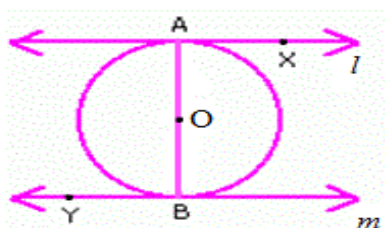


Chapter 10 Circle

Question-1

Prove that the line segment joining the points of contact of two parallel tangents to a circle is a diameter of the circle.

Solution:



Given: l and m are the tangent to a circle such that $l \parallel m$, intersecting at A and B respectively.

To prove: AB is a diameter of the circle.

Proof:

A tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle XAO = 90^\circ$$

$$\text{and } \angle YBO = 90^\circ$$

$$\text{Since } \angle XAO + \angle YBO = 180^\circ$$

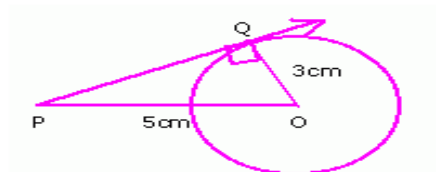
Angles on the same side of the transversal is 180° .

Hence the line AB passes through the centre and is the diameter of the circle.

Question-2

Find the length of the tangent from a point which is at a distance of 5 cm from the centre of the circle of radius 3 cm.

Solution:



Given: PQ is a tangent to the circle intersect at Q. $OP = 5$ cm and $OQ = 3$ cm.

To find: PQ

Proof:

In rt. Δ OQP, by Pythagoras theorem

$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4 \text{ cm}$$

Therefore the length of the tangent from a point is 4 cm.

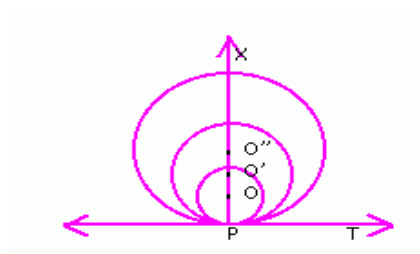
Question-3

Find the locus of centres of circles which touch a given line at a given point.

Solution:

Given: Circles with centres O, O', O'' touching line T at P.

To prove: To find the locus of centres of circles which touch a given line at a given point.



Proof: As OP, O'P, O''P are the radii of the circles touching line T at P, it is perpendicular to the given line.

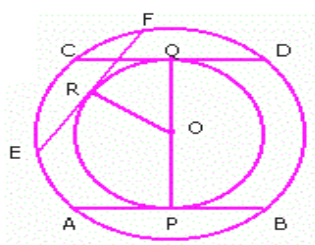
$\therefore$ OP, O'P, O''P represent the same straight line passing through P and $\perp$ to PT.

Hence the locus of the centres of circles which touch a given line at a given point is a straight line to the given line at the given point.

Question-4

In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length.

Solution:



Given: Two concentric circles with centre O. AB, CD and EF are the chords of the outer circle.

To prove: $AB = CD = EF$.

Proof:

OP, OQ and OR are the distances of the chord AB, CD and EF from the centre.

But $OP = OR = OQ = \text{radius}$

Since the chords are at equal distances from the centre they are equal.

Question-5

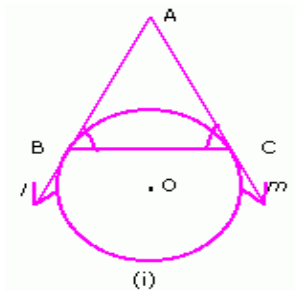
Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Solution:

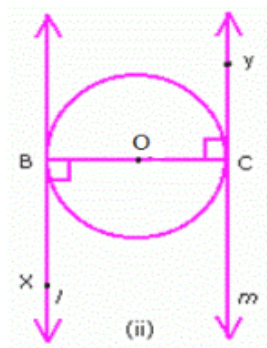
Given: l and m are tangents intersecting at A and touching the circle at B and C.

To prove:

(i) $\angle ABC = \angle ACB$



(ii) $\angle XBO = \angle OCY$

**Proof:**

(i) Using the theorem, the length of the tangents drawn from an external point to a circle are equal.

$\angle ABC = \angle ACB$ ($\because$ Sides opposite to equal angles are equal)

(ii) Using the theorem, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\angle XBO = 90^\circ$

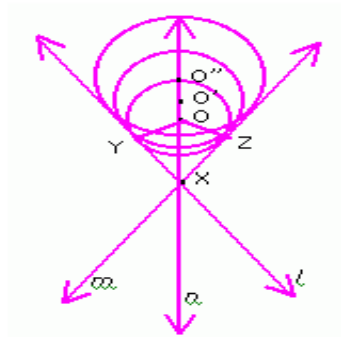
$\angle OCY = 90^\circ$

$\angle XBO = \angle OCY$.

Question-6

Find the locus of centres of circles which touch two intersecting lines.

Solution:



In $\triangle YXO$ and $\triangle ZXO$,

$\angle OYX = \angle OZX (= 90^\circ)$

$XO = XO$ (Common)

$OY = OZ = \text{radius}$

$\triangle YOX \cong \triangle ZOX$ (By SAS congruence)

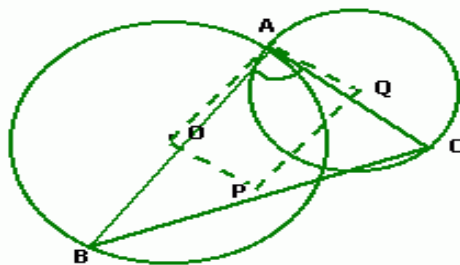
$\therefore \angle YXO = \angle ZXO$

$\therefore$ Locus of the centre is a straight line bisecting them between two intersecting lines.

Question-7

Let A be one point of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C, respectively. Let the point P be located so that AOPQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC in figure.

(Hint: $AQ \perp AB$ and $AQ \parallel OP$. Then $OP \perp AB$ and is also bisector of AB. Similarly, PQ is perpendicular bisector of AC.)



Solution:

Given: Two circles with centre O and Q intersect at A. The tangents at A to the two circles meet the circles again at B and C, respectively. AOQP is a parallelogram.

To prove: P is the circumcentre of the triangle ABC.

Proof:

$AQ \perp AB$ and $AQ \parallel OP$

Therefore $OP \perp AB$

Also OP bisects AB as the line drawn from the centre to the chord bisects the chord.

Hence OP is the perpendicular bisector of AB.

Similarly PQ is the perpendicular bisector of AC.

Since the perpendicular bisectors intersect at P. P is the circumcentre of the triangle ABC.

Question-8

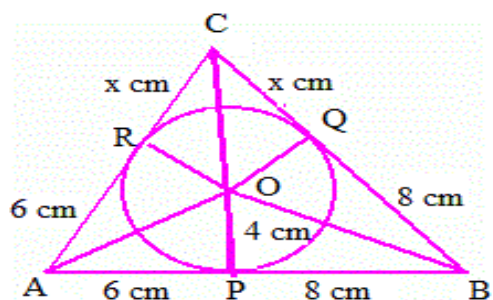
The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.

(Hint: Equate the areas of the triangle found by using the formula $\sqrt{s(s-a)(s-b)(s-c)}$ and also found by dividing it into three triangles.)

Solution:

Given: The radius of the in circle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm.

To find: AC and BC.



Proof:

$$s = \frac{6 + x + x + 8 + 8 + 6}{2}$$

$$= \frac{2x + 28}{2}$$

Area of a triangle

$$= \sqrt{(x+14)(x+14-x-6)(x+14-6-8)(x+14-8-x)}$$

$$= \sqrt{(x+14)(8)(x)(6)}$$

$$= \sqrt{48x(x+14)} \dots\dots\dots(i)$$

$$\text{Area of triangle} = 2 \text{ area of } \Delta AOP + 2 \text{ area of } \Delta COR + 2 \text{ area of } \Delta PBO$$

$$\begin{aligned}
 &= 2 \times \frac{1}{2} \times 4 \times 6 + 2 \times \frac{1}{2} \times 4 \times 8 + 2 \times \frac{1}{2} \times 4 \times x \\
 &= 24 + 32 + 4x \\
 &= 56 + 4x \dots\dots\dots(ii)
 \end{aligned}$$

Equating (i) and (ii)

$$48x(x + 14) = (56 + 4x)^2$$

$$48x(x + 14) = 16(x + 14)^2$$

$$48x = 16(x + 14) \quad \text{or } x + 14 = 0$$

$$48x = 16x + 224$$

$$32x = 224$$

$$x = 7 \quad \text{or } x = -14(\text{which we ignore})$$

$$AC = 6 + 7 = 13 \text{ cm}$$

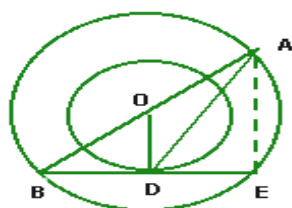
$$BC = 8 + 7 = 15 \text{ cm}$$

The other two sides of the triangles are 13 cm and 15 cm.

Question-9

The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle touching it at D. Find the length AD in figure. [Hint: Let line BD intersect the bigger circle at E. Join AE. AE = 2 × 8 = 16 cm.

$$DE = BD = \sqrt{(169 - 64)} = \sqrt{105} \text{ and } \angle AED = 90^\circ.]$$



Solution:

Given: Two concentric circles with radius 13cm and 8cm. AB is the diameter of the bigger circle. BD is the tangent to the smaller circle touching it at D.

To find: AD

Proof:

In $\triangle BDO$ and $\triangle BEA$,

$\angle DBO = \angle EBA$ (Common)

$$\frac{BD}{BE} = \frac{BO}{BA} \quad (O \text{ is the centre of the circle and } OD \text{ bisects } BE)$$

$\triangle BDO \sim \triangle BEA$

$$AE = 2DO = 2(8 \text{ cm}) = 16 \text{ cm}$$

$$BD = \sqrt{BO^2 - DO^2} = \sqrt{13^2 - 8^2} = \sqrt{169 - 64} = \sqrt{105} \text{ cm}$$

$$DE = BD = \sqrt{105} \text{ cm}$$

$\angle AED = \angle ODB = 90^\circ$ (since $\triangle BDO \sim \triangle BEA$)

In $\triangle DAE$,

$$AD = \sqrt{AE^2 + DE^2} = \sqrt{16^2 + 105} = \sqrt{256 + 105} = \sqrt{361} \text{ cm.}$$

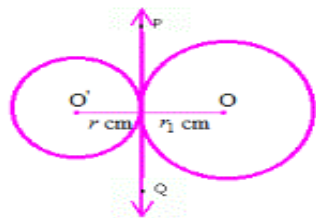
Question-10

Find the locus of the centre of a circle of constant radius (r) which touches a given circle of radius r_1 (i) externally, (ii) internally, given $r_1 > r$.

Solution:

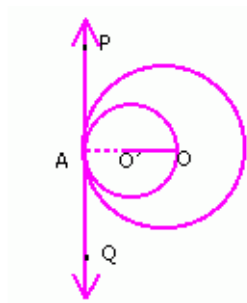
(i) $OO' = r + r_1$

The locus is a circle with centre O' and radius $r_1 + r$.



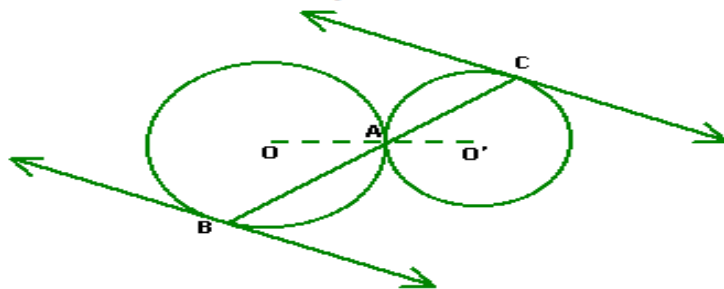
(ii) $OO' = r_1 - r$

The locus is a circle with centre O' and radius $r_1 - r$.



Question-11

In figure, two circles with centres O, O' touch externally at a point A . A line through A is drawn to intersect these circles at B and C . Prove that the tangents at B and C are parallel. [Hint: Prove that $\angle OBA = \angle O'CA$]

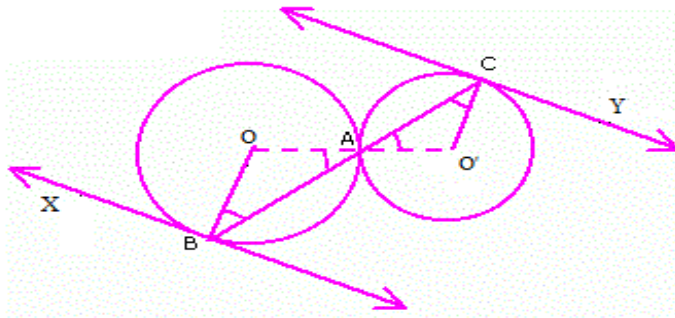


Solution:

Given: Two circles with centers O, O' touch externally at a point A . A line through A is drawn to intersect these circles in B and C .

To prove: Tangents B and C are parallel.

Proof:



In $\triangle AOB$,
 $\angle OBA = \angle OAB$ (OB = OA, opposite angles of equal sides)
 (i)
 In $\triangle AO'C$
 $\angle O'AC = \angle O'CA$ ($O'A = O'C$, opposite angles of equal sides)
(ii)

From (i), (ii) and (iii)

$\angle OBA = \angle O'CA$ (iv)

Since the line joining the centre and the tangent at the point of contact is 90° .

$\angle OBX = \angle O'CY = 90^\circ$ (v)

Adding (iv) and (v)

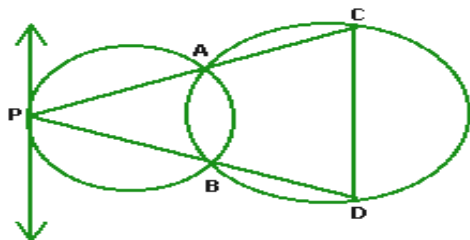
$\angle OBX + \angle OBA = \angle O'CY + \angle O'CA$

$\angle XBA = \angle YCA$

Since alternate angles are equal, tangents at B and C are parallel.

Question-12

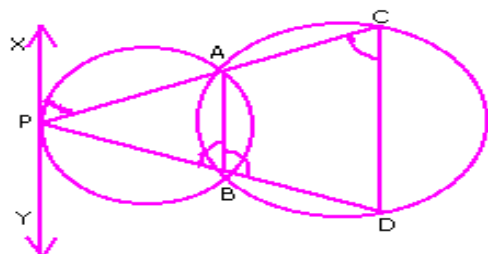
In figure two circles intersect at two points A and B. From a point P on a circle, two line segments PAC and PBD are drawn intersecting the other circle at the points C and D. Prove that CD is parallel to the tangent at P.



Solution:

Given: Two circles intersect at A and B.

To prove: $CD \parallel$ tangent at P.

$$\angle APX = \angle ABP \dots\dots\dots(i)$$
$$\angle ABD + \angle ACD = 180^\circ$$
$$\therefore \angle ACD = \angle ABP \dots\dots\dots(\text{ii})$$

$$\angle ACD = \angle APX$$

Question-13

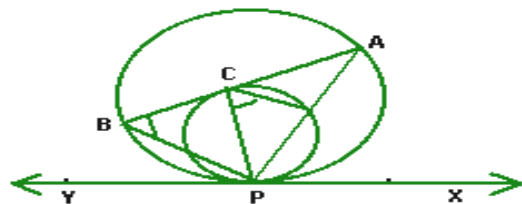
Solution:

Construction: Draw a tangent l at A.

So circumcircles of ΔABC and ΔAPQ touch each other at A.

Question-14

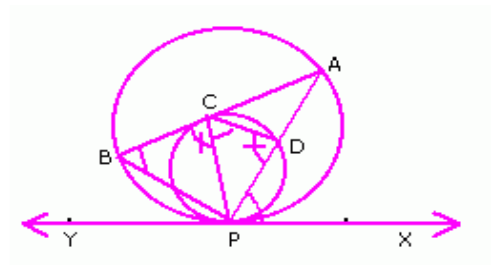
In figure, two circles touch internally at a point P. AB is a chord of the bigger circle touching the other circle at C. Prove that PC bisects the angle APB. [Hint: Draw a tangent at the point P. Joint CD, where D is the point of intersection of AP and the inner circle and prove that $\angle PBC = \angle PCD$.]



Solution:

Given: Two circles touch internally at a point P. AB is chord of the bigger circle touching the other circle at C.

To prove: $\angle BPC = \angle CPA$



Proof:

In $\triangle BCP$ and $\triangle CDP$,

$\angle XPA = \angle PBA$ (Alternate segment angles)

$\angle PCB = \angle CDP$ (Alternate segment angles)

$\triangle BCP \cong \triangle CDP$ (By AA similarity)

$\angle BPC = \angle CPD$

$\therefore \angle BPC = \angle CPA$

$\therefore$ PC bisect $\angle BPA$.

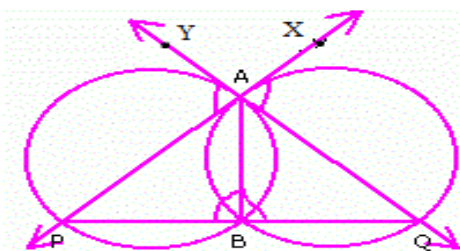
Question-15

Two circles intersect each other at two points A and B. At A, tangents AP and AQ to the two circles are drawn which intersect other circles at the points P and Q respectively. Prove that AB is the bisector of angle PBQ.

Solution:

Given: Two circles intersect each other at two points A and B. At A, tangents AP and AQ to the two circles are drawn which intersect other circles at the points P and Q respectively.

To prove: AB is the bisector of angle PBQ.



Proof:

$\angle ABQ = \angle QAX$ (Alternate interior angles)

$\angle ABP = \angle YAP$ (Alternate interior angles)

But $\angle YAP = \angle XAQ$ (Vertically opposite angles)

$\therefore \angle ABQ = \angle ABP$ $\angle ABQ + \angle ABP = 180^\circ$ (Linear pair)

$\angle ABQ = \angle ABP = 90^\circ$

AB is the bisector of angle PBQ.

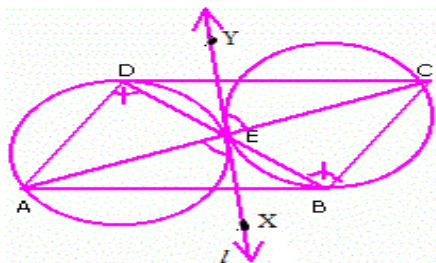
Question-16

The diagonals of a parallelogram ABCD intersect in a point E. Show that the circumcircles of $\triangle ADE$ and $\triangle BCE$ touch each other at E.

Solution:

Given: Diagonals of a parallelogram ABCD intersect in a point E.

To prove: Circumcircles of $\triangle ADE$ and $\triangle BCE$ touch each other at E.



Construction: Let l be a tangent to first circle.

Proof:

$\angle ADB = \angle CBD$ (Alternate interior angles are equal)

$\angle AEX = \angle ADB$ (Alternate segment theorem)

But $\angle AEX = \angle CEY$ (Vertically opposite angles)

$\therefore \angle CBD = \angle CEY$

Therefore by converse of alternate segment theorem,

l is a tangent to second circle with point of contact at E.

Hence the circumcircles of $\triangle ADE$ and $\triangle BCE$ touch each other at E.

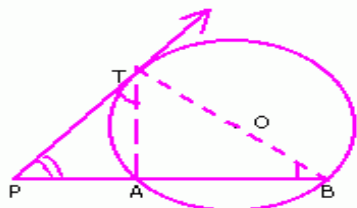
Question-17

If PAB is a secant to a circle intersecting it at A and B, and PT is a tangent, then $PA \cdot PB = PT^2$.

(Prove using alternate segment theorem)

Solution:

Given: A secant PAB to circle with centre O intersecting it at A and B and a tangent PT to the circle.



To prove: $PA \cdot PB = PT^2$

Construction: Join TA and TB.

Proof:

In $\triangle PBT$ and $\triangle PTA$

$\angle BPT = \angle APT$ (Common)

$\angle PBT = \angle PTA$ (Alternate segment theorem)

$\therefore \triangle PBT \sim \triangle PTA$ (AA similarity)

$$\frac{PA}{PT} = \frac{PT}{PB}$$

$$PA \cdot PB = PT^2$$

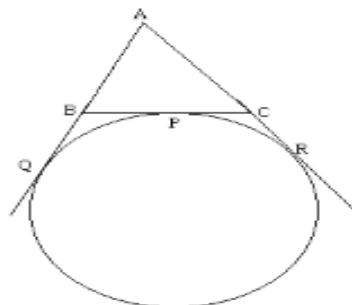
Hence proved.

Question-18

A circle touches the side BC of a triangle ABC at P and touches AB and AC when produced at Q and R respectively. Show that $AQ = \frac{1}{2}$ (perimeter of triangle ABC).

Solution:

Given: A circle touches the side BC of $\triangle ABC$ at P and AB, AC produced at Q and R respectively.



To Prove: AQ is half the perimeter of $\triangle ABC$.

Proof:

Lengths of two tangents from an external points are equal.

$\therefore AQ = AR, BQ = BP$ and $CP = CR$

Perimeter of $\Delta ABC = AB + BC + AC$

$$= AB + (BP + PC) + (AR - CR)$$

$$= (AB + QB) + PC + AQ - PC \quad (\because AQ = AR, BQ = BP \text{ and}$$

$CP = CR)$

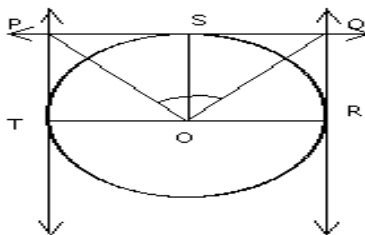
$$= AQ + AQ$$

$$= 2AQ$$

AQ is half the perimeter of ΔABC .

Question-19

Prove that the intercept of a tangent between 2 parallel tangents to a circle subtends a right angle at the centre.

**Solution:**

Given: A circle with centre 'O' has PT and QR as two tangents parallel to each other touching the circles at T and R. PQ is the intercept between 2 tangents touching the circle at S.

To prove : $\angle POQ = 90^\circ$

Construction: Join OS and OT

Proof: Consider ΔPOS and ΔPOT

As OS is the radius at the point of contact where the tangent intercept touches the circle.

$$\angle OSP = 90^\circ,$$

$$\text{Similarly } \angle OTP = 90^\circ$$

$$\text{i.e. } \angle OSP = \angle OTP$$

Hypotenuse OP is common

$$OS = OT \text{ (radii of the same circle)}$$

$$\therefore \Delta POT \cong \Delta SOP \text{ (RHS)}$$

$$\therefore \angle POT = \angle SOP \text{ (cpct)}$$

Similarly $\Delta SOQ, \Delta ROQ$ are congruent

$$\therefore \angle SOQ = \angle ROQ$$

As TOR is a straight line

$$\angle POT + \angle SOP + \angle SOQ + \angle ROQ = 180^\circ$$

$$\text{But } \angle POT = \angle SOP \text{ and } \angle SOQ = \angle ROQ \text{ (already proved)}$$

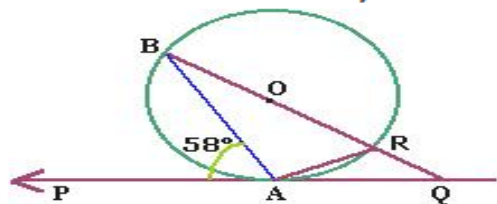
$$\angle SOP + \angle SOP + \angle SOQ + \angle SOQ = 180^\circ$$

$$\therefore 2(\angle SOP + \angle SOQ) = 180^\circ$$

$$\angle SOP + \angle SOQ = \frac{180^\circ}{2} = 90^\circ.$$

Question-20

In the given figure, O is the centre of the circle. PQ is a tangent to the circle at A. If $\angle PAB = 58^\circ$, find $\angle ABQ$ and $\angle AQB$.



Solution:

Given: O is the centre of the circle. PQ is a tangent to the circle at A. $\angle PAB = 58^\circ$.

To find: $\angle ABQ$ and $\angle AQB$.

$\angle BAR = 90^\circ$ (Angle in a semicircle)

$\angle ARB = \angle PAB = 58^\circ$ (Alternate segment theorem)

$\angle ABQ = 180^\circ - (\angle BAR + \angle ARB)$ (Angle sum property of a triangle)
 $= 180^\circ - (90^\circ + 58^\circ)$

$$= 180^\circ - 148^\circ = 32^\circ$$

$\angle QAR = \angle ABR = 32^\circ$ (Alternate segment theorem)

and $\angle AQB = 180^\circ - (\angle ABQ + \angle BAQ)$ (Angle sum property of a triangle)

$$= 180^\circ - (32^\circ + 90^\circ + 32^\circ)$$

$$= 180^\circ - 154^\circ = 26^\circ.$$