GRAVITATION

1. The motion of celestial bodies such as the moon, the earth, the planets etc. has been a subject of great interst for a long time. Famous Indian astronomer and mathernatician, Aryabhata studied these motions in great detail and wrote his conclusions in his book Aryabhata. He established that the earth revolves about its own axis and moves in a circular orbit about the sun, and that the moon moves in a circular orbit about the earth. The saying goes that Newton was sitting under an apple tree when an apple fell down from the tree on the earth. This sparked the idea that the earth attrachts all bodies towards its centre. Newton made several doing assumptions which proved to be turning points in scinece and philosophy.

Newton's Law of Gravitaion (or) Universal Law of Gravitation :

When we drop an object, it falls towards teh earth, which means that earth attracts the various objects towards its centre. The force which pulls the various objects towards the earth is known as force of gravity. Newton foudn that it is not only the earth which attracts the other objects, but every object in this universe attracts every other object. Thefore according to Newton's law of gravitation every body i nthe universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. And the direction of force is along the line joining centres of the two bodies.

(i.e.) $F = \frac{m_1 m_2}{R^2}$

$$F = \frac{G m_1 m_2}{R^2}$$

where F is the force of attraction between the 2 bodies of mases m_1 and m_2 separated by a distance of R. G is the gravitational constant = 6.67×10^{-11} Nm^2/kg^2. If we double the distance between 2 bodies, the gravitational force becomes $1/4^{th}$ and if we halved the distance between teh 2 bodies then the gravitational force becomes 4 times.

The gravitational force between 2 objects of mass 1 kg separated by a distance of 1m is

$$F = \frac{G m_1 m_2}{R^2} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(1)^2} N$$

= 6.67×10^{-11} Newtons.

It is the gravitatinal force between the sun and the earth which keep the earth in uniform circular motion around the sun and it is also responsible for holding the atmosphere above the earth, for rain faling and for flow of rivers.

Example 1 : Calculate the force of gravitation due to earth on a child weighting 20 kg standing on the ground (Mass of earth = 6×10^{24} kg; Radius of earth = 6.4 x 103 km, G = 6.67×10^{-11} Nm^2/kg^2)

Solution : The force of gravitation,

$$F = \frac{G m_1 m_2}{R^2} N$$

Given : Grativational constant,

G = 6.67×10^{-11} Nm^2/Kg^2

Mass of earth, $m_1 = 6 \times 10^{24}$ kg

Mass of child, $m_2 = 20$ kg

Radius of earth, R = $6.4 \times 10^3 \times 10^3$ m

$$F = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 20}{(6.4 \times 10^6)^2}$$

F = 195 N.

Acceleration due to gravity (g) :

When an object is dropped from some height its velocity increases at a constant rae, so an uniform acceleration is produced in it by the gravitational pull of the earth and this acceleration does not depend on the mass of the falling body.

And this uniform acceleration produced in a freely falling body due to grativational pull of the earth is known as acceleration due to gravity. It is denoted by 'g' and its value is 9.8 m/s^2.

Calculation of the value of g

If we drop a body (stone) of mass m from a distance R from the centre of the earth of mass M, then the force exerted by the earth on the stone is given by Newton's law of gravitation.

$$F = G\frac{Mm}{R^2} \quad \text{..... (1)}$$

The force exerted by the earth produces acceleration in the stone due to which the stone moves downwards.
F = ma

Acceleration of stone, $a = \frac{F}{m}$ .....(2)

Substitute (1) in (2) $a = \frac{GMm}{mR^2}$, $a = \frac{GM}{R^2}$. The acceleration produced by the earth is known as acceleration due to gravity (g), $g = \frac{GM}{R^2}$. And the value of 'g' = 9.8 m/s^2.

Example 2 : Calculate the value of g.

Solution : Acceleratio due to gravity, $g = \frac{GM}{R^2}$, where Gravitational constant.

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$. Mass of the earth,
$M = 6 \times 10^{24}$ kg. Radius of the earth,
$R = 6.4 \times 10^5$ m.

$$\therefore\ g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.8\ m/s^2$$

Application of Newton's Law of Gravitation

Newton's law of gravitation helps us to determine the mass of the earth accurately. It is used to detemine the masses of the sun, earth etc. It is used to estimate the masses of the double stars and in discovering new stars and planets.

A double star is a pair of stars revolving around their comon centre of mass :

A double star is shown in Figure. The double star consists of two stars A and B which are orbiting around their common centre of mass. The two stars of a double star system which revolve around each other are held together by the gravitational force between them. And Newton's law of gravitaton helps in estimating their masses.

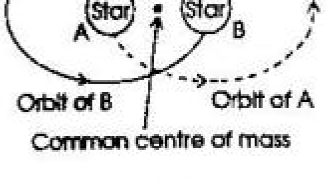


Fig.1

Equations of motion for freely falling bodies :

The freely falling bodies fall with uniformly accelerated motion, the three equations of motions for freely falling bodies are

$v = u + gt$

$h = ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$

Example 3 : A cricket ball is dropped from a height of 20 metres. Calculate teh speed of the ball when it hits the ground. (Take $g = 10\ m/s^2$)

Solution : Initial speed $u = 0$

Final speed, $v = ?$

$g = 10\ m/s^2$

$h = 20\ m$

$v^2 = u^2 + 2gh = 0^2 + 2(10)(20) = 400$

$v = 20\ m/s$

Example 4 : A stone dropped fro the roof of a building takes 8s to reach the ground. Find the height of the building.

Solution : Initial speed, $u = 0$

Time taken, $t = 8s$

$g = 9.0\ m/s^2$

$h = ut + \frac{1}{2}gt^2$

$h = 0(8) + \frac{1}{2}(9.8)(8)^2$

$= \frac{9.8 \times 64}{2} = 313.6\ m$

Mass and weight : The mass (m) of a body is the quantity of matter contained in it. It is a acalar quantity. Its unit iskilogram. The mass of a body is constant and does not change from place to place and it cannot be zero.
On the other hand the weight of a body is the force with which it is attracted towards the centre of the earth.

Force = mass x acceleration due to gravity

$F = m \times g$

By definaiton, the force of attraction of earth on a body is known as weight w of the body $w = mg$. Teh S.I. unit of weight is N (newton). The gravitational unit of force and weight is kilogram - weight (kg. wt). The weight of a body is not a constant and it is a vector quantity.

Example 5 : A man weight 1200 N on the earth. What is his mass ? (take $g = 10\ m/s^2$). If he were taken to the moon, his weigh twould be 200 N. What is his mass on the moon? What is the acceleration due to gravity on the moon?

Solution : $W = m \times g$

Weight of man on earth.

$W = 1200\ N$

$g = 10\ m/s^2$

$W = m \times g$

$1200 = m \times 10$

$m = 120\ kg$

The mass of man on the earth is 120 kg. The mass of a body is same everywhere in the universe. So, the mass of the man on teh mon iwll be 120 kilograms.

g on moon

W = mg

W = 200 N

m = 120 kg

200 = 120 x g

$$g = \frac{200}{120} = 1.67 \text{ m/s}^2$$

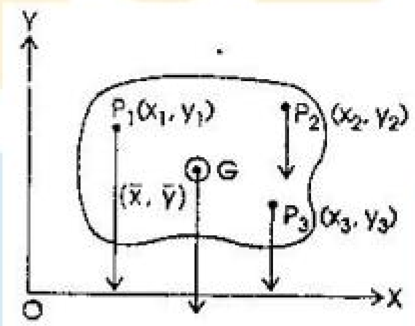
Centre of mass : It is the point at which the whole mass of the body may be supposed to be concentrated. Consider the case of a body of an arbitrary shape in xy plane as shown. Let the body consists of a number of particles P_1, P_2, P_3..... of mases m_1, m_2, m_3... and (x_1, y_1) (x_2, y_2) (x_3, y_3).. if $\overline{x}$, $\overline{y}$ be the coordinates of centre of mass, then

$$\overline{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$$

$$= \frac{\Sigma m_n x_n}{\Sigma m_n}$$

$$\overline{y} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + ...}{m_1 + m_2 + m_3 + ...}$$

$$= \frac{\Sigma m_n y_n}{\Sigma m_n}$$



On the other hand, centre of gravity of a body is a point in the body at which the force of gravity on the whole of the body can be assumed to act. For bodies which are of regular shape and which have uniform density, the centre of gravity lies at the geometrical centre of the body.

CHANGE IN WEIGHT DUE TO ACCELERATION AND DECLERATION

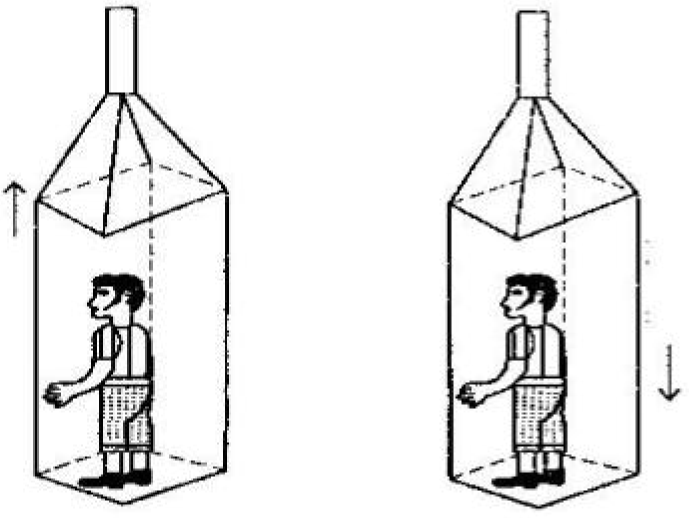


Fig. 2

(a) Lift accelerating upwards man's weight increases (b) Lift accelerating downwards man's weight decreases

When we go up a life which is acclerating upwards, we feel heavier i.e. there is an increase in our weight and when the lift accelerating downwrds, we feel lighter, why?

In the upgoing accelerating lift, the man experiences the same acceleration as the lift say 'a' and hence a force ma in the direction of motion where m is teh mass of the man. This force by Newton's third law of motion exerts on equal downward force on the floor of the lift.

Then there is also the force mg - the normal weight of the man's bodypressing on the floor of the lift where g is the acceleration due to gravity. Thus the totalor resultant force that the man's body exerts on teh floor is ma + mg or m(a + g) which is more than his normal weigh tmg. So he feels heavier.

When the lift accelerates downwards with the same acceleraton 'a', part of his normal weight mg is utilzied in giving him this acceleration and only teh balance is left as his resultant weight which will be mg - ma or m (g - a) and this will be zero when g = a as in free fall.

WEIGHTLESSNESS

Weightlessness is the common experience of the astronaut during most of his journey in space. he fels it during rbital motion, during free fall or where his gravitational weight is balanced or neutralized by an opposing force. In orbital flight, the weight of the astronaut is just sufficient to provide the centripetal force to keep him in orbit leaving no net force to provide his weight. So he feels weightless. In free fall, the acceleration of the astronout and the capsule is the same namely 'g' and in this condition his weight will be zero. See the example of downward accelerating lift with accelerations equal to 'g'.

Example 6 : Calculate the value of g at a point close to the earth.

Solution : $g = g\frac{M}{r^2}$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$.

M = mass of earth = 6×10^{24} kg.

r = radius of earth = 6.4×10^{6} m.

Substituting

$$g = \frac{6.67 \times 10^{11} \times 6 \times 10^{24}}{(6.4 \times 10^{6})^{2}} = 9.77 \ \text{m/s}^2$$

1. PROJECTILES

When a particle is throw obliquely near the earth's surface, it moves along a curved path. Such a particle is called projectile motion. A projectile possesses two motions simultaneously :

(1) Horizontal motion with a constant velocity.

(2) Vertical motion downwards with a constant accelertion (due to gravity)

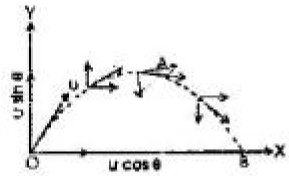


Fig. 3

The **Figure** shows a particle projected from the point O with an initial velocity u at an angle q with the horizontal. It goes throgh the highest point A and falls at B on the horizontal surface thrugh O. The point O is θ is called the angle of projection and the distance OB is the horizontal range. The total times takne by the particle in describing the path OAB is called the time of flight.

The time of flight is given by $T = \frac{2u \sin \theta}{g}$

The horizontal range = $\frac{u^2 \sin^2 \theta}{2g}$

Example 7 : A ball is thrwon from a field with a speed of 12 m/s at an angle of 45° with the horizontal. At what distance will it hit the field again ? Take $g = 10 \ m/s^2$.

Solution : Horizontal Range $= \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(12)^2 \times \sin (2 \times 45°)}{10}$$

$$= \frac{(12)^2 \times \sin 90°}{10}$$

$$= \frac{144}{10}(1) = 14.4 \ \text{m}$$

Therefore the ball hits the field at 14.4 m from the point of projection.

Example 8 : A projectile is fired horizontally from a certain height. Calculate the time taken by it to reach the ground assuming the height to be 1960 m.

Solution : The required time = the time taken by the projectile to reach teh ground, if, instead of firing, it was dropped to the ground from the same height.

Now, for free fall h $= \frac{1}{2} g t^2$ or

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20$$

2. ARTIFICIAL SATELLITES

(i) We have just learnt tw things about projectile motion. i. the motion is not vertical but curved; it. greater the horizontal velocity. greater the horizontal range (see Figure 4). But now, if the speed is sufficiently high, it may so happen that the projectile still continues to fall but never reaches the ground because the surface of the earth curves awya from it. Then we get the conditions for orbital motion of artificial satelites. This velocity depends on the height at which the satelite is released from the booster rocked and the value of 'g' 9acceleration due to gravity at the place). For a height of 160 km, this horizontal velocity is 28,800 km per hours or nearly 8 km/s. (See (ii) below)

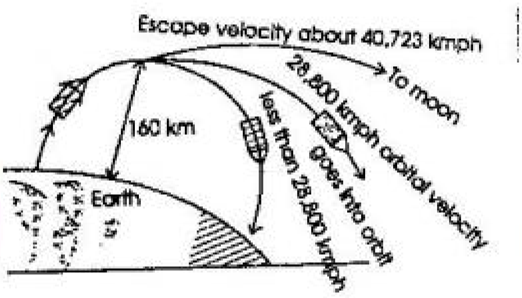


Fig. 4

Orbital and aescape velocity illustrated. At a height of 160 km if a spacecraft reaches a velocity of about 28,800 kmph, it will go into orbit. If the velocity is about 40,000 kmph, it will escepe from the earth overcoming gravity. If the velocity at that height is less than 28,800 kmph, it will not go into orbit, but return to earth. Since the moon weight much less, its gravity is less and therefore the orbital and escape velocities on it are also less for a probe returning from moon.

(ii) Orbital Velocity : The minimum velocity required to put an earth satellite into orbit for a particular height is known as orbital velocity. We have seen for a height of 160 km, it value is 28,800 kmph. As the height increased, the orbital velocity decreases. For a height of 3,84,400 km - the distance of the moon from teh earth - it is only 3629 kmph - the speed with which the moon revolves round the earth.

Thus these satelities - both articial and natural - are projectiles having the right speed to keep orbiting round the earth.

(iii) Excape Velocity : We have mentioned in the figure 28,800 km per hour as the orbital velocity at a height of 160 km. At this velocity the satelite neither goes up into space nor returns to earth but continues circling round the earth. When this velocity is less it returns to earth. When it is more and reaches 40,000 km per hour, it overcomes gravity and escapes into space. This velocity is known as escape velocity for that height. you will not that the escape velocity for a particular height is $\sqrt{2}$ times the orbital velocity.

Example 9 : The orbital velocity for a given height is 28,800 km per hour. How much is the escape velocity for the same height?

Solution : Escape velocity = $\sqrt{2}$ times orbital velocity

$= \sqrt{2} \times 28{,}800$ kmph

$= 40{,}729$ kmph

(iv) To find an expression for the orbital velocity : Let us say the orbital motion takes place at a height of 160 km which is negligible in relation to the radius R of the earth. When the satelite is in circular orbit there must be a force acting on it directed towards the centre of this circle which in this case is the centre of the earth. This force is called the centripetal force. Without this force the circular motion is impossible. The value of this force is equal to $\frac{mv^2}{r}$ where m is mass of the body orbiting, v is linear velocity i.e. orbital velocity and r, the radius of the orbit. Here, the radius is R. So, we have the centripetal force = $\frac{mv^2}{R}$. There is no string or cable pulling the body to the earth. But there is earth's pull on the body equal to mg stepping in to provide this force. Therefore, we have

$$\frac{mv^2}{R} = mg \text{ or } \frac{v^2}{R} = g \quad .....(1)$$

or $v^2 = RG$ or $v = \sqrt{Rg}$

Note : If the satelite's height h above the earth is not negligible compared to the radius of the earth, then for R we substitute (R + h)

GEOTROPISM

Plants are also afected by gravity. Roots are positively geotrophic (i.e.) they always grow downwards. And the shotts are positvely phototrophic and negatively geotrophic (i.e.) the shoots always grows upwards towards light. And this phenomenon is known as Geotropism. From this we understand that plants also recognize gravity and react to it by growing roots downwards and shoots upwards.

ASSIGNMENT 6

SECTION - A

Choose the correct answer :

1. The force of gravitation between two bodies of mass 1 kg each separated by a distance of 1 m in vacuum is
   (1) 6.67×10^{-9} N (2) 6.67×10^{-10} N (3) 6.67×10^{-11} N (4) 6.67×10^{-12} N

2. An artificial satelite revolves around the earth in a circular orbit with a speec v. If m is the mass of the satelite, its total energy is
   (1) $\frac{1}{2}mv^2$ (2) $-\frac{1}{2}mv^2$ (3) $-mv^2$ (4) $\left(\frac{3}{2}\right)mv^2$

3. The unit of position vector of centre of mass is
   (1) metre (2) kg (3) kg-m^2 (4) kg-m

4. A system consists of three particles, each of mass m and located at (1, 1), (2, 2) and (3, 3). The co-ordinates of the centre of mass are
   (1) (1, 1) (2) (2, 2) (3) (3, 3) (4) (6, 6)

5. Which of the following remains constant for a projectile fired from the earth?
   (1) Momentum (2) kinetic energy
   (3) Vertical component of velocity (4) Horizontal component of velocity

6. The force of gravitation between two bodies does not depend on :
   (1) their separation (2) the product of their masses
   (3) the gravitational constant. (4) the sum of their masses

7. The escape velocity from the surface of the earth is
   (1) 1.13 km/s (2) 11.3 km/s (3) 113 km/s (4) 1130 km/s

8. When an object is thrown up, the force of gravity
   (1) is opposite to the direction of motion
   (2) is in the same direction as the direction of motion

(3) becomes zero at the highest point

(4) increases as it rises up

9. What is the weight of a body at a distance 2r from the centre of the earth if the gravitational. P, E of the body at a distance r from teh centre of the earth is u?

(1) u/2r (2) u/3r (3) u/4r (4) ur

10. Two bodies A and B are attracted towards each other due to gravitation. Given that A is much heavier than B, which of the following correctly describes the relative motion of the centre of mass of the bodies?

(1) It moves towards A.

(2) It moves towards B.

(3) It moves perpendicular to teh line joining the particle.

(4) It remains at rest w.r.f. A as well as B.

SECTION -B (NUMERICAL PROBLEMS)

11. A stone is dropped from a cliff. Determine its speed after it has fallen 100 m.

12. Calculate the value of acceleration due to gravity on the surface of the moon. Mass of the moon = 7.4×10^{22} kg, radius of the moon = 1740 km G = 6.7×10^{-11} Nm^2/kg^2)

13. Find the gravitational force between two persons of mass 60 kg each standing one metre apart.

14. To estimate the height of a bridge over a river, a stone is dropped freely in the river from the bridge the stone takes 2 seconds to touch the water surface in the river. Calculate the height of the bridge from the water level (9.8 m/s^2)

15. Determine the gravitational acceleration of a spaceship which is at a distance equal to two earth's radii from the centre of the earth.

16. Find the distance of a point from the earth's centre where the resultant gravitational field due to earth and the moon is zero. The mass of the earth is 6×10^{24} kg and that of the moon is 7.4×10^{22} kg. The distance between the earth and the moon is 4×10^5 km.

17. If a person goes (i) to a height equal to radius of each from its surface (ii) to a depth equal to radius of earth below its surface, what would be his weight in the two cases relative to that on earth?

7. WORK, ENERGY AND POWER

Introduction

In this chapter you will learn about the concepts of work, energy and power, Of these, energy is the most important concept as it represents a fundamental entity common to all forms of matter. A dancing, running man is said to be more energetic compared to a sleeping snoring man. In physics, a moving particle is said to have more energy than an identical particle at rest. Closely associated with energy is the concept of work. This term is used to described the expenditure of one's stored up energy.

WORK

In science the term work has a definite meaning whichdiffers from its everyday one. For instance a man holding a heavy weight on his head is doing no work. Thus mental or physicla work or exertion in which the working bodydoes not move are excluded fromthe domain of work. Another point about the meaning of work is the direction of the motion of body with respect to that of the force.

"Work is said to be done if a force acting upon a body is able to move it through a certain distance along its line of action."

The amount of work done depends on

(i) the mognitude of the force.

(ii) distance throgh which the body moves in the direction of the force.

$W = |\overline{F}||\overline{S}|$

If $\overline{F}$ and $\overline{S}$ are not in the same direction and let the angle between them be θ

$W = |\overline{F}||\overline{S}|\cos\theta$

If $\overline{F}$ and $\overline{S}$ are at right angles $\theta = 90°$ $\cos\theta = 0$ and hence work done is zero.

If $\overline{F}$ and $\overline{S}$ are in the same direction, $\theta = 0$

$W = |\overline{F}||\overline{S}|$ since $\cos\theta = 1$

The work done on a body can be negative if 0 lies between 90° and 270°.

Positive work performed on an object increases the objects energy, while negative work on an object reduces its energy. Work is a scalar quantity. S.I. unit of work is joule.

Example 1 : A force of 10 newtons is applied to a body whch, as a result, moves through a linear distance of 500 cm in the direction of the force. Calculate the work done.

Solution : Distance = 500 cm = 5 m;

Force = 10 N

Work done = Force x distance

= 10 x 5 = 50 J

Example 2 : To a bodyon a smooth surface a steady force is applied making on angle of 60° with the surface. The body moves through a disance of 5.6m, If the work done is 14 J, calculate the magnitude of the force.

Solution :

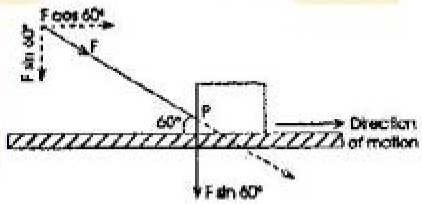


Fig.

The applied force F acts in the directino FP. Its horizontal component is F cos 60° and verticla component is F sin 60°. Only the horizontal component is effective in producing the displacement

Work done = Horizontal force x Distance

Substituting.

Data given : Force applied = F

Horizontal component of force, F = F cos 60°

4 = F cos 60° x 5.6

$= F \times \frac{1}{2} \times 5.6 = 2.8\ F$

Distance moved = 5.6 m $\therefore F = \frac{14}{2.8}$ or 5N

The force applied = 5N

WORK AGAINST GRAVITY

When you lift a body vertically upwards through a certain height, you are doing work agianst gravity. If the mass of the body is m and the height through which it is lifted is h, then we have :

The weight of the body = m x g, where g is acceleration due to gravity

$\therefore$ work done = Weight x Distance

= m x g x h = mgh

If m is in kg g in m/s^2 and h in metre, then work is in joule.

Example 3 : A bucket with water weighting 20 kg is drawn from a well 10 metres deep. Calculate the work done.

Solution : W = mgh

= 20 x 9.8 x 10 = 1960 J

ENERGY

Energy is the capacity for doing work. A body that has energy can do work by using that energy. The energy is utilized in doing that work. Both energy and work are measured in the same unit - joule- in the S.I. system and in ergs in teh C.G.S. System.

Energy is of two kinds viz. kinetic energy and potential energy.

Kinetic Energy

This is the energy possessed by a body by virtue of its motion. Any moving body possesses kinetic energy equal to $\frac{1}{2}$ mv^2 where m is the mass and v, the velocity.

Thus K.E. = $\frac{1}{2}mv^2$

We can arrive at the above formula as below :-

You have seen that energy (here kinetic energy) is produced by doing work. The work is converted into the energy.

Now in making the body move from rest to acquire a velocity v, work has to be done whchis equal to F x S where F is the force and S, the displacement. But F = ma where m is mass of body and a, the acceleratoion. So we have

Workdone W = F x S = maS .....(1)

This work is done on the body whichis turned into KE.

$v^2 = u^2 + 2aS$. Equation of motion

$= 0 + 2aS$

$\therefore aS = \frac{V^2}{2}$

From (1) Work done = maS

(which is equal to the KE) = $m\left(\frac{v^2}{2}\right)$

i.e. K.E. = $\frac{1}{2}mv^2$

Thus K.E. is produced by a force acting on a body and displacing it. Work done by the force appears as K.E. i.e. energy of motion.

Example 4 : A shell of mass 0.5 kg has a velocity of 100 m/s. Calculate its K.E.

Solution : m = 0.5 kg

v = 100 m/s

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 100^2 \text{ J}$$

$$= 0.25 \times 10^4 \text{ J} = \mathbf{2.5 \times 10^3 \text{ J}}$$

POTENTIAL ENERGY

Potential energy is the energy possessed by a body by virtue of its position or state or condition.

A body at an elevated position, a strung bow, a wound spring - all these possess potential energy. We call it potential energy in the sense, energy is stored in the body which it can release in the form of work given the chance. Thus a falling coconut from the tree on falling can break the skull of a perso, a bent bow when released can send on arrow to long distance and a wound spring, given the chance to unwind itself, can turn the hands of a watch. Air in a compessed state has potential energy for when releaed, it can do work say for driling purposes.

We have seen that to lift a body of mass m to a height h, mgh units of work has to be done. This energy is tored in it as it were at that height. hence its potential energy = mgh. It has got gravitational potential energy in this case.

When a coconut is on the tree, all its energy is potential = mgh. As it starts falling, its starts losing this energy because the height becomes less and less but gains and equivalent amount of kinetic energy as it gains velocity. Just before reaching the ground, all is potential energy is lost which appears as kinetic energy = = $\frac{1}{2}mv^2$. We can easily prove that these two kinds of energy are equal i.e. $\frac{1}{2}mv^2 = mgh$.

For $v^2 = u^2 + 2gh = 0 + 2gh = 0 + 2 \times 9.8 \times 2 = 39.2$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 39.2 \text{ or } \mathbf{9.8 \text{ J}}$$

2nd Method : On reaching the ground all the potential energy mgh is converted into kinetic energy.

Potential energy = mgh = 0.5 x 9.8 x 2 or 9.8 J

So, K.E. gained = the P.E. last = 9.8 J

POWER

Power is the rate of doing work i.e. how much work is done every second. In the S.I. system, it is expressed in watts.

Watt : When work is done at the rate of one joule per second, the power developed is one watt. Thus, if the power of an engine is 100 watts, it can do work at the rate of 100 joules per second.

$P = \frac{W}{t}$, where P is the power, W, the work done and t, the time in which it is done. The most practical unit of power is, kilowatt (kW) 1 kW = 1000 watt

Horse Power (H.P.) : In FPS system poer is indicated in tems of horse power. One HP = 746 watts.

Kilowatt - hour : Unit kilowatt - hour means one kilowatt of power supplied for 1 hour, 1 kWh = 1000 x 60 x 60 = 3.6×10^6 J

Example 6 : A force of 2N moves a body to a distance of 500 cm in .5 seconds. What is the power developed ?

Data given : F = 2N, distance = 5000 cm = 5 m, t = 2.5 s

Solution : Work done by the force = F x Distance

= 2 x 5 or 10 J

Time taken to do the work = .5 s

Power developed $= \frac{\text{Work}}{\text{Time}}$

$= \frac{10\text{J}}{2.5\text{s}} = 4\text{W}$

Example 7 : A person weighing 60 kg climbs a staircase with 20 steps, each step being of height 25 cm in just 2 seconds. What is the power developed by him?

Data given : m = 60 kg.

No of steps = 20

Height of each step = 25 cm

Total height (h) = 20 x 25 cm = 5 m

Solution : Height of staircase = 20 x 25 cm = 5 cm

Height climbed by the person = 5 cm

Work done by him against gravity

= mgh

= 60 x 9.8 x 5 J

= 2940 J

$\therefore$ Power $= \frac{2940}{2} = 1470$ W = 1.47 kW

ASSIGNMENT 7

SECTION - A

Choose the correct answer :

1. A body at rest may have :
(1) energy (2) momentum (3) speed (4) velocity

2. Given that $\overline{F}$ is the force and $\overline{S}$ is the displacement. Angle between $\overline{F}$ and $\overline{S}$ is θ. Which of the following is the correct expression for work 'W'?
(1) ($\overline{F}$ cos θ) (2) $\overline{F}$ x $\overline{S}$ cos θ (3) $\overline{F} \cdot \overline{S}$ (4) $\overline{F}$ x $\overline{S}$ cos θ

3. A body pushes a toy box 2.0 m along the floor by means of a force of 10 N direced downward at an angle of 60° to the horizontal. The work done by the boy is
(1) 6 J (2) 8 J (3) 10 J (4) 12 J

4. The work done against gravity in lifting a 10 kg object through a distance of 70 cm is :
(1) 65 J (2) 68.6 J (3) 78.5 J (4) 80 J

5. A moving body may not have
(1) potential energy (2) kinetic energy (3) momentum (4) velocity

6. There will be an increase in potential energy of the system, if the work is done upon the system by:
(1) any conservative or non-conservative force (2) A non-conservative force
(3) A conservative force (4) none of the above

7. A body is falling with velocity 1 m/s at a height 2 m from the ground. The speed at height 2m from the ground will be :
(1) 4.54 m/s (2) 1 m/s (3) 6 m/s (4) 5.32 m/s

8. An engine develops 10 kW of power. How much time will it take to lift a mass of 200 kg to a height of 40 m (g = 10 m/s^2)
(1) 4s (2) 5s (3) 8s (4) 10s

9. A body of mass m accelerates uniformily from rest to a speed v in 1 second. The average power delivered is :

(1) $\frac{mv^2}{t}$ (2) $\frac{mv^2}{2t}$ (3) $\frac{1}{mv^2}$ (4) $\frac{mv^2}{4t}$

10. Radha weighting 500 N climbs the staris from the baseement to her study room 15 m above in 20s, what ower does she develop?

(1) 6.7w (2) 14W (3) 3.8×10^2W. (4) 1.5×10^5W

SECTION - B (NUMERICAL PROBLEMS)

11. 1 g of coal an complete combustion liberates 350 kJ energy. Calculate teh power of a hearth in kilowatt units in which 100 g coal burns in one second?

12. In a house an electric bulb of 60 W is used for 15 hours and an electric heater of 750 W is used for 10 hours everyda. Calculate the cost of using the bulb and heater for 30 days if the cost of 1 unit of electrical energy is Rs. 1.

13. Two bodies of equal masses move with uniform velocities v and 3v respectively. Find the ratio of their kinetic energies.

14. An object of mass 5 kg falls from rest through a vertical distance of 20 m and reaches a velocity of 10 m/s. How much work is done by the push of the air on the bject? ($g = 10$ m/s^2)

15. A metal plate of mass 200 g is balanced in mid air by throwing 40 balls / second each of mass 2 g vertically upwards from table. The balls get rebounded with same speed with which they strike the plate.Calculate the speed of the ball.

SOLUTION TO ASSIGNMENT 6
SECTION - A

1. **(3)** $F = \frac{Gm_1m_2}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1 \times 1}{1^2}$

6.67 x 10^{-11} N

2. **(2)** Kenetic energy of satelliten

$E_k = \frac{1}{2}mv^2$ $\left[\text{where } v = \sqrt{\frac{GM}{r}}\right]$

Potential energy of satellite.

$E_p = -\frac{GMm}{r} = -mv^2$

$\therefore$ total energy = $E_K + E_P$

$= \frac{1}{2}mv^2 - mv^2$

$= -\frac{1}{2}mv^2$

3. **(1)** Position vector of centre of mass is measured in metre.

4. (2) $x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$= \frac{m\,(1+2+3)}{3m} = 2$

Similarly, $y = = \frac{m\,(1+2+3)}{3m} = 2$

5. **(4)**

6. **(4)** The force of gravitation between two bodies does not depend on the sum of their masses.

7. **(2)**

8. **(1)** When an object thrown up, the force of gravity is opposite to the direction of motion.

9. **(3)** $W = mg = m\left(\frac{GM}{R^2}\right)$

$u = \frac{GMm}{r}$ (or) $GM = \frac{ur}{m}$

$W = m\left(\frac{ur}{m}\right) \times \frac{1}{R^2} = m\left(\frac{ur}{m}\right) \times \frac{1}{(2r)^2}$

$= \frac{u}{4r}$

10. **(4)** The two bodies will move towards the common centre of mass. But the location of the centre of mass will remain unchanged.

SECTION - B

11. Given : initial speed, u = 0
Final speed = ?
$g = 9.8\ m/s^2$
h = 100 m.
We known that for a freely falling body.
$V^2 = u^2 + gh$
$v2 = 0 + 2 \times 9.8 \times 100$
= 1960
$v = \sqrt{1960}$ = **44.3 m/s**

12. Acceleration due to gravity, $g = \frac{GM}{R^2}$ .....(1)

Given : $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$
$M = 7.4 \times 10^{22}$ kg
$R = 1740\ km = 1.74 \times 10^6$ m
By substituting the above value in (1)

$$g = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(1.74 \times 10^6)^2}$$

= 1.63 m/s2
Thus the acceleration due to gravity 'g' on the moon is aobut (1/6)th of the value of g on earth (the value of g on earth = $9.8\ m/s^2$)

13. $m_1 = 60$ kg
$m_2 = 60$ kg
d = 1m
$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$$F = G\frac{m_1 m_2}{d^2} = \frac{6.67 \times 10^{-11} \times 60 \times 60}{1^2}$$

$F = 2.40 \times 10^{-7}$ N
= 0.000000240 N

14. Initial velocity of stone, u = 0
Time taken, t = 2 second
g = 9.8 m/s2

$$h = ut + \frac{1}{2}gt^2$$

$$h = (0)\,(2) + \frac{1}{2}(9.8)\,(2)^2$$

= 19.6 m
$\therefore$ The height of bridge above the water level is **19.6 m**.

15. The acceleration due to gravity on the surface of earth.

$$g_e = \frac{GM}{R^2} \qquad ....(1)$$

where R = Radius of the earth.

For the spaceship, the distance is equal to two earths radii (2R)

$$\therefore\ g_s = \frac{GM}{(2R)^2} \qquad ....(2)$$

Divide equation (2) by (1)

$$\frac{g_s}{g_e} = \frac{GMR^2}{GM4R^2}$$

$$\frac{g_s}{g_e} = \frac{1}{4}$$

$$g_s = \frac{1}{4} g_e$$

$g_e = 9.8\ m/s^2$

$$g_s = \frac{1}{4} \times 9.8\ m/s^2 = 2.45\ m/s^2$$

16. The point must be on the line joining the centres of the earth and the moon and in between them. If the distance of the point from the earth is x, the distance from the moon is (4×10^5 km-x)

The magnitude of the gravitational field due to the earth is

$$E_1 = \frac{GM_e}{x^2} = \frac{G \times 6 \times 10^{24}\ kg}{x^2}$$

and magnitude of the gravitational field due to the moon is

$$E_2 = \frac{GM_m}{(4 \times 10^5 - x)^2} = \frac{G\,(7.4) \times 10^{22}}{(4 \times 10^5 - x)^2}$$

These fields are in opposite directions. For the resultant field to be zero. $E_1 = E_2$

$$6 \times \frac{10^{24}}{x^2} = \frac{7.4 \times 10^{22}}{(4 \times 10^5 - x)^2}$$

$$\frac{x}{4 \times 10^5 - x} = \sqrt{\frac{6 \times 10^{24}}{7.4 \times 10^{22}}}$$

$x = 3.6 \times 10^5$ km

17. (i) At earth surface, weight.

$$W = mg = m\frac{GM}{R^2}$$

At height h, weight,

$$W' = mg' = \frac{mGM}{(R+h)^2}$$

$$\therefore \frac{W'}{W} = \frac{R^2}{(R+h)^2}$$

When h = R

$$\frac{W'}{W} = \frac{R^2}{(2R)^2} = \frac{1}{4}$$

So, at height h = R, the weight would be reduced to one-fourth of the weight on the earth's surface.

(ii) At depth h,

$$g'' = \left[1 - \frac{h}{R}\right]g$$

$\therefore$ weight W'' = mg'' = $m'\left[1 - \frac{h}{R}\right]g$

At depth h = R, W'' = 0

So, at depth h = R, from earth's surface, the body has no weight.

SOLUTIONS TO ASSIGNMENT 7
SECTION - A

1. **(1)** A body at rest cannot have speed, velocity and momentum, but it can have energy, (except kinetic energy only).

2. **(3)** W = FS Cosθ = $\overline{F} . \overline{S}$

3. (3) W = FS Cos q
 = 10 x 2 cos 60°
 $= 10 \times 2 \times \frac{1}{2}$
 W = 10 J

4. (2) W = mgh
 $= 10 \times 9.8 \times \frac{70}{100}$
 = 68.6 J

5. (1) A moving body may not have potential energy.

6. (3) Potential energy increases when work is done upon the system by a conservative force.

7. (1) From $v^2 - u^2 = 2as$
 $v^2 - 1 = 2 \times 9.8 (3 - 2)$
 $v^2 - 1 = 19.6$
 $v^2 = 19.6 + 1$
 $v^2 = 20.6$
 $v = \sqrt{20.6} = 4.4$ m/s

8. (3) As, power $= \frac{\text{Work done}}{\text{Time}}$

$$P = \frac{mgh}{t}$$

$$\therefore t = \frac{mgh}{P} = \frac{200 \times 10 \times 40}{10 \times 10^3}$$

 = 8S

9. (2) Power = $\dfrac{\text{Work done}}{\text{Time}} = \dfrac{K.E}{t}$

$= \dfrac{1}{2}\dfrac{mv^2}{t}$

$= \dfrac{mv^2}{2t}$

10 (3) Power = $\dfrac{\text{Work done}}{\text{Time}} = \dfrac{F \times S}{t}$

$= \dfrac{500 \times 15}{20}$

$= 3.8 \times 10^2$ W

SECTION - B

11. Given :

1 g coal produces energy = 350 kilo joules

= 35×10^3 joules

∴ 100g coal produce energy

= $35 \times 10^3 \times 100$ J

= 35×10^5 joules

Time taken = 1 second

Power = $\dfrac{\text{Energy supplied}}{\text{Time taken}}$

$= \dfrac{350 \times 10^5}{1}$ watts

= 350 x 105 watts

= 35000 kilowatts

= 3.5×10^4 kW.

12. Given :

60 watt bulb is used for 15 hours everyday so, electrical energy consumed by it = 60 x 15 = 900 watt-hours

750 watt electric heater is used for 10 hours every day

= 750 x 10 = 7500 watt-hour

Total electrical energy consumed in 1 day = 8400 watt-hour

Electricity consumed in 1 day

$= \dfrac{8400}{1000} = 8.4$ kWh (8.4 units)

Electricity consumed in 30 day

= 8.4 x 30 = 252 units

cost of 1 unit of electrical energy = Re 1

= 252 x 1 = Rs. 252.

13. The masses of the two bodies are equal, so, let the mass of each body be m. The kinetic energies of both the bodies separately are

(i) Mass of the body = m

velocity of first body = v

K.E of first body = $\dfrac{1}{2}mv^2$ ....(1)

(ii) Mass of secodn body = m
velocity of second body = 3v

$$\text{K.E.} = \frac{1}{2} m\,(3v)^2 = 9/2\ mv^2 \qquad ....(2)$$

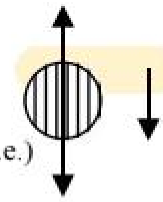
The ratio of kinetic energies of the 2 bodies

$$\frac{\text{Kinetic energy of first body}}{\text{Kinetic energy of second body}} = \frac{\frac{1}{2}mv^2}{\frac{9}{2}mv^2}$$

$$\frac{\text{K.E. of first body}}{\text{K.E. of second body}} = \frac{1}{9} \qquad ....(3)$$

K.E. of second body = 9 x K.E. of first body.
∴ the K.E. of second body is 9 times the K.E. of first body.

14. The motion of the body is shown the following two forces are acting on the body.
(i) Weight mg is acting vertically downward.
(ii) The push of the air is acting downward.
As the body is accelerating downward, the resultant force is (mg - F)
Workdone by the resultant force to fall through a vertical
distance of 20m = (mg - F) x 20 joule



Gain in the kinetic energy = $\frac{1}{2}mv^2$

Now the workdone by the resultant force is equal to the change in K.E. (i.e.)

$$(mg - F)20 = \frac{1}{2}mv^2$$

$$(50 - F)20 = \frac{1}{2}(5)\,(10)^2$$

$$50 - F = \frac{5 \times 100}{2 \times 20}$$

$50 - F = 12.5$
$F = 50 - 12.5$
$\therefore\ F = 37.5$ N
Work done by the force = − 37.5 x 20 = − 750 joule
The negative sign is used because the push at the air is upwards while the displacement is downwards.

15. Let M and m be teh masses of plate and ball respectively. If v be the speed of ball, then change in momentum of ball
= mv − (− mv) = 2 mv
Time of impact = (1/40) second
∴ rote of change of momentum

$$= \frac{2mv}{1/40} = 80mv$$

The rate of change of momentum balances the weight. Hence
80 mv = 200 x 9.8
80 x 2 x v = 200 x 9.8

$$v = \frac{200 \text{ x } 9.8}{80 \text{ x } 2} \qquad = 12.25 \text{ m/s}$$