Government of Karnataka

MATHEMATICS

8

EIGHTH STANDARD

Part-I

विद्यया ऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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Foreword

The National Curriculum Framework, 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

NCERT appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Dr H.K. Dewan for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

Preface

This is the final book of the upper primary series. It has been an interesting journey to define mathematics learning in a different way. The attempt has been to retain the nature of mathematics, engage with the question why learn mathematics while making an attempt to create materials that would address the interest of the learners at this stage and provide sufficient and approachable challenge to them. There have been many views on the purpose of school mathematics. These range from the fully utilitarian to the entirely aesthetic perceptions. Both these end up not engaging with the concepts and enriching the apparatus available to the learner for participating in life. The NCF emphasises the need for developing the ability to mathematise ideas and perhaps experiences as well. An ability to explore the ideas and framework given by mathematics in the struggle to find a richer life and a more meaningful relationship with the world around.

This is not even easy to comprehend, far more difficult to operationalise. But NCF adds to this an even more difficult goal. The task is to involve everyone of that age group in the classroom or outside in doing mathematics. This is the aim we have been attempting to make in the series.

We have, therefore, provided space for children to engage in reflection, creating their own rules and definitions based on problems/tasks solved and following their ideas logically. The emphasis is not on remembering algorithms, doing complicated arithmetical problems or remembering proofs, but understanding how mathematics works and being able to identify the way of moving towards solving problems.

The important concern for us has also been to ensure that all students at this stage learn mathematics and begin to feel confident in relating mathematics. We have attempted to help children read the book and to stop and reflect at each step where a new idea has been presented. In order to make the book less formidable we have included illustrations and diagrams. These combined with the text help the child comprehend the idea. Throughout the series and also therefore in this book we have tried to avoid the use of technical words and complex formulations. We have left many things for the student to describe and write in her own words.

We have made an attempt to use child friendly language. To attract attention to some points blurbs have been used. The attempt has been to reduce the weight of long explanations by using these and the diagrams. The illustrations and fillers also attempt to break the monotony and provide contexts.

Class VIII is the bridge to Class IX where children will deal with more formal mathematics. The attempt here has been to introduce some ideas in a way that is moving towards becoming formal. The tasks included expect generalisation from the gradual use of such language by the child.

The team that developed this textbook consisted teachers with experience and appreciation of children learning mathematics. This team also included people with experience of research in mathematics teaching-learning and an experience of producing materials for children. The feedback on the textbooks for Classes VI and VII was kept in mind while developing this textbook. This process of development also included discussions with teachers during review workshop on the manuscript.

In the end, I would like to express the grateful thanks of our team to Professor Krishna Kumar, *Director*, NCERT, Professor G. Ravindra, *Joint Director*, NCERT and Professor Hukum Singh, *Head*, DESM, for giving us an opportunity to work on this task with freedom and with full support. I am also grateful to Professor J.V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics for his suggestions. I am also grateful for the support of the team members from NCERT, Professor S.K. Singh Gautam, Dr V.P. Singh and in particular Dr Ashutosh K. Wazalwar

who coordinated this work and made arrangements possible. In the end I must thank the Publication Department of NCERT for its support and advice and those from Vidya Bhawan who helped produce the book.

It need not be said but I cannot help mentioning that all the authors worked as a team and we accepted ideas and advice from each other. We stretched ourselves to the fullest and hope that we have done some justice to the challenge posed before us.

The process of developing materials is, however, a continuous one and we would hope to make this book better. Suggestions and comments on the book are most welcome.

H.K. Dewan
Chief Advisor
Textbook Development Committee

A Note for the Teacher

This is the third and the last book of this series. It is a continuation of the processes initiated to help the learners in abstraction of ideas and principles of mathematics. Our students to be able to deal with mathematical ideas and use them need to have the logical foundations to abstract and use postulates and construct new formulations. The main points reflected in the NCF-2005 suggest relating mathematics to development of wider abilities in children, moving away from complex calculations and algorithm following to understanding and constructing a framework of understanding. As you know, mathematical ideas do not develop by telling them. They also do not reach children by merely giving explanations. Children need their own framework of concepts and a classroom where they are discussing ideas, looking for solutions to problems, setting new problems and finding their own ways of solving problems and their own definitions.

As we have said before, it is important to help children to learn to read the textbook and other books related to mathematics with understanding. The reading of materials is clearly required to help the child learn further mathematics. In Class VIII please take stock of where the students have reached and give them more opportunities to read texts that use language with symbols and have brevity and terseness with no redundancy. For this if you can, please get them to read other texts as well. You could also have them relate the physics they learn and the equations they come across in chemistry to the ideas they have learnt in mathematics. These cross-disciplinary references would help them develop a framework and purpose for mathematics. They need to be able to reconstruct logical arguments and appreciate the need for keeping certain factors and constraints while they relate them to other areas as well. Class VIII children need to have opportunity for all this.

As we have already emphasised, mathematics at the Upper Primary Stage has to be close to the experience and environment of the child and be abstract at the same time. From the comfort of context and/or models linked to their experience they need to move towards working with ideas. Learning to abstract helps formulate and understand arguments. The capacity to see interrelations among concepts helps us deal with ideas in other subjects as well. It also helps us understand and make better patterns, maps, appreciate area and volume and see similarities between shapes and sizes. While this is regarding the relationship of other fields of knowledge to mathematics, its meaning in life and our environment needs to be re-emphasised.

Children should be able to identify the principles to be used in contextual situations, for solving problems sift through and choose the relevant information as the first important step. Once students do that they need to be able to find the way to use the knowledge they have and reach where the problem requires them to go. They need to identify and define a problem, select or design possible solutions and revise or redesign the steps, if required. As they go further there would be more to of this to be done. In Class VIII we have to get them to be conscious of the steps they follow. Helping children to develop the ability to construct appropriate models by breaking up the problems and evolving their own strategies and analysis of problems is extremely important. This is in the place of giving them prescriptive algorithms. to solve problems. Learning mathematics is not about remembering solutions or methods but knowing how to solve problems and being able to construct interesting situations to solve.

Cooperative learning, learning through conversations, desire and capacity to learn from each other and the recognition that conversation is not noise and consultation not cheating is an important part of change in attitude for you as a teacher and for the students as well. They should be asked to make presentations as a group with the inclusion of examples from the contexts of their own experiences. They should be encouraged to read the book in groups and formulate and express what they understand from it. The assessment pattern has to recognise and appreciate this and the classroom groups should be such that all children enjoy being with each other and are contributing to the learning of the group. As you would have seen different groups use different strategies. Some of these are not as efficient as others as they reflect the modeling done and reflect the thinking used. All these are appropriate and need to be analysed with children. The exposure to a variety of strategies deepens the mathematical understanding. Each group moves from where it is and needs to be given an opportunity for that.

For conciseness we present the key ideas of mathematics learning that we would like you to remember in your classroom.

1. Enquiry to understand is one of the natural ways by which students acquire and construct knowledge. The process can use generation of observations to acquire knowledge. Students need to deal with different forms of questioning and challenging investigations- explorative, open-ended, contextual and even error detection from geometry, arithmetic and generalising it to algebraic relations etc.
2. Children need to learn to provide and follow logical arguments, find loopholes in the arguments presented and understand the requirement of a proof. By now children have entered the formal stage. They need to be encouraged to exercise creativity and imagination and to communicate their mathematical reasoning both verbally and in writing.
3. The mathematics classroom should relate language to learning of mathematics. Children should talk about their ideas using their experiences and language. They should be encouraged to use their own words and language but also gradually shift to formal language and use of symbols.
4. The number system has been taken to the level of generalisation of rational numbers and their properties and developing a framework that includes all previous systems as sub-sets of the generalised rational numbers. Generalisations are to be presented in mathematical language and children have to see that algebra and its language helps us express a lot of text in small symbolic forms.
5. As before children should be required to set and solve a lot of problems. We hope that as the nature of the problems set up by them becomes varied and more complex, they would become confident of the ideas they are dealing with.
6. Class VIII book has attempted to bring together the different aspects of mathematics and emphasise the commonality. Unitary method, Ratio and proportion, Interest and dividends are all part of one common logical framework. The idea of variable and equations is needed wherever we need to find an unknown quantity in any branch of mathematics.

We hope that the book will help children learn to enjoy mathematics and be confident in the concepts introduced. We want to recommend the creation of opportunity for thinking individually and collectively.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching, to be included in the future editions. This can only happen if you would find time to listen carefully to children and identify gaps and on the other hand also find the places where they can be given space to articulate their ideas and verbalise their thoughts.

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Part-I

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CHAPTER 1

Playing with Numbers

1.1 Introduction

You have studied various types of numbers such as natural numbers, whole numbers, integers and rational numbers. You have also studied a number of interesting properties about them. In Class VI, we explored finding factors and multiples and the relationships among them.

In this chapter, we will explore numbers in more detail. These ideas help in justifying tests of divisibility.

1.2 Numbers in General Form

Let us take the number 52 and write it as

$$52 = 50 + 2 = 10 \times 5 + 2$$

Similarly, the number 37 can be written as

$$37 = 10 \times 3 + 7$$

In general, any two digit number *ab* made of digits *a* and *b* can be written as

$$ab = 10 \times a + b = 10a + b$$

What about *ba*? $ba = 10 \times b + a = 10b + a$

Here *ab* does not mean $a \times b$!

Let us now take number 351. This is a three digit number. It can also be written as

$$351 = 300 + 50 + 1 = 100 \times 3 + 10 \times 5 + 1 \times 1$$

Similarly $497 = 100 \times 4 + 10 \times 9 + 1 \times 7$

In general, a 3-digit number *abc* made up of digits *a*, *b* and *c* is written as

$$abc = 100 \times a + 10 \times b + 1 \times c$$
$$= 100a + 10b + c$$

In the same way,

$$cab = 100c + 10a + b$$
$$bca = 100b + 10c + a$$ and so on.

TRY THESE

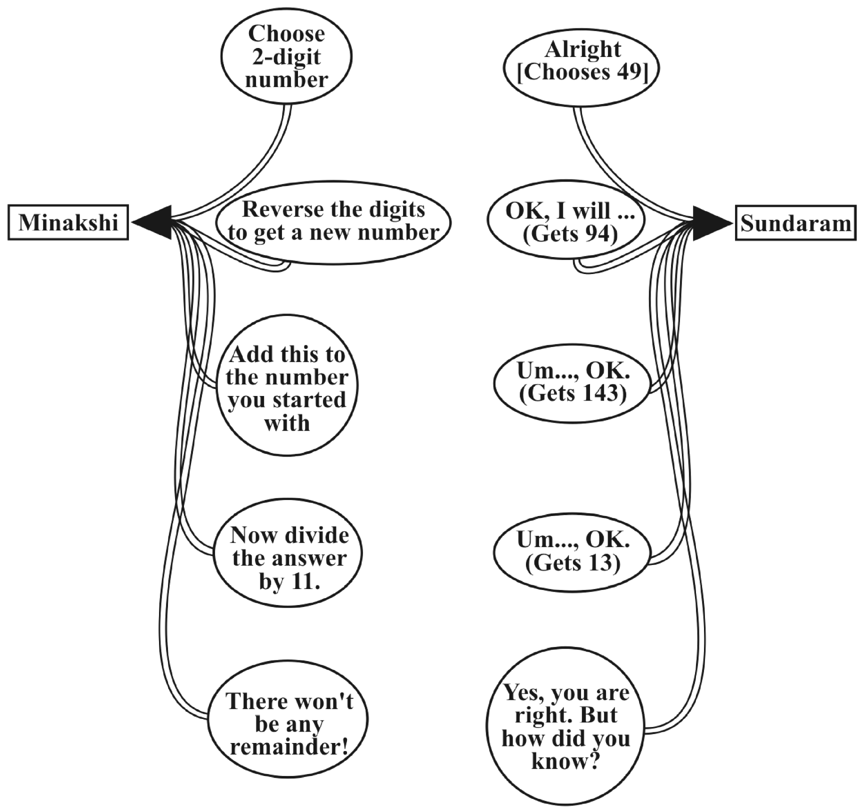
1. Write the following numbers in generalised form.
   (i) 25 (ii) 73 (iii) 129 (iv) 302
2. Write the following in the usual form.
   (i) $10 \times 5 + 6$ (ii) $100 \times 7 + 10 \times 1 + 8$ (iii) $100 \times a + 10 \times c + b$

1.3 Games with Numbers

(i) **Reversing the digits – two digit number**

Minakshi asks Sundaram to think of a 2-digit number, and then to do whatever she asks him to do, to that number. Their conversation is shown in the following figure. **Study the figure carefully before reading on**.

Conversations between Minakshi and Sundaram: First Round ...



It so happens that Sundaram chose the number 49. So, he got the reversed number 94; then he added these two numbers and got 49 + 94 = 143. Finally he divided this number by 11 and got 143 ÷ 11 = 13, with no remainder. This is just what Minakshi had predicted.

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 27 2. 39 3. 64 4. 17

Now, let us see if we can **explain** Minakshi's "trick".

Suppose Sundaram chooses the number *ab*, which is a short form for the 2-digit number $10a + b$. On reversing the digits, he gets the number $ba = 10b + a$. When he adds the two numbers he gets:

$$(10a + b) + (10b + a) = 11a + 11b$$
$$= 11\ (a + b).$$

So, the sum is always a multiple of 11, just as Minakshi had claimed.

Observe here that if we divide the sum by 11, the quotient is $a + b$, which is exactly the sum of the digits of chosen number *ab*.

You may check the same by taking any other two digit number.

The game between Minakshi and Sundaram continues!

Minakshi: Think of another 2-digit number, but don't tell me what it is.

Sundaram: Alright.

Minakshi: Now reverse the digits of the number, and *subtract* the smaller number from the larger one.

Sundaram: I have done the subtraction. What next?

Minakshi: Now divide your answer by 9. I claim that there will be no remainder!

Sundaram: Yes, you are right. There is indeed no remainder! But this time I think I know how you are so sure of this!

In fact, Sundaram had thought of 29. So his calculations were: first he got the number 92; then he got $92 – 29 = 63$; and finally he did $(63 \div 9)$ and got 7 as quotient, with no remainder.

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 17 2. 21 3. 96 4. 37

Let us see how Sundaram explains Minakshi's second "trick". (Now he feels confident of doing so!)

Suppose he chooses the 2-digit number $ab = 10a + b$. After reversing the digits, he gets the number $ba = 10b + a$. Now Minakshi tells him to do a subtraction, the smaller number from the larger one.

- If the tens digit is larger than the ones digit (that is, $a > b$), he does:

$$(10a + b) – (10b + a) = 10a + b – 10b – a$$
$$= 9a – 9b = 9(a – b).$$

- If the ones digit is larger than the tens digit (that is, $b > a$), he does:

  $$(10b + a) - (10a + b) = 9(b - a).$$

- And, of course, if $a = b$, he gets 0.

In each case, the resulting number is divisible by 9. So, the remainder is 0. Observe here that if we divide the resulting number (obtained by subtraction), the quotient is $a - b$ or $b - a$ according as $a > b$ or $a < b$. You may check the same by taking any other two digit numbers.

(ii) **Reversing the digits – three digit number**.

Now it is Sundaram's turn to play some tricks!

Sundaram: Think of a 3-digit number, but don't tell me what it is.

Minakshi: Alright.

Sundaram: Now make a new number by putting the digits in reverse order, and subtract the smaller number from the larger one.

Minakshi: Alright, I have done the subtraction. What next?

Sundaram: Divide your answer by 99. I am sure that there will be no remainder!

In fact, Minakshi chose the 3-digit number 349. So she got:

- Reversed number: 943;
- Difference: 943 – 349 = 594;
- Division: 594 ÷ 99 = 6, with no remainder.

TRY THESE

Check what the result would have been if Minakshi had chosen the numbers shown below. In each case keep a record of the quotient obtained at the end.

1. 132 2. 469 3. 737 4. 901

Let us see how this trick works.

Let the 3-digit number chosen by Minakshi be $abc = 100a + 10b + c$.

After reversing the order of the digits, she gets the number $cba = 100c + 10b + a$. On subtraction:

- If $a > c$, then the difference between the numbers is

  $$(100a + 10b + c) - (100c + 10b + a) = 100a + 10b + c - 100c - 10b - a$$
  $$= 99a - 99c = 99(a - c).$$

- If $c > a$, then the difference between the numbers is

  $$(100c + 10b + a) - (100a + 10b + c) = 99c - 99a = 99(c - a).$$

- And, of course, if $a = c$, the difference is 0.

In each case, the resulting number is divisible by 99. So the remainder is 0. Observe that quotient is $a - c$ or $c - a$. You may check the same by taking other 3-digit numbers.

(iii) **Forming three-digit numbers with given three-digits**.

Now it is Minakshi's turn once more.

Minakshi: Think of any 3-digit number.
Sundaram: Alright, I have done so.
Minakshi: Now use this number to form two more 3-digit numbers, like this: if the number you chose is *abc*, then

- 'the first number is *cab* (i.e., with the ones digit shifted to the "left end" of the number);
- the other number is *bca* (i.e., with the hundreds digit shifted to the "right end" of the number).

Now add them up. Divide the resulting number by 37. I claim that there will be no remainder.
Sundaram: Yes. You are right!

In fact, Sundaram had thought of the 3-digit number 237. After doing what Minakshi had asked, he got the numbers 723 and 372. So he did:

$$\begin{array}{r} 2\ 3\ 7 \\ +\ \ 7\ 2\ 3 \\ +\ \ 3\ 7\ 2 \\ \hline 1\ 3\ 3\ 2 \\ \hline \end{array}$$

Form all possible 3-digit numbers using all the digits 2, 3 and 7 and find their sum. Check whether the sum is divisible by 37! Is it true for the sum of all the numbers formed by the digits *a*, *b* and *c* of the number *abc*?

Then he divided the resulting number 1332 by 37:

$$1332 \div 37 = 36, \quad \text{with no remainder.}$$

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 417
2. 632
3. 117
4. 937

Will this trick always work?

Let us see.

$$abc = 100a + 10b + c$$
$$cab = 100c + 10a + b$$
$$bca = 100b + 10c + a$$
$$abc + cab + bca = 111(a + b + c)$$
$$= 37 \times 3(a + b + c), \text{ which is divisible by } 37$$

1.4 Letters for Digits

Here we have puzzles in which letters take the place of digits in an arithmetic 'sum', and the problem is to find out which letter represents which digit; so it is like cracking a code. Here we stick to problems of addition and multiplication.

Here are two rules we follow while doing such puzzles.

1. *Each letter in the puzzle must stand for just one digit. Each digit must be represented by just one letter.*
2. *The first digit of a number cannot be zero.* Thus, we write the number "sixty three" as 63, and not as 063, or 0063.

A rule that we would *like* to follow is that the puzzle must have just one answer.

Example 1: Find Q in the addition.

$$\begin{array}{r} 3\ 1\ Q \\ +\ 1\ Q\ 3 \\ \hline 5\ 0\ 1 \\ \hline \end{array}$$

Solution:

There is just one letter Q whose value we have to find.

Study the addition in the ones column: from Q + 3, we get '1', that is, a number whose ones digit is 1.

For this to happen, the digit Q should be 8. So the puzzle can be solved as shown below.

$$\begin{array}{r} 3\ 1\ 8 \\ +\ 1\ 8\ 3 \\ \hline 5\ 0\ 1 \\ \hline \end{array}$$

That is, Q = 8

Example 2: Find A and B in the addition.

$$\begin{array}{r} A \\ +\ A \\ +\ A \\ \hline B\ A \\ \hline \end{array}$$

Solution: This has *two* letters A and B whose values are to be found.

Study the addition in the ones column: the sum of *three* A's is a number whose ones digit is A. Therefore, the sum of *two* A's must be a number whose ones digit is 0.
This happens only for A = 0 and A = 5.

If A = 0, then the sum is 0 + 0 + 0 = 0, which makes B = 0 too. We do not want this (as it makes A = B, and then the tens digit of BA too becomes 0), so we reject this possibility. So, A = 5.
Therefore, the puzzle is solved as shown below.

$$\begin{array}{r} 5 \\ +\ 5 \\ +\ 5 \\ \hline 1\ 5 \\ \hline \end{array}$$

That is, A = 5 and B = 1.

Example 3: Find the digits A and B.

$$\begin{array}{r} \text{B A} \\ \times \ \text{B 3} \\ \hline \text{5 7 A} \\ \hline \end{array}$$

Solution:

This also has two letters A and B whose values are to be found.
Since the ones digit of 3 × A is A, it must be that A = 0 or A = 5.

Now look at B. If B = 1, then BA × B3 would *at most* be equal to 19 × 19; that is, it would at most be equal to 361. But the product here is 57A, which is more than 500. So we cannot have B = 1.

If B = 3, then BA × B3 would be more than 30 × 30; that is, more than 900. But 57A is less than 600. So, B can not be equal to 3.

Putting these two facts together, we see that B = 2 only. So the multiplication is either 20 × 23, or 25 × 23.

The first possibility fails, since 20 × 23 = 460. But, the second one works out correctly, since 25 × 23 = 575.

$$\begin{array}{r} 2\ 5 \\ \times \ 2\ 3 \\ \hline 5\ 7\ 5 \\ \hline \end{array}$$

So the answer is A = 5, B = 2.

DO THIS

Write a 2-digit number *ab* and the number obtained by reversing its digits i.e., *ba*. Find their sum. Let the sum be a 3-digit number *dad*

i.e., $ab + ba = dad$

$(10a + b) + (10b + a) = dad$

$11(a + b) = dad$

The sum $a + b$ can not exceed 18 (Why?).

Is *dad* a multiple of 11?

Is *dad* less than 198?

Write all the 3-digit numbers which are multiples of 11 upto 198.

Find the values of *a* and *d*.

EXERCISE 1.1

Find the values of the letters in each of the following and give reasons for the steps involved.

1. $$\begin{array}{r} 3\ \text{A} \\ +\ 2\ 5 \\ \hline \text{B}\ 2 \\ \hline \end{array}$$

2. $$\begin{array}{r} 4\ \text{A} \\ +\ 9\ 8 \\ \hline \text{C B}\ 3 \\ \hline \end{array}$$

3. $$\begin{array}{r} 1\ \text{A} \\ \times\ \text{A} \\ \hline 9\ \text{A} \\ \hline \end{array}$$

4.
$$\begin{array}{r} A\ \ B \\ +\ 3\ \ 7 \\ \hline 6\ \ A \\ \hline \end{array}$$

5.
$$\begin{array}{r} A\ \ B \\ \times\ \ 3 \\ \hline C\ A\ \ B \\ \hline \end{array}$$

6.
$$\begin{array}{r} A\ \ B \\ \times\ \ 5 \\ \hline C\ A\ \ B \\ \hline \end{array}$$

7.
$$\begin{array}{r} A\ \ B \\ \times\ \ 6 \\ \hline B\ B\ \ B \\ \hline \end{array}$$

8.
$$\begin{array}{r} A\ \ 1 \\ +\ 1\ \ B \\ \hline B\ \ 0 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 2\ A\ \ B \\ +\ A\ B\ \ 1 \\ \hline B\ 1\ \ 8 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 1\ \ 2\ \ A \\ +\ 6\ \ A\ \ B \\ \hline A\ \ 0\ \ 9 \\ \hline \end{array}$$

1.5 Tests of Divisibility

In Class VI, you learnt how to check divisibility by the following divisors.

10, 5, 2, 3, 6, 4, 8, 9, 11.

You would have found the tests easy to do, but you may have wondered at the same time *why* they work. Now, in this chapter, we shall go into the "why" aspect of the above.

1.5.1 Divisibility by 10

This is certainly the easiest test of all! We first look at some multiples of 10.

10, 20, 30, 40, 50, 60, ... ,

and then at some non-multiples of 10.

13, 27, 32, 48, 55, 69,

From these lists we see that if the ones digit of a number is 0, then the number is a multiple of 10; and if the ones digit is *not* 0, then the number is *not* a multiple of 10. So, we get a test of divisibility by 10.

Of course, we must not stop with just stating the test; we must also explain *why* it "works". That is not hard to do; we only need to remember the rules of place value.

Take the number. ... cba; this is a short form for

$$\ldots + 100c + 10b + a$$

Here a is the one's digit, b is the ten's digit, c is the hundred's digit, and so on. The dots are there to say that there may be more digits to the left of c.

Since 10, 100, ... are divisible by 10, so are $10b$, $100c$, ... And as for the number a is concerned, it must be a divisible by 10 if the given number is divisible by 10. This is possible only when $a = 0$.

Hence, a number is divisible by 10 when its one's digit is 0.

1.5.2 Divisibility by 5

Look at the multiples of 5.

5, 10, 15, 20, 25, 30, 35, 40, 45, 50,

We see that *the one's digits are alternately* 5 *and* 0, *and no other digit ever appears in this list.*

So, we get our test of divisibility by 5.

If the ones digit of a number is 0 *or* 5, *then it* is *divisible by* 5.

Let us explain this rule. Any number ... *cba* can be written as:

$... + 100c + 10b + a$

Since 10, 100 are divisible by 10 so are $10b$, $100c$, ... which in turn, are divisible by 5 because $10 = 2 \times 5$. As far as number a is concerned it must be divisible by 5 if the number is divisible by 5. So a has to be either 0 or 5.

TRY THESE

(The first one has been done for you.)

1. If the division $N \div 5$ leaves a remainder of 3, what might be the ones digit of N? (The one's digit, when divided by 5, must leave a remainder of 3. So the one's digit must be either 3 or 8.)
2. If the division $N \div 5$ leaves a remainder of 1, what might be the one's digit of N?
3. If the division $N \div 5$ leaves a remainder of 4, what might be the one's digit of N?

1.5.3 Divisibility by 2

Here are the even numbers.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, ... ,

and here are the odd numbers.

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, ... ,

We see that a natural number is even if its one's digit is

2, 4, 6, 8 or 0

A number is odd if its one's digit is

1, 3, 5, 7 or 9

Recall the test of divisibility by 2 learnt in Class VI, which is as follows.

If the one's digit of a number is 0, 2, 4, 6 or 8 then the number is divisible by 2.

The explanation for this is as follows.

Any number *cba* can be written as $100c + 10b + a$

First two terms namely $100c$, $10b$ are divisible by 2 because 100 and 10 are divisible by 2. So far as a is concerned, it must be divisible by 2 if the given number is divisible by 2. This is possible only when $a = 0, 2, 4, 6$ or 8.

TRY THESE

(The first one has been done for you.)

1. If the division $N \div 2$ leaves a remainder of 1, what might be the one's digit of N? (N is odd; so its one's digit is odd. Therefore, the one's digit must be 1, 3, 5, 7 or 9.)

2. If the division N ÷ 2 leaves no remainder (i.e., zero remainder), what might be the one's digit of N?
3. Suppose that the division N ÷ 5 leaves a remainder of 4, and the division N ÷ 2 leaves a remainder of 1. What must be the one's digit of N?

1.5.4 Divisibility by 9 and 3

Look carefully at the three tests of divisibility found till now, for checking division by 10, 5 and 2. We see something common to them: *they use only the one's digit of the given number; they do not bother about the 'rest' of the digits*. Thus, *divisibility is decided just by the one's digit.* 10, 5, 2 are divisors of 10, which is the key number in our place value.

But for checking divisibility by 9, this will not work. Let us take some number say 3573.

Its expanded form is: $3 \times 1000 + 5 \times 100 + 7 \times 10 + 3$

This is equal to $3 \times (999 + 1) + 5 \times (99 + 1) + 7 \times (9 + 1) + 3$

$$= 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 3) \quad \text{... (1)}$$

We see that the number 3573 will be divisible by 9 or 3 if $(3 + 5 + 7 + 3)$ is divisible by 9 or 3.

We see that $3 + 5 + 7 + 3 = 18$ is divisible by 9 and also by 3. Therefore, the number 3573 is divisible by both 9 and 3.

Now, let us consider the number 3576. As above, we get

$$3576 = 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 6) \quad \text{... (2)}$$

Since $(3 + 5 + 7 + 6)$ i.e., 21 is not divisible by 9 but is divisible by 3,

therefore 3576 is not divisible by 9. However 3576 is divisible by 3. Hence,

(i) A number N is divisible by 9 if the sum of its digits is divisible by 9. Otherwise it is not divisible by 9.

(ii) A number N is divisible by 3 if the sum of its digits is divisible by 3. Otherwise it is not divisible by 3.

If the number is '*cba*', then, $100c + 10b + a = 99c + 9b + (a + b + c)$

$$= \underbrace{9(11c + b)}_{\text{divisible by 3 and 9}} + (a + b + c)$$

Hence, divisibility by 9 (or 3) is possible if $a + b + c$ is divisible by 9 (or 3).

Example 4: Check the divisibility of 21436587 by 9.

Solution: The sum of the digits of 21436587 is $2 + 1 + 4 + 3 + 6 + 5 + 8 + 7 = 36$. This number is divisible by 9 (for $36 \div 9 = 4$). We conclude that 21436587 is divisible by 9.

We can double-check:

$$\frac{21436587}{9} = 2381843 \qquad \text{(the division is exact).}$$

Example 5: Check the divisibility of 152875 by 9.

Solution: The sum of the digits of 152875 is 1 + 5 + 2 + 8 + 7 + 5 = 28. This number is **not** divisible by 9. We conclude that 152875 is not divisible by 9.

TRY THESE

Check the divisibility of the following numbers by 9.

1. 108 2. 616 3. 294 4. 432 5. 927

Example 6: If the three digit number 24*x* is divisible by 9, what is the value of *x*?

Solution: Since 24*x* is divisible by 9, sum of it's digits, i.e., $2 + 4 + x$ should be divisible by 9, i.e., $6 + x$ should be divisible by 9.

This is possible when $6 + x = 9$ or 18, ...
But, since *x* is a digit, therefore, $6 + x = 9$, i.e., $x = 3$.

THINK, DISCUSS AND WRITE

1. You have seen that a number 450 is divisible by 10. It is also divisible by 2 and 5 which are factors of 10. Similarly, a number 135 is divisible 9. It is also divisible by 3 which is a factor of 9.
   Can you say that if a number is divisible by any number *m*, then it will also be divisible by each of the factors of *m*?
2. (i) Write a 3-digit number *abc* as $100a + 10b + c$
   $$= 99a + 11b + (a - b + c)$$
   $$= 11(9a + b) + (a - b + c)$$
   If the number *abc* is divisible by 11, then what can you say about $(a - b + c)$?
   Is it necessary that $(a + c - b)$ should be divisible by 11?

   (ii) Write a 4-digit number *abcd* as $1000a + 100b + 10c + d$
   $$= (1001a + 99b + 11c) - (a - b + c - d)$$
   $$= 11(91a + 9b + c) + [(b + d) - (a + c)]$$
   If the number *abcd* is divisible by 11, then what can you say about $[(b + d) - (a + c)]$?

   (iii) From (i) and (ii) above, can you say that a number will be divisible by 11 if the difference between the sum of digits at its odd places and that of digits at the even places is divisible by 11?

Example 7: Check the divisibility of 2146587 by 3.

Solution: The sum of the digits of 2146587 is 2 + 1 + 4 + 6 + 5 + 8 + 7 = 33. This number is divisible by 3 (for 33 ÷ 3 = 11). We conclude that 2146587 is divisible by 3.

Example 8: Check the divisibility of 15287 by 3.

Solution: The sum of the digits of 15287 is 1 + 5 + 2 + 8 + 7 = 23. This number is not divisible by 3. We conclude that 15287 too is not divisible by 3.

TRY THESE

Check the divisibility of the following numbers by 3.

1. 108   2. 616   3. 294   4. 432   5. 927

EXERCISE 1.2

1. If 21y5 is a multiple of 9, where y is a digit, what is the value of y?
2. If 31z5 is a multiple of 9, where z is a digit, what is the value of z?
   You will find that there are *two* answers for the last problem. Why is this so?
3. If 24x is a multiple of 3, where x is a digit, what is the value of x?
   (Since 24x is a multiple of 3, its sum of digits 6 + x is a multiple of 3; so 6 + x is one of these numbers: 0, 3, 6, 9, 12, 15, 18, ... But since x is a digit, it can only be that 6 + x = 6 or 9 or 12 or 15. Therefore, x = 0 or 3 or 6 or 9. Thus, x can have any of four different values.)
4. If 31z5 is a multiple of 3, where z is a digit, what might be the values of z?

WHAT HAVE WE DISCUSSED?

1. Numbers can be written in general form. Thus, a two digit number ab will be written as $ab = 10a + b$.
2. The general form of numbers are helpful in solving puzzles or number games.
3. The reasons for the divisibility of numbers by 10, 5, 2, 9 or 3 can be given when numbers are written in general form.

CHAPTER

2 Rational Numbers

M6M7F9

2.1 Introduction

In Mathematics, we frequently come across simple equations to be solved. For example, the equation

$$x + 2 = 13 \quad (1)$$

is solved when $x = 11$, because this value of x satisfies the given equation. The solution 11 is a **natural number**. On the other hand, for the equation

$$x + 5 = 5 \quad (2)$$

the solution gives the **whole number** 0 (zero). If we consider only natural numbers, equation (2) cannot be solved. To solve equations like (2), we added the number zero to the collection of natural numbers and obtained the whole numbers. Even whole numbers will not be sufficient to solve equations of type

$$x + 18 = 5 \quad (3)$$

Do you see 'why'? We require the number –13 which is not a whole number. This led us to think of **integers, (positive and negative)**. Note that the positive integers correspond to natural numbers. One may think that we have enough numbers to solve all simple equations with the available list of integers. Now consider the equations

$$2x = 3 \quad (4)$$

$$5x + 7 = 0 \quad (5)$$

for which we cannot find a solution from the integers. (Check this)

We need the numbers $\frac{3}{2}$ to solve equation (4) and $\frac{-7}{5}$ to solve equation (5). This leads us to the collection of **rational numbers**.

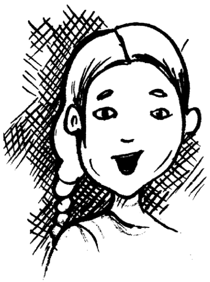
We have already seen basic operations on rational numbers. We now try to explore some properties of operations on the different types of numbers seen so far.

2.2 Properties of Rational Numbers

2.2.1 Closure

(i) Whole numbers

Let us revisit the closure property for all the operations on whole numbers in brief.



Operation	Numbers	Remarks
Addition	0 + 5 = 5, a whole number 4 + 7 = ... Is it a whole number? In general, $a + b$ is a whole number for any two whole numbers a and b.	Whole numbers are closed under addition.
Subtraction	5 – 7 = – 2, which is not a whole number.	Whole numbers are **not** closed under subtraction.
Multiplication	0 × 3 = 0, a whole number 3 × 7 = ... Is it a whole number? In general, if a and b are any two whole numbers, their product ab is a whole number.	Whole numbers are closed under multiplication.
Division	$5 \div 8 = \frac{5}{8}$, which is not a whole number.	Whole numbers are **not** closed under division.

Check for closure property under all the four operations for natural numbers.

(ii) Integers

Let us now recall the operations under which integers are closed.

Operation	Numbers	Remarks
Addition	– 6 + 5 = – 1, an integer Is – 7 + (–5) an integer? Is 8 + 5 an integer? In general, $a + b$ is an integer for any two integers a and b.	Integers are closed under addition.
Subtraction	7 – 5 = 2, an integer Is 5 – 7 an integer? – 6 – 8 = – 14, an integer	Integers are closed under subtraction.

	$-6-(-8)=2$, an integer Is $8-(-6)$ an integer? In general, for any two integers a and b, $a-b$ is again an integer. Check if $b-a$ is also an integer.	
Multiplication	$5\times 8=40$, an integer Is -5×8 an integer? $-5\times(-8)=40$, an integer In general, for any two integers a and b, $a\times b$ is also an integer.	Integers are closed under multiplication.
Division	$5\div 8=\frac{5}{8}$, which is not an integer.	Integers are **not** closed under division.

You have seen that whole numbers are closed under addition and multiplication but not under subtraction and division. However, integers are closed under addition, subtraction and multiplication but not under division.

(iii) Rational numbers

Recall that a number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q\neq 0$ is called a **rational number**. For example, $\frac{-2}{3}, \frac{6}{7}, \frac{9}{-5}$ are all rational numbers. Since the numbers 0, –2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers. (Check it!)

(a) You know how to add two rational numbers. Let us add a few pairs.

$$\frac{3}{8}+\frac{(-5)}{7}=\frac{21+(-40)}{56}=\frac{-19}{56} \quad \text{(a rational number)}$$

$$\frac{-3}{8}+\frac{(-4)}{5}=\frac{-15+(-32)}{40}=... \quad \text{Is it a rational number?}$$

$$\frac{4}{7}+\frac{6}{11}=... \quad \text{Is it a rational number?}$$

We find that sum of two rational numbers is again a rational number. Check it for a few more pairs of rational numbers.

We say that *rational numbers are closed under addition. That is, for any two rational numbers a and b, a + b is also a rational number*.

(b) Will the difference of two rational numbers be again a rational number?

We have,

$$\frac{-5}{7}-\frac{2}{3}=\frac{-5\times 3-2\times 7}{21}=\frac{-29}{21} \quad \text{(a rational number)}$$

$$\frac{5}{8} - \frac{4}{5} = \frac{25-32}{40} = \ldots$$ Is it a rational number?

$$\frac{3}{7} - \left(\frac{-8}{5}\right) = \ldots$$ Is it a rational number?

Try this for some more pairs of rational numbers. We find that *rational numbers are closed under subtraction. That is, for any two rational numbers a and b, a – b is also a rational number*.

(c) Let us now see the product of two rational numbers.

$$\frac{-2}{3} \times \frac{4}{5} = \frac{-8}{15};\ \frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$$ (both the products are rational numbers)

$$\frac{-4}{5} \times \frac{-6}{11} = \ldots$$ Is it a rational number?

Take some more pairs of rational numbers and check that their product is again a rational number.

We say that *rational numbers are closed under multiplication. That is, for any two rational numbers a and b, a × b is also a rational number*.

(d) We note that $\frac{-5}{3} \div \frac{2}{5} = \frac{-25}{6}$ (a rational number)

$\frac{2}{7} \div \frac{5}{3} = \ldots$. Is it a rational number? $\frac{-3}{8} \div \frac{-2}{9} = \ldots$. Is it a rational number?

Can you say that rational numbers are closed under division?

We find that for any rational number a, $a \div 0$ is **not defined**.

So rational numbers are **not closed** under division.

However, if we exclude zero then the collection of, all other rational numbers is closed under division.

TRY THESE

Fill in the blanks in the following table.

Numbers	Closed under			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	Yes	Yes	...	No
Integers	...	Yes	...	No
Whole numbers	...	...	Yes	...
Natural numbers	...	No	...	...

2.2.2 Commutativity

(i) Whole numbers

Recall the commutativity of different operations for whole numbers by filling the following table.

Operation	Numbers	Remarks
Addition	$0 + 7 = 7 + 0 = 7$ $2 + 3 = ... + ... = ...$ For any two whole numbers a and b, $a + b = b + a$	Addition is commutative.
Subtraction	....	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

Check whether the commutativity of the operations hold for natural numbers also.

(ii) Integers

Fill in the following table and check the commutativity of different operations for integers:

Operation	Numbers	Remarks
Addition	....	Addition is commutative.
Subtraction	Is $5 - (-3) = -3 - 5$?	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

(iii) Rational numbers

(a) Addition

You know how to add two rational numbers. Let us add a few pairs here.

$$\frac{-2}{3} + \frac{5}{7} = \frac{1}{21} \text{ and } \frac{5}{7} + \left(\frac{-2}{3}\right) = \frac{1}{21}$$

So, $$\frac{-2}{3} + \frac{5}{7} = \frac{5}{7} + \left(\frac{-2}{3}\right)$$

Also, $$\frac{-6}{5} + \left(\frac{-8}{3}\right) = \text{... and } \frac{-8}{3} + \left(\frac{-6}{5}\right) = \text{...}$$

Is $$\frac{-6}{5} + \left(\frac{-8}{3}\right) = \left(\frac{-8}{3}\right) + \left(\frac{-6}{5}\right) ?$$

Is $\frac{-3}{8}+\frac{1}{7}=\frac{1}{7}+\left(\frac{-3}{8}\right)$?

You find that two *rational numbers can be added in any order. We say that addition is commutative for rational numbers. That is, for any two rational numbers a and b, a + b = b + a.*

(b) Subtraction

Is $\frac{2}{3}-\frac{5}{4}=\frac{5}{4}-\frac{2}{3}$?

Is $\frac{1}{2}-\frac{3}{5}=\frac{3}{5}-\frac{1}{2}$?

You will find that subtraction is not commutative for rational numbers.

Note that subtraction is not commutative for integers and integers are also rational numbers. So, subtraction will not be commutative for rational numbers too.

(c) Multiplication

We have, $\frac{-7}{3}\times\frac{6}{5}=\frac{-42}{15}=\frac{6}{5}\times\left(\frac{-7}{3}\right)$

Is $\frac{-8}{9}\times\left(\frac{-4}{7}\right)=\frac{-4}{7}\times\left(\frac{-8}{9}\right)$?

Check for some more such products.

You will find that *multiplication is commutative for rational numbers.*

In general, a × b = b × a for any two rational numbers a and b.

(d) Division

Is $\frac{-5}{4}\div\frac{3}{7}=\frac{3}{7}\div\left(\frac{-5}{4}\right)$?

You will find that expressions on both sides are not equal.

So division is **not commutative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Commutative for			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	Yes	...	...	...
Integers	...	No	...	...
Whole numbers	...	...	Yes	...
Natural numbers	...	...	...	No

2.2.3 Associativity

(i) Whole numbers

Recall the associativity of the four operations for whole numbers through this table:

Operation	Numbers	Remarks
Addition	....	Addition is associative
Subtraction	....	Subtraction is **not** associative
Multiplication	Is $7 \times (2 \times 5) = (7 \times 2) \times 5$? Is $4 \times (6 \times 0) = (4 \times 6) \times 0$? For any three whole numbers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	....	Division is **not** associative

Fill in this table and verify the remarks given in the last column.
Check for yourself the associativity of different operations for natural numbers.

(ii) Integers

Associativity of the four operations for integers can be seen from this table

Operation	Numbers	Remarks
Addition	Is $(-2) + [3 + (-4)]$ $= [(-2) + 3)] + (-4)$? Is $(-6) + [(-4) + (-5)]$ $= [(-6) + (-4)] + (-5)$? For any three integers a, b and c $a + (b + c) = (a + b) + c$	Addition is associative
Subtraction	Is $5 - (7 - 3) = (5 - 7) - 3$?	Subtraction is **not** associative
Multiplication	Is $5 \times [(-7) \times (-8)$ $= [5 \times (-7)] \times (-8)$? Is $(-4) \times [(-8) \times (-5)]$ $= [(-4) \times (-8)] \times (-5)$? For any three integers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	Is $[(-10) \div 2] \div (-5)$ $= (-10) \div [2 \div (-5)]$?	Division is **not** associative

(iii) Rational numbers

(a) Addition

We have $\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\frac{-2}{3}+\left(\frac{-7}{30}\right)=\frac{-27}{30}=\frac{-9}{10}$

$$\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)=\frac{-1}{15}+\left(\frac{-5}{6}\right)=\frac{-27}{30}=\frac{-9}{10}$$

So, $\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)$

Find $\frac{-1}{2}+\left[\frac{3}{7}+\left(\frac{-4}{3}\right)\right]$ and $\left[\frac{-1}{2}+\frac{3}{7}\right]+\left(\frac{-4}{3}\right)$. Are the two sums equal?

Take some more rational numbers, add them as above and see if the two sums are equal. We find that *addition is associative for rational numbers. That is, for any three rational numbers a, b* and *c*, $a + (b + c) = (a + b) + c$.

(b) Subtraction

You already know that subtraction is not associative for integers, then what about rational numbers.

Is $\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]=\left[\frac{2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$?

Check for yourself.

Subtraction is **not associative** for rational numbers.

(c) Multiplication

Let us check the associativity for multiplication.

$$\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\frac{-7}{3}\times\frac{10}{36}=\frac{-70}{108}=\frac{-35}{54}$$

$$\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}=\ldots$$

We find that $\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}$

Is $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{4}{5}\right)=\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{4}{5}$?

Take some more rational numbers and check for yourself.

We observe that *multiplication is associative for rational numbers. That is for any three rational numbers a, b* and *c*, $a \times (b \times c) = (a \times b) \times c$.

(d) Division

Recall that division is not associative for integers, then what about rational numbers?

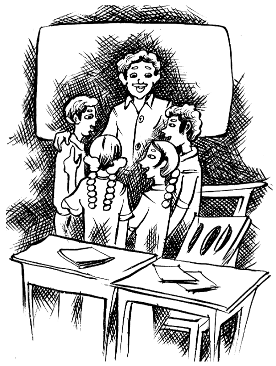
Let us see if $\frac{1}{2} \div \left[\frac{-1}{3} \div \frac{2}{5}\right] = \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$

We have, LHS $= \frac{1}{2} \div \left(\frac{-1}{3} \div \frac{2}{5}\right) = \frac{1}{2} \div \left(\frac{-1}{3} \times \frac{5}{2}\right)$ (reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$)

$= \frac{1}{2} \div \left(-\frac{5}{6}\right) = \ldots$

RHS $= \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$

$= \left(\frac{1}{2} \times \frac{-3}{1}\right) \div \frac{2}{5} = \frac{-3}{2} \div \frac{2}{5} = \ldots$



Is LHS = RHS? Check for yourself. You will find that division is **not associative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Associative for			
	addition	subtraction	multiplication	division
Rational numbers	...	...	...	No
Integers	...	...	Yes	...
Whole numbers	Yes	...	...	...
Natural numbers	...	No	...	...

Example 1: Find $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

Solution: $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

$= \frac{198}{462} + \left(\frac{-252}{462}\right) + \left(\frac{-176}{462}\right) + \left(\frac{105}{462}\right)$ (Note that 462 is the LCM of 7, 11, 21 and 22)

$= \frac{198 - 252 - 176 + 105}{462} = \frac{-125}{462}$

We can also solve it as.

$$\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\frac{5}{22}$$

$$=\left[\frac{3}{7}+\left(\frac{-8}{21}\right)\right]+\left[\frac{-6}{11}+\frac{5}{22}\right] \quad \text{(by using commutativity and associativity)}$$

$$=\left[\frac{9+(-8)}{21}\right]+\left[\frac{-12+5}{22}\right] \quad \text{(LCM of 7 and 21 is 21; LCM of 11 and 22 is 22)}$$

$$=\frac{1}{21}+\left(\frac{-7}{22}\right)=\frac{22-147}{462}=\frac{-125}{462}$$

Do you think the properties of commutativity and associativity made the calculations easier?

Example 2: Find $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

Solution: We have

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(-\frac{4\times 3}{5\times 7}\right)\times\left(\frac{15\times(-14)}{16\times 9}\right)$$

$$=\frac{-12}{35}\times\left(\frac{-35}{24}\right)=\frac{-12\times(-35)}{35\times 24}=\frac{1}{2}$$

We can also do it as.

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(\frac{-4}{5}\times\frac{15}{16}\right)\times\left[\frac{3}{7}\times\left(\frac{-14}{9}\right)\right] \quad \text{(Using commutativity and associativity)}$$

$$=\frac{-3}{4}\times\left(\frac{-2}{3}\right)=\frac{1}{2}$$

2.2.4 The role of zero (0)

Look at the following.

$2 + 0 = 0 + 2 = 2$ (Addition of 0 to a whole number)

$-5 + 0 = ... + ... = -5$ (Addition of 0 to an integer)

$\frac{-2}{7} + ... = 0 + \left(\frac{-2}{7}\right) = \frac{-2}{7}$ (Addition of 0 to a rational number)

You have done such additions earlier also. Do a few more such additions.

What do you observe? You will find that when you add 0 to a whole number, the sum is again that whole number. This happens for integers and rational numbers also.

In general, $a + 0 = 0 + a = a$, where a is a whole number

$b + 0 = 0 + b = b$, where b is an integer

$c + 0 = 0 + c = c$, where c is a rational number

Zero is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers as well.

2.2.5 The role of 1

We have,

$$5 \times 1 = 5 = 1 \times 5 \quad \text{(Multiplication of 1 with a whole number)}$$

$$\frac{-2}{7} \times 1 = \ldots \times \ldots = \frac{-2}{7}$$

$$\frac{3}{8} \times \ldots = 1 \times \frac{3}{8} = \frac{3}{8}$$

What do you find?

You will find that when you multiply any rational number with 1, you get back the same rational number as the product. Check this for a few more rational numbers. You will find that, $a \times 1 = 1 \times a = a$ for any rational number a.

We say *that* 1 *is the multiplicative identity for rational numbers.*

Is 1 the multiplicative identity for integers? For whole numbers?

THINK, DISCUSS AND WRITE

If a property holds for rational numbers, will it also hold for integers? For whole numbers? Which will? Which will not?

1.2.6 Negative of a number

While studying integers you have come across negatives of integers. What is the negative of 1? It is – 1 because 1 + (– 1) = (–1) + 1 = 0

So, what will be the negative of (–1)? It will be 1.

Also, 2 + (–2) = (–2) + 2 = 0, so we say 2 is the **negative or additive inverse** of –2 and vice-versa. In general, for an integer a, we have, $a + (-a) = (-a) + a = 0$; so, a is the negative of $-a$ and $-a$ is the negative of a.

For the rational number $\frac{2}{3}$, we have,

$$\frac{2}{3} + \left(-\frac{2}{3}\right) = \frac{2 + (-2)}{3} = 0$$

Also, $\left(-\frac{2}{3}\right)+\frac{2}{3}=0$ (How?)

Similarly, $\frac{-8}{9}+\ldots=\ldots+\left(\frac{-8}{9}\right)=0$

$\ldots+\left(\frac{-11}{7}\right)=\left(\frac{-11}{7}\right)+\ldots=0$

In general, for a rational number $\frac{a}{b}$, we have, $\frac{a}{b}+\left(-\frac{a}{b}\right)=\left(-\frac{a}{b}\right)+\frac{a}{b}=0$. We say that $-\frac{a}{b}$ *is the additive inverse of* $\frac{a}{b}$ *and* $\frac{a}{b}$ *is the additive inverse of* $\left(-\frac{a}{b}\right)$.

2.2.7 Reciprocal

By which rational number would you multiply $\frac{8}{21}$, to get the product 1? Obviously by $\frac{21}{8}$, since $\frac{8}{21}\times\frac{21}{8}=1$.

Similarly, $\frac{-5}{7}$ must be multiplied by $\frac{7}{-5}$ so as to get the product 1.

We say that $\frac{21}{8}$ is the reciprocal of $\frac{8}{21}$ and $\frac{7}{-5}$ is the reciprocal of $\frac{-5}{7}$.

Can you say what is the reciprocal of 0 (zero)?

Is there a rational number which when multiplied by 0 gives 1? Thus, zero has no reciprocal.

We say that a rational number $\frac{c}{d}$ is called the **reciprocal** or **multiplicative inverse** of another non-zero rational number $\frac{a}{b}$ if $\frac{a}{b}\times\frac{c}{d}=1$.

2.2.8 Distributivity of multiplication over addition for rational numbers

To understand this, consider the rational numbers $\frac{-3}{4}$, $\frac{2}{3}$ and $\frac{-5}{6}$.

$$\frac{-3}{4}\times\left\{\frac{2}{3}+\left(\frac{-5}{6}\right)\right\}=\frac{-3}{4}\times\left\{\frac{(4)+(-5)}{6}\right\}$$

$$=\frac{-3}{4}\times\left(\frac{-1}{6}\right)=\frac{3}{24}=\frac{1}{8}$$

Also $\frac{-3}{4}\times\frac{2}{3}=\frac{-3\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$

And $\frac{-3}{4} \times \frac{-5}{6} = \frac{5}{8}$

Therefore $\left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right) = \frac{-1}{2} + \frac{5}{8} = \frac{1}{8}$

Thus, $\frac{-3}{4} \times \left\{\frac{2}{3} + \frac{-5}{6}\right\} = \left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right)$

Distributivity of Multiplication over Addition and Subtraction.
For all rational numbers a, b and c,
$a\,(b + c) = ab + ac$
$a\,(b - c) = ab - ac$

TRY THESE

Find using distributivity. (i) $\left\{\frac{7}{5} \times \left(\frac{-3}{12}\right)\right\} + \left\{\frac{7}{5} \times \frac{5}{12}\right\}$ (ii) $\left\{\frac{9}{16} \times \frac{4}{12}\right\} + \left\{\frac{9}{16} \times \frac{-3}{9}\right\}$

Example 3: Write the additive inverse of the following:

(i) $\frac{-7}{19}$ (ii) $\frac{21}{112}$

When you use distributivity, you split a product as a sum or difference of two products.

Solution:

(i) $\frac{7}{19}$ is the additive inverse of $\frac{-7}{19}$ because $\frac{-7}{19} + \frac{7}{19} = \frac{-7+7}{19} = \frac{0}{19} = 0$

(ii) The additive inverse of $\frac{21}{112}$ is $\frac{-21}{112}$ (Check!)

Example 4: Verify that $-(-x)$ is the same as x for

(i) $x = \frac{13}{17}$ (ii) $x = \frac{-21}{31}$

Solution: (i) We have, $x = \frac{13}{17}$

The additive inverse of $x = \frac{13}{17}$ is $-x = \frac{-13}{17}$ since $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$.

The same equality $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$, shows that the additive inverse of $\frac{-13}{17}$ is $\frac{13}{17}$

or $-\left(\frac{-13}{17}\right) = \frac{13}{17}$, i.e., $-(-x) = x$.

(ii) Additive inverse of $x = \frac{-21}{31}$ is $-x = \frac{21}{31}$ since $\frac{-21}{31} + \frac{21}{31} = 0$.

The same equality $\frac{-21}{31} + \frac{21}{31} = 0$, shows that the additive inverse of $\frac{21}{31}$ is $\frac{-21}{31}$, i.e., $-(-x) = x$.

Example 5: Find $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}$

Solution: $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5} = \frac{2}{5}\times\frac{-3}{7}-\frac{3}{7}\times\frac{3}{5}-\frac{1}{14}$ (by commutativity)

$$= \frac{2}{5}\times\frac{-3}{7}+\left(\frac{-3}{7}\right)\times\frac{3}{5}-\frac{1}{14}$$

$$= \frac{-3}{7}\left(\frac{2}{5}+\frac{3}{5}\right)-\frac{1}{14} \quad \text{(by distributivity)}$$

$$= \frac{-3}{7}\times 1-\frac{1}{14} = \frac{-6-1}{14} = \frac{-1}{2}$$

EXERCISE 2.1

1. Using appropriate properties find.

   (i) $-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$ (ii) $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

2. Write the additive inverse of each of the following.

   (i) $\frac{2}{8}$ (ii) $\frac{-5}{9}$ (iii) $\frac{-6}{-5}$ (iv) $\frac{2}{-9}$ (v) $\frac{19}{-6}$

3. Verify that $-(-x)=x$ for.

   (i) $x=\frac{11}{15}$ (ii) $x=-\frac{13}{17}$

4. Find the multiplicative inverse of the following.

   (i) -13 (ii) $\frac{-13}{19}$ (iii) $\frac{1}{5}$ (iv) $\frac{-5}{8}\times\frac{-3}{7}$

   (v) $-1\times\frac{-2}{5}$ (vi) -1

5. Name the property under multiplication used in each of the following.

   (i) $\frac{-4}{5}\times 1=1\times\frac{-4}{5}=-\frac{4}{5}$ (ii) $-\frac{13}{17}\times\frac{-2}{7}=\frac{-2}{7}\times\frac{-13}{17}$

   (iii) $\frac{-19}{29}\times\frac{29}{-19}=1$

6. Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.

7. Tell what property allows you to compute $\frac{1}{3}\times\left(6\times\frac{4}{3}\right)$ as $\left(\frac{1}{3}\times 6\right)\times\frac{4}{3}$.

8. Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

9. Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

10. Write.

(i) The rational number that does not have a reciprocal.

(ii) The rational numbers that are equal to their reciprocals.

(iii) The rational number that is equal to its negative.

11. Fill in the blanks.

(i) Zero has ________ reciprocal.

(ii) The numbers ________ and ________ are their own reciprocals

(iii) The reciprocal of – 5 is ________.

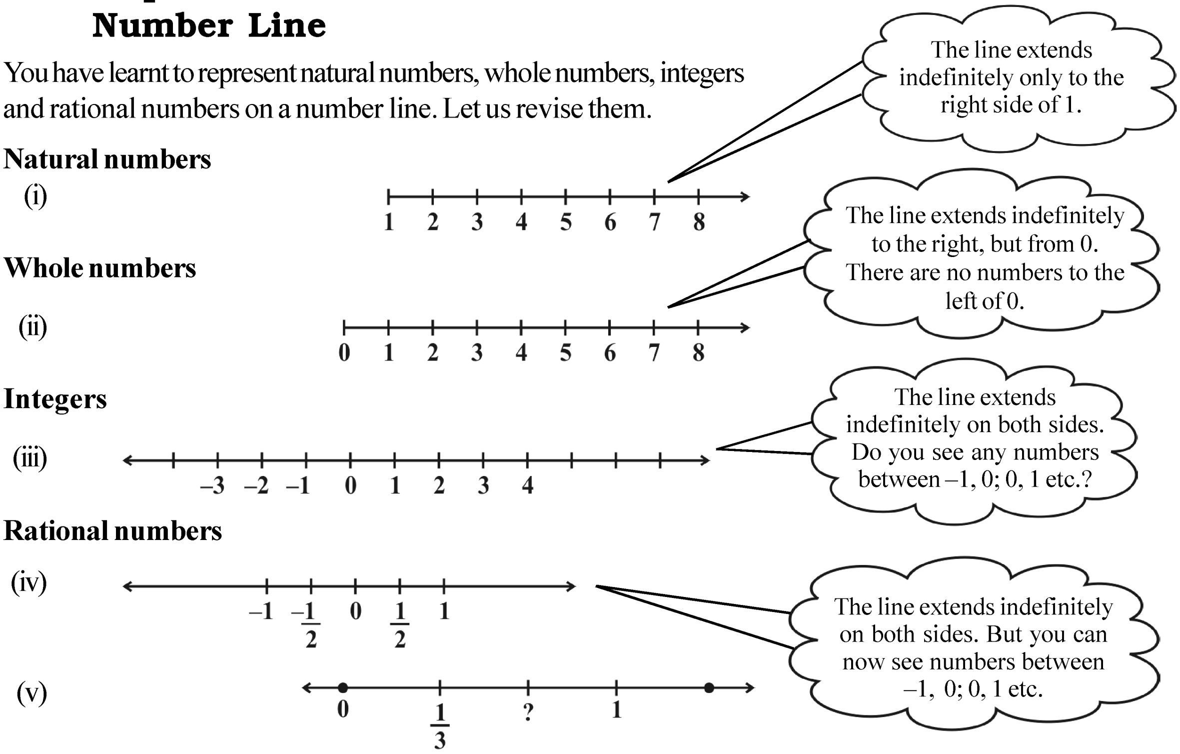
(iv) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is ________.

(v) The product of two rational numbers is always a _______.

(vi) The reciprocal of a positive rational number is ________.

2.3 Representation of Rational Numbers on the Number Line

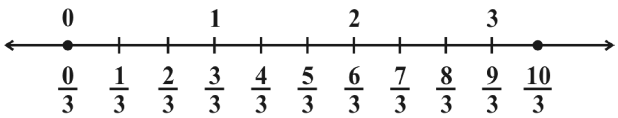
You have learnt to represent natural numbers, whole numbers, integers and rational numbers on a number line. Let us revise them.

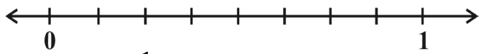


The point on the number line (iv) which is half way between 0 and 1 has been labelled $\frac{1}{2}$. Also, the first of the equally spaced points that divides the distance between 0 and 1 into three equal parts can be labelled $\frac{1}{3}$, as on number line (v). How would you label the second of these division points on number line (v)?

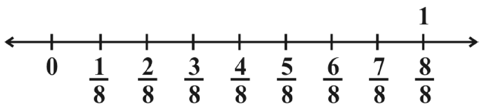
The point to be labelled is twice as far from and to the right of 0 as the point labelled $\frac{1}{3}$. So it is two times $\frac{1}{3}$, i.e., $\frac{2}{3}$. You can continue to label equally-spaced points on the number line in the same way. In this continuation, the next marking is 1. You can see that 1 is the same as $\frac{3}{3}$.

Then comes $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}$ (or 2), $\frac{7}{3}$ and so on as shown on the number line (vi)

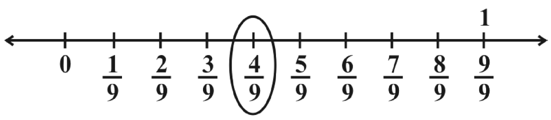
(vi)


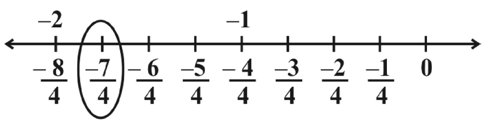
Similarly, to represent $\frac{1}{8}$, the number line may be divided into eight equal parts as shown:


We use the number $\frac{1}{8}$ to name the first point of this division. The second point of division will be labelled $\frac{2}{8}$, the third point $\frac{3}{8}$, and so on as shown on number line (vii)

(vii)


Any rational number can be represented on the number line in this way. In a rational number, the numeral below the bar, i.e., the denominator, tells the number of equal parts into which the first unit has been divided. The numeral above the bar i.e., the numerator, tells 'how many' of these parts are considered. So, a rational number such as $\frac{4}{9}$ means four of nine equal parts on the right of 0 (number line viii) and for $\frac{-7}{4}$, we make 7 markings of distance $\frac{1}{4}$ each on the *left* of zero and starting from 0. The seventh marking is $\frac{-7}{4}$ [number line (ix)].

(viii)


(ix)


TRY THESE

Write the rational number for each point labelled with a letter.

(i)

	A			B	C			D	E			
$\frac{0}{5}$	?	$\frac{2}{5}$	$\frac{3}{5}$	?	?	$\frac{6}{5}$	$\frac{7}{5}$	?	?	$\frac{10}{5}$	$\frac{11}{5}$	$\frac{12}{5}$

(ii)

	J			I	H		G			F		
$\frac{-12}{6}$	?	$\frac{-10}{6}$	$\frac{-9}{6}$	?	?	$\frac{-6}{6}$	?	$\frac{-4}{6}$	$\frac{-3}{6}$	?	$\frac{-1}{6}$	$\frac{0}{6}$

2.4 Rational Numbers between Two Rational Numbers

Can you tell the natural numbers between 1 and 5? They are 2, 3 and 4.
How many natural numbers are there between 7 and 9? There is one and it is 8.
How many natural numbers are there between 10 and 11? Obviously none.
List the integers that lie between –5 and 4. They are – 4, – 3, –2, –1, 0, 1, 2, 3.
How many integers are there between –1 and 1?
How many integers are there between –9 and –10?

You will find a definite number of natural numbers (integers) between two natural numbers (integers).

How many rational numbers are there between $\frac{3}{10}$ and $\frac{7}{10}$?

You may have thought that they are only $\frac{4}{10}, \frac{5}{10}$ and $\frac{6}{10}$.

But you can also write $\frac{3}{10}$ as $\frac{30}{100}$ and $\frac{7}{10}$ as $\frac{70}{100}$. Now the numbers, $\frac{31}{100}, \frac{32}{100}, \frac{33}{100}, \ldots \frac{68}{100}, \frac{69}{100}$, are all between $\frac{3}{10}$ and $\frac{7}{10}$. The number of these rational numbers is 39.

Also $\frac{3}{10}$ can be expressed as $\frac{3000}{10000}$ and $\frac{7}{10}$ as $\frac{7000}{10000}$. Now, we see that the rational numbers $\frac{3001}{10000}, \frac{3002}{10000}, \ldots, \frac{6998}{10000}, \frac{6999}{10000}$ are between $\frac{3}{10}$ and $\frac{7}{10}$. These are 3999 numbers in all.

In this way, we can go on inserting more and more rational numbers between $\frac{3}{10}$ and $\frac{7}{10}$. So unlike natural numbers and integers, the number of rational numbers between two rational numbers is not definite. Here is one more example.

How many rational numbers are there between $\frac{-1}{10}$ and $\frac{3}{10}$?

Obviously $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}$ are rational numbers between the given numbers.

If we write $\frac{-1}{10}$ as $\frac{-10000}{100000}$ and $\frac{3}{10}$ as $\frac{30000}{100000}$, we get the rational numbers $\frac{-9999}{100000}, \frac{-9998}{100000}, ..., \frac{-29998}{100000}, \frac{29999}{100000}$, between $\frac{-1}{10}$ and $\frac{3}{10}$.

You will find that *you get countless rational numbers between any two given rational numbers*.

Example 6: Write any 3 rational numbers between –2 and 0.

Solution: –2 can be written as $\frac{-20}{10}$ and 0 as $\frac{0}{10}$.

Thus we have $\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, ..., \frac{-1}{10}$ between –2 and 0.

You can take any three of these.

Example 7: Find any ten rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.

Solution: We first convert $\frac{-5}{6}$ and $\frac{5}{8}$ to rational numbers with the same denominators.

$$\frac{-5 \times 4}{6 \times 4} = \frac{-20}{24} \quad \text{and} \quad \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

Thus we have $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, ..., \frac{14}{24}$ as the rational numbers between $\frac{-20}{24}$ and $\frac{15}{24}$.

You can take any ten of these.

Another Method

Let us find rational numbers between 1 and 2. One of them is 1.5 or $1\frac{1}{2}$ or $\frac{3}{2}$. This is the **mean** of 1 and 2. You have studied mean in Class VII.

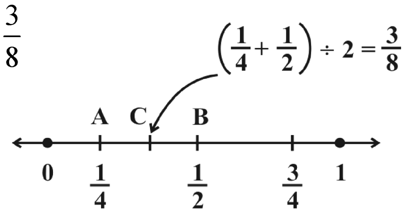
We find that *between any two given numbers, we need not necessarily get an integer but there will always lie a rational number*.

We can use the idea of mean also to find rational numbers between any two given rational numbers.

Example 8: Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$.

Solution: We find the mean of the given rational numbers.

$$\left(\frac{1}{4} + \frac{1}{2}\right) \div 2 = \left(\frac{1+2}{4}\right) \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$



$\frac{3}{8}$ lies between $\frac{1}{4}$ and $\frac{1}{2}$.

This can be seen on the number line also.

We find the mid point of AB which is C, represented by $\left(\frac{1}{4}+\frac{1}{2}\right) \div 2 = \frac{3}{8}$.

We find that $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.

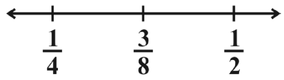
If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

This again shows that there are countless number of rational numbers between any two given rational numbers.

Example 9: Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

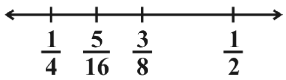
Solution: We find the mean of the given rational numbers.

As given in the above example, the mean is $\frac{3}{8}$ and $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.



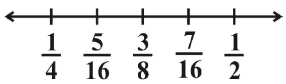
We now find another rational number between $\frac{1}{4}$ and $\frac{3}{8}$. For this, we again find the mean of $\frac{1}{4}$ and $\frac{3}{8}$. That is, $\left(\frac{1}{4}+\frac{3}{8}\right) \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$

$$\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{1}{2}$$



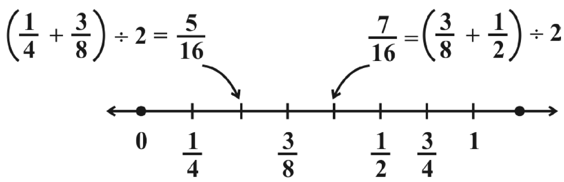
Now find the mean of $\frac{3}{8}$ and $\frac{1}{2}$. We have, $\left(\frac{3}{8}+\frac{1}{2}\right) \div 2 = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$

Thus we get $\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{7}{16} < \frac{1}{2}$.



Thus, $\frac{5}{16}, \frac{3}{8}, \frac{7}{16}$ are the three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

This can clearly be shown on the number line as follows:



In the same way we can obtain as many rational numbers as we want between two given rational numbers . You have noticed that there are countless rational numbers between any two given rational numbers.

EXERCISE 2.2

1. Represent these numbers on the number line. (i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$
2. Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ on the number line.
3. Write five rational numbers which are smaller than 2.
4. Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.
5. Find five rational numbers between.
   (i) $\frac{2}{3}$ and $\frac{4}{5}$ (ii) $\frac{-3}{2}$ and $\frac{5}{3}$ (iii) $\frac{1}{4}$ and $\frac{1}{2}$
6. Write five rational numbers greater than –2.
7. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

WHAT HAVE WE DISCUSSED?

1. Rational numbers are **closed** under the operations of addition, subtraction and multiplication.
2. The operations addition and multiplication are
   (i) **commutative** for rational numbers.
   (ii) **associative** for rational numbers.
3. The rational number 0 is the **additive identity** for rational numbers.
4. The rational number 1 is the **multiplicative identity** for rational numbers.
5. The **additive inverse** of the rational number $\frac{a}{b}$ is $-\frac{a}{b}$ and vice-versa.
6. The **reciprocal** or **multiplicative inverse** of the rational number $\frac{a}{b}$ is $\frac{c}{d}$ if $\frac{a}{b} \times \frac{c}{d} = 1$.
7. **Distributivity** of rational numbers: For all rational numbers *a*, *b* and *c*,
   $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
8. Rational numbers can be represented on a number line.
9. Between any two given rational numbers there are countless rational numbers. The idea of **mean** helps us to find rational numbers between two rational numbers.

CHAPTER

3 Linear Equations in One Variable

Q6K2R6

3.1 Introduction

In the earlier classes, you have come across several **algebraic expressions** and **equations**. Some examples of expressions we have so far worked with are:

$$5x,\ 2x - 3,\ 3x + y,\ 2xy + 5,\ xyz + x + y + z,\ x^2 + 1,\ y + y^2$$

Some examples of equations are: $5x = 25$, $2x - 3 = 9$, $2y + \frac{5}{2} = \frac{37}{2}$, $6z + 10 = -2$

You would remember that equations use the *equality* (=) sign; it is missing in expressions.

Of these given expressions, many have more than one variable. For example, $2xy + 5$ has two variables. We however, restrict to expressions with only one variable when we form equations. Moreover, the expressions we use to form equations are linear. This means that the highest power of the variable appearing in the expression is 1.

These are linear expressions:

$$2x,\ 2x + 1,\ 3y - 7,\ 12 - 5z,\ \frac{5}{4}(x - 4) + 10$$

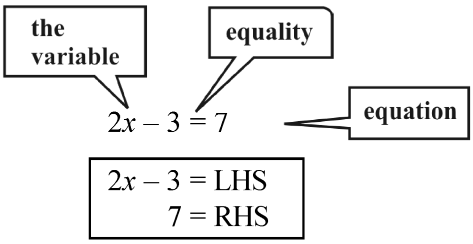
These are **not** linear expressions:

$x^2 + 1$, $y + y^2$, $1 + z + z^2 + z^3$ (since highest power of variable > 1)

Here we will deal with equations with linear expressions in one variable only. Such equations are known as **linear equations in one variable**. The simple equations which you studied in the earlier classes were all of this type.

Let us briefly revise what we know:

(a) *An algebraic equation is an equality involving variables*. It has an *equality sign*. The expression on the left of the equality sign is the *Left Hand Side* (LHS). The expression on the right of the equality sign is the *Right Hand Side* (RHS).

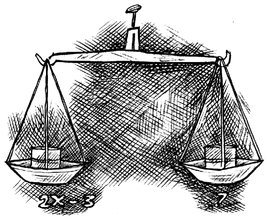


(b) In an equation the *values of the expressions on the LHS and RHS are equal*. This happens to be *true* only for certain values of the variable. These values are the **solutions** of the equation.

$x = 5$ is the solution of the equation $2x - 3 = 7$. For $x = 5$,
LHS $= 2 \times 5 - 3 = 7 =$ RHS
On the other hand $x = 10$ is not a solution of the equation. For $x = 10$, LHS $= 2 \times 10 - 3 = 17$. This is not equal to the RHS

(c) *How to find the solution of an equation?*

We assume that the two sides of the equation are balanced. We perform the same mathematical operations on both sides of the equation, so that the balance is not disturbed. A few such steps give the solution.



3.2 Solving Equations which have Linear Expressions on one Side and Numbers on the other Side

Let us recall the technique of solving equations with some examples. Observe the solutions; they can be any rational number.

Example 1: Find the solution of $2x - 3 = 7$

Solution:

Step 1 Add 3 to both sides.

$$2x - 3 + 3 = 7 + 3 \qquad \text{(The balance is not disturbed)}$$

or $\quad 2x = 10$

Step 2 Next divide both sides by 2.

$$\frac{2x}{2} = \frac{10}{2}$$

or $\quad x = 5 \qquad$ (required solution)

Example 2: Solve $2y + 9 = 4$

Solution: Transposing 9 to RHS

$$2y = 4 - 9$$

or $\quad 2y = -5$

Dividing both sides by 2, $\quad y = \frac{-5}{2} \qquad$ (solution)

To check the answer: LHS $= 2\left(\frac{-5}{2}\right) + 9 = -5 + 9 = 4 =$ RHS $\qquad$ (as required)

Do you notice that the solution $\left(\frac{-5}{2}\right)$ is a rational number? In Class VII, the equations we solved did not have such solutions.

Example 3: Solve $\frac{x}{3}+\frac{5}{2} = -\frac{3}{2}$

Solution: Transposing $\frac{5}{2}$ to the RHS, we get $\frac{x}{3} = \frac{-3}{2}-\frac{5}{2}=-\frac{8}{2}$

or $\frac{x}{3} = -4$

Multiply both sides by 3, $x = -4 \times 3$

or $x = -12$ (solution)

Check: LHS $= -\frac{12}{3}+\frac{5}{2} = -4+\frac{5}{2} = \frac{-8+5}{2} = \frac{-3}{2} =$ RHS (as required)

Do you now see that the coefficient of a variable in an equation need not be an integer?

Example 4: Solve $\frac{15}{4} - 7x = 9$

Solution: We have $\frac{15}{4} - 7x = 9$

or $-7x = 9 - \frac{15}{4}$ (transposing $\frac{15}{4}$ to R H S)

or $-7x = \frac{21}{4}$

or $x = \frac{21}{4\times(-7)}$ (dividing both sides by – 7)

or $x = -\frac{3\times 7}{4\times 7}$

or $x = -\frac{3}{4}$ (solution)

Check: LHS $= \frac{15}{4} - 7\left(\frac{-3}{4}\right) = \frac{15}{4}+\frac{21}{4} = \frac{36}{4} = 9 =$ RHS (as required)

EXERCISE 3.1

Solve the following equations.

1. $x - 2 = 7$
2. $y + 3 = 10$
3. $6 = z + 2$
4. $\frac{3}{7} + x = \frac{17}{7}$
5. $6x = 12$
6. $\frac{t}{5} = 10$
7. $\frac{2x}{3} = 18$
8. $1.6 = \frac{y}{1.5}$
9. $7x - 9 = 16$

10. $14y - 8 = 13$ **11.** $17 + 6p = 9$ **12.** $\frac{x}{3} + 1 = \frac{7}{15}$

3.3 Some Applications

We begin with a simple example.

Sum of two numbers is 74. One of the numbers is 10 more than the other. What are the numbers?

We have a puzzle here. We do not know either of the two numbers, and we have to find them. We are given two conditions.

(i) One of the numbers is 10 more than the other.

(ii) Their sum is 74.

We already know from Class VII how to proceed. If the smaller number is taken to be x, the larger number is 10 more than x, i.e., $x + 10$. The other condition says that the sum of these two numbers x and $x + 10$ is 74.

This means that $x + (x + 10) = 74$.

or $2x + 10 = 74$

Transposing 10 to RHS, $2x = 74 - 10$

or $2x = 64$

Dividing both sides by 2, $x = 32$. This is one number.

The other number is $x + 10 = 32 + 10 = 42$

The desired numbers are 32 and 42. (Their sum is indeed 74 as given and also one number is 10 more than the other.)

We shall now consider several examples to show how useful this method is.

Example 5: What should be added to twice the rational number $\frac{-7}{3}$ to get $\frac{3}{7}$?

Solution: Twice the rational number $\frac{-7}{3}$ is $2 \times \left(\frac{-7}{3}\right) = \frac{-14}{3}$. Suppose x added to this number gives $\frac{3}{7}$; i.e.,

$$x + \left(\frac{-14}{3}\right) = \frac{3}{7}$$

or $$x - \frac{14}{3} = \frac{3}{7}$$

or $$x = \frac{3}{7} + \frac{14}{3} \qquad \text{(transposing } \frac{14}{3} \text{ to RHS)}$$

$$= \frac{(3 \times 3) + (14 \times 7)}{21} = \frac{9 + 98}{21} = \frac{107}{21}.$$

Thus $\frac{107}{21}$ should be added to $2 \times \left(\frac{-7}{3}\right)$ to give $\frac{3}{7}$.

Example 6: The perimeter of a rectangle is 13 cm and its width is $2\frac{3}{4}$ cm. Find its length.

Solution: Assume the length of the rectangle to be x cm.

The perimeter of the rectangle = 2 × (length + width)

$$= 2 \times (x + 2\frac{3}{4})$$

$$= 2\left(x + \frac{11}{4}\right)$$

The perimeter is given to be 13 cm. Therefore,

$$2\left(x + \frac{11}{4}\right) = 13$$

or $$x + \frac{11}{4} = \frac{13}{2}$$ (dividing both sides by 2)

or $$x = \frac{13}{2} - \frac{11}{4}$$

$$= \frac{26}{4} - \frac{11}{4} = \frac{15}{4} = 3\frac{3}{4}$$

The length of the rectangle is $3\frac{3}{4}$ cm.

Example 7: The present age of Sahil's mother is three times the present age of Sahil. After 5 years their ages will add to 66 years. Find their present ages.

Solution: Let Sahil's present age be x years.

We could also choose Sahil's age 5 years later to be x and proceed. Why don't you try it that way?

	Sahil	Mother	Sum
Present age	x	$3x$	
Age 5 years later	$x + 5$	$3x + 5$	$4x + 10$

It is given that this sum is 66 years.

Therefore, $$4x + 10 = 66$$

This equation determines Sahil's present age which is x years. To solve the equation,