Set No-1

Series DA2AB/2

Q.P.Code **30/2/1**

Roll No. | | | | | | | | |

Candidates must write the Q.P. Code on the title page of the answer-book.

(i) Please check that this question paper contains 26 printed pages.
(ii) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(iii) Please check that this question paper contains 38 questions.
(iv) Please write down the serial number of the question in the answer book before attempting it.
(v) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS (STANDARD) THEORY

HINTS & SOLUTIONS

Time allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This Question Paper is divided into FIVE Sections - Section A, B, C, D and E.
(iii) In Section-A question number 1 to 18 are Multiple Choice Questions (MCQs) and question number 19 & 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section-B question number 21 to 25 are Very Short-Answer-I (SA-I) type questions of 2 marks each.
(v) In Section-C question number 26 to 31 are Short Answer-II (SA-II) type questions carrying 3 marks each.
(vi) In Section-D question number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section-E question number 36 to 38 are Case Study / Passage based integrated units of assessment questions carrying 4 marks each. Internal choice is provided in 2 marks question in each case-study.
(viii) There is no overall choice. However: an internal choice has been provided in 2 questions in Section-B, 2 questions in Section-C, 2 questions in Section-D and 3 question in Section-E.
(ix) Draw neat figures wherever required. Take π =22/7 wherever required if not stated.
(x) Use of calculator is NOT allowed

SECTION-A **20 × 1 = 20**

(This section consists of 20 questions of 1 mark each.)

1. The value of k for which the system of equations 3x – y + 8 = 0 and 6x – ky + 16 = 0 has infinitely many solutions, is **1**

(A) – 2 (B) 2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Ans. (B) 2

Sol. $\frac{3}{6} = \frac{-1}{-K} = \frac{8}{16}$

$\frac{1}{K} = \frac{1}{2}$

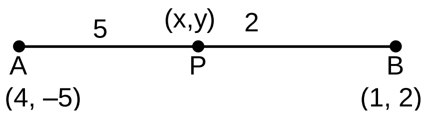
K = 2

2. Point P divides the line segment joining the points A(4,–5) ad B (1,2) in the ratio 5 : 2. Co-ordinates of point P are **1**

(A) $\left(\frac{5}{2}, \frac{-3}{2}\right)$ (B) $\left(\frac{11}{7}, 0\right)$ (C) $\left(\frac{13}{7}, 0\right)$ (D) $\left(0, \frac{13}{7}\right)$

Ans. (C) $\left(\frac{13}{7}, 0\right)$

Sol.



$x = \frac{8+5}{7} = \frac{13}{7}$

$y = \frac{-10+10}{7} = \frac{0}{7} = 0$

$P \equiv \left(\frac{13}{7}, 0\right)$

3. The common difference of an A.P. in which $a_{15} - a_{11} = 48$, is **1**

(A) 12 (B) 16 (C) –12 (D) –16

Ans. (A) 12

Sol. $\because a_{15} - a_{11} = 48$

Let 'a' is first term

'd' is common difference

$\therefore a_{15} = a + 14d$

$a_{11} = a + 10d$

$a_{15} - a_{11} = a + 14d - a - 10d = 48$

4d = 48

d = 12

4. The quadratic equation $x^2 + x + 1 = 0$ has _____ roots. **1**

(A) real and equal (B) irrational (C) real and distinct (D) not-real

Ans. (D) Not-real

Sol. $x^2 + x + 1 = 0$

$D = b^2 - 4ac$

$D = (1)^2 - 4(1)(1)$

$D = 1 - 4 = -3$

D is negative

so root are not-real

5. If the HCF (2520, 6600) = 40 and LCM (2520, 6600) = 252 × k, then the value of k is **1**

(A) 1650 (B) 1600 (C) 165 (D) 1625

Ans. (A) 1650

Sol. We know that for two numbers (a, b)

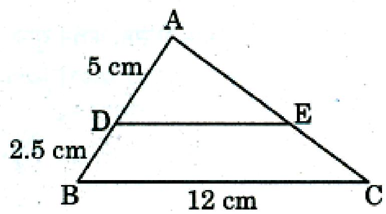
a × b = H.C.F (a, b) × L.C.M (a, b)

2520 × 6600 = 40 × 252 × K

$K = \frac{2520 \times 6600}{40 \times 252}$

K = 1650

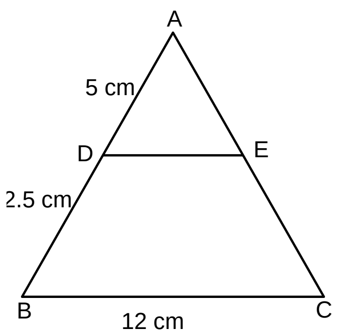
6. In the given figure ΔABC is shown. DE is parallel to BC. If AD = 5 cm, DB = 2.5 cm and BC = 12 cm, then DE is equal to **1**



(A) 10cm (B) 6 cm (C) 8 cm (D) 7.5 cm

Ans. (C) 8 cm

Sol.



$\because$ DE || BC

$\therefore \Delta ABC \sim \Delta ADE$ (by AA similarity)

$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

$\frac{7.5}{5} = \frac{12}{DE}$

$DE = \frac{12 \times 5}{7.5} = 8$ cm

7. If $\sin\theta = \cos\theta$, $(0^o < \theta < 90^o)$, then value of $(\sec\theta . \sin\theta)$ is : **1**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 1 (D) 0

Ans. (C) 1

Sol. If $\sin\theta = \cos\theta$ $(0° < \theta < 90°)$

$\therefore \theta = 45°$

$\Rightarrow \sec\theta \cdot \sin\theta = \frac{1}{\cos\theta} \times \sin\theta$

$= \tan\theta$

now $\tan 45° = 1$

8. Two dice are rolled together. The probability of getting the sum of the two numbers to be more than 10, is **1**

(A) $\frac{1}{9}$ (B) $\frac{1}{6}$ (C) $\frac{7}{12}$ (D) $\frac{1}{12}$

Ans. (D) $\frac{1}{12}$

Sol. Total outcomes = 36

Favourable outcomes $\Rightarrow$ sum will be 11 or 12

$11 \rightarrow (5, 6), (6, 5)$

$12 \rightarrow (6, 6)$

Favourable outcomes = 3

Probability (getting sum more than 10) = $\frac{3}{36} = \frac{1}{12}$

9. If α and β are zeroes of the polynomial $5x^2 + 3x - 7$, the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is **1**

(A) $-\frac{3}{7}$ (B) $\frac{3}{5}$ (C) $\frac{3}{7}$ (D) $-\frac{5}{7}$

Ans. (C) $\frac{3}{7}$

Sol. $P(x) = 5x^2 + 3x - 7$

α, β are zeroes

$\alpha + \beta = \frac{-3}{5}$, $\alpha\beta = \frac{-7}{5}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{3}{5}}{-\frac{7}{5}} = \frac{3}{7}$$

10. The perimeters of two similar triangles ABC and PQR are 56 cm and 48 cm respectively. PQ/AB is equal to **1**

(A) $\frac{7}{8}$ (B) $\frac{6}{7}$ (C) $\frac{7}{6}$ (D) $\frac{8}{7}$

Ans. (B) $\frac{6}{7}$

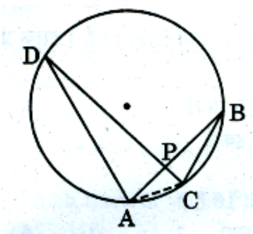
Sol. $\because \Delta ABC \sim \Delta PQR$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$

$\frac{AB}{PQ} = \frac{56}{48} = \frac{7}{6}$

So, $\frac{PQ}{AB} = \frac{6}{7}$

11. AB and CD are two chords of a circle intersecting at P. choose the correct statement from the following : **1**



(A) $\Delta ADP \sim \Delta CBA$ (B) $\Delta ADP \sim \Delta BPC$ (C) $\Delta ADP \sim \Delta BCP$ (D) $\Delta ADP \sim \Delta CBP$

Ans. (D) $\Delta ADP \sim \Delta CBP$

Sol. AB and CD are chords which intersect at P.

In ΔADP and ΔCBP

$\angle ADP = \angle CBP$ [Angle in same segment]

$\angle APD = \angle CPB$ [Vertically opposite angle]

By angle sum property in triangle. We can say that

$\angle DAP = \angle BCP$

$\Delta ADP \sim \Delta CBP$ By (AAA) similar

12. If value of each observation in a data is increased by 2, then median of the new data **1**

(A) increases by 2 (B) increases by 2n (C) remains same (D) decreases by 2

Ans. (A) increases by 2

Sol. We know that if value of each observation in data is increased by 2, then our median of data is also increased by 2.

13. A box contains cards numbered 6 to 55. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square, is **1**

(A) $\frac{7}{50}$ (B) $\frac{7}{55}$ (C) $\frac{1}{10}$ (D) $\frac{5}{49}$

Ans. (C) $\frac{1}{10}$

Sol. Total number of cards in box

= [55 – 6] + 1

= 49 – 1 = 50 cards

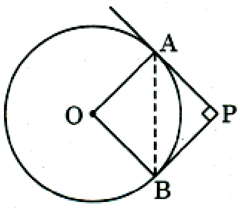
Number of cards which has perfect square number = 9, 16, 25, 36, 49

Total cards = 7 cards

Probability = $\frac{\text{Total number of perfect square cards}}{\text{Total cards in box}}$

$= \frac{5}{50} = \frac{1}{10}$

14. In the given figure, tangents PA and PB to the circle centred at O, from point P are perpendicular to each other. If PA = 5 cm, then length of AB is equal to **1**



(A) 5 cm (B) $5\sqrt{2}$ cm (C) $2\sqrt{2}$ cm (D) 10 cm

Ans. (B) $5\sqrt{2}$ cm

Sol. We know length of tangent which are drawn from external point are equal

PA = PB = 5cm

PA $\perp$ PB, ΔAPB are right angle triangle

$AB^2 = AP^2 + PB^2$

$AB = (5)^2 + (5)^2$

AB = 25 + 25

AB = $\sqrt{50}$

AB = $5\sqrt{2}$ cm

15. XOYZ is a rectangle with vertices X(–3, 0), O(0, 0), Y(O,4) and Z(x, y). The length of its each diagonal is **1**

(A) 5 units (B) $\sqrt{5}$ units (C) $x^2 + y^2$ units (D) 4 units

Ans. (A) 5 units

Sol. We know that XOYZ is a rectangle

We know XY = OZ

XY is distance between two points which is

XY = $\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$

XY = $\sqrt{(0+3)^2 + (4-0)^2}$

XY = $\sqrt{9+16}$

XY = $\sqrt{25}$

XY = 5 Units

16. Which term of the A.P. –29,–26, –23, ..., 61 is 16 ? 1

(A) 11^{th} (B) 16^{th} (C) 10^{th} (D) 31^{St}

Ans. (B) 16^{th}

Sol. AP $\rightarrow$ –29, –26, –23, ...61

Let the n^{th} term is 16, $a_n = 16$

We know $a_n = a + (n–1)d$ $\quad$ d = –26+29

d = 3

16 = –29 + (n–1)3

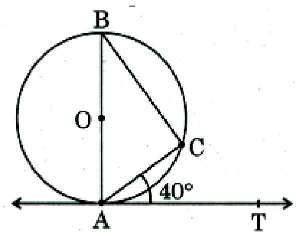
16 + 29 = (n–1)3

45 = (n–1)3

15 = n–1

[n = 16^{th} term]

17. In the given figure, AT is tangent to a circle centred at O. If $\angle$CAT = 40°, then $\angle$CBA is equal to **1**



(A) 70º (B) 50º (C) 65° (D) 40º

Ans. (D) 40º

Sol. $\angle$CAT = 40° (given)

$\angle$OAT = 90° (we know OA $\perp$ AT)

then $\angle$BAC = 90°–40°

$\angle$BAC = 50°

We know $\angle$BCA = 90° (Angle in semi circle)

Applying angle sum property in ΔBCA

$\angle$CBA + $\angle$BCA + $\angle$BAC = 180°

$\angle$CBA + 90° + 50° = 180°

$\angle$CBA = 180°–140°

$\angle$CBA = 40°

18. After an examination, a teacher wants to know the marks obtained by maximum number of the students in her class. She requires to calculate __________ of marks. **1**

(A) median (B) mode (C) mean (D) range

Ans. (B) mode

Sol. If she want to know marks obtained by maximum number of the students.

She have to calculate mode of marks.

Directions : In Question **19** and **20**, **Assertion (A)** and **Reason (R)** are given. Select the correct option from the following:

(A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A)

(B) Both Assertion (A) and Reason (R) are true. Reason (R) does not give correct explanation of (A).

(C) Assertion (A) is true but Reason (R) is not true.

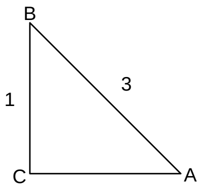
(D) Assertion (A) is not true but Reason (R) is true.

19. **Assertion (A) :** If sin A = $\frac{1}{3}$ ($0^\circ < A < 90^\circ$), then the value of cos A is $\frac{2\sqrt{2}}{3}$

Reason (R) : For every angle θ, $\sin^2\theta + \cos^2\theta = 1$ **1**

Ans. (A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A)

Sol. $\text{SinA} = \frac{1}{3}$



We know, $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

In our triangle ABC, Base AC

$AC = \sqrt{(AB)^2 - (BC)^2}$

$= \sqrt{(3)^2 - (1)^2}$

$= \sqrt{9-1}$

$= \sqrt{8}$

$AC = 2\sqrt{2}$

$\cos A = \frac{2\sqrt{2}}{3}$

Assertion is true

Reason is also true $\sin^2\theta + \cos^2\theta = 1$

Both Assertion and Reason are true. Reason is the correct explanation of Assertion.

20. **Assertion (A) :** Two cubes each of edge length 10 cm are joined together. The total surface area of newly formed cuboid is 1200 cm^2.

Reason (R) : Area of each surface of a cube of side 10 cm is 100 cm^2. **1**

Ans. (D) Assertion (A) is not true but Reason (R) is true.

Sol. We two cubes are joined then length of cuboid is 20 cm.

L = 20 cm

b = 10 cm

H = 10 cm

TSA of cuboid = 2 [L×b+b×H+H×L]

= 2[20×10+10×10+10×20]

= 2[200+100+200]

= 2[500]

= 1000 cm^3

Assertion is wrong

Reason

Each surface of cube is square then area is side × side

= 10cm × 10cm

= 100 cm^2

Reason is true

SECTION-B

(In this section, there are 5. questions of 2 marks each.)

21. Can the number $(15)^n$, n being a natural number, end with the digit 0 ? Give reasons. **2**

Sol. Any positive integer ending with 0 is divided by both 2 and 5. So prime factorisation of given number must contain 2 as well as 5 in its prime factorisation

So $15^n = (3\times5)^n = 3^n \times 5^n$

Here 2 does not accure in prime factorisation of number.

So 15^n does not end with 0.

22. Find the type of triangle ABC formed whose vertices are A(1, 0), B(-5, 0) and C(-2, 5). **2**

Sol. Given A(1, 0), B(–5,0), C(–2,5)

So $AB = \sqrt{(1+5)^2 + (0+0)^2} = 6$

$BC = \sqrt{(-5+2)^2 + (0-5)^2} = \sqrt{9+25} = \sqrt{34}$

$AC = \sqrt{(1+2)^2 + (0-5)^2} = \sqrt{9+25} = \sqrt{34}$

Here BC = AC $\Rightarrow$ 2 sides are equal.

So it is an isosceles triangle.

23. (a) Evaluate: $2 \sin^2 30° \sec 60° + \tan^2 60°$. **2**

Sol. $2 \sin^2 30° \sec 60° + \tan^2 60°$.

$$2\left(\frac{1}{2}\right)^2 (2) + (\sqrt{3})^2$$

$$2 \times \frac{1}{4} \times 2 + 3$$

1+3=4

OR

(b) If $2 \sin (A + B) = \sqrt{3}$ and $\cos (A - B) = 1$, then find the measures of angles A and B. $0 \le A, B, (A + B) \le 90°$. **2**

Sol. $2 \sin (A + B) = \sqrt{3}$

$\Rightarrow \sin (A + B) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow A + B = 60°$ …..(1)

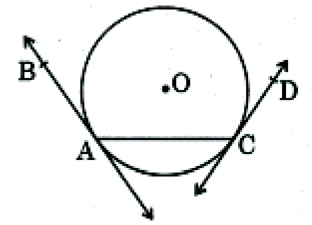
Also $\cos (A - B) = 1$

$\Rightarrow A - B = 0°$ ….(2)

From (1) & (2) $A = 30°$

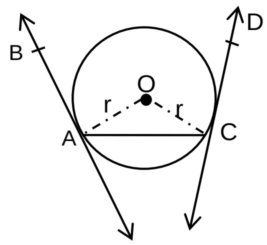
$B = 30°$

24. In the given figure, AB and CD are tangents to a circle centred at O. Is $\angle BAC = \angle DCA$? Justify your answer. **2**



Sol. Given AB and CD are tangents.

O is a center of circle and AC is a chord



To prove $\angle BAC = \angle DCA$

Costruction : Join OA & OC so that OA = OC = r

Proof : $\angle OAB = \angle OCD = 90°$ …(1)

{A tangent to a circle is perpendicular to the radius through the point of contact}

Now in ΔOAC

OA = OC = r (construction)

So, ΔOAC is an isosceles triangle

$\Rightarrow \angle OAC = \angle OCA = x°$(Let) ……(2)

Angles opposite to equal sides of a triangle are equal.

So By adding equation (1) & (2)

$\angle OAB + \angle OAC = \angle OCD + \angle OCA$

$\Rightarrow \angle BAC = \angle DCA$

25. (a) In what ratio is the line segment joining the points (3, -5) and (-1, 6) divided by the line y = x ? **2**

Sol. (a) Given : Point A(3, –5)

B(–1, 6)

Intersecting line ⇒ y = x

Let given line intersect the line by joining the point A and B at the ratio K : 1 and at $p(x_1, y_1)$

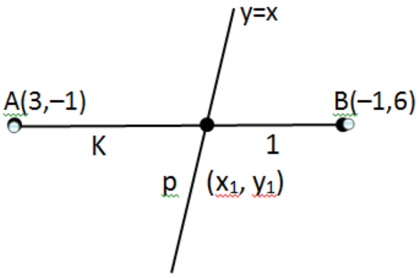
So, $x_1 = \frac{K(-1)+1(3)}{K+1}$

$\Rightarrow x_1 = \frac{-K+3}{K+1}$ ….(1)

Also $y_1 = \frac{K(6)+1(-5)}{K+1}$

$\Rightarrow y_1 = \frac{6K-5}{K+1}$ …..(2)

Point p lies on line y = x



$\frac{6k-5}{k+1} = \frac{-k+3}{k+1}$

$6K-5 = -K + 3 \ (K \neq -1)$

$7K = 8$

$K = \frac{8}{7}$

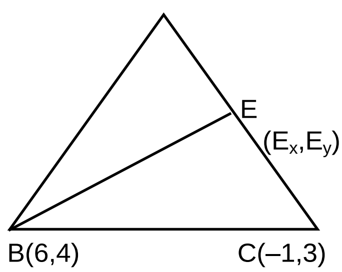
So the ratio is K : 1 ⇒ $\frac{8}{7}$ **: 1**

⇒ 8 : 7

OR

(b) A(3, 0), B(6, 4) and C(–1, 3) are vertices of a triangle ABC. Find length of its median BE. **2**

Sol. Given : A (3, 0), B (6, 4) and C(–1, 3)



Now median BE is a line joining vertex B to the middle point of line AC

So $E_x = \frac{A_x + C_x}{2}$ & $E_y = \frac{A_y + C_y}{2}$

X= $E_x = \frac{3-1}{2} = 1$, Y = $E_y = \frac{0+3}{2} = \frac{3}{2}$

So point $E\left(1, \frac{3}{2}\right)$

$$BE = \sqrt{(1-6)^2 + \left(\frac{3}{2} - y\right)^2}$$

$$= \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{125}{4}} = \frac{5}{2}\sqrt{5}$$

SECTION- C

(This section consists of 6 questions of 3 marks each.)

26 (a) If the sum of first m terms of an A.P. is same as sum of its first n terms ($m \neq n$), then show that the sum of its first(m + n) terms is zero. **3**

Sol. (a) $s_m = s_n$

$\Rightarrow \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 2am + (m^2 - m)d = 2an + (n^2 - n)d$

$\Rightarrow 2am - 2an + (m^2 - m)d - (n^2 - n)d = 0$

$\Rightarrow 2a(m-n) + d\,[m^2 - m - n^2 + n] = d$

$\Rightarrow 2a(m-n) + d\,[(m-n)(m+n) - (m-n)] = 0$

$\Rightarrow (m-n)\,[2a + d(m+n-1)] = 0$

As $m \neq n$

$\therefore$ 2a + (m + n –1) d = 0

Now $S_{n+n} = \frac{m+n}{2}$ [2a + (m+n–1)d]

$= \frac{m+n}{2}[0] = 0$

OR

(b) In an A.P., the sum of three consecutive terms is 24 and the sum of their squares is 194. Find the numbers. **3**

Sol. (b) Let the three terms be

a–d, a, a + d

$\therefore$ a–d + a + a + d = 24

3a = 24

a = 8

Now, $(a-d)^2 + a^2 + (a+d)^2 = 194$

$a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 194$

$3a^2 + 2d^2 = 194$

$3(8)^2 + 2d^2 = 194$

$3\times64 + 2d^2 = 194$

$2d^2 = 194 - 192$

$2d^2 = 2$

$d^2 = 1$

$\therefore$ d = ± 1

So, If d = 1

then a – d, a, a + d

8–1, 8, 8+1

7,8,9

for d = –1

then a – d, a, a +d

8–(–1), 8, 8 + (–1)

9, 8, 7

27. Prove that $\sqrt{5}$ is an irrational number. **3**

Sol. Let $\sqrt{5}$ is an irrational number.

So, $\sqrt{5} = \frac{p}{q}$ [where p,q, are integers and co-prime also $q \neq 0$]

$\Rightarrow p = \sqrt{5}q$

Squaring both sides

$p^2 = 5q^2$ …….(1)

$\Rightarrow p^2$ is divisible by 5 then p is also divisible by 5

$\therefore$ p = 5m $\quad m \in z$

Now put it in (1)

$(5m)^2 = 5q^2$

$25m^2 = 5q^2$

$5m^2 = q^2$

Now q^2 is divisible by 5

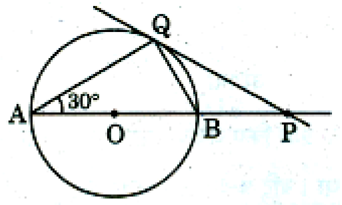
q is also divisible by 5

But this contradicts our assumption that p and q are co-prime so $\sqrt{5}$ is an rational number.

Hence $\sqrt{5}$ is irrational

28. **(a)** In the given figure PQ is tangent to a circle centred at O and $\angle BAQ = 30°$; show that BP = BQ.

3



Sol. (a) Given $\angle BAQ = 30°$

PQ is tangent

AB is diameter

To show : BP = BQ

construction : Join OQ

Now, In $\angle AQB = 90°$ (As, $\angle AQB$ is formed in semicircle)

In ΔAQB

$\angle BAQ + \angle AQB + \angle ABQ = 180°$

$30° + 90° + \angle ABQ = 180°$

$\angle ABQ = 180° - 120°$

$\angle ABQ = 60°$

Now, In ΔAOQ

AO = OQ = radius

$\therefore \angle AOQ = \angle AQO = 30°$

[Angles opposite to equal sides of a triangle are equal]

Similarly, In ΔBOQ

OB = OQ = Radius

$\angle OBQ = \angle OQB = 60°$

$\angle OQP = 90°$ [line joining from centre to point of contact is perpendicular to the tangent]

$\therefore \quad \angle BQP = \angle OQP - \angle OQB$

$= 90° - 60°$

$= 30°$

Also $\angle ABQ + \angle PBQ = 180°$ (linear pair)

$60° + \angle PBQ = 180°$

$\angle PBQ = 180° - 60°$

$\angle PBQ = 120°$

In ΔPBQ

$\angle PBQ + \angle BQP + \angle BPQ = 180°$

$120° + 30° + \angle BPQ = 180°$

$\angle BPQ = 180° - 150°$

$\angle BPQ = 30°$

So, In ΔPBQ

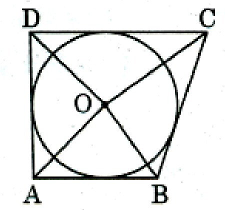
$\angle BPQ = \angle BQP = 30°$

$\therefore \quad$ BP = BQ [Sides opposite to equal angles of a triangle are equal]

OR

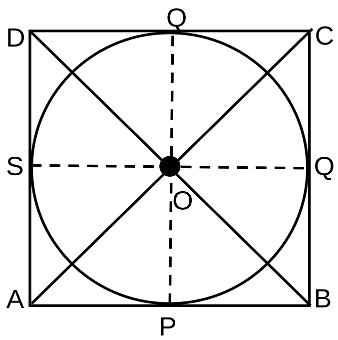
(b) In the given figure, AB, BC, CD and DA are tangents to the circle with centre O forming a quadrilateral ABCD. **3**

Show that $\angle AOB + \angle COD = 180^o$



Sol. (b) Given AB, BC, CD

DA are tangents



To show, $\angle AOB + \angle COD = 180°$

Construction : Let p, d, R, S one point of contancts

Join OP, OD, OR, OD

In ΔBOP and ΔBOD

BP = BD (Tangents drawn from on external point to a circle are equal)

OB = OB (common)

OP = OD (radii)

$\Rightarrow \Delta BOP \cong \Delta BOD$ (SSS)

$\therefore \angle BOP = \angle BOD = n$ (Let)

By CPCT

Similarly, $\angle COD = \angle COR = y$ (Let)

$\angle DOR = \angle DOS = z$ (Let)

$\angle AOS = \angle AOP = w$ (Let)

Now, $\angle AOB + \angle BOP + \angle BOQ + \angle COQ + \angle COR + \angle DOR + \angle DOS + \angle AOS = 360°$

Angle formed around a point

$\Rightarrow w + x + x + y + y + z + z + x = 360°$

$\Rightarrow 2(x + y + z + w) = 360°$

$\Rightarrow x + y + z + w = \frac{360^o}{2}$

$\Rightarrow x + y + z + w = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

29. Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$. **3**

Sol. $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$

by taking L.H.S.

$\Rightarrow \quad \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$\Rightarrow$ We know that

$\Rightarrow \quad 1+\tan^2\theta = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

Value of 1 put in expression

$\Rightarrow \quad \frac{(\sec^2\theta-\tan^2\theta)+\sec\theta+\tan\theta}{1+\sec\theta+\tan\theta}$

$\Rightarrow \quad \frac{(\sec\theta-\tan\theta)[\sec\theta+\tan\theta+1]}{[1+\sec\theta+\tan\theta]}$ $\therefore$ [(secθ + tanθ) (secθ –tanθ) = 1]

$\Rightarrow \quad (\sec\theta - \tan\theta)$

$\Rightarrow \quad \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}$

$\Rightarrow \quad \left[\frac{1-\sin\theta}{\cos\theta}\right]$

Here $\boxed{\text{L.H.S. = R.H.S.}}$

$\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$

Hence proved.

30. In a test, the marks obtained by 100 students (out of 50) are given below : **3**

Marks obtained:	0–10	10–20	20–30	30–40	40–50
Number of students:	12	23	34	25	6

Find the mean marks of the students.

Sol.

Marks Obtained Interval	No. of Student f_i	Class mark x_i	f_ix_i
0-10	**12**	**5**	**60**
10-20	**23**	**15**	**345**
20-30	**34**	**25**	**850**
30-40	**25**	**35**	**875**
40-50	**6**	**45**	**270**
	Σf_i = 100		**Σf_ix_i = 2400**

Class mark = $\frac{\text{Upper class limit} + \text{Lower class limit}}{2}$

e.g. = $\frac{0+10}{2} = \frac{10}{2} = 5$

Mean ($\overline{X}$) = $\frac{\sum f_i x_i}{\sum f_i}$

$= \frac{2400}{100}$

Mean $\overline{X}$ = 24

31. In a 2-digit number, the digit at the unit's place is 5 less than the digit at the ten's place. The product of the digits is 36. Find the number. **3**

Sol. Let 2 digit number is 10x+y

x = tan's place digit

y = one's place digit

According to question :

$y = x - 5$ ....(1)

Product of digits is 36. Then

$xy = 36$ ....(2)

Value of y put in equation (1)

$x(x-5) = 36$

$x^2 - 5x - 36 = 0$

$x^2 - 9x + 4x - 36 = 0$

$x(x-9) + 4(x-9) = 0$

$(x-9)(x+4) = 0$

$x = 9,\ x = -4$

digit is positive so x = 9 is correct

$y = x - 5$

$y = 9 - 5$

$y = 4$

if x = 9 and y = 4 so the number is

$\Rightarrow$ $10x + y$

$90 + 4 = 94$

SECTION-D

(This section consists of 4 questions of 5 marks each)

32. **(a)** Using graphical method, solve the following system of equations: $3x + y + 4 = 0$ and $3x - y + 2 = 0$ **5**

Sol. **(a)** $3x + y + 4 = 0$

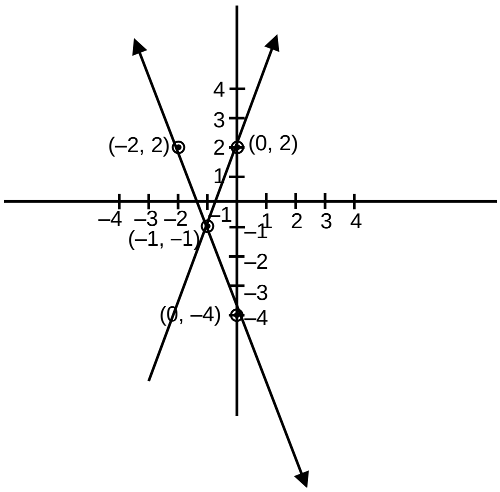
$y = -3x - 4$

x	0	–2
y	–4	2

$3x - y + 2 = 0$

$y = 3x + 2$

x	0	–1
y	2	–1



OR

(b) Tara scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Tara would have scored 50 marks. Assuming that Tara attempted all questions, find the total number of questions in the test. **5**

Sol. **(b)** Case-I:

Let No. of correct question = x

& no of wrong question = y

Marks obtained for each correct questions = +3

Marks obtained for each wrong questions = –1

According to question,

$3x + (-1)y = 40$

$3x - y = 40$ ....(i)

Case-II:

Marks obtained for each correct questions = +4

Marks obtained for each wrong questions = –2

According to question,

$4x + (-2)y = 50$

$4x - 2y = 50$

$2x - y = 25$ ....(ii)

$$\begin{array}{r} 3x - y = 40 \\ 2x - y = 25 \\ - \quad + \quad - \\ \hline x = 15 \end{array}$$

put value of x in equation (i)

$3(15) - y = 40$

$y = 5$

So total question = x + y

= 15 + 5

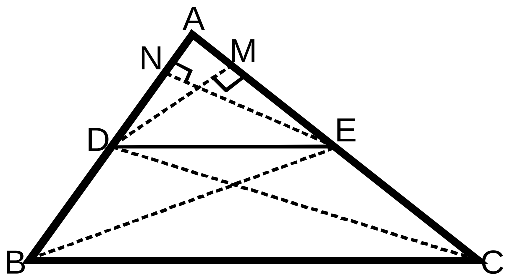
= 20

33. (a) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. **5**

Sol. (a)

Given: A $\triangle ABC$ in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$.



Construction: Join BE and CD and draw DM $\perp$ AC and EN $\perp$ AB.

Proof: Area of Δ ADE $= \frac{1}{2}$ (base × height) $= \frac{1}{2}$ AD × EN.

Area of Δ ADE is denoted as ar(ADE).

So, ar(ADE) $= \frac{1}{2} \times AD \times EN$ and ar(BDE) $= \frac{1}{2} \times DB \times EN$.

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ .... (i)

Similarly, ar(ADE) $= \frac{1}{2} \times AE \times DM$ and ar(DEC) $= \frac{1}{2} \times EC \times DM$.

And $\quad \dfrac{ar(ADE)}{ar(DEC)} = \dfrac{\frac{1}{2}AE \times DM}{\frac{1}{2}EC \times DM} = \dfrac{AE}{EC}$ ..... (ii)

Note that Δ BDE and Δ DEC are on the same base DE and between the two parallel lines BC and DE.

So, ar(BDE) = ar(DEC) ..... (iii)

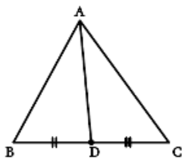
Therefore, from (i), (ii) and (iii), we have :

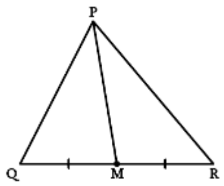
$\dfrac{AD}{DB} = \dfrac{AE}{EC}$ Hence Proved.

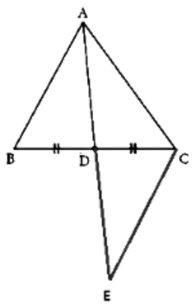
OR

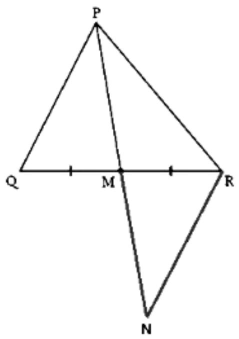
(b) Sides AB and AC and median AD to ΔABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC – ΔPQR. **5**

Sol. **Given :** $\dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{AD}{PM}$









Produce AD to E so that AD = DE. Join CE

Similarly, produce PM to N such that PM = MN , and join RN.

In ΔABD and ΔCDE

AD = DE [By Construction]

BD = DC [AD is the median]

$\angle$ADB = $\angle$CDE [Vertically opposite angles]

Therefore, $\Delta ABD \cong \Delta ECD$ [By SAS criterion of congruence]

$\Rightarrow$ AB = CE [by cpct] .....(i)

Also, in ΔPQM and ΔMNR

PM = MN [By Construction]

QM = MR [PM is the median]

$\angle$PMQ = $\angle$NMR [Vertically opposite angles]

Therefore, ΔPQM = ΔNRM [By SAS criterion of congruence]

$\Rightarrow$ PQ = RN [by cpct] .....(ii)

Now,

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ [Given]

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$ (from (i) and (ii))

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$

Therefore, $\Delta ACE \sim \Delta PRN$ [By SSS similarity criterion]

Therefore, $\angle CAE = \angle RPN$

Similarly, $\angle BAE = \angle QPN$

Hence, $\angle CAE + \angle BAE = \angle RPN + \angle QPN$

$\angle BAC = \angle QPR$

$\therefore \angle A = \angle P$ ......(iii)

Now, In ΔABC and ΔPQR

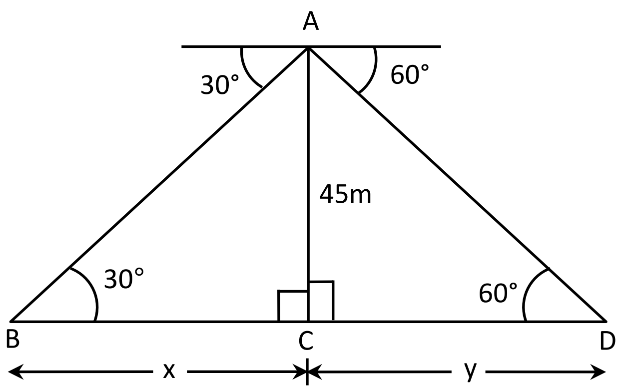
$\frac{AB}{PQ} = \frac{AC}{PR}$

$\angle A = \angle P$[from (iii)]

Therefore, $\Delta ABC \sim \Delta PQR$ [By SAS similarity criterion]

34. From the top of a 45 m high light house, the angles of depression of two ships, on the opposite side of it, are observed to be 30° and 60°. If the line joining the ships passes through the foot of the light house, find the distance between the ships. (Use $\sqrt{3}$ = 1.73) **5**

Ans.



Let the distance between one ship and light house is x and light house and other ship is y.

In ΔABC

$\tan 30° = \frac{P}{B}$

$\frac{1}{\sqrt{3}} = \frac{45}{x}$

$x = 45\sqrt{3}$ m ....(i)

In ΔACD

$\tan 60° = \frac{P}{B}$

$\sqrt{3} = \frac{45}{y}$

$y = \frac{45}{\sqrt{3}}$ m ....(ii)

equation (i) + equation (ii)

Distance both ships = x + y

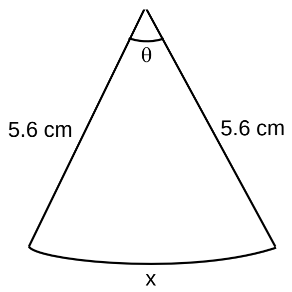
Distance both ships is $45\sqrt{3} + \frac{45}{\sqrt{3}}$

$\frac{45 \times 3 + 45}{\sqrt{3}} = \frac{180 \times \sqrt{3}}{3}$

= 60 × 1.732 = 103.92 m

35. The perimeter of a certain sector of a circle of radius 5.6 m is 20.0 m. Find the area of the sector. **5**

Ans.



Let arc length = x

Perimeter = x + 5.6 + 5.6 = 20.0 m

x + 11.2 = 20

x = 8.8 m.

arc length = $\frac{\theta}{360} \times 2\pi r$

$8.8 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 5.6$

$\frac{1}{4} = \frac{\theta}{360}$

$\theta = 90°$

Area of miror sector = $\frac{\theta}{360} \times \pi r^2$

Area of sector = $\frac{90}{360} \times \frac{22}{7} \times \frac{56}{10} \times \frac{56}{10}$

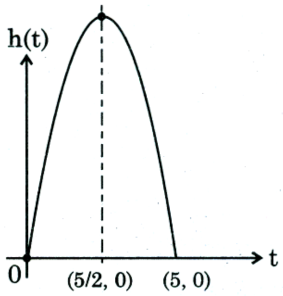
$= \frac{22 \times 2 \times 56}{100}$

$= \frac{44 \times 56}{100} = 24.64 \text{ m}^2$

SECTION-E

(This section consists of 3 case based questions of 4 marks each)

36. A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$.



Observe the graph of the polynomial and answer the following questions:

(i) Write zeroes of the given polynomial. **1**

(ii) Find the maximum height achieved by ball. **1**

(iii) (a) After throwing upward, how much time did the ball take to reach to the height of 30 m?

Sol. (i) given polynomial is $h = 25t - 5t^2$

for zero of the polynomial h = 0

$\therefore\ 25t - 5t^2 = 0$

$5t(5 - t) = 0$

t = 0, t = 5 are zeros of the polynomial

(ii) Given polynomial is $h = 25t - 5t^2$

for maximum height put $t = \frac{5}{2}$ in above equation

$$h_{max} = 25\left(\frac{5}{2}\right) - 5\left(\frac{5}{2}\right)^2$$

$$= \frac{125}{2} - \frac{125}{4}$$

$$= \frac{125}{4} = 31.25 \text{ m}$$

(iii) h = 30m. put in the given polynomial.

$30 = 25t - 5t^2$

$5t^2 - 25t + 30 = 0$

$t^2 - 5t + 6 = 0$

$t^2 - (2 + 3)t + 6 = 0$

$t^2 - 2t - 3t + 6 = 0$

$t(t - 2) - 3(t - 2) = 0$

t = 2, 3 second.

OR

(iii) (b) Find the two different values of t when the height of the ball was 20 m. **2**

Sol. (iii) (b) When height of the ball is 20m.

put h = 20m in given polynomial

$h = 25t - 5t^2$

$20 = 25t - 5t^2$

$5t^2 - 25t + 20 = 0$

$t^2 - 5t + 4 = 0$

$t^2 - (4 + 1)t + 4 = 0$

$t^2 - 4t - t + 4 = 0$

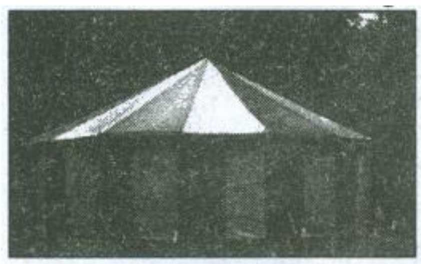
$t(t - 4) - 1(t - 4) = 0$

$(t - 4)(t - 1) = 0$

t = 1 second and t = 4 second.

37. The word 'circus' has the same root as 'circle'. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd.

A circus tent is cylindrical upto a height of 8 m and conical above it.The diameter of the base is 28 m and total height of tent is 18.5 m.



Based on the above, answer the following questions:

(i) Find slant height of the conical part.

(ii) Determine the floor area of the tent.

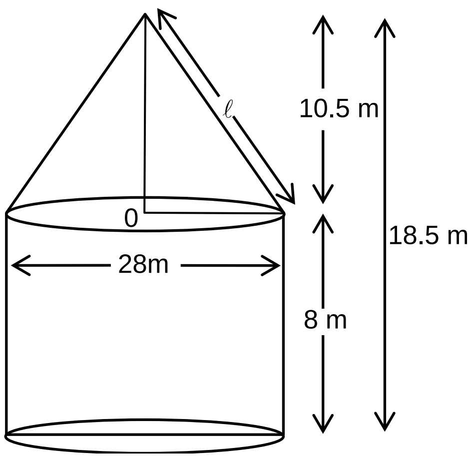
(iii) (a) Find area of the cloth used for making tent.

Sol. Given total height of tent = 18.5m

let height of cylinder is h_1 and height of cone is h_2

$\therefore$ $h_1 = 8$ m given

$h_2 = 18.5 \text{ m} - 8\text{m} = 10.5$ m



(i) Slant height of the conical part (let slant height of cone is ℓ)

height of conical part = total height of tent – height of cylinder

= 18.5 m – 8 m

= 10.5 m

$\therefore$ slant height = $\sqrt{h_{cone}^2 + r_{base}^2}$

= $\sqrt{(10.5)^2 + (14)^2}$

= $\sqrt{110.25 + 196}$

= $\sqrt{306.25}$

= 17.5 m

(ii) floor area of tent = area of base (base is circular)

= area of circle

= πr^2

= $\pi(14)^2$

= $\frac{22}{7} \times 14 \times 14$

= 22 × 2 × 14

= 616 m^2

(iii) Area of cloth used for making tent = curved surface area of cylinder + curved surface are of conc.

= $2\pi rh_1 + \pi r\ell$

= $\pi r(2h_1 + \ell)$

= $\frac{22}{7} \times 14(2 \times 8 + 17.5)$

= 22 × 2 × (16 + 17.5)

= 44 × 33.5

= 1474 m^2

OR

(iii) (b) Find total volume of air inside an empty tent.

Sol. Total volume of air inside the empty tent = Volume of cylinder + Volume of cone.

= $\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

= $\pi r^2\left(h_1 + \frac{h_2}{3}\right)$

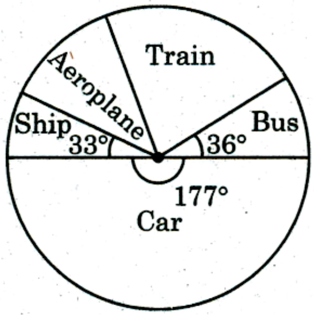
= $\pi r^2\left(8 + \frac{10.5}{3}\right)$

= $\frac{22}{7} \times 14 \times 14\ (8 + 3.5)$

= 22 × 2 × 14(11.5)

= 7084 m^3

38. In a survey on holidays, 120 people were asked to state which type of transport they used on their last holiday. The following pie chart shows the results of the survey.



Observe the pie chart and answer the following questions:

(i) If one person is selected at random, find the probability that he/she travelled by bus or ship. **1**

(ii) Which is most favourite mode of transport and how many people used it? **1**

(iii) (a) A person is selected at random. If the probability that he did not use train is 4/5, find the number of people who used train. **2**

Sol. Given 120 people in 360°

$\therefore$ No of person used bus as a transport = $\frac{36^\circ}{360^\circ} \times 120 = 12$

No. of person used ship as a transport = $\frac{30^\circ}{360^\circ} \times 120 = 11$

(i) The probability of one person is selected at random that he/she travelled by bus or ship

$= \frac{\text{Total no. of person used bus or ship}}{\text{total no. of persons}}$

$= \frac{12+11}{120} = \frac{23}{120}$

(ii) No of person used can as a transport = $\frac{177^\circ}{360^\circ} \times 120$

$= \frac{177}{3} = 59$

$\therefore$ most favorite mode of transport is car and No of person used car as a transport = 59 persons.

(iii) (a) Probability a person not use train = 4/5

$\therefore$ probability a person use train as a transport = 1 – 4/5 = 1/5 total no of person = 120

$\therefore$ No of person use train = 120 × $\frac{1}{5}$ = 24

OR

(iii) (b) The probability that randomly selected person used aeroplane is 7/60. Find the revenue collected by air company at the rate of Rs. 5,000 per person. **2**

Sol. (iii) (b) probability a person used aeroplane = 7/60

$\therefore$ No of person for aeroplane = $\frac{7}{60} \times 120 = 14$

total revenue collected by air company = 14 × rate of per person
= 14 persons × 5000 Rs./ person
=70,000 Rs.