20. Area of Trapezium and a Polygon

EXERCISE 20 (A)

Question 1.
Find the area of a triangle, whose sides are :
(i) 10 cm, 24 cm and 26 cm
(ii) 18 mm, 24 mm and 30 mm
(iii) 21 m, 28 m and 35 m
Solution:
(i) Sides of Δ are
a = 10 cm
b = 24 cm
c = 26 cm

$$S = \frac{a+b+c}{2} = \frac{10+24+26}{2}$$

$$= \frac{60}{2} = 30$$

$$\text{area of } \Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{30(30-10)(30-24)(30-26)}$$

$$= \sqrt{30 \times 20 \times 6 \times 4}$$

$$= \sqrt{10 \times 3 \times 10 \times 2 \times 2 \times 3 \times 2 \times 2}$$

$$= \sqrt{10 \times 10 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}$$

$$= 10 \times 3 \times 2 \times 2 = 120 \text{ cm}^2 \textbf{ Ans.}$$

(ii) Sides of Δ are
a = 18 mm
b = 24 mm
c = 30 mm

$$S = \frac{a+b+c}{2} = \frac{18+24+30}{2}$$

$$= \frac{72}{2} = 36$$

$$\text{area of } \Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{36(36-18)(36-24)(36-30)}$$

$$= \sqrt{36 \times 18 \times 12 \times 6}$$

$$= \sqrt{18 \times 2 \times 18 \times 2 \times 6 \times 6}$$

$$= \sqrt{18 \times 18 \times 2 \times 2 \times 6 \times 6}$$

$= 18 \times 2 \times 6 = 216\ mm^2$ **Ans.**

(iii) Sides of Δ are
a = 21 m
b = 28 m
c = 35 m

$$S = \frac{a+b+c}{2} = \frac{21+28+35}{2}$$

$$= \frac{84}{2} = 42$$

$$\text{area of } \Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{42(42-21)(42-28)(42-35)}$$

$$= \sqrt{42 \times 21 \times 14 \times 7}$$

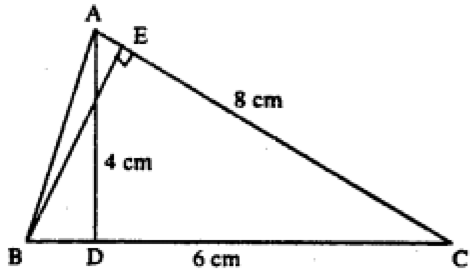
$$= \sqrt{7 \times 3 \times 2 \times 3 \times 7 \times 2 \times 7 \times 7}$$

$$= \sqrt{7 \times 7 \times 7 \times 7 \times 3 \times 3 \times 2 \times 2}$$

$= 7 \times 7 \times 3 \times 2$

$= 294\ m^2$ **Ans.**

Question 2.
Two sides of a triangle are 6 cm and 8 cm. If height of the triangle corresponding to 6 cm side is 4 cm ; find :
(i) area of the triangle
(ii) height of the triangle corresponding to 8 cm side.
Solution:



$$BC = 6 \text{ cm}$$
$$\text{height } AD = 4 \text{ cm}$$
$$\text{area of } \Delta = \frac{1}{2} \text{base} \times \text{height}$$
$$= \frac{1}{2} \times BC \times AD$$
$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$$
$$\text{Again area of } \Delta = \frac{1}{2} AC \times BE$$
$$12 = \frac{1}{2} \times 8 \times BE$$
$$\therefore \quad BE = \frac{12 \times 2}{8}$$
$$BE = 3 \text{ cm}$$
$\therefore$ (*i*) 12 cm^2 (*ii*) 3 cm **Ans.**

Question 3.
The sides of a triangle are 16 cm, 12 cm and 20 cm. Find :
(i) area of the triangle ;
(ii) height of the triangle, corresponding to the largest side ;
(iii) height of the triangle, corresponding to the smallest side.
Solution:
Sides of Δ are
a = 20 cm
b = 12 cm

c = 16 cm

$$S = \frac{a+b+c}{2}$$

$$= \frac{20+12+16}{2}$$

$$= \frac{48}{2} = 24$$

$$\text{area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-20)(24-12)(24-16)}$$

$$= \sqrt{24 \times 4 \times 12 \times 8}$$

$$= \sqrt{12 \times 2 \times 4 \times 12 \times 2 \times 4}$$

$$= \sqrt{12 \times 12 \times 4 \times 4 \times 2 \times 2}$$

$$= 12 \times 4 \times 2 = 96 \text{ cm}^2$$

AD is height of Δ corresponding to largest side.

$$\therefore \quad \frac{1}{2} \times BC \times AD = 96$$

$$\frac{1}{2} \times 20 \times AD = 96$$

$$AD = \frac{96 \times 2}{20}$$

$$AD = 9{\cdot}6 \text{ cm}$$

BE is height of Δ corresponding to smallest side.

$$\therefore \quad \frac{1}{2} AC \times BE = 96$$

$$\frac{1}{2} \times 12 \times BE = 96$$

$$BE = \frac{96 \times 2}{12}$$

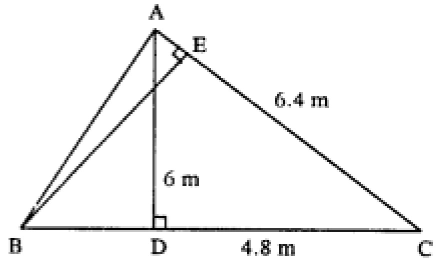
$$BE = 16 \text{ cm}$$

(*i*) 96 cm^2 (*ii*) 9·6 cm (*iii*) 16 cm **Ans.**

Question 4.

Two sides of a triangle are 6.4 m and 4.8 m. If height of the triangle corresponding to 4.8 m side is 6 m; find :
(i) area of the triangle ;
(ii) height of the triangle corresponding to 6.4 m side.

Solution:



ABC is the Δ in which BC = 4·8 m

AC = 6·4 m and AD = 6 m

$\therefore$ area of $\Delta ABC = \frac{1}{2} BC \times AD$

$= \frac{1}{2} \times 4 \cdot 8 \times 6$

$= 14 \cdot 4 \text{ m}^2$

BE is height of Δ corresponding to 6·4 m

$\therefore \frac{1}{2} AC \times BE = 14 \cdot 4$

$\frac{1}{2} \times 6 \cdot 4 \times BE = 14 \cdot 4$

$BE = \frac{14 \cdot 4 \times 2}{6 \cdot 4}$

$BE = \frac{14 \cdot 4}{3 \cdot 2}$

= 9/2 = 4.5 m

Hence (i) 14.4 m^2 (ii) 4.5 m

Question 5.

The base and the height of a triangle are in the ratio 4 : 5. If the area of the triangle is 40 m^2; find its base and height.

Solution:

Let base of Δ = 4x m

and height of Δ = 5x m

area of Δ =40 m^2

$$\because \quad \frac{1}{2}\text{base} \times \text{height} = \text{area of } \Delta$$

$$\frac{1}{2} \times 4x \times 5x = 40$$

$$10x^2 = 40$$

$$x^2 = 4$$

$$x = \sqrt{4}$$

$$x = 2$$

$$\therefore \quad \text{base} = 4x = 4 \times 2 = 8 \text{ m}$$

$$\text{height} = 5x = 5 \times 2 = 10 \text{ m}$$

∴ 8 m; 10 m **Ans.**

Question 6.

The base and the height of a triangle are in the ratio 5 : 3. If the area of the triangle is 67.5 m^2; find its base and height.

Solution:

Let base = 5x m

height = 3x m

$$\text{Area of } \Delta = \frac{1}{2}\text{base} \times \text{height}$$

$$\therefore \quad \frac{1}{2} \times 5x \times 3x = 67{\cdot}5$$

$$x^2 = \frac{67{\cdot}5 \times 2}{15}$$

$$x^2 = 4{\cdot}5 \times 2$$

$$x^2 = 9{\cdot}0$$

$$x = \sqrt{9}$$

$$x = 3$$

base = 5x = 5 x 3 = 15 m

height = 3x = 3 x 3 = 9 m

Question 7.

The area of an equilateral triangle is 144√3 cm^2; find its perimeter.

Solution:

Let each side of an equilateral triangle = x cm

$\therefore$ Its area $= \frac{\sqrt{3}}{4}$ (side)^2

$= \frac{\sqrt{3}}{4} x^2 = 144\sqrt{3}$ (given)

$\Rightarrow x^2 = 144\sqrt{3} \times \frac{4}{\sqrt{3}}$

$\Rightarrow x^2 = 144 \times 4 \quad \Rightarrow x^2 = 576$

$\Rightarrow x = \sqrt{576} = 24$ cm.

$\Rightarrow$ Each side = 24 cm

Hence perimeter = 3(24) = 72 cm.

Question 8.

The area of an equilateral triangle is numerically equal to its perimeter. Find its perimeter correct to 2 decimal places.

Solution:

Let each side of the equilateral traingle = x

$\therefore$ Its area $= \frac{\sqrt{3}}{4} x^2$

Area perimeter = $3x$

By the given condition $= \frac{\sqrt{3}}{4} x^2 = 3x$

$x^2 = 3x \times \frac{4}{\sqrt{3}}$

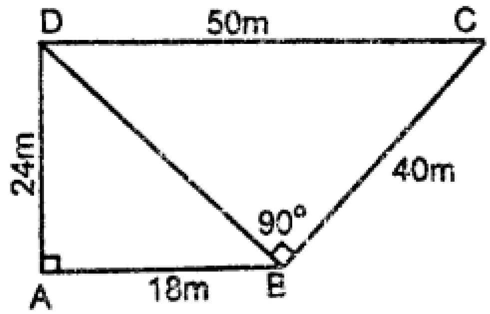
$x^2 = \frac{3x \times 4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3x \times 4 \times \sqrt{3}}{3} = 4x\sqrt{3}$

$\Rightarrow x^2 = \sqrt{3}\ (4x) \quad \Rightarrow x = 4\sqrt{3} \qquad [\because x \neq 0]$

$\therefore$ Perimeter $= 12\sqrt{3}$ units

$= 12\ (1.732) = 20.784 = 20.78$ units

Question 9.
A field is in the shape of a quadrilateral ABCD in which side AB = 18 m, side AD = 24 m, side BC = 40m, DC = 50 m and angle A = 90°. Find the area of the field.
Solution:
Since ∠A = 90°
By Pythagorus Theorem,
In △ABD,



$$BD = \sqrt{AB^2 + AD^2} = \sqrt{18^2 + 24^2}$$

$$= \sqrt{324 + 576} = \sqrt{900} = 30 \text{ m}.$$

Now, area of $\Delta ABD = \frac{1}{2}(18)(24)$

$$= (18)(12) = 216 \text{ m}^2$$

Again in ΔBCD; sides are 30, 40, 50

⇒ By Pythagoras Theorem $\angle CBD = 90°$

[$\therefore$ $DC^2 = BD^2 + BC^2$, Since $(50)^2 = (30)^2 + (40)^2$]

$\therefore$ Area of $\Delta BCD = \frac{1}{2}(40)(30) = 600 \text{ m}^2$

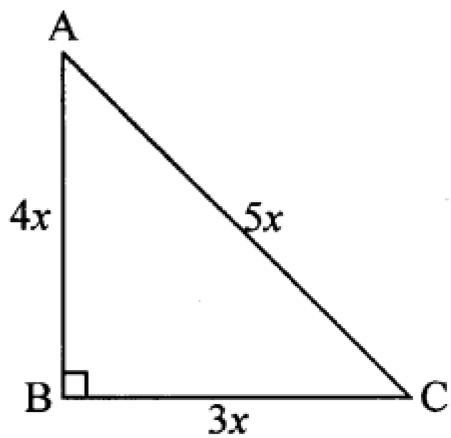
Hence, area of quadrilateral ABCD = Area of ΔABD + area of ΔBCD

$= 216 + 600 = 816 \text{ m}^2$.

Question 10.
The lengths of the sides of a triangle are in the ratio 4 : 5 : 3 and its perimeter is 96 cm. Find its area.
Solution:
Let the sides of the triangle ABC be 4x, 5x and 3x
Let AB = 4x, AC = 5x and BC = 3x

Perimeter = 4x + 5x + 3x = 96
=> 12x = 96

$$\Rightarrow \quad x = \frac{96}{12}$$



$\therefore x = 8$

$\therefore$ Sides are

BC = 3(8) = 24 cm, AB = 4 (8) = 32 cm,

AC = 5(8) = 40 cm

Since $(AC)^2 = (AB)^2 + (BC)^2$

$$[\because (5x)^2 = (3x)^2 + (4x)^2]$$

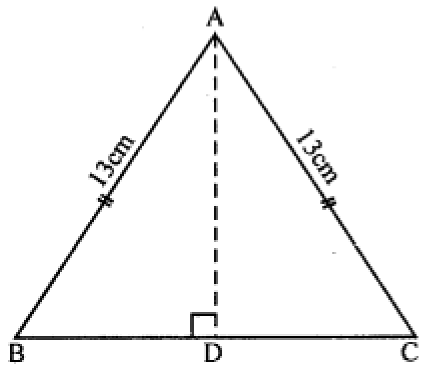
$\therefore$ By Pythagorus Theorem, $\angle B = 90°$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} (BC)(AB) = \frac{1}{2} (24)(32)$$

$= 12 \times 32 = 384 \text{ cm}^2$

Question 11.
One of the equal sides of an isosceles triangle is 13 cm and its perimeter is 50 cm. Find the area of the triangle.
Solution:
In isosceles ΔABC

AB = AC = 13 cm But perimeter = 50 cm



$\therefore$ BC = 50 – (13 + 13) cm

= 50 – 26 = 24 cm

AD ⊥ BC

$\therefore$ AD = DC = $\frac{24}{2}$ = 12 cm.

In right ΔABD,

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem)

$(13)^2 = AD^2 + (12)^2$

$\Rightarrow$ $169 = AD^2 + 144$

$\Rightarrow$ $AD^2 = 169 - 144$

$= 25 = (5)^2$

$\therefore$ AD = 5 cm.

Now area of ΔABC = $\frac{1}{2}$ Base × Altitude

$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 24 \times 5 = 60 \text{ cm}^2$

Question 12.

The altitude and the base of a triangular field are in the ratio 6 : 5. If its cost is ₹ 49,57,200 at the rate of ₹ 36,720 per hectare and 1 hectare = 10,000 sq. m, find (in metre) dimensions of the field,

Solution:

Total cost = ₹ 49,57,200

Rate = ₹ 36,720 per hectare

Total area of the triangular field

$$= \frac{4957200}{36720} \times 10000 \text{ m}^2 = 1350000 \text{ m}^2$$

Ratio in altitude and base of the field = 6 : 5

Let altitude = $6x$

and base = $5x$

$$\therefore \text{ Area} = \frac{1}{2} \text{ Base} \times \text{Altitude}$$

$$\Rightarrow 1350000 = \frac{1}{2} \times 5x \times 6x$$

$$\Rightarrow 15x^2 = 1350000 \quad \Rightarrow x^2 = \frac{1350000}{15}$$

$$\Rightarrow x^2 = 90000 = (300)^2$$

$$\therefore x = 300$$

$$\therefore \text{ Base} = 5x = 5 \times 300 = 1500 \text{ m}$$

and altitude $= 6x = 6 \times 300 = 1800$ m

Question 13.

Find the area of the right-angled triangle with hypotenuse 40 cm and one of the other two sides 24 cm.

Solution:

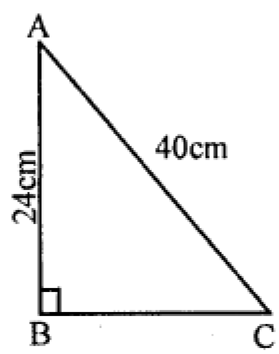
In right angled triangle ABC Hypotenuse AC = 40 cm

One side AB = 24 cm

$$\therefore \text{ BC} = \sqrt{AC^2 - AB^2}$$

$$= \sqrt{40^2 - 24^2} = \sqrt{1600 - 576}$$

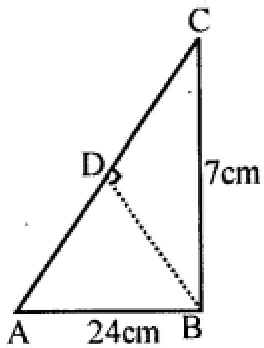
$$= \sqrt{1024} = 32 \text{ cm}$$



$\therefore$ Area

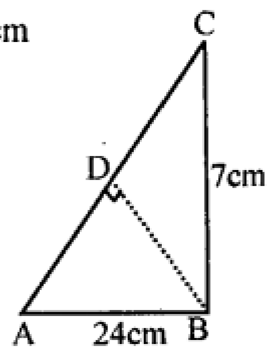
$$= \frac{1}{2} AB \times BC = \frac{1}{2} \times 24 \times 32 \text{ cm}^2 = 384 \text{ cm}^2$$

Question 14.
Use the information given in the adjoining figure to find :
(i) the length of AC.
(ii) the area of a ΔABC
(iii) the length of BD, correct to one decimal place.



Solution:

Sol. AB = 24 cm, BC = 7 cm



(*i*) $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{24^2 + 7^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625} = 25$ cm

(*ii*) Area of $\Delta ABC = \frac{1}{2} AB \times BC$

$= \frac{1}{2} \times 24 \times 7 = 84$ cm^2

(*iii*) $BD \perp AC$

Area $\Delta ABC = \frac{1}{2} AC \times BD$

$84 = \frac{1}{2} \times 25 \times BD$

$\Rightarrow BD = \frac{84 \times 2}{25} = \frac{168}{25} = 6.72$ cm

= 6.7 cm

EXERCISE 20(B)

Question 1.
Find the length and perimeter of a rectangle, whose area = 120 cm^2 and breadth = 8 cm
Solution:
area of rectangle = 120 cm^2
breadth, b = 8 cm
Area = l x b
l x 8 = 120
l = 120/8 = 15 cm
Perimeter = 2 (l+b) = 2(15+8) = 2 x 23 = 46 cm
Length = 15 cm; Perimeter = 46 cm

Question 2.
The perimeter of a rectangle is 46 m and its length is 15 m. Find its :
(i) breadth
(ii) area
(iii) diagonal.
Solution:
(i) Perimeter of rectangle = 46 m
length, l = 15 m
2 (l+b) = 46
2(15 + b) = 46
15+b = 46/2 = 23
b = 23 – 15
b = 8 m

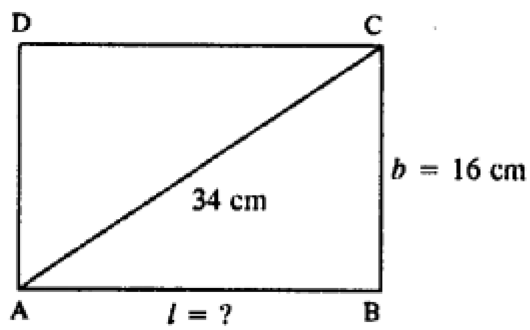
(*ii*) area $= l \times b = 15 \times 8 = 120 \text{ m}^2$

(*iii*) diagonal $= \sqrt{l^2 + b^2} = \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64} = \sqrt{289} = 17 \text{ m}$

Hence (*i*) 8 m (*ii*) 120 m^2 (*iii*) 17 m **Ans.**

Question 3.
The diagonal of a rectangle is 34 cm. If its breadth is 16 cm; find its :
(i) length
(ii) area
Solution:



$AC^2 = AB^2 + BC^2$ (By Pythagoras theorem)
$(34)^2 = l^2 + (16)^2$
$1156 = l^2 + 256$
$l^2 = 1156 - 256$
$l^2 = 900$
$l = \sqrt{900} = 30$ cm
area = l x b = 30 x 16 = 480 cm^2
(i) 30 cm (ii) 480 cm^2

Question 4.
The area of a small rectangular plot is 84 m^2. If the difference between its length and the breadth is 5 m; find its perimeter.
Solution:
Area of a rectangular plot = 84 m^2
Let breadth = x m
Then length = (x + 5) m
Area = l x b
x(x + 5) = 84
$x^2 + 5x - 84 = 0$
=> $x^2 + 12x - 7x - 84 = 0$
=> x(x + 12) – 7(x + 12) = 0
=> (x + 12) (x – 7) = 0
Either x + 12 = 0, then x = -12 which is not possible being negative
or x – 7 = 0, then x = 7
Length = x + 5 = 7 + 5 = 12m
and breadth = x = 7 m
Perimeter = 2(l + b) = 2(12+ 7) = 2 x 19 m = 38 m

Question 5.
The perimeter of a square is 36 cm; find its area
Solution:

Perimeter of Square = 36 cm

$$\text{Side} = \frac{\text{Perimeter}}{4} = \frac{36}{4} = 9 \text{ cm}$$

$$\therefore \quad \text{Area of Square} = \text{side} \times \text{side}$$
$$= 9 \times 9$$
$$= 81 \text{ cm}^2$$

Question 6.
Find the perimeter of a square; whose area is : 1.69 m^2
Solution:
Area of square= 1.69 m^2
Side = √area = √1.69 = 1.3 m
Perimeter = 4 x side = 4 x 1.3 = 5.2 m

Question 7.
The diagonal of a square is 12 cm long; find its area and length of one side.
Solution:
Let side of square = a cm
diagonal = 12 cm
By Pythagoras Theorem, $a^2 + a^2 = (12)^2$
$2a^2 = 144$
$a^2 = 72$
Area of square = a^2 = 72 cm^2
$a^2 = 72$
a = √72 = 8.49 cm

Question 8.
The diagonal of a square is 15 m; find the length of its one side and perimeter.
Solution:
Diagonal of square = 15 m
Let side of square = a
$a^2 + a^2 = (15)^2 = 225$
a^2 = 225/2 = 112.50
a = √112.50 = 10.6 m
Perimeter = 4 x a = 10.6 x 4 = 42.4 m

Question 9.
The area of a square is 169 cm^2. Find its:
(i) one side
(ii) perimeter
Solution:
Let each side of the square be x cm.
Its area = x^2 = 169 (given)

$x = \sqrt{169}$
x = 13 cm
(i) Thus, side of the square = 13 cm
(ii) Again perimeter = 4 (side) = 4 x 13 = 52 cm

Question 10.
The length of a rectangle is 16 cm and its perimeter is equal to the perimeter of a square with side 12.5 cm. Find the area of the rectangle.
Solution:
Length of the rectangle = 16 cm
Let its breadth be x cm
Perimeter = 2 (16 + x) = 32 + 2x
Also perimeter = 4(12.5) = 50 cm.
According to statement,
32 + 2x = 50
=> 2x = 50 – 32 = 18
=> x = 9
Breadth of the rectangle = 9 cm.
Area of the rectangle (l x b)= 16 x 9 = 144 cm^2

Question 11.
The perimeter of a square is numerically equal to its area. Find its area.
Solution:
Let each side of the square be x cm.
Its perimeter = 4x,
Area $= x^2$
By the given condition $4x = x^2$
=> $x^2 - 4x = 0$
=> x (x – 4) = 0
=> x = 4 [$x \neq 0$]
Area = $x^2 = (4)^2$ = 4 x 4 = 16 sq.units.

Question 12.
Each side of a rectangle is doubled. Find the ratio between :
(i) perimeters of the original rectangle and the resulting rectangle.
(ii) areas of the original rectangle and the resulting rectangle.
Solution:
Let length of the rectangle = x
and breadth of the rectangle = y
(i) Perimeter P = 2(x + y)
Again, new length = 2x

New breadth = 2y

$\therefore$ New perimeter $P' = 2(2x + 2y)$

$= 4(x + y) = 2.2(x + y) = 2P$

$\therefore \frac{P}{P'} = \frac{1}{2}$ i.e. $P : P' = 1 : 2$

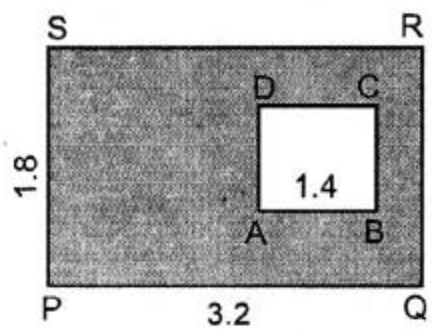
(ii) Area $A = xy$

New Area $A' = (2x)(2y) = 4xy = 4A$

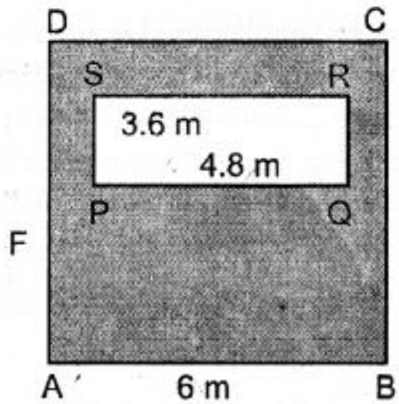
$\therefore \frac{A}{A'} = \frac{1}{4}$ i.e. $A : A' = 1 : 4$

Question 13.
In each of the following cases ABCD is a square and PQRS is a rectangle. Find, in each case, the area of the shaded portion.
(All measurements are in metre).

(i)



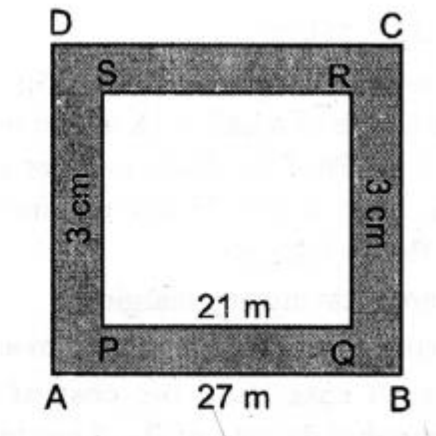
(ii)



Solution:
(i) Area of the shaded portion

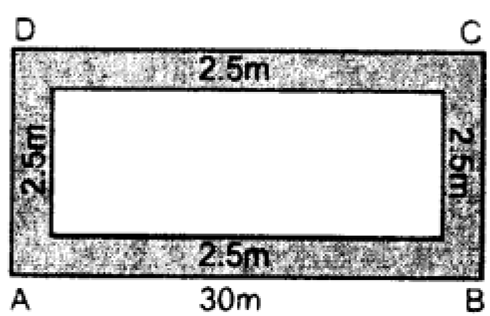
= Area of the rectangle PQRS – Area of square ABCD
= 3.2 x 1.8 – $(1.4)^2$ (∵ PQ = 3.2 and PS = 1.8) Side of square AB = 1.4
= 5.76 – 1.96 = 3.80 = 3.8 m^2
(ii) Area of the shaded portion = Area of square ABCD – Area of rectangle PQRS
= 6 x 6 – (3.6) (4.8) = 36 – 17.28 = 18.72 m^2

Question 14.
A path of uniform width, 3 m, runs around the outside of a square field of side 21 m. Find the area of the path.
Solution:
According to the given information the figure will be as shown alongside.
Clearly, length of the square field excluding path = 21 m.



Area of the square side excluding the path = 21 x 21 = 441 m^2
Again, length of the square field including the path = 21 + 3 + 3 = 27 m
Area of the square field including the path = 27 x 27 = 729 m^2
Area of the path = 729 – 441 = 288 m^2

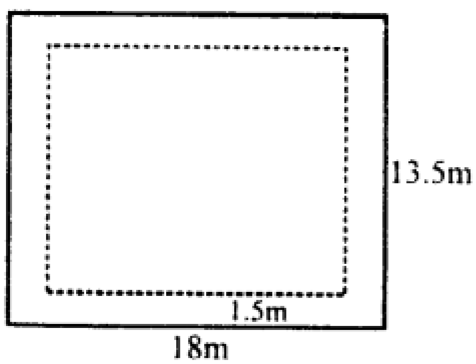
Question 15.
A path of uniform width, 2.5 m, runs around the inside of a rectangular field 30 m by 27 m. Find the area of the path.
Solution:
According to the given statement the figure will be as shown alongside.



Clearly, the length of the rectangular field including the path = 30 m.
Breadth = 27 m.

Its Area = 30 x 27 = 810 m^2
Width of the path = 2.5 m
Length of the rectangular field including the path = 30 – 2.5 – 2.5 = 25 m.
Breadth = 27 – 2.5 – 2.5 = 22m
Area of the rectangular field including the path = 25 x 22 = 550 m^2
Hence, area of the path = 810 – 550 = 260 m^2.

Question 16.
The length of a hall is 18 m and its width is 13.5 m. Find the least number of square tiles, each of side 25 cm, required to cover the floor of the hall,
(i) without leaving any margin.
(ii) leaving a margin of width 1.5 m all around. In each case, find the cost of the tiles required at the rate of Rs. 6 per tile
Solution:
(i) Length of hall (l) = 18 m and breadth (b) = 13.5 m



$\therefore$ Area of the floor = $l \times b$

$= 18 \times 13.5 \text{ m}^2 = 243.0 \text{ m}^2$

Side of each square tiles (a) = 25 cm

$= \frac{25}{100} = \frac{1}{4}$ m

$\therefore$ Area of one tile = $a^2 = \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16} \text{ m}^2$

No. of tiles required = $243 \div \frac{1}{16}$

$= \frac{243 \times 16}{1} = 3888$

Rate of tiles = Rs. 6 per tile

$\therefore$ Total cost = Rs. 3888×6

= Rs. 23328

ii) Width of margin left in side = 1.5 m

$\therefore$ Inner length = $18 - 2 \times 1.5 = 18 - 3 = 15$ m

and breadth = $13.5 - 2 \times 1.5 = 13.5 - 3$
$= 10.5$ m

$\therefore$ Inner area = $15 \times 10.5 \text{ m}^2$

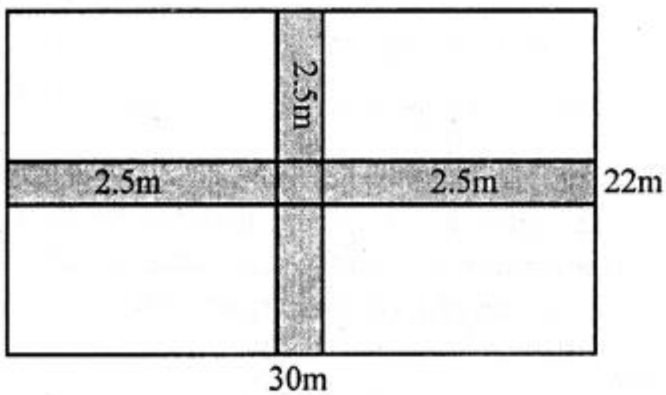
$= 157.5 \text{ m}^2$

$\therefore$ No. of tiles = $157.5 \div \frac{1}{16}$

$= 157.5 \times 16 = 2520$

$\therefore$ Cost of tiles = 2520×6 = Rs. 15120

Question 17.
A rectangular field is 30 m in length and 22m in width. Two mutually perpendicular roads, each 2.5 m wide, are drawn inside the field so that one road is parallel to the length of the field and the other road is parallel to its width. Calculate the area of the crossroads.
Solution:
Length of rectangular field (l) = 30 m and breadth (b) = 22m
width of parallel roads perpendicular to each other inside the field =2.5m



Area of cross roads = width of roads (Length + breadth) – area of middle square
= 2.5 (30 + 22) – $(2.5)^2$
= 2.5 x 52 – 6.25 m^2
= (130 – 6.25) m = 123.75 m^2

Question 18.
The length and the breadth of a rectangular field are in the ratio 5 : 4 and its area is 3380 m^2. Find the cost of fencing it at the rate of ₹75 per m.
Solution:
Ratio in length and breadth = 5 : 4
Area of rectangular field = 3380 m^2
Let length = 5x and breadth = 4x
5x x 4x = 3380
=> $20x^2$= 3380
x^2 = 3380/20 = 169 = (13)2
x = 13
Length = 13 x 5 = 65 m
Breadth =13 x 4 = 52 m
Perimeter = (l + b) = 2 x (65 + 52) m = 2 x 117 = 234 m
Rate of fencing = ₹ 75 per m
Total cost = 234 x 75 = ₹ 17550

Question 19.
The length and the breadth of a conference hall are in the ratio 7 : 4 and its perimeter is 110 m. Find:
(i) area of the floor of the hall.

(ii) number of tiles, each a rectangle of size 25 cm x 20 cm, required for flooring of the hall.
(iii) the cost of the tiles at the rate of ₹ 1,400 per hundred tiles.

Solution:

Ratio in length and breadth = 7 : 4
Perimeter = 110 m

$\therefore$ Length + Breadth $= \frac{110}{2} = 55$ m

Sum of ratios = 7 + 4 = 11

$\therefore$ Length $= \frac{55 \times 7}{11} = 35$ m

and breadth $= \frac{55 \times 4}{11} = 20$ m

(*i*) Area of floor $= l \times b$

$= 35 \times 20 = 700 \text{ m}^2$

(*ii*) Size of tile $= 25 \text{ cm} \times 20 \text{cm}$

$= \frac{25 \times 20}{100 \times 100}$

$= \frac{1}{20} \text{m}^2$

$\therefore$ Number of tiles

$= \frac{\text{Area of floor}}{\text{Area of one tile}}$

$= \frac{700 \times 20}{1} = 14000$

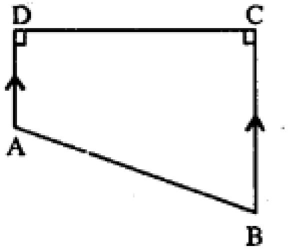
(*iii*) Cost of tiles = ₹ 1400 per 100 tiles

$\therefore$ Total cost $= \frac{14000 \times 1400}{100}$

= ₹ 196000

EXERCISE 20(C)

Question 1.
The following figure shows the cross-section ABCD of a swimming pool which is trapezium in shape.
If the width DC, of the swimming pool is 6.4cm, depth (AD) at the shallow end is 80 cm and depth (BC) at deepest end is 2.4m, find Its area of the cross-section.



Solution:
Area of the cross-section = Area of trapezium ABCD

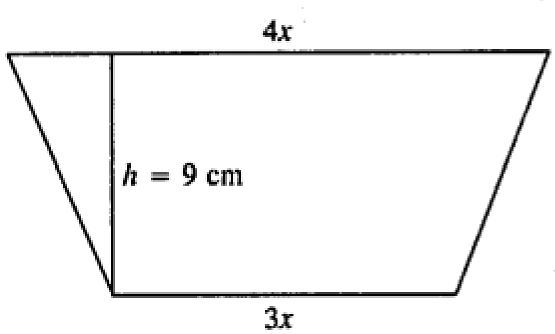
$$= \frac{1}{2} \text{(Sum of parallel sides)} \times \text{height}$$

$$= \frac{1}{2}(80 + 240) \times 6.4$$

$$= (320)(3.2) = (32)\ (32)$$

= 1024 cm^2 or = 10.24 sq.m.

Question 2.
The parallel sides of a trapezium are in the ratio 3 : 4. If the distance between the parallel sides is 9 dm and its area is 126 dm^2 ; find the lengths of its parallel sides.
Solution:



Let parallel sides of trapezium be
a = 3x
b = 4x
Distance between parallel sides, h = 9 dm

area of trapezium = 126 dm^2

$$\frac{1}{2}(a + b) \times h = 126$$

$$\frac{1}{2}(3x + 4x) \times 9 = 126$$

$$7x \times 9 = 126 \times 2$$

$$x = \frac{126 \times 2}{7 \times 9}$$

$$x = 4$$

$$a = 3x = 3 \times 4 = 12 \text{ dm}$$

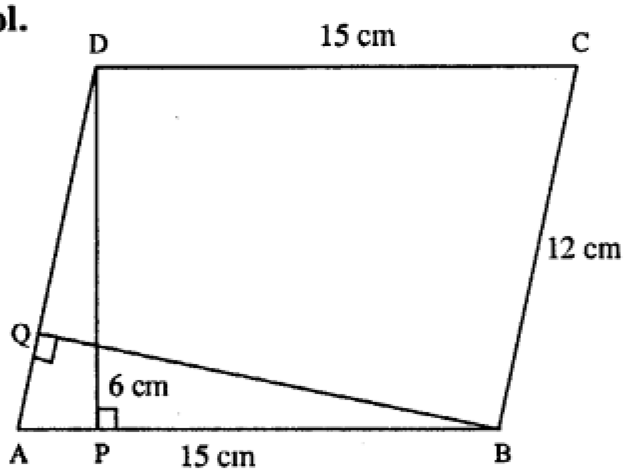
$$b = 4x = 4 \times 4 = 16 \text{ dm}$$

12 dm , 16 dm Ans.

Question 3.
The two parallel sides and the distance between them are in the ratio 3 : 4 : 2. If the area of the trapezium is 175 cm^2, find its height.
Solution:
Let the two parallel sides and the distance between them be 3x, 4x, 2x cm respectively
Area = $\frac{1}{2}$ (sum of parallel sides) x (distance between parallel sides)
= $\frac{1}{2}$ (3x + 4x) x 2x = 175 (given)
=> 7x x x = 175
=> $7x^2 = 175$
=> $x^2 = 25$
=> x = 5
Height i.e. distance between parallel sides = 2x = 2 x 5 = 10 cm

Question 4.
A parallelogram has sides of 15 cm and 12 cm; if the distance between the 15 cm sides is 6 cm; find the distance between 12 cm sides.
Solution:

Sol.



$$\text{Base, } AB = 15 \text{ cm}$$

Distance between 15 cm sides

i.e. $\text{height } DP = 6 \text{ cm}$

$$\therefore \quad \text{Area of } ||\text{gm} = \text{Base} \times \text{height}$$
$$= AB \times DP$$
$$= 15 \times 6 = 90 \text{ cm}^2$$

Let BQ be distance between 12 cm sides

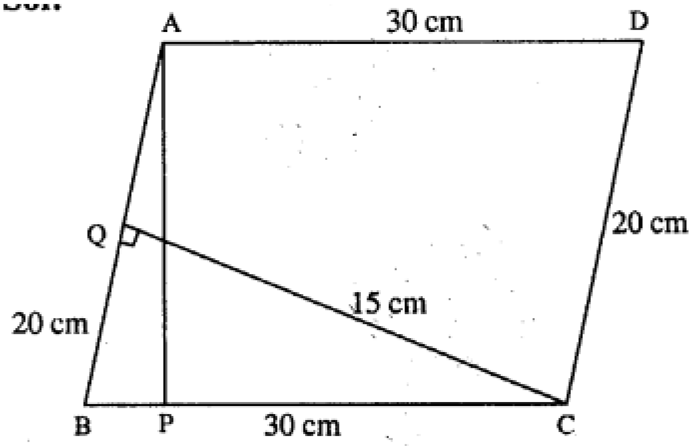
$$\therefore \quad AD \times BQ = \text{area of } ||\text{gm } ABCD$$
$$\therefore \quad 12 \times BQ = 90$$
$$BQ = \frac{90}{12}$$

BQ = $\frac{15}{2}$ = 7.5 cm

Question 5.

A parallelogram has sides of 20 cm and 30 cm. If the distance between its shorter sides is 15 cm; find the distance between the longer sides.

Solution:



Let ABCD be the ||gm in which BC = 30 cm and CD = 20 cm

Distance between shorter sides,

i.e. CQ = 15 cm

$\therefore$ area of ||gm = $AB \times CQ$

$= 20 \times 15$

$= 300 \text{ cm}^2$

Again $BC \times AP$ = Area of || gm

$30 \times AP = 300$

$$AP = \frac{300}{30}$$

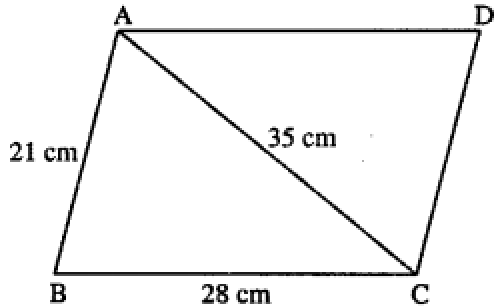
AP = 10 cm

$\therefore$ Distance between larger sides is = 10 cm **Ans.**

Question 6.

The adjacent sides of a parallelogram are 21 cm and 28 cm. If its one diagonal is 35 cm; find the area of the parallelogram.

Solution:



First we find area of ΔABC.

$$\text{Sides are, } a = 28 \text{ cm}$$
$$b = 35 \text{ cm}$$
$$\text{and } c = 21 \text{ cm}$$
$$S = \frac{a+b+c}{2} = \frac{28+35+21}{2} = \frac{84}{2} = 42 \text{ cm}$$

$$\text{area of } \Delta ABD = \sqrt{S(S-a)(S-b)(S-c)}$$
$$= \sqrt{42(42-28)(42-35)(42-21)}$$
$$= \sqrt{42 \times 14 \times 7 \times 21}$$
$$= \sqrt{2 \times 21 \times 2 \times 7 \times 7 \times 21}$$
$$= \sqrt{2 \times 2 \times 21 \times 21 \times 7 \times 7}$$
$$= 2 \times 21 \times 7$$
$$= 294 \text{ cm}^2$$

$\because$ Diagonal of ||gm divides it into two equal parts.

$$\therefore \text{ area of ||gm} = 2 \times \text{area of } \Delta ABC = 2 \times 294 = 588 \text{ cm}^2 \text{ Ans.}$$

Question 7.
The diagonals of a rhombus are 18 cm and 24 cm. Find:
(i) its area ;
(ii) length of its sides.

(iii) its perimeter;

Solution:

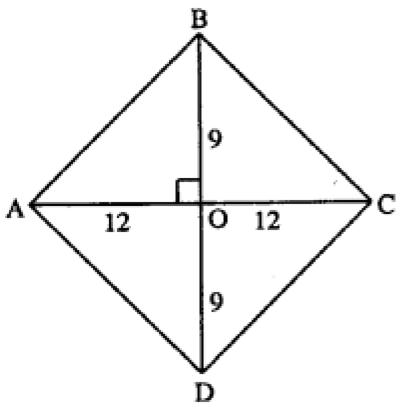
Diagonal of rhombus are 18 cm and 24 cm.

area of rhombus = $\frac{1}{2}$ x Product of diagonals

= $\frac{1}{2}$ x 18 x 24

= 216 cm^2

(ii) Diagonals of rhombus bisect each other at right angles.



$$\therefore \quad OA = \frac{1}{2} \times 24 = 12 \text{ cm}$$

$$OB = \frac{1}{2} \times 18 = 9 \text{ cm}$$

In right $\angle$d Δ AOB

$$AB = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{(12)^2 + (9)^2}$$

$$= \sqrt{144 + 81}$$

$$= \sqrt{225} = 15 \text{ cm}$$

$\therefore$ Side of rhombus = 15 cm

(*iii*) Perimeter of rhombus = $4 \times$ side

$= 4 \times 15 = 60$ cm

(*i*) 216 cm^2 (*ii*) 15 cm (*iii*) 60 cm **Ans.**

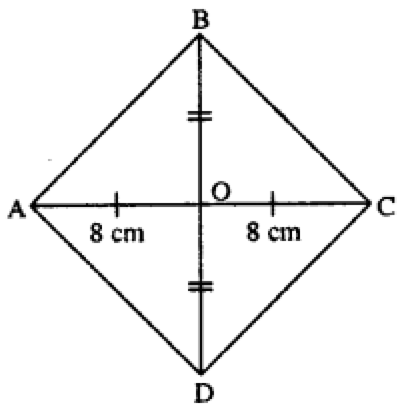
Question 8.

The perimeter of a rhombus is 40 cm. If one diagonal is 16 cm; find :

(i) its another diagonal

(ii) area

Solution:
(i) Perimeter of rhombus = 40 cm
side = $\frac{1}{4}$ x 40 = 10 cm
One diagonal = 16 cm
Diagonals of rhombus bisect each other at right angles.



$$OB = \sqrt{AB^2 - OA^2}$$
$$= \sqrt{(10)^2 - (8)^2}$$
$$= \sqrt{100 - 64}$$
$$= \sqrt{36}$$
$$= 6 \text{ cm}$$
$$\therefore \quad \text{diagonal BD} = 6 \times 2 = 12 \text{ cm}$$

(*ii*) Area of rhombus

$$= \frac{1}{2} \times \text{product of diagonals}$$
$$= \frac{1}{2} \times 12 \times 16$$
$$= 96 \text{ cm}^2$$

∴ (*i*) 12 cm (*ii*) 96 cm^2 **Ans.**

Question 9.
Each side of a rhombus is 18 cm. If the distance between two parallel sides is 12 cm, find its area.
Solution:
Each side of the rhombus = 18 cm
base of the rhombus = 18 cm

Distance between two parallel sides = 12 cm
Height = 12 cm
Area of the rhombus = base x height = 18 x 12 = 216 cm²

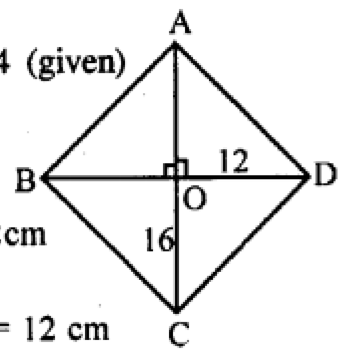
Question 10.
The length of the diagonals of a rhombus is in the ratio 4 : 3. If its area is 384 cm², find its side.
Solution:
Let the lengths of the diagonals of rhombus are 4x, 3x.

$\therefore$ Area of the rhombus $= \frac{1}{2}$ (Product of its diagonals)

$= \frac{1}{2}(4x \times 3x) = 384$ (given)

$\Rightarrow 6x^2 = 384 \Rightarrow x^2 = 64$

$\Rightarrow x = 8$ cm



$\therefore$ Diagonals are $4 \times 8 = 32$ cm and $3(8) = 24$ cm.

$\therefore$ OC = 16 cm and OD = 12 cm

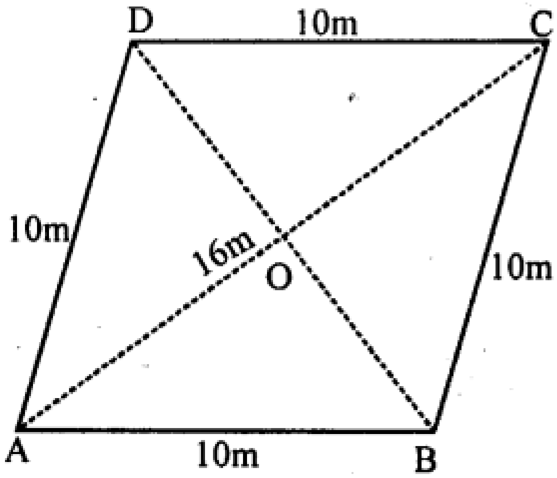
$\therefore$ Side DC = $\sqrt{OC^2 + OD^2}$

$\therefore$ Side DC = $\sqrt{16^2 + 12^2}$ [By Pythagoras Theorem in ΔDOC]

$= \sqrt{256 + 144} = \sqrt{400} = 20$ cm

Hence, side of the rhombus = 20 cm.

Question 11.
A thin metal iron-sheet is rhombus in shape, with each side 10 m. If one of its diagonals is 16 m, find the cost of painting its both sides at the rate of ₹ 6 per m².
Also, find the distance between the opposite sides of this rhombus.
Solution:
Side of rhombus shaped iron sheet = 10 m and one diagonals (AC) = 16 m
Join BD diagonal which bisects AC at O
The diagonals of a rhombus bisect each other at right angle



$\therefore$ $AO = OC = \frac{16}{2} = 8$ m.

Now in right ΔAOB

$AB^2 = AO^2 + BO^2 \Rightarrow (10)^2 = (8)^2 + BO^2$

$\Rightarrow 100 = 64 + BO^2 \Rightarrow BO^2 = 100 - 64 = 36$

$= (6)^2$

$\therefore$ $BO = 6$ m

$\therefore$ $BD = 2 \times BO = 2 \times 6 = 12$ m

Now, area of rhombus $= \frac{d_1 \times d_2}{2}$

$= \frac{16 \times 12}{2} = 96$ m^2

Rate of painting = ₹ 6 per m^2

$\therefore$ Total cost of painting both sides,

$= 2 \times 96 \times 6 =$ ₹ 1152

Distance between two opposite sides,

$= \frac{\text{Area}}{\text{Base}} = \frac{96}{10} = 9.6$ m

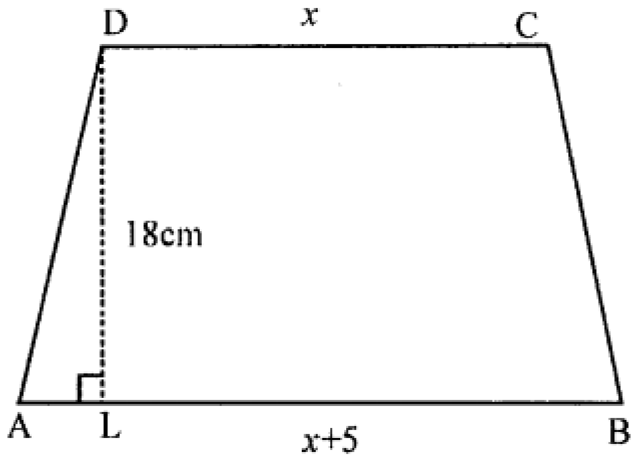
Question 12.

The area of a trapezium is 279 sq.cm and the distance between its two parallel sides is 18 cm. If one of its parallel sides is longer than the other side by 5 cm, find the lengths of its parallel sides.

Solution:

Area of trapezium = 279 sq.cm

Distance between two parallel lines (h) = 18 cm



$\therefore$ Sum of parallel sides $= \frac{\text{Area} \times 2}{\text{Height}}$

$= \frac{279 \times 2}{18} = 31$ m

Let shorter side, CD = x

Then longer side = $x + 5$

$\therefore x + x + 5 = 31$

$\Rightarrow 2x = 31 - 5 = 26$

$\Rightarrow x = \frac{26}{2} = 13$

$\therefore$ Shorter side = 13 cm

and longer side = 13 + 5 = 18 cm

Question 13.

The area of a rhombus is equal to the area of a triangle. If base of Δ is 24 cm, its corresponding altitude is 16 cm and one of the diagonals of the rhombus is 19.2 cm. Find its other diagonal.

Solution:

Area of a rhombus = Area of a triangle Base of triangle = 24 cm and altitude = 16 cm

$\therefore$ Area = $\frac{1}{2}$ base × altitude

$= \frac{1}{2} \times 24 \times 16 = 192 \text{ cm}^2$

$\therefore$ Area of rhombus = 192 cm^2

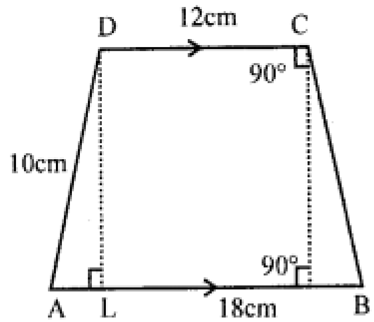
One diagonal = 19.2 cm

$\therefore$ Second diagonal = $\frac{\text{Area} \times 2}{\text{One diagonal}}$

$= \frac{192 \times 2}{19.2} = \frac{192 \times 10 \times 2}{192} = 20 \text{ cm}$

Question 14.
Find the area of the trapezium ABCD in which AB//DC, AB = 18 cm, ∠B = ∠C = 90°, CD = 12 cm and AD = 10 cm.

Solution: In trapezium ABCD,



AB ∥ DC, AB = 18 cm

∠B = ∠C = 90°, CD = 12 cm and AD = 10 cm

Area of trapezium ABCD

Draw DL ⊥ AB

$\therefore$ AL = 18 – 12 = 6 cm

AL = BC

$AL = \sqrt{AD^2 - AL^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{100 - 36}$

$= \sqrt{64} = 8 \text{ cm}$

Now area of trapezium = $\frac{1}{2}(AB + CD) \times AL$

$= \frac{1}{2}(18 + 12) \times 8 \text{ cm}^2$

$= \frac{1}{2} \times 30 \times 8 = 120 \text{ cm}^2$

EXERCISE 20 (D)

Question 1.

Find the radius and area of a circle, whose circumference is :
(i) 132 cm
(ii) 22 m

Solution:

(i) Circumference of circle = 132 cm
$2\pi r = 132$
$2 \times 22/7 \times r = 132$

$$r = \frac{132 \times 7}{2 \times 22}$$
$$r = 21 \text{ cm}$$
$$\therefore \quad \text{Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 21 \times 21$$
$$= 1386 \text{ cm}^2 \textbf{ Ans.}$$

(ii) Circumference of circle = 22 m

$$\therefore \quad 2\pi r = 22$$
$$2 \times \frac{22}{7} \times r = 22$$
$$r = \frac{22 \times 7}{2 \times 22}$$
$$r = \frac{7}{2}$$
$$r = 3{\cdot}5 \text{ m}$$
$$\text{Area of circle} = \pi r^2$$
$$= \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$$
$$= 38{\cdot}5 \text{ m}^2 \textbf{ Ans.}$$

Question 2.

Find the radius and circumference of a circle, whose area is :
(i) 154 cm^2
(ii) 6.16 m^2

(*i*) Area of circle $= 154 \text{ cm}^2$

$$\pi r^2 = 154$$

$$r^2 = \frac{154}{\pi}$$

$$r^2 = \frac{154}{22} \times 7$$

$$r^2 = 7 \times 7$$

$$r = 7 \text{ cm}$$

$\therefore$ circumference $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 7$$

$$= 44 \text{ cm}$$

Hence 7 cm ; 44 cm **Ans.**

(*ii*) Area of circle $= 6{\cdot}16 \text{ m}^2$

$$\pi r^2 = 6{\cdot}16$$

$$\frac{22}{7} r^2 = \frac{616}{100}$$

$$r^2 = \frac{616}{100} \times \frac{7}{22}$$

$$r^2 = \frac{196}{100}$$

$$r^2 = 1{\cdot}96$$

$$r = \sqrt{1{\cdot}96}$$

$$r = 1{\cdot}4 \text{ m}$$

Circumference $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 1{\cdot}4$$

$$= 8{\cdot}8 \text{ m}$$

$\therefore$ 1·4 m; 8·8 m **Ans.**

Question 3.

The circumference of a circular table is 88 m. Find its area.

Solution:

Circumference of circle = 88 m

$2\pi r = 88$ m

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{2 \times 22}$$

$$r = 14 \text{ m}$$

$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times 14 \times 14$$

$$= 616 \text{ m}^2 \textbf{ Ans.}$$

Question 4.
The area of a circle is 1386 sq.cm ; find its circumference.
Solution:
Area of circle = 1386 cm^2
$\pi r^2 = 1386$

$$\frac{22}{7} r^2 = 1386$$

$$r^2 = 1386 \times \frac{7}{22}$$

$$r^2 = 441$$

$$r = \sqrt{441}$$

$$r = 21 \text{ cm}$$

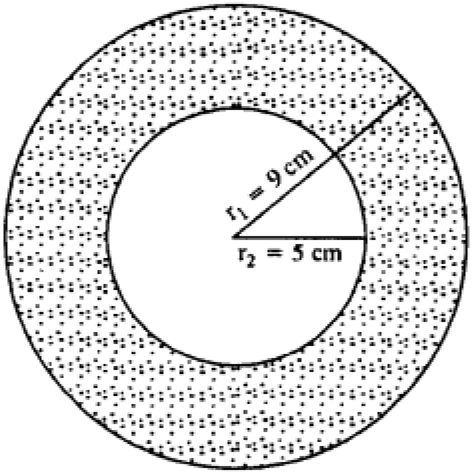
$$\text{Circumference} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 21$$

$$= 132 \text{ m } \textbf{Ans.}$$

Question 5.
Find the area of a flat circular ring formed by two concentric circles (circles with same centre) whose radii are 9 cm and 5 cm.

Solution:



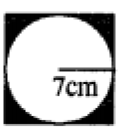
External radius, $r_1 = 9$ cm

Internal radius, $r_2 = 5$ cm

$$\text{Area of ring} = \pi r_1^2 - \pi r_2^2$$
$$= \pi(r_1^2 - r_2^2)$$
$$= \pi(9^2 - 5^2)$$
$$= \frac{22}{7}[81 - 25]$$
$$= \frac{22}{7} \times 56$$
$$= 176 \text{ cm}^2 \textbf{ Ans.}$$

Question 6.

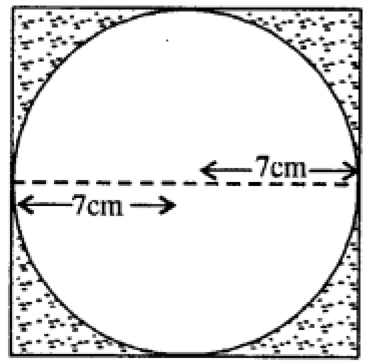
Find the area of the shaded portion in each of the following diagrams :

(i) 

(ii) 

Solution:

(*i*)



Radius of circle, $r = 7$ cm

$\therefore$ Side of Square $= 7+7 = 14$ cm

Area of circle $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7$

$= 154 \text{ cm}^2$

Area of Square $= 14 \times 14$

$= 196 \text{ cm}^2$

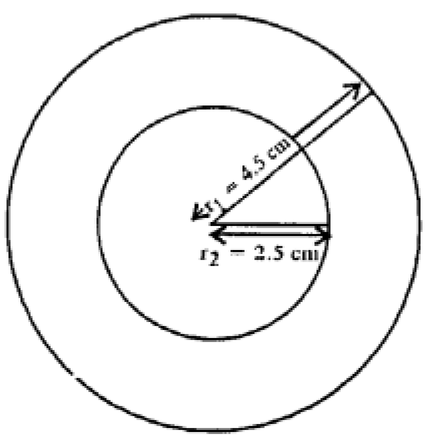
$\therefore$ Area of Shaded portion $= 196 - 154$

$= 42 \text{ cm}^2$ **Ans.**

(*ii*) Radii of concentric circles are

$r_1 = 4{\cdot}5$ m

$r_2 = 2{\cdot}5$ m



$\therefore$ Area of shaded portion $= \pi r_1^2 - \pi r_2^2$

$= \pi[r_1^2 - r_2^2] = \frac{22}{7}\left[(4 \cdot 5)^2 - (2 \cdot 5)^2\right]$

$= \frac{22}{7} \times (4.5 + 2.5)(4.5 - 2.5)$

$= \frac{22}{7} \times 14 = 44 \text{ cm}^2$ **Ans.**

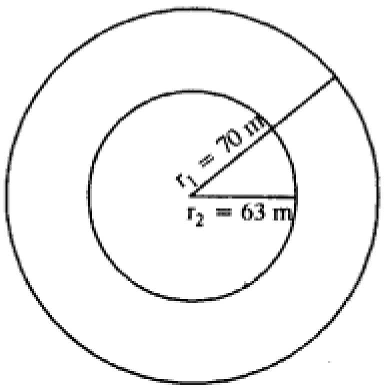
Question 7.

The radii of the inner and outer circumferences of a circular running track are 63 m and 70 m respectively. Find :

(i) the area of the track ;

(it) the difference between the lengths of the two circumferences of the track.

Solution:



Outer radius, $r_1 = 70$ m

Inner radius, $r_2 = 63$ m

$\therefore$ Area of track $= \pi r_1^2 - \pi r_2^2$

$= \frac{22}{7}\left[(70)^2 - (63)^2\right]$

$= \frac{22}{7}(70 + 63)(70 - 63)$

$= \frac{22}{7} \times 133 \times 7$

$= 2926 \text{ m}^2$

Length of outer edge *i.e.* circumference

$= 2\pi r_1$

$= 2 \times \frac{22}{7} \times 70 = 440\text{m}$

Length of inner edge $= 2\pi r_2$

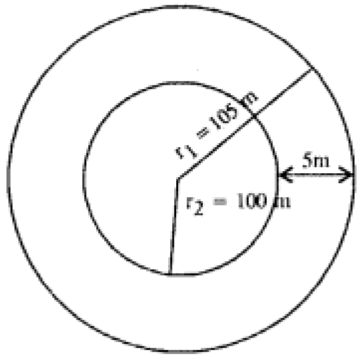
= 2x 22/7 x 63 = 396 m

Difference between lengths of two circumferences = 440 – 396 = 44 m

Hence (i) 2926 m^2 (ii) 44 m

Question 8.
A circular field cf radius 105 m has a circular path of uniform width of 5 m along and inside its boundary. Find the area of the path.

Solution:



Radius of circular field, $r_1 = 105$ m

Width of path $= 50$ m

$\therefore$ Radius of inner circle, $r_2 = 105 - 5 = 100$m

$\therefore$ Area of path $= \pi r_1^2 - \pi r_2^2$

$$= \frac{22}{7}[(105)^2 - (100)^2]$$

$$= \frac{22}{7}(105 + 100)(105 - 100)$$

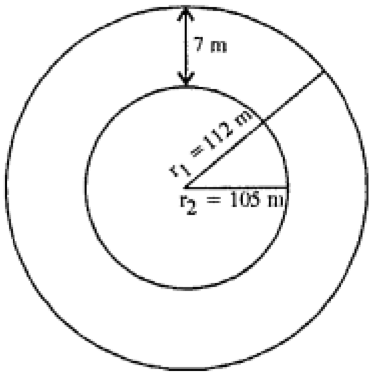
$$= \frac{22}{7} \times 205 \times 5$$

$$= \frac{22550}{7} \text{ m}^2$$

$$= 3221\frac{3}{7} \text{ m}^2 \textbf{ Ans.}$$

Question 9.
There is a path of uniform width 7 m round and outside a circular garden of diameter 210 m. Find the area of the path.

Solution:



$$\text{Diameter} = 210 \text{ m}$$
$$\text{Radius of inner circule } r_2 = 105 \text{ m}$$
$$\text{Width} = 7\text{m}$$
$$\text{Radius of outer circle } r_1 = 105 + 7 = 112 \text{ m}$$
$$\therefore \quad \text{Area of path} = \pi r_1^2 - \pi r_2^2$$
$$= \pi[r_1^2 - r_2^2]$$
$$= \frac{22}{7}(r_1 + r_2)(r_1 + r_2)$$
$$= \frac{22}{7}(112 + 105)$$
$$(112 - 105)$$
$$= \frac{22}{7} \times 217 \times 7$$
$$= 4774 \text{ m}^2 \text{ **Ans.**}$$

Question 10.
A wire, when bent in the form of a square encloses an area of 484 cm^2. Find :
(i) one side of the square ;
(ii) length of the wire ;
(iii) the largest area enclosed; if the same wire is bent to form a circle.
Solution:
(i) Area of Square = 484 cm^2
Side of Square = √Area = √484 = 22 cm
(ii) Perimeter, i.e. length of wire = 4 x 22 = 88 cm
(iii) Circumference of circle = 88 cm

$2\pi r = 88$

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{2 \times 22}$$

$$r = 14 \text{ cm}$$

$\therefore$ The largest area enclosed $= \pi r^2$

$$= \frac{22}{7} \times 14 \times 14$$

$$= 616 \text{ cm}^2$$

Hence (*i*) 22 cm (*ii*) 88 cm (*iii*) 616 cm^2 **Ans.**

Question 11.

A wire, when bent in the form of a square; encloses an area of 196 cm^2. If the same wire is bent to form a circle; find the area of the circle.

Solution:

Area of Square = 196 cm^2

Side of Square = √Area = √196 = 14 cm

Perimeter of Square = 4 x 14 cm

i.e. length of wire = 56 cm

Circumference of circle = 56 cm

$2\pi r = 56$

$$2 \times \frac{22}{7} \times r = 56$$

$$r = \frac{56 \times 7}{2 \times 22}$$

$$r = \frac{98}{11} \text{ cm}$$

$\therefore$ Area of circle enclosed $= \pi r^2$

$$= \frac{22}{7} \times \frac{98}{11} \times \frac{98}{11}$$

= 2744/11

249.45 cm^2

Question 12.

The radius of a circular wheel is 42 cm. Find the distance travelled by it in :

(i) 1 revolution ;

(ii) 50 revolutions ;

(iii) 200 revolutions ;

Solution:
(i) Radius of wheel, r = 42 cm
Circumference i.e. distance travelled in 1 revolution = $2\pi r = 2 \times 22/7 \times 42 = 264$ cm
(ii) Distance travelled in 50 revolutions = 264 x 50 = 13200 cm = 132 m
(iii) Distance travelled in 200 revolutions = 264 x 200 = 52800 cm = 528 m
Hence (i) 264 cm (ii) 132 m (iii) 528 m

Question 13.
The diameter of a wheel is 0.70 m. Find the distance covered by it in 500 revolutions. If the wheel takes 5 minutes to make 500 revolutions; find its speed in :
(i) m/s
(ii) km/hr.
Solution:
Diameter = 0.70 m
Radius, r = 0.35 m
Distance covered in 1 revolution, i.e. circumference = $2\pi r = 2 \times 22/7 \times 0.35 = 2.20$ m
Distance covered in 500 revolutions = 2.20 x 500 = 1100 m
Time taken = 5 minutes = 5 x 60 = 300 sec.

$$\therefore \quad \text{Speed in m/s} = \frac{1100}{300} = \frac{11}{3} = 3\frac{2}{3} \text{ m/s}$$

$$\text{Again, Distance} = 1100 \text{ m} = \frac{1100}{1000} = \frac{11}{10} \text{ km}$$

$$\text{Time} = 5 \text{ minutes} = \frac{5}{60} \text{ hr.}$$

$$\text{Speed in km/hr} = \frac{\frac{11}{10}}{\frac{5}{60}} = \frac{11}{10} \times \frac{60}{5} = \frac{66}{5} = 13{\cdot}2 \text{ km/hr.}$$

Hence 1100 m, (*i*) $3\frac{2}{3}$ m/s (*ii*) 13·2 km/hr **Ans.**

Question 14.
A bicycle wheel, diameter 56 cm, is making 45 revolutions in every 10 seconds. At what speed in kilometre per hour is the bicycle travelling ?
Solution:

Sol. Diameter = 56 cm

$\therefore$ Radius, r = 28 cm

$\therefore$ Distance travelled in 1 revolution

i.e. circumference $= 2\pi r = 2 \times \frac{22}{7} \times 28 = 176$cm

$\therefore$ Distance travelled in 45 revolutions

$= 176 \times 45 = 7920 \text{ cm} = \frac{7920}{100 \times 1000}$ km

Time $=$ 10 sec $= \frac{10}{60 \times 60}$ hr.

$$\text{Speed} = \frac{\frac{7920}{100 \times 1000}}{\frac{10}{60 \times 60}} \text{ m}$$

$$= \frac{7920}{100 \times 1000} \times \frac{60 \times 60}{10} = \frac{28512}{1000} \text{ km/hr}$$

$= 28 \cdot 512$ km/hr **Ans.**

Question 15.
A roller has a diameter of 1.4 m. Find :
(i) its circumference ;
(ii) the number of revolutions it makes while travelling 61.6 m.
Solution:

Diameter = 1.4 m

$r = \frac{1 \cdot 4}{2} = 0 \cdot 7$ m

$\therefore$ Circumference of roller $= 2\pi r$

$= 2 \times \frac{22}{7} \times 0 \cdot 7 = 4 \cdot 4$ m

Revolutions made in $4 \cdot 4$ m distance $= 1$

Revolutions made in 1 m distance $= \frac{1}{4 \cdot 4}$

Revolutions made in $61 \cdot 6$ m distance

$= \frac{1}{4 \cdot 4} \times 61 \cdot 6 = \frac{616}{44} = 14$

Hence (*i*) $4 \cdot 4$ m (*ii*) 14 **Ans.**

Question 16.
Find the area of the circle, length of whose circumference is equal to the sum of the lengths of the circumferences with radii 15 cm and 13 cm.
Solution:
In a circle
Circumference = Sum of circumferences of two circle of radii 15 cm and 13 cm
Now circumference of first smaller circle = 2πr

$$= 2 \times \frac{22}{7} \times 15 = \frac{660}{7} \text{ cm}$$

Circumference of second smaller circle

$$= 2 \times \frac{22}{7} \times 13 = \frac{572}{7} \text{ cm}$$

∴ Circumference of bigger circle

$$= \frac{660}{7} + \frac{572}{7} = \frac{1232}{7} \text{ cm}$$

Let R be its radius, then

$$2\pi R = \frac{1232}{7} \Rightarrow \frac{2 \times 22}{7} R = \frac{1232}{7}$$

$$\Rightarrow R = \frac{1232}{7} \times \frac{7}{44} = 28 \text{ cm}$$

∴ Area of the circle = πR^2

$$= \frac{22}{7} \times 28 \times 28 \text{ cm}^2 = 2464 \text{ cm}^2$$

Question 17.
A piece of wire of length 108 cm is bent to form a semicircular arc bounded by its diameter. Find its radius and area enclosed.
Solution:
Length of wire = 108 cm
Let r be the radius of the semicircle
πr+ 2r = 108

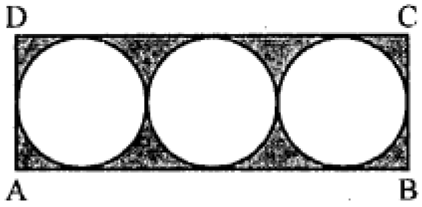
$$\Rightarrow r(\pi + 2) = 108 \Rightarrow r\left(\frac{22}{7} + 2\right) = 108$$

$$\Rightarrow \frac{36}{7} r = 108 \Rightarrow r = \frac{108 \times 7}{36} = 21 \text{ cm}$$

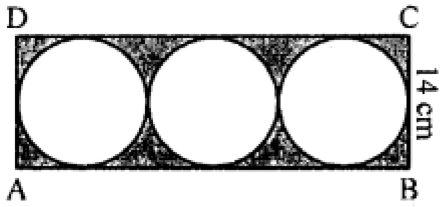
$$\text{Area} = \frac{\pi r^2}{2} = \frac{22}{7 \times 2} \times 21 \times 21 = \frac{1386}{2} \text{ cm}^2$$

$$= 693 \text{ cm}^2$$

Question 18.
In the following figure, a rectangle ABCD enclosed three circles. If BC = 14 cm, find the area of the shaded portion (Take π = 22/7)



In rectangle ABCD, BC = 14 cm



$\therefore$ Radius of each circle $= \frac{14}{2} = 7$ cm

$\therefore$ Length of rectangle $= (7 + 7 + 7) \times 2$

$= 42$ cm

Now area of rectangle $= l \times b = 42 \times 14$

$= 588 \text{ cm}^2$

and area of 3 circles $= 3 \times \pi r^2$

$= 3 \times \frac{22}{7} \times 7 \times 7 = 462 \text{ cm}^2$

$\therefore$ Area of shaded portion

$= 588 - 462 = 126 \text{ cm}^2$