C.

(Ans : $\frac{3F}{8}$)

6. In fig four point charges are placed at the four corners of a square of side 2 cm. Determine the magnitude and direction of electric field at the centre O of the square $Q = 0.02\ \mu C$.

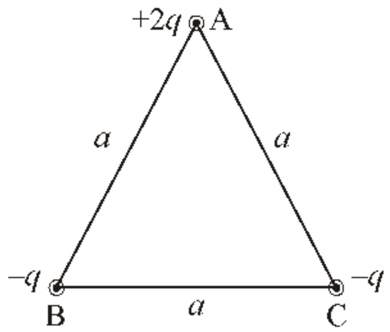


(Ans : $9\sqrt{2} \times 10^5\ N/C$ parallel to $\vec{BA}$)

7. An electric charge Q is divided into two part Q_1 and Q_2 which then are kept at a distance r apart. What will be the condition for on force between them to be maximum.

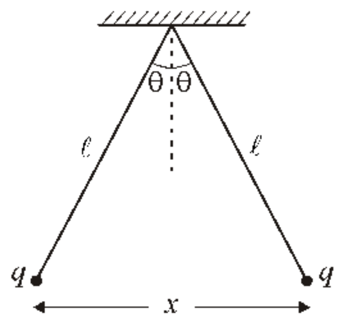
(Ans : $Q_1 = Q_2 = Q/2$)

8. Three charges +2q, $-q$ are $-q$ placed at the vertices A, B and C respectively of a equilateral triangle ABC of side a. Find the magnetude of dipole moment of this system.



(Ans : $\sqrt{3}\,qa$)

9. Two small identical balls each of mass and charge q are suspended same point by silk cords (each card is of length l) as shown in fig. separation between charges is x and angle between cords $(2\theta \approx 10^\circ)$. Calculate the value of x assuming system to be in equilibrium.



10. In a system two charges $q_A = 2.5 \times 10^{-7} C$ and $q_B = -2.5 \times 10^{-7} C$ are situated at point A(0, 0, -15 cm) and B (0, 0, + 15 cm). find the electric dipole moment of the system.

(Ans : $7.5 \times 10^{-8}\ Cm(-\hat{z})$)

11. An electric dipole having dipole moment 4×10^{-9} Cm is oriented at 30^o from the direction of a uniform electric field of magenitude 5×10^4 NC^{-1} . Calculate the magenitude of the torque on the dipole.

(Ans : 10^{-4} Nm)

12. The separation between two point charges q_1 and q_2 is 3 cm. The sum of the two charges is 20 μC and they repel each other by a force of 0.075 N. Find the value of the two chages.

(Ans : 15 μC and 5 μC)