





- a) Both A and R are true and R is the correct explanation of A.      b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.      d) A is false but R is true.

14. **Assertion:** In an isolated system, the entropy increases. [1]

**Reason:** The processes in an isolated system are adiabatic.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.      b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.      d) Assertion is wrong statement but reason is correct statement.

15. **Assertion:** The time period of revolution of a satellite close to surface of earth is smaller than that revolving away from surface of earth. [1]

**Reason:** The square of time period of revolution of a satellite is directly proportional to cube of its orbital radius.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.      b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.      d) Assertion is wrong statement but reason is correct statement.

16. **Assertion (A):** The trajectory of projectile is quadratic in x and linear in y. [1]

**Reason (R):** y component of trajectory is independent of x-component.

- a) Both A and R are true and R is the correct explanation of A.      b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.      d) A is false but R is true.

### Section B

17. A stone is dropped into a well and its splash is heard at the mouth of the well after an interval of 1.45 s. Find the depth of the well. Given that velocity of sound in air at room temperature is equal to  $332 \text{ ms}^{-1}$ . [2]

18. State the limitations of the method of dimensional analysis. [2]

19. In the expression  $P = El^2m^{-5}G^{-2}$ , E, l, m and G denote energy, angular momentum, mass and gravitational constant respectively. Show that P is a dimensionless quantity. [2]

20. A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of  $12 \text{ ms}^{-1}$ . If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball) [2]

21. The mass of moon is  $\frac{M}{81}$  (where M is mass of earth). Find the distance of the point where the gravitational field due to earth and moon cancel each other. Given distance of moon from earth is 60 R, where R is radius of earth. [2]

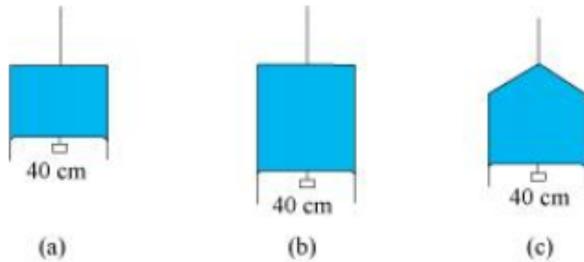
OR

If earth has a mass 9 times and radius twice that of a planet Mars, calculate the minimum velocity required by a rocket to pull out of the gravitational force of Mars. Take the escape velocity on the surface of earth to be  $11.2 \text{ kms}^{-1}$ .

### Section C

22. In a given Figure (a) shows a thin liquid film supporting a small weight  $= 4.5 \times 10^{-2} \text{ N}$ . What is the weight [3]

supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.



23. Define the terms [3]

- i. absorptive power,
- ii. emissive power and
- iii. emissivity.

Write their SI units, if any.

24. A parachutist bails out from an aeroplane and after dropping through a distance of 40 m, he opens the parachute and decelerates at  $2 \text{ ms}^{-2}$ . If he reaches the ground with a speed of  $2 \text{ ms}^{-1}$ , how long is he in the air? At what height did he bailout from the plane? [3]

25. A railway car of mass 20 tonnes moves with an initial speed of 54 km/hr. On applying brakes, a constant negative acceleration of  $0.3 \text{ m/s}^2$  is produced. [3]

- i. What is the breaking force acting on the car?
- ii. In what time it will stop?
- iii. What distance will be covered by the car before it finally stops?

26. An ideal refrigerator runs between  $-23^\circ\text{C}$  and  $27^\circ\text{C}$  [3]

- i. Find the heat rejected to atmosphere for every joule of work input.
- ii. Also, find heat extracted from cold body.
- iii. Find coefficient of performance of the refrigerator.

27. A small body tied to one end of the string is whirled in a vertical circle. Represent the forces on a diagram when the string makes an angle  $\theta$  with initial position below the fixed point. Find an expression for the tension in the string. Also, find the tension and velocity at the lowest and highest points respectively. [3]

28. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are  $70 \text{ ms}^{-1}$  and  $63 \text{ ms}^{-1}$  respectively. What is the lift on the wing if its area is  $2.5 \text{ m}^2$ ? Take the density of air to be  $1.3 \text{ kg m}^{-3}$ . [3]

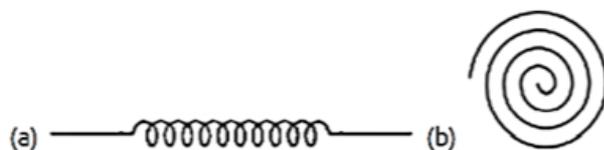
OR

Calculate the radius of new bubble formed when two bubbles of radius  $r_1$  and  $r_2$  coalesce?

#### Section D

29. Read the text carefully and answer the questions: [4]

There are many types of spring. Important among these are helical and spiral springs as shown in the figure.



Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of the

elastic potential energy of the spring. Thus, the potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

- (i) The potential energy of a spring increases in which of the following cases?
- |   |   |
|---|---|
| a) If work is done against conservative force | b) If work is done by non-conservative force      |
| c) If work is done by conservative force      | d) If work is done against non-conservative force |
- (ii) The potential energy, i.e.  $U(x)$  can be assumed zero when
- |  |                 |
|--|-----------------|
| a) gravitational force is constant                 | b) $x = 0$      |
| c) infinite distance from the gravitational source | d) All of these |
- (iii) The ratio of spring constants of two springs is 2 : 3. What is the ratio of their potential energy, if they are stretched by the same force?
- |          |          |
|----------|----------|
| a) 3 : 2 | b) 9 : 4 |
| c) 2 : 3 | d) 4 : 9 |

**OR**

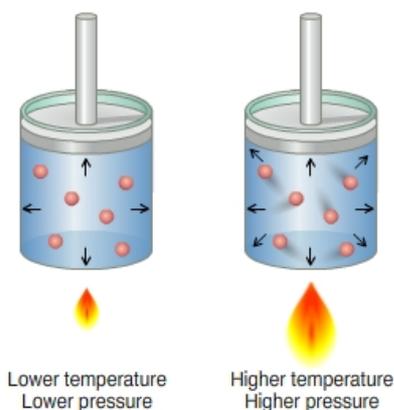
The potential energy of a spring when stretched through a distance  $x$  is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance  $x$ ?

- |         |         |
|---------|---------|
| a) 40 J | b) 10 J |
| c) 30 J | d) 20 J |
- (iv) The potential energy of a spring increases by 15 J when stretched by 3 cm. If it is stretched by 4 cm, the increase in potential energy is
- |         |         |
|---------|---------|
| a) 36 J | b) 30 J |
| c) 27 J | d) 33 J |

30. **Read the text carefully and answer the questions:**

[4]

In a gas the particles are always in a state of random motion, all the particles move at different speed constantly colliding and changing their speed and direction, as speed increases it will result in an increase in its kinetic energy.



- (i) If the temperature of the gas increases from 300 K to 600 K then the average kinetic energy becomes:
- |         |                   |
|---------|-------------------|
| a) same | b) becomes double |
|---------|-------------------|



OR

A tube of length  $L$  is filled completely with an incompressible liquid of mass  $M$  and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity  $\omega$ . Determine the force exerted by the liquid at the other end.

## Solution

### Section A

- (d)  $[ML^2T^{-3}I^{-2}]$

**Explanation:**  $[R] = \frac{V}{I} = \frac{W}{qI} = \frac{[ML^2T^{-2}]}{[IT][I]}$

$= [ML^2T^{-3}I^{-2}]$
- (d) 12 sec

**Explanation:** On the superposition of the two waves, beats are produced.

Beat frequency  $= \frac{1}{T_2} - \frac{1}{T_1} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$
- (a)  $\frac{100}{\sqrt{3}}$

**Explanation:** Moment of inertia of the rod about an axis passing through its centre of gravity and perpendicular to its length

$$I = \frac{Ml^2}{3}$$

Moment of inertia of rod in terms of the radius of gyration

$$I = Mk^2$$
$$M = 100 \text{ gm}$$
$$l = 100 \text{ cm}$$
$$Mk^2 = \frac{Ml^2}{3}$$
$$k = \sqrt{\frac{l^2}{3}} = \sqrt{\frac{100 \times 100}{3}}$$
$$k = \frac{100}{\sqrt{3}} \text{ cm}$$
- (b) Constant terminal speed

**Explanation:** The rain drops acquire terminal velocity after falling long distance due to balance of buoyant force and weight of the drop due to gravity.
- (b) kinetic energy

**Explanation:** K.E. changes due to the change in the speed of celestial body around the sun.
- (c)  $2\pi A$

**Explanation:** Maximum particle velocity = Wave velocity

$$\omega A = \frac{\omega}{k}$$

or  $k = \frac{2\pi}{\lambda} = \frac{1}{A}$

$\therefore \lambda = 2\pi A$
- (a) 20.0

**Explanation:** Initial velocity,  $u = 100 \text{ m/s}$

As it stops so final velocity,  $v = 0$

Acceleration  $a = -5 \text{ m/s}^2$

We know,  $v - u = at$

$$\Rightarrow t = \frac{v-u}{a}$$
$$\Rightarrow t = \frac{0-100}{-5} = \frac{-100}{-5}$$
$$\Rightarrow t = 20.0 \text{ s}$$
- (a) 196 Hz

**Explanation:** Let the frequency of the first fork =  $\nu$

Frequency of second fork =  $\nu + 8$

Frequency of third fork =  $\nu + 2 \times 8$

Frequency of 16th fork =  $\nu + 15 \times 8$

But the frequency of the last is the octave of the first.

$$2\nu = \nu + 49 \times 8$$

$$\nu = 196 \text{ Hz}$$

9.

(c) surface tension

**Explanation:** Rain drops are spherical due to surface tension.

10.

(d)  $m^0$

**Explanation:** Escape velocity ( $v_e$ ) =  $\sqrt{2gR_e}$ . Therefore it is independent of the mass of the particle or it will depend on  $m^0$ .

11.

(c)  $\frac{1}{2}I(\omega_1 + \omega_2)^2$

**Explanation:** By conservation of angular momentum,

$$I\omega_1 + I\omega_2 = 2I\omega$$

$$\Rightarrow \omega = \frac{\omega_1 + \omega_2}{2}$$

Loss of K.E. =  $K_i - K_f$

$$= \frac{1}{2}I(\omega_1^2 + \omega_2^2) - \frac{1}{2} \times 2I\left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

$$= \frac{1}{2}I\left[\left(\omega_1^2 + \omega_2^2\right) - \frac{\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2}{2}\right]$$

$$= \frac{1}{2}I\left[\frac{2\omega_1^2 + 2\omega_2^2 - \omega_1^2 - \omega_2^2 - 2\omega_1\omega_2}{2}\right]$$

$$= \frac{1}{4}I(\omega_1 - \omega_2)^2$$

12. (a) blue shines like brighter red compared to the red piece

**Explanation:** According to Stefan's law,

$$E \propto T^4$$

As the temperature of blue glass is more than that of red glass, so it will appear brighter than red glass.

13. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

14.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

**Explanation:** Assertion and reason both are correct statements but reason is not correct explanation for assertion.

15. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

**Explanation:** The time period of satellite,  $T \propto r^{3/2}$

$$\text{or } T \propto (R_e + h)^{3/2}$$

For a satellite revolving close to surface of the earth,  $h = 0$ .

$$\therefore T \propto (R_e)^{3/2}$$

It is evident that the period of revolution of a satellite depends upon its height above the earth's surface. Greater is the height of a satellite above the earth's surface greater is its period of revolution.

16.

(c) A is true but R is false.

**Explanation:** The equation of the trajectory of a projectile is  $y = x \tan \theta - \frac{1}{2} \frac{g}{u^2 \cos^2 \theta} x^2$ . Thus y component depends on x component.

### Section B

17. Let h be the depth of the well. Then time  $t_1$  taken by the stone to fall into well under gravity is given by

$$h = 0 + \frac{1}{2}gt_1^2 \text{ or } t_1 = \sqrt{\frac{2h}{g}}$$

Time taken for the splash to travel height h is given by  $t_2 = \frac{h}{v}$

where v = velocity of sound

But  $t_1 + t_2 = 1.45$  s

$$\therefore \sqrt{\frac{2h}{g}} + \frac{h}{v} = 1.45$$

$$\text{or } \sqrt{\frac{2h}{9.8}} + \frac{h}{332} = 1.45$$

On solving,  $h = 9.9$  m.

18. Method of dimensional analysis suffers from the following limitations :

- i. It does not enable us to determine the value of the constant of proportionality which may be a pure number or a dimensionless ratio.
- ii. Since many physical quantities have the same dimensions (e.g., momentum and impulse, work and torque, etc.), the method of dimensions is not a full-proof method to check the correctness of physical relation.
- iii. The use of the method of dimensions requires a sound background of the subject, otherwise, the method may lead to incorrect results.
- iv. It does not test whether a physical quantity is a scalar or a vector.
- v. It cannot derive relation or formula if a physical quantity depends upon more than three factors having dimensions.
- vi. It cannot derive a formula containing trigonometric function, exponential function, and logarithmic function.
- vii. It cannot derive a relation having more than one part in an equation.

19. The formulae or expressions that indicate how and which fundamental quantities are there in a physical quantity are called as the Dimensional Formula of the Physical Quantity.

$$\text{Dimension of } E = [ML^2T^{-2}]$$

$$\text{Dimension of } l = [ML^2T^{-1}]$$

$$\text{Dimension of } G = [M^{-1}L^3T^{-2}]$$

Hence,  $P = El^2m^{-5}G^{-2}$  will have dimensions:

$$[P] = \frac{[E][l]^2}{[m^5][G^2]} = \frac{[ML^2T^{-2}][ML^2T^{-1}]^2}{[M]^5[M^{-1}L^3T^{-2}]^2} = [M^0L^0T^0]$$

Thus, P is a dimensionless quantity.

20. Impulse = change in linear momentum

initial momentum of ball =  $mu$

$$= 0.15 \times 12 = 1.8 \text{ kg.m/sec}$$

$$\text{final momentum of ball} = -mu$$

$$= -0.15 \times 12 = -1.8 \text{ kgm/sec}$$

now,

change in momentum of ball = final - initial

$$= -1.8 - 1.8 = -3.6 \text{ kgm/sec}$$

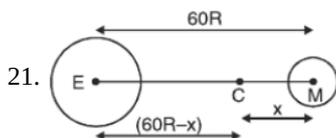
so,

change in momentum of bat = -change in momentum of ball

$$\text{change in momentum of bat} = 3.6 \text{ kgm/s}$$

now,

$$\text{impulse imparted to ball} = \text{change in momentum of bat} = 3.6 \text{ kgm/s} = 3.6 \text{ Ns}$$



Gravitational field at C due to earth

= Gravitational field at C due to moon

$$\frac{GM}{(60R-x)^2} = \frac{Gm/81}{x^2}$$

$$81x^2 = (60R - x)^2$$

$$9x = 60R - x$$

$$X = 6 R$$

OR

Here,  $M_e = 9M_m$ , and  $R_e = 2R_m$

$v_e$  (escape speed on surface of Earth) = 11.2 km/s

Let  $V_m$  be the speed required to pull out of the gravitational force of mars.

We know that

$$v_e = \sqrt{\frac{2GM_e}{R_e}} \text{ and } v_m = \sqrt{\frac{2GM_m}{R_m}}$$

$$\text{Dividing, we get } \frac{v_m}{v_e} = \sqrt{\frac{2GM_m}{R_m} \times \frac{R_e}{2GM_e}}$$

$$= \sqrt{\frac{M_m}{M_e} \times \frac{R_e}{R_m}} = \sqrt{\frac{1}{9} \times 2} = \frac{\sqrt{2}}{3}$$

$$\Rightarrow v_m = \frac{\sqrt{2}}{3} (11.2 \text{ km/s}) = 5.3 \text{ km/s}$$

### Section C

22. In case "a"-

The length of the liquid film supported by the weight is given by,  $l = 40 \text{ cm} = 0.4 \text{ m}$

The weight supported by the film is given by,  $W = 4.5 \times 10^{-2} \text{ N}$

A liquid film has two free surfaces thus

$$\text{Surface tension is given by } S = \frac{W}{2l} = \frac{4.5 \times 10^{-2}}{2 \times 0.4} = 5.625 \times 10^{-2} \text{ Nm}^{-1}$$

In all the three figures, the liquid is the same. Temperature is also the same for each case.

Hence, the surface tension in figure (b) and figure (c) is the same as in figure (a) i.e.  $S = 5.625 \times 10^{-2} \text{ Nm}^{-1}$

Since the length of the film in all the cases is 40 cm, hence the total weight supported in each case is given by  $4.5 \times 10^{-2} \text{ N}$ .

23. **Absorptive power:** The absorptive power of a body for a given wavelength  $\lambda$  is defined as the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength  $\lambda$ . It is denoted by  $a_\lambda$ . A perfect black body absorbs all the heat radiations incident upon it. So its absorptive power is unity.

If the radiant energy  $dQ$  in wavelength range  $\lambda$  and  $\lambda + d\lambda$  is incident on a body of absorptive power  $a_\lambda$ , then amount of radiant energy absorbed by the body  $= a_\lambda dQ$

The absorptive power is a dimensionless quantity.

**Emissive power:** The amount of heat energy radiated by a body per second depends upon

- the area of its surface,
- the temperature of its surface and
- the nature of its surface. The strength of emission is measured by a quantity called emissive power. The emissive power of a body at a given temperature and for a given wavelength  $\lambda$  is defined as the amount of radiant energy emitted per unit time per unit surface area of the body

If heat radiation of wavelength range  $\lambda$  to  $\lambda + d\lambda$  is incident on the surface of a body of emissive power  $e_\lambda$ , then the amount of radiant energy emitted per second per unit area  $= e_\lambda d\lambda$

The SI unit of emissive power is  $\text{Js}^{-1} \text{ m}^{-2}$  or  $\text{Wm}^{-2}$ .

**Emissivity:** The emissivity of a body is defined as the ratio of the heat energy radiated per unit time per unit area by the given body to the amount of heat energy radiated per unit time per unit area by a perfect black body of the same temperature i.e., it is the ratio of the emissive power ( $e$ ) of a body to the emissive power ( $E$ ) of a black body at the same temperature. It is denoted by  $\epsilon$ .

$$\text{Thus } \epsilon = \frac{e}{E}$$

It is dimensionless quantity. Its value lies between 0 and 1. The emissivity of a perfect black body is 1. The emissivities of polished copper, polished aluminium and lamp black are 0.018, 0.05 and 0.95 respectively.

24. When the parachutist falls freely :

$$u = 0, v = 9.8 \text{ ms}^{-2}, s = 40 \text{ m}, t = ?, u = ?$$

$$\text{As } s = ut + \frac{1}{2}gt^2$$

$$\therefore 40 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$\text{or } t = \sqrt{\frac{80}{9.8}} = \frac{20}{7} \text{ s} = 2.86 \text{ s}$$

$$\text{Also, } v = u + gt = 0 + 9.8 \times \frac{20}{7} = 28 \text{ ms}^{-1}$$

When the parachutist decelerates uniformly:

$$u = 28 \text{ ms}^{-1}, a = -2 \text{ ms}^{-2}, v = 2 \text{ ms}^{-1}$$

$$\text{Time taken, } t = \frac{v - u}{a} = \frac{2 - 28}{-2} = 13 \text{ s}$$

$$\text{Distance, } s = ut + \frac{1}{2}at^2 = 28 \times 13 - \frac{1}{2} \times 2 \times (13)^2$$

$$= 364 - 169 = 195 \text{ m}$$

Total time of parachutist in air

$$= 2.86 + 13 = 15.86 \text{ s}$$

Height at which parachutist bails out

$$= 40 + 195 = 235 \text{ m}$$

25. Mass of the railway car,  $m = 20 \text{ tonnes} = 20 \times 1000 \text{ kg} = 20 \times 10^4 \text{ kg}$ , Initial speed,  $u = 54 \text{ km/hr} = 54 \times \frac{5}{18} = 15 \text{ m/s}$

Negative acceleration,  $a = -0.3 \text{ m/s}^2$

a. Breaking force acting on the car,  $F = -ma$

$$F = -(2 \times 10^4 \text{ kg}) \times (-0.3 \text{ m/s}^2)$$

$$F = 6000 \text{ N}$$

b. When the railway car stops, its final velocity is zero.

$$\text{i.e. } v = 0$$

Using the relation:  $v - u = at$

$$\Rightarrow 0 = 15 + (-0.3)t$$

$$\Rightarrow t = 50 \text{ s}$$

c. Using the relation:  $v^2 - u^2 = 2as$

$$\Rightarrow 0 - (15)^2 = 2(-0.3)s$$

$$\Rightarrow s = 375 \text{ m}$$

26. Let heat rejected  $Q_1 = x$  and  $W = 1 \text{ J}$

$$\text{Now, } Q_2 = Q_1 - W = x - 1$$

$$\text{Given, } T_1 = 273 + 27 = 300 \text{ K}, T_2 = 273 - 23 = 250 \text{ K}$$

$$\text{i. For an ideal process, } \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Rightarrow \frac{x-1}{x} = \frac{250}{300}$$

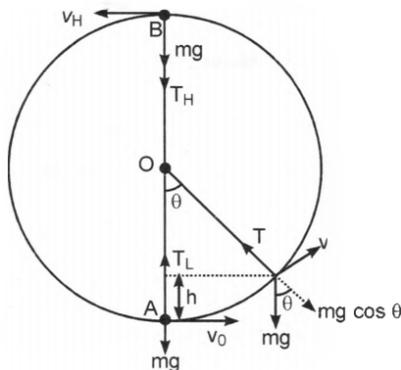
$$Q_1 = x = 6 \text{ J}$$

$$\text{ii. } Q_2 = 5 \text{ J}$$

iii. Coefficient of performance

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{250}{300 - 250} = 5$$

27. Consider a small body of mass  $m$  attached to one end of a string (of length  $l$ ) and whirled in a vertical circle of radius ' $r$ '. Let body starts motion from its initial position A, just below the fixed point O, with a speed  $v_0$ .



The forces acting on the body, when the string makes an angle  $\theta$  with the initial position are shown in the figure. Here,  $mg$  is the weight of body and  $T$  the tension in the string. If  $v$  be the instantaneous velocity at this point, then a centripetal force  $F = \frac{mv^2}{l}$  is required radially inward. From figure, it is clear that in equilibrium the centripetal force is provided by resultant of two forces i.e.,

$$T - mg \cos \theta = \frac{mv^2}{l}$$

$$\text{or } T = mg \cos \theta + \frac{mv^2}{l} \dots(1)$$

If the body has covered a vertical distance  $h$ , then from law of conservation of mechanical energy, we have

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh$$

$$\Rightarrow v^2 = v_0^2 - 2gh \dots(ii)$$

which is the required expression for the velocity of a particle at any point.

At the lowest point  $\theta = 0^\circ$  and  $h = 0$ , hence we have

$$v_L = v = v_0 \dots[\text{from (i) putting } h = 0]$$

Thus,

$$T_L = mg \cos 0^\circ + \frac{m}{l}v_L^2 = mg + \frac{mv_0^2}{l}$$

and at the highest point  $\theta = 180^\circ$  and  $h = 2l$ . Hence,

$$v_H^2 = v^2 = v_0^2 - 4gl \text{ [from (i) putting } h = 2l]$$

$$\text{or } v_H = \sqrt{v_0^2 - 4gl}$$

$$\text{and } T_H = mg \cos 180^\circ + \frac{mv_H^2}{l} = mg(-1) + \frac{m}{l}(v_0^2 - 4gl) = \frac{mv_0^2}{l} - 5mg$$

which is the required expression for the Tension.

28. Speed of wind on the lower surface of the wing,  $V_2 = 63 \frac{m}{s}$

Area of the wing,  $A = 2.5 \text{ m}^2$

Density of air,  $\rho = 1.3 \text{ kg m}^{-3}$

According to Bernoulli's theorem, we have the relation:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho (V_1^2 - V_2^2)$$

Where,

$P_1$  = Pressure on the upper surface of the wing

$P_2$  = Pressure on the lower surface of the wing

The pressure difference between the upper and lower surfaces of the wing provides lift to the aeroplane.

Lift on the wing =  $(P_2 - P_1)A$

$$= \frac{1}{2}\rho (V_1^2 - V_2^2) A$$

$$= \frac{1}{2} \cdot 1.3 ((70)^2 - (63)^2) \times 2.5$$

$$= 1512.87$$

$$= 1.51 \times 10^3 \text{ N}$$

Therefore, the lift on the wing of the aeroplane is  $1.51 \times 10^3 \text{ N}$ .

OR

Consider two soap bubbles of radii  $r_1$  and  $r_2$  and volumes as  $V_1$  and  $V_2$ . Thus  $V_1 = \frac{4\pi r_1^3}{3}$  and  $V_2 = \frac{4\pi r_2^3}{3}$ . Let  $S$  be the surface tension of the soap solution. If  $P_1$  and  $P_2$  are excess pressure inside the two soap bubbles then  $P_1 = \frac{4S}{r_1}$ ;  $P_2 = \frac{4S}{r_2}$ . Let  $r$  be the radius of the new soap bubble formed when the two soap bubble coalesces under isothermal conditions. If  $V$  and  $P$  are volume and excess of pressure inside the new soap bubble then  $V = \frac{4}{3}\pi r^3$   $P = \frac{4S}{r}$ . As the new bubble is formed under isothermal condition, so Boyle's law holds good and hence

$$P_1V_1 + P_2V_2 = PV$$

$$\left(\frac{4S}{r_1} \times \frac{4}{3}\pi r_1^3\right) + \left(\frac{4S}{r_2} \times \frac{4}{3}\pi r_2^3\right) = \frac{4S}{r} \times \frac{4}{3}\pi r^3$$

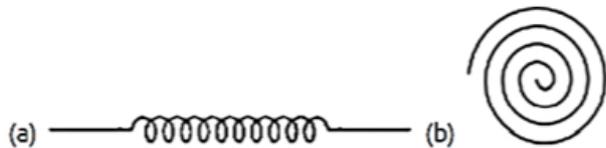
$$(16 \times S \times \pi \times r_1^2) + (16 \times S \times \pi \times r_2^2) = 16S\pi r$$

$$r = \sqrt{r_1^2 + r_2^2}$$

### Section D

29. Read the text carefully and answer the questions:

There are many types of spring. Important among these are helical and spiral springs as shown in the figure.



Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of the elastic potential energy of the spring. Thus, the potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

(i) (a) If work is done against conservative force

**Explanation:** If work is done against conservative force

(ii) (d) All of these

**Explanation:** All of these

(iii) (a) 3 : 2

**Explanation:** 3 : 2

OR

(c) 30 J

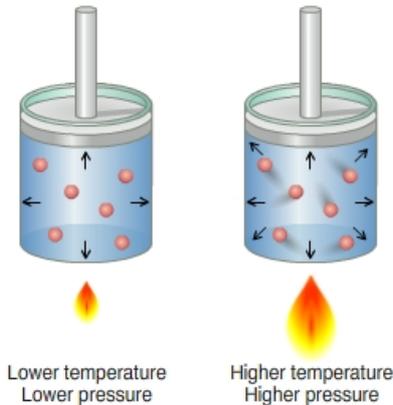
**Explanation:** 30 J

(iv) (c) 27 J

**Explanation:** 27 J

30. Read the text carefully and answer the questions:

In a gas the particles are always in a state of random motion, all the particles move at different speed constantly colliding and changing their speeds and direction, as speed increases it will result in an increase in its kinetic energy.



(i) (b) becomes double

**Explanation:** becomes double

(ii) (d) Zero

**Explanation:** Zero

(iii) (d) remains same

**Explanation:** remains same

(iv) (a) 1:1

**Explanation:** 1:1

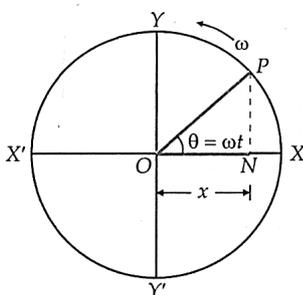
OR

(c) 4.08 v

**Explanation:** 4.08 v

**Section E**

31. Relation between S.H.M. and uniform circular motion. As shown in figure, consider a particle P moving along a circle of radius A with uniform angular velocity  $\omega$ . Let N be the foot of the perpendicular drawn from the point P to the diameter XX'. Then N is called the projection of P on the diameter XX'. As P moves along the circle from X to Y, Y to X', X' to Y' and Y' to X; N moves from X to O, O to X', X' to O and O to X. Thus, as P revolves along the circumference of the circle, N moves to and fro about the point O along the diameter XX'. The motion of N about O is said to be simple harmonic. Hence **simple harmonic motion** may be defined as the projection of uniform circular motion upon a diameter of a circle. The particle P is called the reference particle or generating particle and the circle along which the particle P revolves is called the circle of reference.



Displacement in simple harmonic motion. As shown in Figure, consider a particle moving in the anticlockwise direction with uniform angular velocity  $\omega$  along a circle of radius A and centre O. Suppose at time  $t = 0$ , the reference particle is at point A such that  $\angle XOA = \phi_0$ . At any time t, suppose the particle reaches the point P such that  $\angle AOP = \omega t$ . Draw  $PN \perp XX'$ .

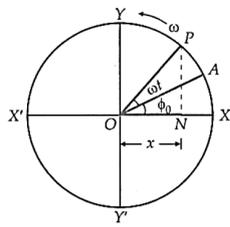


Fig. Displacement in S.H.M., epoch  $(+\phi_0)$

Clearly, displacement of projection N from centre O at any instant  $t$  is  $x = ON$ .

In right-angled  $\triangle ONP$ ,

$$\angle PON = \omega t + \phi_0$$

$$\therefore \frac{ON}{OP} = \cos(\omega t + \phi_0)$$

$$\text{or } \frac{x}{A} = \cos(\omega t + \phi_0)$$

$$\text{or } x = A \cos(\omega t + \phi_0)$$

This equation gives the displacement of a particle in S.H.M. at any instant  $t$ . The quantity  $\omega t + \phi_0$  is called the phase of the particle and  $\phi_0$  is called the initial phase or phase constant or epoch of the particle. The quantity  $A$  is called the amplitude of the motion. It is a positive constant whose value depends on how the motion is initially started. Thus

$$\begin{array}{ccccccc}
 & & & \text{Phase} & & & \\
 x & = & A & \cos(\omega t + \phi_0) & & & \\
 \uparrow & & \uparrow & \uparrow & + & \uparrow & \\
 \text{Displacement} & & \text{Amplitude} & \text{Angular frequency} & & \text{Initial phase} & 
 \end{array}$$

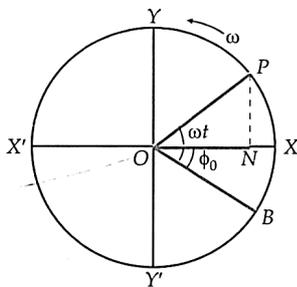


Fig. Epoch  $(-\phi_0)$

As shown in Figure, if the reference particle starts motion from the point P such that  $\angle BOX = \phi_0$  and  $\angle BOP = \omega t$ , then

$$\angle PON = \omega t - \phi_0$$

$$\therefore x = A \cos(\omega t - \phi_0)$$

Here  $-\phi_0$  is the initial phase of the S.H.M.

OR

Assuming the standard equation

$$x(t) = A \sin(\omega t + \phi)$$

i. When  $t = 0$ ,  $x = 0$  [mean position]

$$\Rightarrow 0 = A \sin(\omega \times 0 + \phi)$$

$$A \sin \phi = 0 \text{ [as } A \neq 0]$$

$$\text{or } \sin \phi = 0 \therefore \phi = 0$$

$\therefore$  Required function is

$$x(t) = A \sin(\omega t + 0) \text{ or } x(t) = A \sin \omega t$$

$$\text{where, } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3.0}} = 20 \text{ rad/s}$$

$$\therefore x(t) = A \sin 20t \text{ or } x(t) = 2 \sin 20t$$

ii. When  $t = 0$ ,  $x = +A$  (maximum stretched position)

$$x(t) = A \sin(\omega t + \phi) \text{ at } t = 0 \text{ and } x = +A$$

$$+A = A \sin(\omega \times 0 + \phi) \text{ or } 1 = \sin \phi \Rightarrow \phi = \frac{\pi}{2}$$

$$\therefore x(t) = A \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= A \cos \omega t = A \cos 20t = 2 \cos 20t$$

iii. When the spring is at maximum compressed position.

$$\text{At } t = 0, x(t) = -A$$

$$\Rightarrow -A = A \sin(\omega \times 0 + \phi) \text{ or } -1 = \sin \phi \text{ or } \phi = \frac{3\pi}{2}$$

So, the equations only differ in initial phase and in no other factors.

32. a. Area of a parallelogram =  $|\vec{A} \times \vec{B}| = AB \sin \theta$  ( $\therefore$  Applying cross product)

$$\text{Given, area of parallelogram} = \frac{1}{2} AB$$

$$\text{So, } \frac{1}{2} AB = AB \sin \theta$$

$$\frac{1}{2} = \sin \theta$$

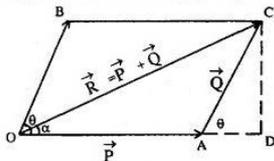
$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 30^\circ$$

b. Triangular law of vector addition states that if two vectors can be represented both in magnitude and direction by the sides of a triangle taken in order then their resultant is given by the third side of the triangle taken in opposite order.

Proof: in  $\triangle ADC$

$$(OC)^2 = (OD)^2 + (DC)^2$$



$$(OC)^2 = (OA + AD)^2 + (DC)^2$$

$$(OC)^2 = (OA)^2 + (AD)^2 + 2(OA)(AD) + (DC)^2$$

$$(OC)^2 = (P^2) + (Q \cos \theta)^2 + 2(P)(Q \cos \theta) + (Q \sin \theta)^2$$

$$(OC)^2 = P^2 + Q^2 (\sin^2 \theta + \cos^2 \theta) + 2PQ \cos \theta \quad \left( \because \frac{CD}{AC} = \sin(\theta), \frac{AD}{AC} = \cos(\theta) \right)$$

$$(R)^2 = P^2 + Q^2 + 2PQ \cos \theta \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

OR

i. Incorrect

In order to make  $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$ , it is not necessary to have all the four given vectors to be null vectors. There are many other combinations that can give the sum zero.

ii. Correct

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$$

$$\vec{a} + \vec{c} = -(\vec{b} + \vec{d})$$

Taking modulus on both the sides, we get:

$$|\vec{a} + \vec{c}| = |-(\vec{b} + \vec{d})| = |\vec{b} + \vec{d}|$$

Hence, the magnitude of  $(\vec{a} + \vec{c})$  is the same as the magnitude of  $(\vec{b} + \vec{d})$ .

iii. Correct

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$$

$$\vec{a} = -(\vec{b} + \vec{c} + \vec{d})$$

Taking modulus both sides, we get:

$$|\vec{a}| = |\vec{b} + \vec{c} + \vec{d}|$$

$$|\vec{a}| \leq |\vec{b}| + |\vec{c}| + |\vec{d}| \dots (i)$$

Equation (i) shows that the magnitude of a is equal to or less than the sum of the magnitudes of  $\vec{b}$ ,  $\vec{c}$ , and  $\vec{d}$ .

Hence, the magnitude of a vector can never be greater than the sum of the magnitudes of b, c, and d.

iv. Correct

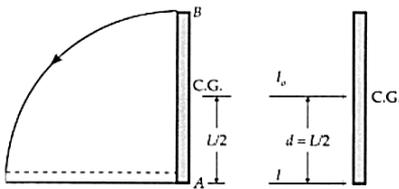
$$\text{For } \vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$$

$$\vec{a} + (\vec{b} + \vec{c}) + \vec{d} = 0$$

The resultant sum of the three vectors  $\vec{a}$ ,  $(\vec{b} + \vec{c})$ , and  $\vec{d}$  can be zero only if  $(\vec{b} + \vec{c})$  lie in a plane containing  $\vec{a}$  and  $\vec{d}$ , assuming that these three vectors are represented by the three sides of a triangle.

If  $\vec{a}$  and  $\vec{d}$  are collinear, then it implies that the vector  $(\vec{b} + \vec{c})$  is in the line of  $\vec{a}$  and  $\vec{d}$ . This implication holds only then the vector sum of all the vectors will be zero.

33. Let M be the mass and L be the length of the metre scale. When the upper end of the rod strikes the floor, its centre of gravity falls through height  $\frac{L}{2}$ .



M.I. of the scale about the lower end A,  $I = \text{M.I. of the scale about the parallel axis through CG} + Md^2$

$$= I_0 + Md^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3} \quad \left[ \because d = \frac{L}{2} \right]$$

Also,  $\omega = \frac{v}{r} = \frac{v}{L}$

Gain in rotational K.E.

$$= \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{ML^2}{3} \cdot \frac{v^2}{L^2} = \frac{Mv^2}{6}$$

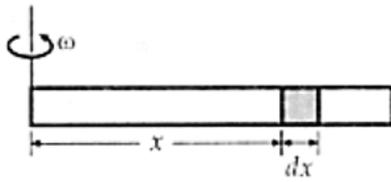
Now, Gain in rotational K.E. = Loss in P.E.

$$\frac{Mv^2}{6} = Mg \cdot \frac{L}{2} \quad \text{or} \quad v^2 = 3gl$$

$$\text{or } v = \sqrt{3gL} = \sqrt{3 \times 9.8 \times 1} = 5.4 \text{ ms}^{-1}$$

OR

Consider a small element of the liquid of length  $dx$  at a distance  $x$  from one end.



Mass of the small element =  $\frac{M}{L} dx$

Centripetal force associated with the element

$$dF = \left( \frac{M}{L} dx \right) x \omega^2 \quad \left[ \because F = m r \omega^2 \right]$$

Force exerted by the liquid = Total centripetal force at the other end

$$F = \int dF = \int_0^L \frac{M}{L} \omega^2 x dx = \frac{M}{L} \omega^2 \left[ \frac{x^2}{2} \right]_0^L$$

$$= \frac{M}{L} \omega^2 \frac{L^2}{2} = \frac{1}{2} M \omega^2 L$$