Work, Energy and Power

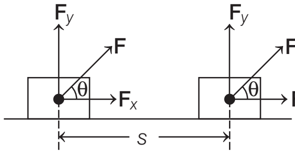
Quick Revision

1. **Work** Work is said to be done by a force, when the body is displaced actually through some distance in the direction of the applied force. Thus, work is done on a body only if the following two conditions are satisfied
   - A force acts on the body.
   - The point of application of the force moves in the direction of the force.
2. **Work Done by a Constant Force** Work done by the force (constant force) is the product of component of force in the direction of the displacement and the magnitude of the displacement. Then, the work done on the body by the force is given by

   Work done, $W = \mathbf{F} \cdot \mathbf{s}$

   SI unit of work is joule(J).

   Its dimensions are $[M^1L^2T^{-2}]$.
3. **Work Done when Force and Displacement are Inclined to Each Other**



   Work done, $W = \mathbf{F} \cdot \mathbf{s} = (F\cos\theta) \cdot s = Fs\cos\theta$

   Two cases can be considered as given below for the maximum and minimum work

   ***Case* I** When $\mathbf{F}$ and $\mathbf{s}$ are in the same direction, i.e. $\theta = 0°$, then work done is $W = Fs\cos 0° = Fs(1) = Fs$ i.e. maximum work done by the force.

   ***Case* II** When $\mathbf{F}$ and $\mathbf{s}$ are perpendicular to each other, i.e. then $W = \mathbf{F} \cdot \mathbf{s} = Fs\cos 90° = Fs(0) = 0$, i.e. no work done by the force, when a body moves in a direction perpendicular to the force acting.
4. **Work Done by a Variable Force** Work done by variable force is given as,

   $$W_{x_i \to x_f} = \int_{x_i}^{x_f} \mathbf{F} \cdot d\mathbf{x} = \int_{x_i}^{x_f} (F\cos\theta)\, dx$$

   = Area under force-displacement curve

   When the magnitude and direction of a force vary in three dimensions, then it can be expressed in terms of rectangular components.

   So, work done from x_i to x_f,

   $$W = \int_{x_i}^{x_f} F_x\, dx + \int_{x_i}^{x_f} F_y\, dy + \int_{x_i}^{x_f} F_z\, dz$$

   where, F_x, F_y and F_z are the rectangular components of force in x, y and z-directions, respectively.
5. **Conservative Force** If the work done by the force in displacing an object depends only on the initial and final positions of the object and not on the nature of the path followed between the initial and final positions, such a force is known as conservative force. e.g. Gravitational force is a conservative force.

6. **Non-Conservative Force** If the work done by a force in displacing an object from one position to another depends upon the path between the two positions. Such a force is known as non-conservative force. e.g. Friction is a non-conservative force.
7. **Energy** The energy of a body is defined as its capacity or ability for doing work.
   - The dimensions of energy are the same as the dimensions of work, i.e. $[M^1 L^2 T^{-2}]$.
   - It is measured in the same unit as work, i.e. joule in SI system and erg in CGS system.
8. **Kinetic Energy** The energy possessed by a body by virtue of its motion is called kinetic energy. In other words, the amount of work done, by a moving object before coming to rest is equal to its kinetic energy.

   $\therefore$ Kinetic energy, $KE = \frac{1}{2} mv^2$

   where, m is a mass and v is the velocity of a body.
   - **Relation between Kinetic Energy and Linear Momentum**

     $$p = \sqrt{2mK}$$
9. **Work Energy Theorem or Work Energy Principle** It states that, work done by the net force acting on a body is equal to the change produced in the kinetic energies of the body.

   $$K_f - K_i = \int_i^f \mathbf{F}_{\text{net}} \cdot dx$$

   $$\therefore K_f - K_i = W$$

   where, K_f and K_i are the final and initial kinetic energies of the body.
10. **Potential Energy** The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration. So, if configuration of the system changes, then its potential energy changes.

    Dimensions = $[ML^2T^{-2}]$

    SI unit = Joule
11. **Gravitational Potential Energy** Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth. Gravitational potential energy, $U = mgh$
12. **Potential Energy of a Spring** For a small stretch or compression, spring obeys Hooke's law, i.e. restoring force $\propto$ stretch or compression

    $$-F_s \propto x \quad \Rightarrow \quad F_s = -kx$$

    where, k is called **spring constant**. Its SI unit is Nm^{-1}.The negative sign shows F_s acts in the opposite direction of displacement x.

    If the block is moved from an initial displacement x_i to final displacement x_f, then work done by spring force is

    $$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

    $\therefore$ Change in potential energy of a spring

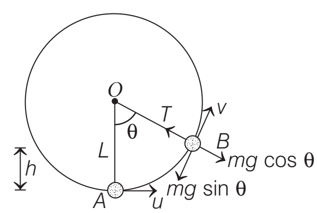
    $$\Delta U = -W_s = \frac{1}{2}k(x_f^2 - x_i^2)$$

    If $\quad x_i = 0$, then $\Delta U = \frac{1}{2}kx_f^2$
13. **Conservation of Mechanical Energy** This principle states that, if only the conservative forces are doing work on a body, then its mechanical energy (KE + PE) remains constant.

    i.e. $K + U = \text{constant} = E$

    $\therefore \quad K_i + U_i = K_f + U_f$

    The quantity $K + U$, is called the total **mechanical energy** of the system.
14. **Motion in a Vertical Circle** A particle of mass m is attached to an inextensible string of length L and is moving in a vertical circle about fixed point O (as shown)

    - Minimum velocity at highest point, so that particle complete the circle, $v_{\min} = \sqrt{gL}$, at this velocity, tension in the string is zero.
    - Minimum velocity at lowest point, so that particle complete the circle, $v_{\min} = \sqrt{5gL}$, at this velocity, tension in the string is 6 mg.
    - When string is horizontal, then minimum velocity is $\sqrt{3Rg}$ and tension in this condition is 3 mg.

15. **Power** Power of a person or machine is defined as the rate at which work is done or energy is transferred.

Average power (P_{av}) = rate of doing work

$$= \frac{\text{work done } (W)}{\text{time taken } (t)}$$

Thus, the **average power** of a force is defined as the ratio of the work (W) to the total time (t).

16. The **instantaneous power** of an agent at any instant is equal to the dot product of its force and velocity vectors at that instant.

$$P = \mathbf{F} \cdot \mathbf{v}$$

17. Power is a scalar quantity and its dimensional formula is $[ML^2T^{-3}]$.

The SI unit of power is watt (W).

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} = 1 \text{ Js}^{-1}$$

Another popular units of power are kilowatt and horse power.

1 kilowatt = 1000 watt or 1 kW = 10^3 W

1 horse power = 746 watt or 1 HP = 746 W

This unit is used to describe the output of automobiles, motorbikes, engines, etc.

18. **Collision** A collision is an isolated event in which two or more colliding bodies exert strong forces on each other for a relatively short time. For a collision to take place, the actual physical contact is not necessary.

Collision between particles have been divided into two types which can be differentiated as

Elastic Collision	Inelastic Collision
A collision in which there is absolutely no loss of kinetic energy.	A collision in which there occurs some loss of kinetic energy.
Forces involved during elastic collision must be conserved in nature.	Some or all forces involved during collision may be non- conservative in nature.
The mechanical energy is not converted into heat, light, sound, etc.	A part of the mechanical energy is converted into heat, light, sound, etc.
e.g. Collision between subatomic particles, collision between glass balls, etc.	e.g. Collision between two vehicles, collision between a ball and floor, etc.

19. **Conservation of Linear Momentum in Collision** Total linear momentum is conserved at each instant during collision.

$\therefore\ p_1 + p_2 = \text{constant}$

20. **Elastic Collision in One Dimension**

In one-dimensional elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.

$$u_1 - u_2 = v_2 - v_1$$

Velocities of the Bodies After the Collision

Velocity of Ist body after collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2 \qquad \ldots\text{(i)}$$

Velocity of IInd body after collision,

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1 \qquad \ldots\text{(ii)}$$

Eqs. (i) and (ii) give the final velocities of the colliding bodies in terms of their initial velocities.

The two cases under the action of same and different masses can be considered as given below

***Case* I** When two bodies of equal masses collide.

i.e. $m_1 = m_2 = m$ (say)

From Eq. (i), we get

$v_1 = \frac{2mu_2}{2m} = u_2$ = velocity of body of mass m_2 before collision

From Eq. (ii), we get

$v_2 = \frac{2mu_1}{2m} = u_1$ = velocity of body of mass m_1 before collision.

***Case* II** When a light body collides against a massive stationary body.

Here, $m_1 << m_2$ and $u_2 = 0$

Neglecting m_1 in Eq. (i), we get

$$v_1 = -\frac{m_2 u_1}{m_2} = -u_1$$

From Eq. (ii), we get

$$v_2 \simeq 0$$

21. **Perfectly Inelastic Collision in One Dimension**

When the two colliding bodies together move as a single body with a common velocity after the collision, then the collision is perfectly inelastic.

In perfectly inelastic collision between two bodies of masses m_1 and m_2, the body of mass m_2 happens to be initially at rest ($u_2 = 0$). After the collision, the two bodies move together with common velocity v. The change in their kinetic energies is $KE = \dfrac{m_1 m_2 u_1^2}{2(m_1 + m_2)}$

$\therefore$ ΔKE is a positive quantity.

Therefore, kinetic energy is lost mainly in the form of light, sound and heat.

22. **Elastic Collision in Two Dimensions** When the collision between two bodies is not head-on (the force during the collision is not along the initial velocity). The bodies move along different lines, then the collision is called elastic collision in two dimensions.

The three cases can be considered as given below

***Case* I Glancing Collision** In a glancing collision, the incident particle does not lose any kinetic energy and is scattered almost undeflected. Thus, for such collision, when $\theta = 0°$, $\phi = 90°$, $u_1 = v_1$ and $v_2 = 0$.

KE of the target particle $= \dfrac{1}{2} m_2 v_2^2 = 0$

***Case* II Head-on Collision** In this type of collision, the target particle moves in the direction of the incident particle, i.e. $\phi = 0°$.

$$m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \quad \text{and} \quad 0 = m_1 v_1 \sin\theta$$

So, the kinetic energy remains unchanged.

***Case* III Elastic Collision of Two Identical Particles** When two particles of same mass undergo perfectly elastic collision in two dimensions, i.e. $m_1 = m_2$.

$$\therefore \qquad \theta + \phi = 90°$$

Thus, after collision the two particles will move at right angle to each other.

23. **Inelastic Collision in Two Dimensions** When two bodies travelling initially along the same straight line collide involving some loss of kinetic energy and move after collision along different directions in a plane, then it is called inelastic collision in two dimensions.

24. **Coefficient of Restitution or Coefficient of Resilience**

It is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by e.

$$e = \frac{\text{Relative velocity of separation (after collision)}}{\text{Relative velocity of approach (before collision)}}$$

$$e = \frac{|v_2 - v_1|}{|u_2 - u_1|}$$

where, u_1 & u_2 are velocities of two bodies before collision and v_1 & v_2 are their respective velocities after collision.

25. **Comparison between Different Types of Collisions**

Collision	Kinetic Energy	Coefficient of Restitution	Main Domain
Elastic	Conserved	$e = 1$	Between atomic particles
Inelastic	Non-conserved	$0 < e < 1$	Between ordinary objects
Perfectly inelastic	Maximum loss of KE	$e = 0$	During shooting
Super elastic	KE increases	$e > 1$	In explosions

Objective Questions

Multiple Choice Questions

1. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is *(NCERT Exemplar)*

(a) +2000 J (b) −200 J
(c) zero (d) −20,000 J

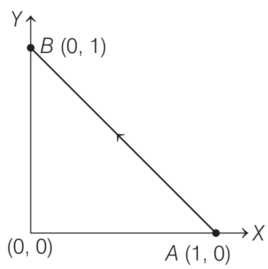
2. Force of 50 N acting on a body at an angle θ with horizontal. If 150 J work is done by displacing it 3 m, then θ is

(a) 60° (b) 30°
(c) 0° (d) 45°

3. A particle is pushed by forces $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ simultaneously and it is displaced from point $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ to point $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. The work done is

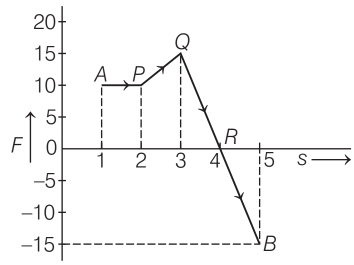
(a) 7 units (b) −7 units
(c) 10 units (d) −10 units

4. Consider a force $\mathbf{F} = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$. The work done by this force in moving a particle from point $A(1, 0)$ to $B(0, 1)$ along the line segment is (all quantities are in SI units)



(a) $\frac{3}{2}$ (b) 2
(c) 1 (d) $\frac{1}{2}$

5. A body moves from point A to B under the action of a force varying in magnitude as shown in figure, then the work done is (force is expressed in newton and displacement in metre)



(a) 30 J (b) 22.5 J
(c) 25 J (d) 27 J

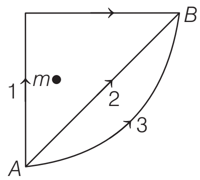
6. A string of length L and force constant k is stretched to obtain extension l. It is further stretched to obtain extension l_1. The work done in second stretching is

(a) $\frac{1}{2}kl_1(2l + l_1)$ (b) $\frac{1}{2}kl_1^2$
(c) $\frac{1}{2}k(l^2 + l_1^2)$ (d) $\frac{1}{2}k(l_1^2 - l^2)$

7. A uniform chain of length l and mass m is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, work required to pull the hanging part on to the table is

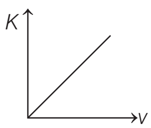
(a) mgl (b) $\frac{mgl}{3}$
(c) $\frac{mgl}{9}$ (d) $\frac{mgl}{18}$

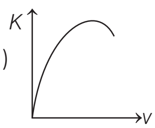
8. If W_1, W_2 and W_3 are the work done in moving a particle from A and B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, the relation between W_1, W_2 and W_3 is

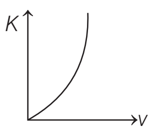


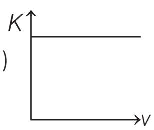
(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$
(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

9. Amongst the given graphs which one correctly represents the variation of the kinetic energy (K) of a body with velocity (v)?

(a)


(b)


(c)


(d)


10. The kinetic energy of a body of mass 4 kg and momentum 6 N-s will be

(a) 3.5 J (b) 5.5 J
(c) 2.5 J (d) 4.5 J

11. For a moving particle (mass m, velocity v) having a momentum p, which one of the following correctly describes the kinetic energy of the particle?

(a) $\frac{p^2}{2m}$ (b) $\frac{p}{2m}$ (c) $\frac{v^2}{2m}$ (d) $\frac{v}{2m}$

12. Two bodies of masses 4 kg and 5 kg are moving with equal momentum. Then, the ratio of their respective kinetic energies is

(a) 4 : 5 (b) 2 : 1
(c) 1 : 3 (d) 5 : 4

13. A heavy body and a light body have same kinetic energy. Which will have larger linear momentum?

(a) Heavy body
(b) Light body
(c) Both have same linear momenta
(d) None of the above

14. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 rev/min, its kinetic energy (in J) would be

(a) $250\pi^2$ (b) $100\pi^2$ (c) $5\pi^2$ (d) zero

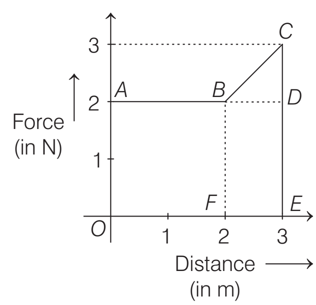
15. Two moving objects $(m_1 > m_2)$ having same kinetic energy are stopped by application of equal retarding force. Which object will come to rest at shorter distance?

(a) Bigger
(b) Smaller
(c) Both at same distance
(d) Cannot say

16. A particle which is experiencing a force, is given by $\mathbf{F} = 3\hat{\mathbf{i}} - 12\hat{\mathbf{j}}$, undergoes a displacement of $\mathbf{d} = 4\hat{\mathbf{i}}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

(a) 9 J (b) 15 J (c) 12 J (d) 10 J

17. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is



(a) 4 J (b) 2.5 J
(c) 6.5 J (d) 5 J

18. When a person lifts a brick above the surface of the earth, then its potential energy

(a) increases (b) decreases
(c) remains same (d) None of these

19. A massless spring of spring constant k, has extension y and potential energy E. It is now stretched from y to $2y$. The increase in its potential energy is

(a) $3E$ (b) $2E$
(c) E (d) $4E$

20. A bread gives 5 kcal of energy to a boy. How much height he can climbs by using this energy, if his efficiency is 28% and mass is 60 kg?

(a) 15m (b) 5m
(c) 2.5 m (d) 10 m

21. A body is falling freely under the action of gravity alone in vaccum. Which of the following quantities remain constant during the fall? *(NCERT Exemplar)*

(a) Kinetic energy
(b) Potential energy
(c) Total mechanical energy
(d) Total linear momentum

22. A stone is projected vertically up to reach maximum height h. The ratio of its kinetic energy to its potential energy at a height $\frac{4}{5}h$, will be

(a) 5 : 4 (b) 4 : 5
(c) 1 : 4 (d) 4 : 1

23. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 16 J (b) 8 J
(c) 32 J (d) 24 J

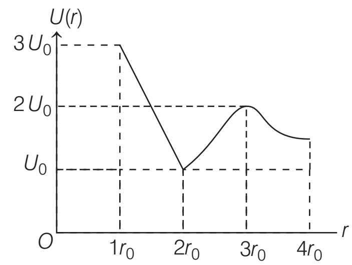
24. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10000 N/m. The spring compresses by

(a) 5.5 cm (b) 2.5 cm
(c) 11.0 cm (d) 8.5 cm

25. 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m (taking, $g = 10\ \text{ms}^{-2}$). Work done against friction is

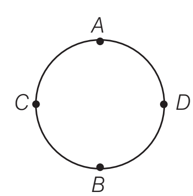
(a) 200 J (b) 100 J
(c) zero (d) 1000 J

26. The graph below represents the potential energy U as a function of position r for a particle of mass m. If the particle is released from rest at position r_0, what will its speed be at position $3r_0$?



(a) $\sqrt{\frac{3U_0}{m}}$ (b) $\sqrt{\frac{4U_0}{m}}$
(c) $\sqrt{\frac{2U_0}{m}}$ (d) $\sqrt{\frac{6U_0}{m}}$

27. A pebble is attached to one end of a string and rotated in a vertical circle. If string breaks at the position of maximum tension, so from the figure shown below, it will break at



(a) A (b) B (c) C (d) D

28. What is the ratio of kinetic energy of a particle at the bottom to the kinetic energy at the top, when it just loops a vertical loop of radius r?

(a) 5 :1 (b) 2 : 3
(c) 5 :2 (d) 7 : 2

29. A man can do work of 600 J in 2 min, then man's power is

(a) 7.5 W (b) 10 W
(c) 5 W (d) 15 W

30. A particle is acted by a constant power. Then, which of the following physical quantity remains constant?

(a) Speed
(b) Rate of change of acceleration
(c) Kinetic energy
(d) Rate of change of kinetic energy

31. An object of mass m moves horizontally, increasing in speed from 0 to v in a time t. The power necessary to accelerate the object during this time period is

(a) $\frac{mv^2 t}{2}$ (b) $\frac{mv^2}{2}$
(c) $2mv^2$ (d) $\frac{mv^2}{2t}$

32. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (take, 1 HP = 746 W and $g = 10\ \text{ms}^{-2}$)

(a) 2.0 ms^{-1} (b) 1.5 ms^{-1}
(c) 1.9 ms^{-1} (d) 1.7 ms^{-1}

33. A car of mass m starts from rest and accelerates, so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to

(a) $t^2 P_0$ (b) $t^{1/2}$
(c) $t^{-1/2}$ (d) $t/\sqrt{m}$

34. For a system to follow the law of conservation of linear momentum during a collision, the condition is

I. total external force acting on the system is zero
II. total external force acting on the system is finite and time of collision is negligible
III. total internal force acting on the system is zero

(a) Only I (b) Only II (c) Only III (d) I or II

35. Two identical balls A and B having velocities of $0.5\ \text{ms}^{-1}$ and $-0.3\ \text{ms}^{-1}$ respectively, collide elastically in one dimension. The velocities of B and A after the collision respectively will be

(a) $-0.5\ \text{ms}^{-1}$ and $0.3\ \text{ms}^{-1}$
(b) $0.5\ \text{m/s}^{-1}$ and $-0.3\ \text{ms}^{-1}$
(c) $-0.3\ \text{ms}^{-1}$ and $0.5\ \text{ms}^{-1}$
(d) $0.3\ \text{ms}^{-1}$ and $0.5\ \text{ms}^{-1}$

36. A particle of mass 1g moving with a velocity $\mathbf{v}_1 = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\ \text{ms}^{-1}$ experiences a perfectly elastic collision with another particle of mass 2 g and velocity $\mathbf{v}_2 = (4\hat{\mathbf{j}} - 6\hat{\mathbf{k}})\ \text{ms}^{-1}$. The velocity of the particle is

(a) 2.3 ms^{-1} (b) 4.6 ms^{-1}
(c) 9.2 ms^{-1} (d) 6 ms^{-1}

37. A particle of mass m_1 moves with velocity v_1 collides with another particle at rest of equal mass. The velocity of second particle after the elastic collision is

(a) $2v_1$ (b) v_1 (c) $-v_1$ (d) zero

38. During inelastic collision between two bodies, which of the following quantities always remain conserved? *(NCERT Exemplar)*

(a) Total kinetic energy
(b) Total mechanical energy
(c) Total linear momentum
(d) Speed of each body

39. Two objects of mass m each moving with speed u ms^{-1} collide at 90°, then final momentum is (assume collision is inelastic)

(a) mu (b) $2\,mu$
(c) $\sqrt{2}\,mu$ (d) $2\sqrt{2}\,mu$

40. A body of mass 5×10^3 kg moving with speed 2 ms^{-1} collides with a body of mass 15×10^3 kg inelastically and sticks to it. Then, loss in kinetic energy of the system will be

(a) 7.5 kJ (b) 15 kJ (c) 10 kJ (d) 5 kJ

41. If the linear momentum of a body is increased by 50%, then the kinetic energy of that body increases by ....

(a) 100% (b) 125% (c) 225% (d) 25%

42. A ball of mass m moves with speed v and strikes a wall having infinite mass and it returns with same speed, then the work done by the ball on the wall is ....

(a) zero (b) mv J
(c) m/v J (d) v/mJ

43. A body of mass 5 kg is thrown vertically up with a kinetic energy of 490 J. The height at which the kinetic energy of the body becomes half of the original value is ....

(a) 12.5 m (b) 10 m
(c) 2.5 m (d) 5 m

44. If two persons A and B take 2 s and 4 s, respectively to lift an object to the same height h, then the ratio of their powers is ....

(a) 1:2 (b) 1:1
(c) 2:1 (d) 1:3

45. At time $t = 0$, particle starts moving along the x-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to t^n, where the value of n is ....

(a) 1 (b) – 1/2
(c) 2 (d) 1/2

46. A man of mass m, standing at the bottom of the staircase, of height L climbs it and stands at its top. Which amongst the following statement is correct? *(NCERT Exemplar)*

(a) Work done by all forces on man is equal to the rise in potential energy mgL.
(b) Work done by all forces on man is zero.
(c) Work done by the gravitational force on man is mgL.
(d) The reaction force from a step does some work because the point of application of the force does not move while the force exists.

47. Which of the following statement is correct about non-conservative force?

(a) It depends on velocity of the object.
(b) It depends on the particular path taken by the object.
(c) It depend on the initial and final positions of the object.
(d) Both (a) and (b)

48. Which of the following statement is correct?

(a) Conservation of mechanical energy does not consider only conservative force.
(b) Conservation of energy consider both conservative and non-conservative forces.
(c) Conservation of energy consider only conservative force.
(d) Mass converted into energy in nuclear reaction is called mass-defect.

49. Which of the following statement does not specify an example of perfectly inelastic collision?

(a) A bullet fired into a block if bullet gets embedded into block.
(b) Capture of electrons by an atom.
(c) A man jumping on to moving boat.
(d) A ball bearing striking another ball bearing.

50. Match the Column I (angle) with Column II (work done) and select the correct option from the codes given below.

	Column I		Column II
A.	$\theta < 90°$	p.	Friction
B.	$\theta = 90°$	q.	Satellite rotating around the earth
C.	$\theta > 90°$	r.	Coolie is lifting a luggage

Codes

	A	B	C
(a)	p	q	r
(b)	r	q	p
(c)	p	r	q
(d)	r	p	q

51. A body is moved along a straight line by a machine delivering a power proportional to time ($P \propto t$). Then, match the Column I with Column II and select the correct option from the codes given below.

	Column I		Column II
A.	Velocity is proportional to	p.	t
B.	Displacement is proportional to	q.	t^2
C.	Work done is proportional to	r.	t^3

Codes

	A	B	C
(a)	p	q	r
(b)	r	q	p
(c)	p	q	q
(d)	r	p	p

Assertion-Reasoning MCQs

*For question numbers 52 to 60, two statements are given-one labelled **Assertion** (A) and the other labelled **Reason** (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) are as given below*

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

52. Assertion Stopping distance

$$= \frac{\text{Kinetic energy}}{\text{Stopping force}}$$

Reason Work done in stopping a body is equal to change in kinetic energy of the body.

53. Assertion A spring of force constant k is cut into two pieces having lengths in the ratio 1 : 2. The force constant of series combination of the two parts is $2k/3$.

Reason The spring connected in series are represented by $k = k_1 + k_2$.

54. Assertion According to the law of conservation of mechanical energy, change in potential energy is equal and opposite to the change in kinetic energy.

Reason Mechanical energy is not conserved.

55. Assertion Decrease in mechanical energy is more in case of an object sliding up a relatively less inclined plane due to friction.

Reason The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

56. **Assertion** For looping a vertical loop of radius r, the minimum velocity at lowest point should be $\sqrt{5gr}$.

Reason In this event, the velocity at the highest point will be zero.

57. **Assertion** Kilowatt-hour is the unit of energy.

Reason One kilowatt hour is equal to 3.6×10^6 J.

58. **Assertion** There is no loss in energy in elastic collision.

Reason Linear momentum is conserved in elastic collision.

59. **Assertion** Quick collision between two bodies is more violent than a slow collision; even when the initial and final velocities are identical.

Reason The momentum is greater in first case.

60. **Assertion** Two particles are moving in the same direction do not lose all their energy in completely inelastic collision.

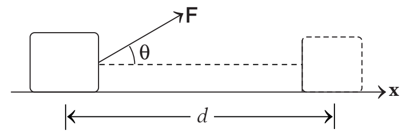
Reason Principle of conservation of momentum does not holds true for all kinds of collisions.

Case Based MCQs

Direction *Answer the questions from 61-65 on the following case.*

Work

A farmer ploughing the field, a construction worker carrying bricks, a student studying for a competitive examination, an artist painting a beautiful landscape, all are said to be working. In physics, however, the word 'Work' covers a definite and precise meaning. Work refers to the force and the displacement over which it acts. Consider a constant force F acting on an object of mass m. The object undergoes a displacement d in the positive x-direction as shown in figure.



The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement, thus
$W = (F\cos\theta)\, d = \mathbf{F} \cdot \mathbf{d}$.

61. The earth is moving around the sun in a circular orbit, is acted upon by a force and hence work done on the earth by the force is

(a) zero
(b) positive
(c) negative
(d) None of the above

62. In which case, work done will be zero?

(a) A weight-lifter while holding a weight of 100 kg on his shoulders for 1 min
(b) A locomotive against gravity is running on a level plane with a speed of 60 kmh^{-1}
(c) A person holding a suitcase on his head and standing at a bus terminal
(d) All of the above

63. Find the angle between force $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$ unit and displacement $\mathbf{d} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ unit.

(a) $\cos^{-1}(0.49)$ (b) $\cos^{-1}(0.32)$
(c) $\cos^{-1}(0.60)$ (d) $\cos^{-1}(0.90)$

64. Which of the following statement(s) is/are correct for work done to be zero?

I. If the displacement is zero.

II. If force applied is zero.

III. If force and displacement are mutually perpendicular to each other.

(a) Only I (b) I and II
(c) Only II (d) I, II and III

65. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In same time t, the work done on the two moving charged particles is

(a) same as the same force law is involved in the two experiments
(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens
(c) more for the case of a positron, as the positron moves away a larger distance
(d) same as the work is done by charged particle on the stationary proton

Direction *Answer the questions from 66-70 on the following case.*

Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy. In other words, the amount of work done, a moving object can do before coming to rest is equal to its kinetic energy.

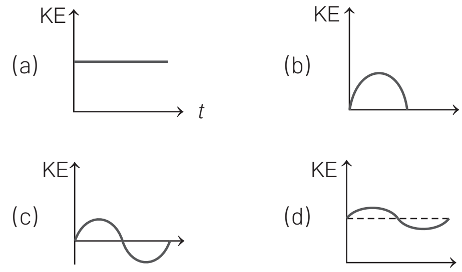
$\therefore$ Kinetic energy, $KE = \frac{1}{2} mv^2$

where, m is a mass and v is the velocity of a body.

The units and dimensions of KE are Joule (in SI) and $[ML^2T^{-2}]$, respectively.

Kinetic energy of a body is always positive. It can never be negative.

66. Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?



67. A force which is inversely proportional to the speed is acting on a body. The kinetic energy of the body starting from rest is

(a) a constant
(b) inversely proportional to time
(c) directly proportional to time
(d) directly proportional to square of time

68. The kinetic energy of an air molecule (10^{-21} J) in eV is

(a) 6.2 meV (b) 4.2 meV
(c) 10.4 meV (d) 9.7 meV

69. Two masses of 1 g and 4 g are moving with equal kinetic energy. The ratio of the magnitudes of their momentum is

(a) 4 : 1 (b) $\sqrt{2}$:1
(c) 1 : 2 (d) 1 : 16

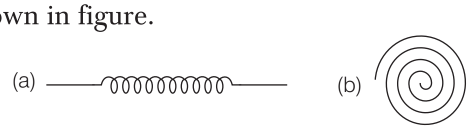
70. An object of mass 10 kg is moving with velocity of 10 ms^{-1}. Due to a force, its velocity become 20 ms^{-1}. Percentage increase in its KE is

(a) 25% (b) 50%
(c) 75% (d) 300%

Direction *Answer the questions from 71-75 on the following case.*

PE of Spring

There are many types of spring. Important among these are helical and spiral springs as shown in figure.



Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of elastic potential energy of the spring. Thus, potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

71. The potential energy of a body is increases in which of the following cases?

(a) If work is done by conservative force
(b) If work is done against conservative force
(c) If work is done by non-conservative force
(d) If work is done against non- conservative force

72. The potential energy, i.e. $U(x)$ can be assumed zero when

(a) $x=0$
(b) gravitational force is constant
(c) infinite distance from the gravitational source
(d) All of the above

73. The ratio of spring constants of two springs is 2 : 3. What is the ratio of their potential energy, if they are stretched by the same force?

(a) 2 : 3
(b) 3 : 2
(c) 4 : 9
(d) 9 : 4

74. The potential energy of a spring increases by 15 J when stretched by 3 cm. If it is stretched by 4 cm, the increase in potential energy is

(a) 27 J (b) 30 J
(c) 33 J (d) 36 J

75. The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x?

(a) 10 J (b) 20 J
(c) 30 J (d) 40 J

Direction *Answer the questions from 76-80 on the following case.*

Principle of Conservation of Energy

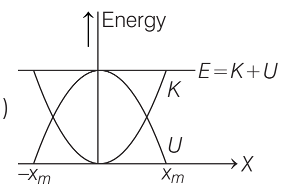
Total energy of an isolated system always remains constant. Since, the universe as a whole may be viewed as an isolated system, total energy of the universe is constant. If one part of the universe loses energy, then other part must gain an equal amount of energy.

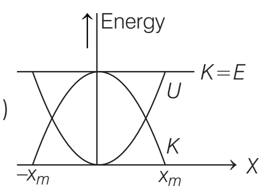
The principle of conservation of energy cannot be proved as such. However, no violation of this principle has been observed.

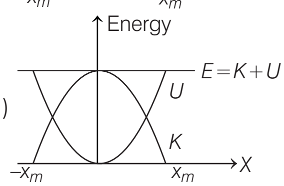
76. When we rub two flint stones together, got them to heat up and to ignite a heap of dry leaves in the form of

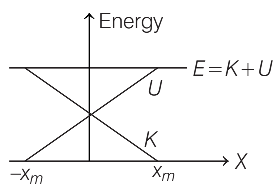
(a) chemical energy (b) sound energy
(c) heat energy (d) electrical energy

77. Which graph represents conservation of total mechanical energy?

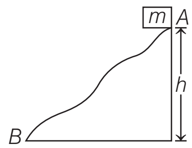
(a)


(b)


(c)


(d)


78. In the given curved road, if particle is released from A, then



(a) kinetic energy at B must be mgh
(b) kinetic energy at B must be zero
(c) kinetic energy at B must be less than mgh
(d) kinetic energy at B must not be equal to potential energy

79. U is the potential energy, K is the kinetic energy and E is the mechanical energy. Which of the following is not possible for a stable system?

(a) $U > E$ (b) $U < E$ (c) $E > K$ (d) $K > E$

80. A body of mass 5 kg is thrown vertically up with a kinetic energy of 490 J. The height at which the kinetic energy of the body becomes half of the original value is

(a) 12.5 m (b) 10
(c) 2.5 m (d) 5 m

ANSWERS

Multiple Choice Questions

1. (c)	2. (c)	3. (b)	4. (c)	5. (b)	6. (d)	7. (d)	8. (b)	9. (c)	10. (d)
11. (a)	12. (d)	13. (a)	14. (a)	15. (c)	16. (b)	17. (c)	18. (a)	19. (a)	20. (d)
21. (c)	22. (c)	23. (b)	24. (a)	25. (b)	26. (c)	27. (b)	28. (a)	29. (c)	30. (d)
31. (d)	32. (c)	33. (b)	34. (a)	35. (b)	36. (b)	37. (b)	38. (c)	39. (c)	40. (a)
41. (b)	42. (a)	43. (d)	44. (c)	45. (b)	46. (b)	47. (d)	48. (b)	49. (d)	50. (b)
51. (c)									

Assertion-Reasoning MCQs

52. (a)	53. (c)	54. (d)	55. (c)	56. (c)	57. (b)	58. (b)	59. (a)	60. (c)

Case Based MCQs

61. (a)	62. (d)	63. (b)	64. (d)	65. (c)	66. (d)	67. (c)	68. (a)	69. (c)	70. (d)
71. (b)	72. (d)	73. (b)	74. (a)	75. (c)	76. (a)	77. (c)	78. (a)	79. (a)	80. (d)

SOLUTIONS

1. Here, work is done by the frictional force on the cycle $= -200 \times 10 = -2000$ J.

As the road is not moving, hence work done by the cycle on the road is zero.

2. Given, $F = 50$ N, $W = 150$ J and $s = 3$ m

Work done, $W = Fs\cos\theta$

$$150 = 50 \times 3 \times \cos\theta$$

$$\cos\theta = \frac{150}{150} = 1$$

$$\Rightarrow \qquad \theta = 0°$$

3. Net force, $\mathbf{F} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}} + 5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$$= 7\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

Displacement, $\mathbf{d} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}} - \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$$= \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

Work done $= \mathbf{F}\cdot\mathbf{d} = (7\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$$= 7 - 4 - 10 = -7 \text{ units}$$

4. Work done by a variable force on the particle,

$$W = \int \mathbf{F}\cdot d\mathbf{r} = \int \mathbf{F}\cdot(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$

$\therefore$ In two dimension, $d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$

and it is given $\mathbf{F} = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$$\therefore \quad W = \int (-x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$$
$$= \int -x\,dx + y\,dy$$
$$= \int -x\,dx + \int y\,dy$$

As particle is displaced from $A(1, 0)$ to $B(0, 1)$, so x varies from 1 to 0 and y varies from 0 to 1.

So, with limits, work will be

$$W = \int_1^0 -x\,dx + \int_0^1 y\,dy$$
$$= \left[\frac{-x^2}{2}\right]_1^0 + \left[\frac{y^2}{2}\right]_0^1$$
$$= \frac{1}{2}[0 - (-1)^2 + (1)^2 - 0] = 1\,\text{J}$$

5. Work done = Area under F-s curve

$$W_{AB} = W_{12} + W_{23} + W_{34} + W_{45}$$
$$= \text{Area under } AP + \text{Area under } PQ + \text{Area under } QR - \text{Area above } RB$$
$$= 10 \times 1 + \frac{1}{2}(10 + 15) \times 1 + \frac{1}{2} \times 1 \times 15 - \frac{1}{2} \times 1 \times 15$$
$$= 10 + 12.5 = 22.5\,\text{J}$$

6. Work done in stretching a string to obtain an extension l,

$$W_1 = \frac{1}{2}kl^2$$

Similarly, work done in stretching a string to obtain an extension l_1 is

$$W_2 = \frac{1}{2}kl_1^2$$

$\therefore$ Work done in second case,

$$W = W_2 - W_1 = \frac{1}{2}k(l_1^2 - l^2)$$

7. The weight of hanging part $\left(\frac{l}{3}\right)$ of chain is $\left(\frac{1}{3}mg\right)$. This weight acts at the centre of gravity of the hanging part, which is at a distance of $\left(\frac{l}{6}\right)$ from the table.

Hence, work required to pull hanging part,

$$W = \text{force} \times \text{displacement}$$
$$\therefore \quad W = \frac{mg}{3} \times \frac{l}{6} = \frac{mgl}{18}$$

8. Gravitational force is a conservative force and work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed by it.

So, $W_1 = W_2 = W_3$

9. As we know that, KE $= \frac{1}{2}mv^2$

So, kinetic energy is directly proportional to the square of velocity.

$$K \propto v^2$$

As this equation resembles equation of parabola as m is constant, hence option (c) represents a parabola.

10. The kinetic energy K and momentum p of a body are related as

$K = \frac{p^2}{2m}$, where m is the mass of the body

Here, $p = 6$ N-s and $m = 4$ kg

$$K = \frac{(6)^2}{2 \times 4} = 4.5\,\text{J}$$

11. The kinetic energy of the particle is

$$K = \frac{1}{2}mv^2$$

As, momentum, $p = mv$

or $\quad p^2 = m^2v^2$

or $\quad v^2 = \frac{p^2}{m^2}$

$$\therefore \quad K = \frac{1}{2}m\left(\frac{p^2}{m^2}\right) = \frac{p^2}{2m}$$

12. Kinetic energy of a body, $K = \frac{p^2}{2m}$

where, p is the momentum and m is the mass of the body.

As, $\quad p_1 = p_2 \quad$ (given)

$$\therefore \quad \frac{K_1}{K_2} = \frac{m_2}{m_1} = \frac{5}{4}$$

13. Kinetic energy of a body, $K = \frac{p^2}{2m}$

or $\quad p = \sqrt{2mK}$

Since, $K_H = K_L$ (given)

where, subscripts H and L represents for heavy and light bodies.

$\therefore$ $$\frac{p_H}{p_L} = \sqrt{\frac{m_H}{m_L}}$$

So, $m_H > m_L$

and $p_H > p_L$

14. Given, mass, $m = 5$ kg

Radius, $R = 1$ m

Revolution per minute, $\omega = 300$ rev/min

$$= (300 \times 2\pi) \text{ rad/min}$$

$$= \frac{300 \times 2 \times \pi}{60} \text{ rad/s} = 10\,\pi \text{ rad/s}$$

Linear speed, $v = \omega R = 10\,\pi \times 1$

$= 10\pi$ m/s

$$\therefore \quad \text{KE} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 5 \times (10\pi)^2$$

$$= 100\pi^2 \times 5 \times \frac{1}{2}$$

$$= 250\,\pi^2 \text{ J}$$

15. Applying work-energy theorem on both moving objects,

$$\frac{1}{2} m_1 v_1^2 = Fx_1$$

and $$\frac{1}{2} m_2 v_2^2 = Fx_2$$

Since, both moving objects have same kinetic energy,

i.e. $$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 \Rightarrow Fx_1 = Fx_2$$

$$\Rightarrow \quad x_1 = x_2$$

Therefore, both the objects will come to rest at the same distance.

16. Work done in $\mathbf{F}$ is given by $\Delta W = \mathbf{F} \cdot \mathbf{d}$

By substituting given values, we get

$$\Rightarrow \quad \Delta W = (3\hat{\mathbf{i}} - 12\hat{\mathbf{j}}) \cdot (4\hat{\mathbf{i}})$$

$$\Rightarrow \quad \Delta W = 12 \text{ J} \quad \dots \text{(i)}$$

Now, using work-energy theorem, we get work done, ΔW = change in kinetic energy, ΔK

or $$\Delta W = K_2 - K_1 \quad \dots \text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$K_2 - K_1 = 12 \text{ J} \;\text{ or }\; K_2 = K_1 + 12 \text{ J}$$

Given, initial kinetic energy, $K_1 = 3$ J

$\therefore$ Final kinetic energy, $K_2 = 3 \text{ J} + 12 \text{ J} = 15 \text{ J}$

17. $\therefore$ Work done on the particle

= Area under the curve ABC

W = Area of square $ABFO$ + Area of ΔBCD + Area of rectangle $BDEF$

$$= 2 \times 2 + \frac{1}{2} \times 1 \times 1 + 2 \times 1 = 6.5 \text{ J}$$

Now, from work-energy theorem,

$$\Delta W = K_f - K_i$$

$$\Rightarrow \quad K_f = \Delta W = 6.5 \text{ J} \qquad (\because K_i = 0)$$

18. Potential energy of brick above the earth's surface is given by

$$U = mgh$$

i.e. $U \propto h$

Hence, when a brick is lifted above the surface of the earth, then its potential energy increases.

19. The potential energy of the spring is

$$E = \frac{1}{2} ky^2 \quad \dots \text{(i)}$$

Now, it is stretched from y to $2y$, so its potential energy becomes

$$E' = \frac{1}{2} k(2y)^2$$

$$= 2ky^2 = 4E \qquad \text{[using Eq. (i)]}$$

$\therefore$ The increase in its potential energy is

$$\Delta E = E' - E = 4E - E = 3E$$

20. Energy received by the boys from bread

$$= 5000 \text{ cal} = 5000 \times 4.2$$

$$= 21 \times 10^3 \text{ J}$$

According to law of conservation of mechanical energy,

$$mgh = \frac{28}{100} \times 21 \times 10^3$$

$$\therefore \quad h = \frac{28 \times 21 \times 10^3}{100 \times 9.8 \times 60} = 10 \text{ m}$$

21. As the body is falling freely under gravity, the potential energy decreases and kinetic energy increases but total mechanical energy (PE + KE) of the body and earth system will be constant as external force on the system is zero.

22. At a height $\frac{4}{5}h$,

Potential energy $= mg \times \frac{4}{5}h = \frac{4}{5}\,mgh$

Total energy = mgh

$\therefore$ Kinetic energy at that height

$$= mgh - \frac{4}{5}mgh = \frac{1}{5}\,mgh$$

$\therefore$ At a height $\frac{4}{5}h$, the ratio of

$$\frac{\text{KE}}{\text{PE}} = \frac{\frac{1}{5}mgh}{\frac{4}{5}mgh} = \frac{1}{4}$$

23. The work done on the spring is stored as the PE of the body and is given by

$$U = \int_{x_1}^{x_2} F_{\text{ext}}\,dx$$

or $$U = \int_{x_1}^{x_2} kx\,dx$$

$$= \frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{800}{2}[(0.15)^2 - (0.05)^2]$$

$$= 400\,(0.2 \times 0.1) = 8\text{ J}$$

24. According to work-energy theorem,
loss in kinetic energy = work done against friction + potential energy of spring

$$\frac{1}{2}mv^2 = f \cdot x + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times 2(4)^4 = 15\,x + \frac{1}{2} \times 10000\,x^2$$

$$\Rightarrow \quad 5000\,x^2 + 15x - 16 = 0$$

$$\therefore \quad x = 0.055\text{ m} = 5.5\text{ cm}$$

25. Net work done in sliding a body up to a height h on an inclined plane
= Work done against the gravitational force + Work done against the frictional force

$$\Rightarrow \quad W = W_g + W_f \quad \ldots\text{(i)}$$

But $W = 300$ J

Given, $m = 2$ kg and $h = 10$ m

$$W_g = mgh = 2 \times 10 \times 10 = 200\text{ J}$$

Putting these values in Eq. (i), we get

$$300 = 200 + W_f$$

$$\Rightarrow \quad W_f = 300 - 200 = 100\text{ J}$$

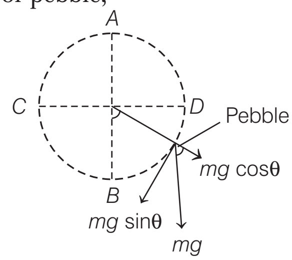
26. According to the law of conservation of energy, $U_i + K_i = U_f + K_f$ ...(i)

So, by putting the values in Eq. (i),

$$3U_0 + 0 = 2U_0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2U_0}{m}}$$

27. FBD of pebble,



$$\therefore \quad T - mg\cos\theta = \frac{mv^2}{l} \Rightarrow T = mg\cos\theta + \frac{mv^2}{l}$$

Tension is maximum, when $\cos\theta = 1$ and velocity is maximum. Both conditions are satisfied at $\theta = 0°$, i.e. at lowest point B.

28. At top point, the tension (T_H) in string becomes zero, so velocity of the particle is

$$v_H = \sqrt{gr}$$

At the bottom, the velocity of the particle is

$$v_L = \sqrt{5gr}$$

Therefore, the ratio of kinetic energies at bottom and top is

$$\frac{K_L}{K_H} = \frac{\frac{1}{2}mv_L^2}{\frac{1}{2}mv_H^2} = \left(\frac{v_L}{v_H}\right)^2$$

$$= \frac{5\,gr}{gr} = \frac{5}{1} = 5:1$$

Hence, the ratio of kinetic energies is 5 : 1.

29. Given, $W = 600$ J

and $t = 2\text{ min} = 2 \times 60 = 120$ s

$$\therefore \quad \text{Power, } P = \frac{W}{t} = \frac{600}{120} = 5\text{ W}$$

30. By definition, $P = \frac{dW}{dt}$

$\because$ Work done = Kinetic energy

$$\Rightarrow \quad P = \frac{dW}{dt} = \frac{d(\text{KE})}{dt} = \text{constant}$$

31. Power, $P = \dfrac{W}{t}$

Since, K_i = initial KE = 0

and K_f = final KE = $\dfrac{1}{2}mv^2$

From work-energy theorem,
work done = change in KE

$$\therefore \quad P = \frac{K_f - K_i}{t} = \frac{\frac{1}{2}mv^2 - 0}{t}$$

$$\Rightarrow \quad P = \frac{mv^2}{2t}$$

32. At maximum load, force provided by motor to pull the lift,

$$F = \text{weight carried} + \text{friction} = mg + f$$
$$= (2000 \times 10) + 4000 = 24000 \text{ N}$$

Power delivered by motor at speed v of load, $P = F \times v$

$$\Rightarrow \quad v = \frac{P}{F} = \frac{60 \times 746}{24000} = 1.865 = 1.9 \text{ ms}^{-1}$$

33. As, $P_0 = Fv = \left(m\dfrac{dv}{dt}\right)v = mv\dfrac{dv}{dt}$

$\Rightarrow \quad P_0 \cdot dt = mvdv$

Integrating both sides, we get

$$\int_0 P_0 \, dt = \int mv \, dv$$

$$\Rightarrow \quad P_0 t = \frac{mv^2}{2}$$

$$\Rightarrow \quad v^2 = \frac{2P_0 t}{m}$$

$$\Rightarrow \quad v \propto t^{1/2}$$

34. From Newton's second law, $F = \dfrac{dp}{dt}$

If $F = 0$, then $\dfrac{dp}{dt} = 0$

$\Rightarrow \quad p$ = constant

Thus, if total external force acting on the system is zero during collision, then the linear momentum of the system remains conserved.

35. As, we know in a elastical collision of two identical bodies, i.e. $m_A = m_B$, the particles mutually exchange their velocities.

So, $(v_i)_A = 0.5 \text{ ms}^{-1}$ and $(v_i)_B = -0.3 \text{ ms}^{-1}$.

After collision,

$$(v_f)_A = -0.3 \text{ ms}^{-1} \text{ and } (v_f)_B = 0.5 \text{ ms}^{-1}.$$

36. From conservation of momentum,

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}$$

$$1 \times (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2 \times (4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}) = (1 + 2)\ \mathbf{v}$$

$$\Rightarrow \quad 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}} = 3\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

$$\therefore \text{ Velocity,} \quad v = |\mathbf{v}| = \sqrt{1 + 4 + 16} = 4.6 \text{ ms}^{-1}$$

37. Given, mass, $m_1 = m_2 = m$ and velocity, $v = v_1$

For elastic collision,

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \frac{2m_1v_1}{m_1 + m_2}$$

After putting given values, we will get

$$v_2 = \frac{2m_1v_1}{2m_1} \Rightarrow v_2 = v_1$$

38. When we are considering the two bodies as system, the total external force on the system will be zero.

Hence, total linear momentum of the system remain conserved.

39. Speed of objects = u ms^{-1}

Since, both objects collide with 90°.

According to the law of conservation of momentum,

Total moment before collision

= Total momentum after collision

$$|mu\hat{\mathbf{i}} + mu\hat{\mathbf{j}}| = p_f$$

$$\sqrt{m^2u^2 + m^2u^2} = p_f \Rightarrow p_f = \sqrt{2}\ mu$$

40. Given, mass of body, $m_1 = 5 \times 10^3$ kg

and mass of another body

$$m_2 = 15 \times 10^3 \text{ kg}$$

Velocity, $v_1 = 2 \text{ ms}^{-1}$

For perfectly inelastic collision ($e = 0$),

Loss in kinetic energy of system,

$$\Delta E_K = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2} \times v_1^2$$

$$= \frac{1}{2} \times \frac{5 \times 10^3 \times 15 \times 10^3}{5 \times 10^3 + 15 \times 10^3} \times 2^2$$

$$= 7.5 \times 10^3 \text{ J} = 7.5 \text{ kJ}$$

41. Kinetic energy of the body,

$$K = \frac{p^2}{2m}$$

Since, the mass remains constant, so $K \propto p^2$.

$$\therefore \qquad \frac{K_2}{K_1} = \frac{p_2^2}{p_1^2} = \left[\frac{150}{100}\right]^2 = \frac{9}{4}$$

Thus, $\left(\frac{K_2}{K_1} - 1\right) \times 100 = \left(\frac{9}{4} - 1\right) \times 100$

$$= 125\%$$

42. As, work done = force × displacement

As, there is no displacement produced in the wall, so work done by the ball on the wall is zero.

Alternative Method As, there is no change in kinetic energy of the ball, so according to work-energy theorem, work done should be zero.

43. According to the law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) + mgh$$

$$\Rightarrow \qquad 490 = 245 + 5 \times 9.8 \times h,$$

$$h = \frac{245}{49} = 5\,\text{m}$$

44. Given, $t_1 = 2$ s, $t_2 = 4$ s

and $h_1 = h_2 = h$

As, $P_A = \frac{mgh_1}{t_1}$ and $P_B = \frac{mgh_2}{t_2}$ ...(i)

$$\Rightarrow P_A : P_B = \frac{mgh_1/t_1}{mgh_2/t_2} = \left(\frac{h_1}{h_1}\right)\left(\frac{t_2}{t_1}\right) = \frac{t_2}{t_1} = \frac{4}{2} = \frac{2}{1}$$

$$\Rightarrow \qquad P_A : P_B = 2:1$$

45. Given, $k \propto t \Rightarrow \frac{dk}{dt} = \text{constant}$

$$\Rightarrow \qquad K \propto t$$

$$\frac{1}{2}mv^2 \propto t \quad \Rightarrow \quad v \propto \sqrt{t}$$

Also, $P = Fv = \frac{dK}{dt} = \text{constant}$

$$\Rightarrow \qquad F \propto \frac{1}{v} \quad \Rightarrow \quad F \propto \frac{1}{\sqrt{t}}$$

$$\Rightarrow \qquad F \propto t^{-1/2}$$

46. When a man of mass m climbs up the staircase of height L, work done by the gravitational force on the man is mgl, work done by internal muscular forces will be mgL as the change in kinetic energy is almost zero.

Hence, total work done $= -mgL + mgL = 0$.

As the point of application of the contact forces does not move, hence work done by reaction forces will be zero.

47. If the work done or the kinetic energy depend on other factors such as the velocity or the particular path taken by the object, the force would be called non-conservative.

Thus, the statements given in options (a) and (b) are correct, rest is incorrect.

48. In elastic collision, the conservation of mechanical energy consider only conservative force while conservation of energy consider both conservative and non-conservative force.

Mass converted into energy in nuclear reaction is called nuclear energy.

Thus, the statement given in option (b) is correct, rest are incorrect.

49. Whenever there is a collision between two bodies, the total momentum of the bodies remains conserved. If after the collision of two bodies, the total kinetic energy of the bodies remains the same as it was before collision, then it a perfectly elastic collision.

A ball bearing striking another ball bearing is an example of elastic collision. If two bodies strick together after the collision, then the collision is said to be perfectly inelastic collision.

Options (a), (b) and (c) are examples of perfectly inelastic collisions.

50. Work done by an agent is given by

$$W = \mathbf{F} \cdot \mathbf{s} = Fs\cos\theta$$

where, F is the applied force, s is the displacement and θ is the smaller angle between F and s.

A. If $\theta < 90°$, i.e. acute angle, then work done is positive, as in case of coolie lifting luggage.

B. If $\theta = 90°$, i.e. right angle, then work done is zero, as in case of satellite rotation around the earth.

C. If $\theta > 90°$, i.e. obtuse angle, work done is negative, as in case of friction.

Hence, A $\rightarrow$ r, B $\rightarrow$ q and C $\rightarrow$ p.

51. As, power, $P \propto t$

So, $W = \int P dt = \int \alpha t\, dt$

or $W \propto t^2$

Since, work done is equal to change is KE.

Hence, $v^2 \propto t^2$ or $v \propto t$

Further, $v = \dfrac{ds}{dt}$

$\therefore \quad \dfrac{ds}{dt} \propto t$ or $ds \propto t\, dt$

or $s \propto t^2$ (by integration)

Hence, A $\rightarrow$ p, B $\rightarrow$ q and C $\rightarrow$ q.

52. According to work-energy theorem, work done by a body is equal to change in kinetic energy of the body.

$$\Rightarrow \quad W = \Delta \text{KE} = \frac{1}{2} mv^2 \quad \ldots\text{(i)}$$

But, W = stopping force $\times$ stopping distance

$$W = F \cdot d \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

Stopping distance (d)

$$= \frac{\text{Kinetic energy}\left(\frac{1}{2}mv^2\right)}{\text{Stopping force}\,(F)}$$

Therefore, both A and R are true and R is the correct explanation of A.

53. As we know, $k = \dfrac{F}{l} \Rightarrow k \propto \dfrac{1}{l}$

$$\Rightarrow \quad \frac{k_2}{k_1} = \frac{l_1}{l_2} = \frac{1}{2}$$

$$k_1 = 2k,\ k_2 = k$$

In series, $\dfrac{1}{k'} = \dfrac{1}{k_1} + \dfrac{1}{k_2} = \dfrac{1}{2k} + \dfrac{1}{k} = \dfrac{3}{2k}$

$$\because \quad k' = \frac{2k}{3}$$

Therefore, A is true but R is false.

54. According to the law of conservation of mechanical energy, for conservative forces, the sum of kinetic energy and potential energy remains constant and throughout the motion it is independent of time.

This is the law of conservation of mechanical energy, i.e. KE + PE = total energy = constant.

Therefore, A is false and R is also false.

55. Mechanical energy consists of both PE and KE. In the given cases, some of the mechanical energy is converted into heat energy and it is more in the case when inclination is less due to increased (as θ decreases, value of $\cos\theta$ will increases) friction force on an inclined plane.

$$f_r = \mu mg \cos\theta$$

The coefficient of friction does not depend on the angle of inclination of the plane. It depends only on the nature of surfaces in contact.

Therefore, A is true but R is false.

56. At the lowest point of a vertical circle, the minimum velocity at bottom,

$$v_{\min} = \sqrt{5gr}$$

Velocity at highest point, $v = \sqrt{gr}$

Therefore, A is true but R is false.

57. Power $= \dfrac{\text{Work done (or energy)}}{\text{Time}}$

$\Rightarrow$ Work done = Power $\times$ Time

$$W = P \times t$$

If $P = 1$ kilowatt, $t = 1$ hour, then

$$\begin{aligned} W &= 1 \text{ kilowatt} \times 1 \text{ hour} \\ &= 1 \text{ kilowatt-hour} \\ &= 10^3 \text{ watt} \times 60 \times 60 \text{ s} \\ &= 3.6 \times 10^6 \text{ J} \end{aligned}$$

Therefore, both A and R are true but R is not the correct explanation of A.

58. In elastic collision, total energy, kinetic energy and momentum remain conserved, therefore no loss in energy occurs in elastic collision.

Therefore, both A and R are true but R is not the correct explanation of A.

59. As momentum, $p = mv$ or $p \propto v$, i.e. momentum is directly proportional to its velocity, so the momentum is greater in a quicker collision between two bodies than in slower one.

Hence, due to greater momentum quicker collision between two bodies will be more violent even initial and final velocities are identical.

Therefore, both A and R are true and R is the correct explanation of A.

60. If two particles are initially moving in the same direction, then their resultant momentum will not be zero. Therefore, their resultant momentum cannot be zero after a completely inelastic collision.

As, kinetic energy is directly proportional to the square of the momentum, hence kinetic energy cannot be zero. This implies, not all the energy in inelastic collision is lost.

Therefore, A is true but R is false.

61. When earth is moving around the sun in a circular orbit, then gravitational attraction on earth due to the sun provides required centripetal force, which is in radially inward direction, i. e. in a direction perpendicular to the direction of motion of the earth in its circular orbit around the sun.

As a result, the work done on the earth by the force will be zero. i.e. $W = Fd\cos 90° = 0$.

62. Work done by weight-lifter is zero, because there is no displacement.

In a locomotive, work done is zero because force due to gravity and displacement are mutually perpendicular to each other.

In case of a person holding a suitcase on his head and standing at a bus terminal, work done is zero because there is no displacement.

Hence, options (a), (b) and (c) are correct.

63. Given, $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$ unit

and $\mathbf{d} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ unit

$$\therefore \quad \mathbf{F}\cdot\mathbf{d} = F_x d_x + F_y d_y + F_z d_z$$
$$= 3(5) + 4(4) + (-5)(3)$$
$$= 16 \text{ units}$$

Now, $$\mathbf{F}\cdot\mathbf{F} = F^2 = F_x^2 + F_y^2 + F_z^2$$
$$= 9 + 16 + 25$$
$$= 50 \text{ units}$$
$$\Rightarrow \quad F = \sqrt{50} \text{ units}$$

and $$\mathbf{d}\cdot\mathbf{d} = d^2 = d_x^2 + d_y^2 + d_z^2$$
$$= 25 + 16 + 9$$
$$= 50 \text{ units}$$
$$\Rightarrow \quad d = \sqrt{50} \text{ units}$$
$$\therefore \quad \cos\theta = \frac{16}{\sqrt{50}\,\sqrt{50}}$$
$$= \frac{16}{50} = 0.32 \qquad \left(\because \cos\theta = \frac{\mathbf{F}\cdot\mathbf{d}}{Fd}\right)$$
$$\Rightarrow \quad \theta = \cos^{-1}(0.32)$$

64. The work done in displacing an object by applying force F is given by

$$W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta$$

So, work done will be zero, when

(i) either applied force F or displacement s is zero.

(ii) the force and displacement are mutually perpendicular to each other. i.e. $\theta = 90°$.

So, all statements are correct.

65. Force between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton.

We know that, work done = force × distance. As, forces are same in case of proton and positron but distance moved by positron is larger, hence work done will be more in case of positron.

66. When the earth is closest to the sun, speed of the earth is maximum, hence KE is maximum. When the earth is farthest from the sun, speed is minimum, hence KE is minimum but never zero and negative.

This variation is correctly represented by option (d).

67. $F = \dfrac{K}{v}$ (given)

$$m\frac{dv}{dt} = \frac{K}{v}$$
$$\Rightarrow \quad \int mv\,dv = \int K\,dt$$
$$\Rightarrow \quad m\frac{v^2}{2} = Kt$$
$$\Rightarrow \quad \text{KE} \propto t$$

68. The kinetic energy of an air molecule is

$$= \frac{10^{-21}\text{ J}}{1.6\times10^{-19}\text{ J/eV}} \simeq 0.0062 \text{ eV}$$

This is the same as 6.2 meV.

69. As we know that, linear momentum, p

$$= \sqrt{2mK} \qquad \left(\because K = \frac{p^2}{2m}\right)$$

For same kinetic energy, $p \propto \sqrt{m}$

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 1:2$$

70. Initial velocity $= 10 \text{ ms}^{-1}$

Final velocity $= 20 \text{ ms}^{-1}$

Initial KE $= \frac{1}{2} \times 10 \times 10 \times 10 = 5 \times 10^2$ J

Final KE $= \frac{1}{2} \times 10 \times 20 \times 20 = 20 \times 10^2$ J

% increase $= \frac{(20-5) \times 10^2}{5 \times 10^2} \times 100 = 300\%$

71. Potential energy of a body increases when work is done against a conservative force, e.g. if we raise the height of an object, its potential energy increases because work is done against gravitational force which is a conservative force.

72. The zero of the potential energy is arbitrary. It is set according to convenience. For the spring force, we took $U(x) = 0$, at $x = 0$, i.e. the unstretched spring had zero potential energy.

For the constant gravitational force mg, we took $U = 0$ on the earth's surface.

Also, for the force due to the universal law of gravitation, the zero is best defined at an infinite distance from the gravitational source.

73. $F = k_1 x_1$, $F = k_2 x_2$

$$k_1 x_1 = k_2 x_2 \Rightarrow \frac{k_1}{k_2} = \frac{x_2}{x_1}$$

$$\frac{\text{PE (1)}}{\text{PE (2)}} = \frac{k_1 \, x_1^2}{k_2 \, x_2^2}$$

$$= \frac{k_1}{k_2} \times \left(\frac{k_2}{k_1}\right)^2 = \frac{k_2}{k_1} = \frac{3}{2}$$

74. PE of spring $= \frac{1}{2} kx^2 \Rightarrow$ PE $\propto x^2$

$$\therefore \quad \text{PE} = 15 \times \frac{(4)^2}{(3)^2} = 15 \times \frac{16}{9} \simeq 27 \text{ J}$$

75. Potential energy of the spring when stretched through a distance x,

$$U = \frac{1}{2} kx^2 = 10 \text{ J}$$

When x becomes $2x$, the potential energy will be

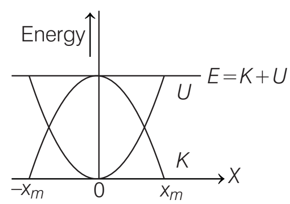
$$U' = \frac{1}{2} k(2x)^2 = 4 \times \frac{1}{2} kx^2$$

$$= 4 \times 10 = 40 \text{ J}$$

$\therefore$ Work done $= U' - U = 40 - 10 = 30$ J

76. One of the greatest technical achievements of human kind occurred when we discovered how to ignite and control fire . We learnt to rub two flint stones together (mechanical energy), got them to heat up and to ignite a heap of dry leaves (chemical energy), which then provided sustained warmth.

77. Parabolic plots of the potential energy U and kinetic energy K of a block attached to a spring obey in a Hooke's law. The two plots are complementary, one decreasing as the other increases. The total mechanical energy $E = K + U$ remains constant.



78. In a conservative field loss of PE or gain of KE depends only on initial and final point and not on path covered, i.e. at B, KE $= mgh$.

79. We know that, PE + KE = Mechanical energy

$$U + K = E$$

$$\Rightarrow \quad U = E - K$$

Now, K can never be negative, so

$$U < E$$

$$K = E - U$$

Now, U can be negative, so $K > E$ is possible.

80. According to the law of conservation of energy,

$$\frac{1}{2} mu^2 = \frac{1}{2}\left(\frac{1}{2} mu^2\right) + mgh$$

$$\Rightarrow \quad 490 = 245 + 5 \times 9.8 \times h$$

$$h = \frac{245}{49} = 5 \text{ m}$$