Cubes and cube roots

If m and n are two natural numbers, n is a perfect cube when n = m x m x m = m^3

Hence m is the cube root of n ; m = $\sqrt[3]{n}$ or m = $n^{\frac{1}{3}}$

n is a perfect cube or a cube number (ex. 8, 27, 64, 125…)

In the prime factorization, of a perfect cube: each prime factor appears 3 times and the factors can be grouped into triples

To convert a number that is not a perfect cube into a perfect cube

- first resolve the given number into its prime factors
- form triples; groups of three of similar factors
- Check which factor(group) does not have triples; multiply or divide the given number by this factor (or factors so that we can form triples)

ones digit of number	Ones digit of cube
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

Properties of cubes and cube roots:

- Cubes of all even natural numbers is even
- Cubes of all odd natural numbers is odd
- Sum of cubes of first n natural numbers = square of their sums :
  $1^3 + 2^3 + 3^3 + \ldots\ldots\ n^3 = (1 + 2 + 3 \ldots . n)^2$
- Cubes of numbers 1, 4, 6, 9 are numbers are ending in same digit
- If a and b are any two integers then :
  $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$
  $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$
  B $\neq$ 0
- Cubes of numbers ending in digit 8 and 2 are numbers ending in digits 2 and 8 respectively
- Cubes of numbers ending in digit 7 and 3 are numbers ending in digits 3 and 7 respectively
- Cubes of numbers ending in digit 7 and 3 are numbers ending in digits 3 and 7 respectively
- Cubes of numbers ending in digit 0 are numbers ending in digit 0

Number	cube	Number	cube
1	1	11	1331
2	8	12	1728
3	27	13	2197
4	64	14	2744
5	125	15	3375
6	216	16	4096
7	343	17	4913
8	512	18	5832
9	729	19	6859
10	1000	20	8000

Cube root is the inverse operation of finding a cube

If $a^3 = x$, then $\sqrt[3]{x} = a$

Cube root of a cube through prime factorization

- Resolve the given number into prime factors
- make triplets; groups of similar factors
- Take one factor from each group (triplet) and multiply
- product gives cube root of given number

Cube root of 3375 through prime factorization:

$$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$$

$$= 3^3 \times 5^3 = (3 \times 5)^3$$

$$\text{cube root of } 3375 = \sqrt[3]{3375}$$

$$= 3 \times 5 = 15$$

Cube root of a cube number (estimation method)

- take any cube number start forming groups of 3 digits starting from rightmost number

Group 2	Group1

- Look at group 1 : from group 1 we get the ones place (units) digit of the required cube root
- The other group (group 2) say lies between n^3 and $(n+1)^3$
- $n^3 <$ group2 $< (n+1)^3$
- Take n as the tenths place of required cube root

Finding the cube root of 17576 through estimation:

- Form groups of three starting from the rightmost digit of 17576 $\underline{17}$ $\underline{576}$.
- Group 1: 576 has three digits whereas 17 has only two digits
- Annex zeroes to make 3 digits we get $\underline{017}$ $\underline{576}$.
- Take 576. The digit 6 is at its one's place.
- We take the one's place of the required cube root as 6.
- Take the other group, i.e., 17.
- Cube of 2 is 8 and cube of 3 is 27; 8 < 17 < 27
- The smaller number among 2 and 3 is 2.

The one's place of 2 is 2 itself. Take 2 as ten's place of the cube root of 17576.

$$\text{Thus, } \sqrt[3]{17576} = 26$$

Hardy Ramanujan's number : numbers that can be expressed as the sum of 2 cubes in two different ways (3 of them are given below)

1729 ; (1, 12) and (9,10)

4101; (2, 16) and (9, 15)

13832; (18, 20) and (2, 24)