Linear equation in two variables

$a_1x + b_1y + c_1 = 0$ ..(1)

$a_2x + b_2y + c_2 = 0$ ..(2)

Methods to solve

Algebraic method

Graphical method

Substitution method

$a_1x + b_1y + c_1 = 0$ ...(i)

$a_2x + b_2y + c_2 = 0$ ...(ii)

From eq. (i) $x = \frac{-c_1 - b_1y}{a_1}$

Substitute x in eq. (ii) and solve.

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Elimination method

$a_1x + b_1y + c_1 = 0$ ...(i)

$a_2x + b_2y + c_2 = 0$ ...(ii)

Multiply b_2 in (i) & b_1 in (ii)

$a_1b_2x + b_1b_2y + c_1b_2 = 0$ ..(iii)

$b_1a_2x + b_2b_1y + c_2b_1 = 0$ ..(iv)

(3) -(4) ....

$(a_1b_2 - b_1a_2)x + (c_1b_2 - c_2b_1) = 0$

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Cross multiplication method

	x		y		1	
b_1		c_1		a_1		b_1
b_2		c_2		a_2		b_2

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

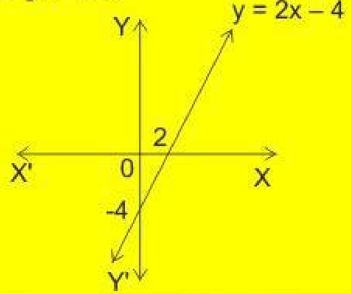
$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

Equation of a straight line

$ax + by + c = 0$

$\{a \neq 0, b \neq 0 \ \& \ a, b, c \in R\}$

Solution (x,y) → point lying on straight line.



Condition of Solvability of System of Linear Equations

Intersecting (intersect at 1 point)	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ →	Unique solution (consistent)
Coincident (Coincide)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ →	Infinite solution (consistent)
Parallel (No intersection)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ →	No solution (inconsistent)

e.g. Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. Find their present ages.

Sol. Let man's present age be 'x' yrs & son's present age be 'y' yrs.

According to problem

$x + 6 = 3(y + 6)$

$x - 3y = 12$... (i)

and $x - 3 = 9(y - 3)$

$x - 9y = -24$... (ii)

On solving equation (i) & (ii)

$x = 30$ and $y = 6$.

So, the present age of man = 30 years and present age of son = 6 years.

Equations reducible to a pair of linear equations

$\frac{2}{x} + \frac{3}{y} = 13, \quad \frac{5}{x} - \frac{4}{y} = -2$

Let

$\frac{1}{x} = p, \ \frac{1}{y} = q$

$2p + 3q = 13$

$5p - 4q = -2$

e.g. Solve the following system of linear equations graphically : $x - y = 1, \ 2x + y = 8$.

Sol.(i) $x - y = 1$

$x = y + 1$

x	0	1	2
y	– 1	0	1

(ii) $2x + y = 8$

$y = 8 - 2x$

x	0	1	2
y	8	6	4

Solution is x = 3 and y = 2