Differential Equations

Que 1: $If\ the\ integrating\ factor\ of \frac{dy}{dx} + P(x)y = Q(x) is\ x.\ Find\ P(x)$

Marks :(2)

Ans:

$$e^{\int pdx} = x = e^{logx}$$

$$so \int pdx = logx$$

$$hence\ p = \frac{dlogx}{dx} = \frac{1}{x}$$

Que 2: $Find\ the\ solution\ of \frac{dy}{dx} = 1 + y + y^2 + x + xy + xy^2$

Marks :(6)

Ans:

$$\frac{dy}{dx} = (1+x)(1+y+y^2) \Rightarrow \frac{dy}{1+y+y^2} = (1+x)dx$$

$$\frac{dy}{(y+\frac{1}{2})^2 + \left(\frac{\sqrt{3}}{4}\right)^2} = (1+x)dx$$

$$integrating, \frac{1}{\sqrt{\frac{3}{4}}} tan^{-1}(\frac{y+\frac{1}{2}}{\sqrt{\frac{3}{4}}}) = x + \frac{x^2}{2} + C$$

$$\frac{2}{\sqrt{3}} tan^{-1}(\frac{2y+1}{\sqrt{3}}) = \frac{2x+x^2}{2} + C$$

$$4tan^{-1}\frac{2y+1}{\sqrt{3}} = \sqrt{3}(2x+x^2) + C$$

Que 3: $Solve\ \frac{dy}{dx} tany = sin(x+y) + sin(x-y)$

Marks :(2)

Ans:

$$\frac{dy}{dx}tany = 2sinxcosy$$
$$2sinxdx = secytanydy$$
$$integrating$$
$$-2cosx = secy + C$$

Que 4: $Find\ the\ solution\ of\ \frac{dy}{dx} + 1 = cosec(x+y)$

Marks :(2)

Ans:

$$\frac{dy}{dx}tany = 2sinxcosy$$
$$2sinxdx = secytanydy$$
$$integrating$$
$$-2cosx = secy + C$$

Que 5: $Find\ the\ general\ solution\ of\ \frac{dy}{dx} + ycotx = cosecx$

Marks :(2)

Ans:

$$P = cotx, Q = cosecx$$
$$I.F = e^{\int cotxdx}$$
$$Solution$$
$$y.sinx = \int sinxcosecxdx = x + c$$